**EXPONTENTIAL GROWTH AND DECAY**

1. If you started a retirement account this year with $1000 that paid 6% compounded interest, how much could you expect to have after:

1. 10 years if the interest is compounded annually? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Since the interest is given annually (1 time per year) your b term is now 1.06 (106%). The *x* term is the 10 times the interest is given.

$$y=1000∙1.06^{10}=1790.85$$

1. 10 years if the interest is compounded quarterly? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Since the interest is compounded quarterly, you would first need to divide the interest rate, 6%, by the 4 times interest is given (quarterly) to determine the *b* term. You would have interest of 1.5% given each time. This needs to be changed to the decimal equivalent of 0.015, then added to the 100% that you had to start with at the beginning. In other words, you will get 100% of your money PLUS 1.5% interest, or 1.015.

The second thing you need to do is to determine the number of times you receive interest. In this case the money is in the account for 10 years and the interest is given 4 times per year. You need to multiply 10 by 4 to find that you will be applying the *b* term 40 times over the life of the account.

Now that the *b* and *x* terms have been determined, you can apply them in the exponential equation:

$$y=1000∙(1.015)^{40}$$

$$y=1814.02$$

1. 25 years if the interest is compounded monthly? \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Although this would be nice, it is not very realistic. The *b* term has now been changed since you are getting interest more times. You would divide the 6% by 12 (monthly) to get 0.5%. This is then added to the 100% of money you had to start with to get a *b* term of 1.005.

The *x* term has also changed. You are getting interest for 25 years. Since you are getting interest every month, or 12 times a year, you need to multiply 25 by 12 to determine the number of times you get interest, or 300.

Now substitute these into the formula to determine the total amount of money:

$$y=1000∙(1.005)^{300}$$

$$y=4464.97$$

2. Andy is looking to buy a used car. Andy knows that cars lose 20% of their value each year. The car that Andy is interested in is a 2006 Ford Explorer. The owner is asking $6000 for the car. Should Andy buy the car? Explain your reasoning for why or why not.

This problem could be answered either yes or no depending on the reasoning used. The first thing to do would be to determine, based on the $6000 asking price, how much the owners are saying the car sold for the first time. We will need to look back in time to do this. In order to have the exponential formula work in this case, the *x* term will be a negative number, or the number of years in the past. Since it is 2013 and this is a 2006 vehicle, the *x* term is -7.

The *b* term is determined by SUBTRACTING 20% from 100%, or is 0.8. The vehicle is losing value, or is worth less than the previous year.

Substituting into the formula you would have:

$$y=6000∙(0.8)^{-7}$$

$$y=28,610$$

Should Andy buy the car? If this vehicle originally sold for $28,610 then the owners are asking a fair price. However, the standard issue Ford Explorer sold for around $22,000, so the owners are probably asking a bit too much. Andy would then need to consider the condition of the vehicle, the mileage, extras, etc. before making his decision.

NOTE: By using math, Andy could negotiate a better price for the vehicle.

3. The Department of Natural Resources (DNR) has determined that the moose population in Michigan is approximately 50,000. Due to many factors, the population of moose has been steadily increasing by about 3% each year. If this trend continues, how many moose would the DNR expect to find in Michigan after:

1. 10 years \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

For this problem, you need to determine what the *b* term is equal to. The population is increasing, so the population is equal to 100% of the previous year PLUS a 3% increase, or 103%.

The *x* term is the number of years since the population increase is happening on an annual basis.

$$y=50,000 ∙(1.03)^{10}$$

$$y=67,196$$

1. 20 years \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Only the *x* term is changed for this problem.

$$y=50,000 ∙(1.03)^{20}$$

$$y=90,306$$

4. You are looking at purchasing a used car. The car is 7 years old and is for sale for $4000. What could you expect the price to have been when the car was new? Assume that the car loses 20% of its value each year.

This problem is very similar to number 2 above. The *b* term is 100% - 20%, or 80%.

The *x* term is -7 since you are trying to determine the price 7 years in the past.

$$y=4000 ∙(0.8)^{-7}$$

$$y=19,073$$

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6. Mr. Marvel’s retirement account has been gaining about 4.5% in value each quarter for the last two years. If Mr. Marvel’s account had $40,000 in it at the beginning of 2007, what can he expect it to have in it at the end of 2015?

In this problem, you would divide the interest rate (4.5%) by the number of times the interest is given each year (4) to get 1.125% each time. This then needs to be added to the 100% of the money you had the previous year for a *b* term of 1.01125.

The *x* term is found by determining the number of years (2015 – 2007 = 8) and multiplying that by the number of times the interest is given in one year (4). The *x* term is then 8 X 4 or 32.

$$y=40000 ∙ (1.0125)^{32}$$

$$y=59,525.22$$

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7. A biologist discovers a strain of a certain bacteria that will double its number every 30 minutes. At 9:00 am, there were 150 bacteria in the petri dish. How many bacteria could be expected to be present at the following times?

 a. 10:00 am \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Since the bacteria DOUBLES in size, the b term is 2

The bacteria doubles every 30 minutes, so since we started at 9:00 am and are trying to determine the count at 10:00 am, there have been TWO 30 minutes periods. The x term is 2.

$$y=150 ∙ (2)^{2}$$

$$y=600$$

 b. Noon \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

The only change in this problem is that the amount of time (*x*) has changed. Since there are 3 hours between 9:00 and noon, there have been 6 times that the population has doubled.

$$y=150 ∙ \left(2\right)^{6}$$

$$y=9600$$

 c. 5:30 pm \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

The number of times the population has doubled is equal to the number of hours that have passed (8 ½) divided by the 30 minute period. There are 17 times the population doubles in this problem.

$$y=150 ∙ \left(2\right)^{17}$$

$$y=19,660,800$$

8. A tree nursery advertises a tree that will grow 18% each year for the first ten years after it is planted. How tall, to the nearest foot, would you expect a two-year-old tree that is 5 ½ feet tall to be when it is 10 years old?

The *b* term is found by adding 100% and 18%, or 1.18

The *x* term is the number of years that have passed. Since the *a* term (5 ½) is when the tree is when the tree is two years old, 8 years have passed before it is 10.

$$y=5.5(1.18)^{8}$$

$$y=20.67 or 21 ft$$

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