Similarity

DIAGNOSING READINESS page 414

1. $12 : 18 = 2 \cdot 6 : 3 \cdot 6 = 2 : 3$, or $\frac{2}{3}$ **2.** $\frac{55}{11} = \frac{5 \cdot 11}{1 \cdot 11} = \frac{5}{1} = 5$ **3.** $\frac{20a^2}{15a^5} = \frac{4 \cdot 5a^2}{3 \cdot 5a^2a^3} = \frac{4}{3a^3}$ **4.** $\frac{3x^2 - 12x}{x^3 - x^2} = \frac{3x(x - 4)}{x^2(x - 1)} = \frac{3(x - 4)}{x(x - 1)} = \frac{3x - 12}{x^2 - x}$ **5.** A pentagon has 5 sides, so each exterior $\angle$ measures $\frac{360}{5}$, or 72. Then the measure of each interior $\angle$ is $180 - 72$, or 108. **6.** An octagon has eight sides, so each exterior $\angle$ measures $\frac{360}{8}$, or 45. Then the measure of each interior $\angle$ is $180 - 45$, or 135. **7.** A decagon has 10 sides, so each exterior $\angle$ measures $\frac{360}{10}$, or 36. Then the measure of each interior $\angle$ is $180 - 36$, or 144. **8.** Each exterior $\angle$ of a 27-gon measures $\frac{360}{27}$, or $\frac{40}{3}$. Then each interior $\angle$ measures $180 - \frac{40}{3}$, or $166\frac{2}{3}$. **9.** The letters in the $\cong$ segment hold the same position in the $\triangle$ name as the letters in the given segment, so the $\cong$ segment is $\overline{DL}$. **10.** H is the middle letter in the $\triangle$ name, so it is $\cong$ to $\angle A$ in $\triangle PAC$. **11.** Name the $\cong$ $\angle$ according to the order of the letters in the given $\triangle$ names, so $\angle PCA \cong \angle DLH$. **12.** Name the $\triangle$ according to the order of the letters in the given $\triangle$ names, so $\triangle HDL \cong \triangle APC$. **13.** In a 30°-60°-90° $\triangle$, the longer leg is $\sqrt{3}$ the length of the shorter leg, so $x = 10\sqrt{3}$. **14.** In an isosc. rt. $\triangle$, the hyp. is $\sqrt{2}$ times the length of a leg, so $x = \frac{6}{\sqrt{2}} = 3\sqrt{2}$. **15.** Since $\triangle ABC$ is equilateral, all its $\angle$s measure 60. So, the altitude divides the $\triangle$ into two 30°-60°-90° $\triangle$s whose sides are in the ratio of $1 : \sqrt{3} : 2$. Since 3 is opp. the 60° $\angle$, the side opp. the 30° $\angle$ is $\frac{3}{\sqrt{3}}$, or $\sqrt{3}$. So, $x = 2\sqrt{3}$. **16.** The perimeter is 24 units, so each side measures $24 \div 6$, or 4 units. The radii of a regular hexagon divide it into 6 $\cong$ equilateral $\triangle$s. The $\triangle$ altitude divides it into two 30°-60°-90° $\triangle$s whose hyp. is 4, so the alt. is $2\sqrt{3}$. The total area is $6 \cdot \frac{1}{2}(4)(2\sqrt{3}) = 24\sqrt{3}$, or $24\sqrt{3}$ units. **17.** The perimeter is 8(9), or 72 units. The area is $\frac{1}{2}ap = \frac{1}{2}(10.9)(72) = 392.4$, or 392.4 units^2.

8-1 Ratios and Proportions pages 416–421

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or Presentation Pro CD-ROM.

1. $\frac{1}{2}$ **2.** $\frac{2}{3}$ **3.** $\frac{3}{4}$ **4.** 1 **5.** $\frac{2}{3}$ **6.** 4 **7.** $\frac{1}{4}$ **8.** $\frac{4}{3}$ **9.** Each side of the smaller $\triangle$ is $\frac{1}{2}$ the length of a side of the larger $\triangle$.

Check Understanding **1.** The height of the photo is $5\frac{1}{3}$ in., and the height of the poster is $1\frac{1}{3}$ ft, which is $12 + 4$, or 16 in. The ratio is $5\frac{1}{3} : 16 = 3(5\frac{1}{3}) : 3(16) = 16 : 48 = 16(1) : 16(3) = 1 : 3$. **2.** Use the Properties of Proportions. Answers may vary. Samples: $\frac{m}{n} = \frac{4}{11}, \frac{m + n}{n} = \frac{4 + 11}{11}, \frac{4}{m} = \frac{11}{n}, \frac{m + 4}{4} = \frac{n + 11}{11}$ **3a.** By the Properties of Proportions, $\frac{5}{z} = \frac{20}{3}$ is equivalent to $\frac{z}{5} = \frac{3}{20}$, so $z = 5 \cdot \frac{3}{20} = 0.75$. **3b.** By the Properties of Proportions, $\frac{18}{n + 6} = \frac{6}{n}$ is equivalent to $\frac{n + 6}{18} = \frac{n}{6}$. By the Cross-Product Property, $18n = 6(n + 6)$; $18n = 6n + 36$; $12n = 36$; $n = 3$. **4.** It would be $\frac{14}{4}$ in. by $\frac{10}{4}$ in., or $3\frac{1}{2}$ in. by $2\frac{1}{2}$ in.

Exercises **1.** $\frac{0.675}{675} = \frac{675}{675{,}000} = \frac{1}{1000}$, or $1 : 1000$ **2.** 185 ft is 12(185) in. The ratio is $\frac{6}{12(185)} = \frac{1}{2(185)} = \frac{1}{370}$, or $1 : 370$. **3.** By the Cross-Product Property, $4a = 3b$. **4.** By Properties of Proportions ②, $\frac{b}{a} = \frac{4}{3}$. **5.** By Properties of Proportions ③, $\frac{a}{3} = \frac{b}{4}$. **6.** By Properties of Proportions ①, $4a = 3b$. Then by the Division Property of Equality, $\frac{4a}{3a} = \frac{3b}{3a}$, so $\frac{4}{3} = \frac{b}{a}$. **7.** By Properties of Proportions ①, $4a = 3b$. Then by the Division Property of Equality, $\frac{4a}{ab} = \frac{3b}{ab}$, so $\frac{4}{b} = \frac{3}{a}$. **8.** By the Cross-Product Property, $3b = 4a$. **9.** By Properties of Proportions ④, $\frac{a + b}{b} = \frac{3 + 4}{4} = \frac{7}{4}$. **10.** By Properties of Proportions ④, $\frac{a + b}{b} = \frac{7}{4}$. By Properties of Proportions ①, $\frac{b}{a + b} = \frac{4}{7}$. **11.** By Properties of Proportions ③, $\frac{a}{3} = \frac{b}{4}$. By Properties of Proportions ④, $\frac{a + 3}{3} = \frac{b + 4}{4}$. **12.** $\frac{x}{2} = \frac{8}{4}$; $x = 2\left(\frac{8}{4}\right) = 4$ **13.** $\frac{9}{5} = \frac{3}{x}$; $\frac{5}{9} = \frac{x}{3}$; $x = 3\left(\frac{5}{9}\right) = \frac{5}{3} = 1\frac{2}{3}$ **14.** $\frac{1}{3} = \frac{x}{12}$. Multiplying both sides by 12 results in $x = \frac{12}{3} = 4$. **15.** $\frac{5}{x} = \frac{8}{11}$; $\frac{x}{5} = \frac{11}{8}$; $x = 5\left(\frac{11}{8}\right) = \frac{55}{8} = 6.875$ **16.** $\frac{4}{x} = \frac{5}{9}$; $\frac{x}{4} = \frac{9}{5}$; $x = 4\left(\frac{9}{5}\right) = \frac{36}{5} = 7.2$ **17.** $\frac{5}{6} = \frac{6}{x}$; $\frac{6}{5} = \frac{x}{6}$; $\frac{x}{6} = \frac{6}{5}$; $x = 6\left(\frac{6}{5}\right) = \frac{36}{5} = 7.2$ **18.** $\frac{x + 3}{3} = \frac{10 + 4}{4}$; $\frac{x}{3} = \frac{10}{4}$; $x = 3\left(\frac{10}{4}\right) = 3(2.5) = 7.5$ **19.** $\frac{x + 7}{7} = \frac{15}{5}$; $\frac{x + 7}{7} = \frac{10 + 5}{5}$; $\frac{x}{7} = \frac{10}{5}$; $x = 7\left(\frac{10}{5}\right) = 7(2) = 14$ **20.** $\frac{3}{5} = \frac{6}{x + 3}$; $3(x + 3) = 5(6)$; $x + 3 = 5(2)$; $x + 3 = 10$; $x = 7$ **21.** $\frac{3\frac{1}{8}}{1} = \frac{d}{40}$; $d = 40\left(3\frac{1}{8}\right) = 40\left(\frac{25}{8}\right) = 5(25) = 125$. The actual distance is about 125 mi. **22.** Morgan City is located in the southeast corner of the map, and Rayne is on Highway 10 in the middle of the map. They are about $1\frac{7}{8}$ in. apart on the map. $\frac{1\frac{7}{8}}{1} = \frac{d}{40}$; $d = 40\left(1\frac{7}{8}\right) = 40\left(\frac{15}{8}\right) = 5(15) = 75$. The distance is about 75 mi. **23.** Vinton is just off Highway 10 on the west side of the map. New Roads is in the northeast portion of the map just west-northwest of Baton Rouge. They are about $3\frac{3}{8}$ in. apart on the map. $\frac{3\frac{3}{8}}{1} = \frac{d}{40}$; $d = 40\left(\frac{3}{8}\right) = 40\left(\frac{27}{8}\right) = 5(27) = 135$. The distance is about 135 mi. **24.** Kaplan is halfway across the map on Highway 14. Plaquemine is on the east side of the map on Highway 1 south of Highway 10. They are about $1\frac{11}{16}$ in. apart on the map. $\frac{1\frac{11}{16}}{1} = \frac{d}{40}$; $d = 40\left(1\frac{11}{16}\right) = 40\left(\frac{27}{16}\right) = 2.5(27) = 67.5$. The actual distance is about 67.5 mi. **25.** For the width, $\frac{w}{12} = \frac{3}{2}$, so $w = 12\left(\frac{3}{2}\right) = 18$. For the length, $\frac{\ell}{15} = \frac{3}{2}$, so $\ell = 15\left(\frac{3}{2}\right) = 22.5$. The dimensions of the scale drawing should be 18 in. by 22.5 in. **26.** $65 : 30 = 5(13) : 5(6) = 13 : 6$ **27.** $50 : 40 = 10(5) : 10(4) = 5 : 4$ **28.** The ratio 18 in. to 2 ft is the same as 18 in. to 24 in. $18 : 24 = 6(3) : 6(4) = 3 : 4$ **29.** Since 1 ft = 12 in., the ratio is $1\frac{1}{4} : 12$, or $\frac{5}{4} : 12$. Multiply both parts by 4 to eliminate the fraction, so the ratio becomes $5 : 48$, or $\frac{5}{48}$. **30.** By the Properties of Proportions ③, $\frac{x}{y} = \frac{7}{3}$. **31.** Dividing both sides by $4n$ results in $\frac{m}{n} = \frac{9}{4}$. **32.** By Properties of Proportions ②, $\frac{t}{r} = \frac{30}{18}$. **33.** By Properties of Proportions ④, $\frac{a}{5} = \frac{b}{2}$. **34.** Check students' work. **35.** $\frac{y}{10} = \frac{15}{25}$; $y = 10\left(\frac{15}{25}\right) = \frac{150}{25} = 6$ **36.** $\frac{9}{24} = \frac{12}{n}$; $9n = 24(12)$; $n = \frac{24(12)}{9} = 8(4) = 32$ **37.** $\frac{11}{14} = \frac{b}{21}$; $b = 21\left(\frac{11}{14}\right) = 3\left(\frac{11}{2}\right) = \frac{33}{2} = 16.5$ **38.** $\frac{5}{x-3} = \frac{10}{x}$; $5x = 10(x-3)$; $x = 2(x-3)$; $x = 2x - 6$; $-x = -6$; $x = 6$ **39.** $\frac{8}{n+4} = \frac{4}{n}$; $8n = 4(n+4)$; $2n = n + 4$; $n = 4$ **40.** $\frac{2b-1}{5} = \frac{b}{12}$; $12(2b-1) = 5b$; $24b - 12 = 5b$; $19b - 12 = 0$; $19b = 12$; $b = \frac{12}{19}$ **41.** $\frac{2}{7} = \frac{x-5}{x}$; $2x = 7x - 35$; $-5x = -35$; $x = 7$ **42.** $\frac{3y-5}{y} = \frac{12}{5}$; $5(3y-5) = 12y$; $15y - 25 = 12y$; $3y - 25 = 0$; $y = \frac{25}{3}$ **43.** $\frac{h}{2} = \frac{40}{5}$; $h = 2\left(\frac{40}{5}\right) = 2(8) = 16$. The milk bottle is 16 cm tall. **44.** $\frac{h}{29{,}000} = \frac{1}{1{,}000{,}000}$; $h = 29{,}000\left(\frac{1}{1{,}000{,}000}\right) = \frac{29}{1000} = 0.029$ Mount Everest is 0.029 ft, or 0.348 in. **45.** 6 is $\frac{3}{4}$ of 8, so the denominator of the ratio having a numerator of 6 is $\frac{3}{4}$ of 12, or 9. 12 is twice 6, so the denominator of the ratio having a numerator of 12 is 2(9), or 18. **46.** Since 15 is $\frac{3}{5}$ of 25, the numerator of the ratio having 15 in the denominator is $\frac{3}{5}(15)$, or 9. Since 20 is $\frac{4}{5}$ of 25, the numerator of the ratio having 20 in the denominator is $\frac{4}{5}(15)$, or 12. **47.** Since 14 is $\frac{2}{5}$ of 35, the denominator of the ratio having 14 in the numerator is $\frac{2}{5}(20)$, or 8. Since 12 is $\frac{3}{5}$ of 20, the numerator of the ratio having 12 in the denominator is $\frac{3}{5}(35)$, or 21.

48. Choose a convenient scale for which the numbers are easy to multiply and divide. Answers may vary. Sample:

Scale 1 in. = 5 ft

49. Choose a convenient scale for which the numbers are compatible. Answers may vary. Sample:

Scale 1 cm = 10 ft

50. Choose a convenient scale for which the numbers are approximately compatible. Answers may vary. Sample:

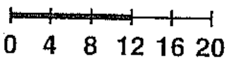
Scale 1 in. = 46 ft

51. Choose a convenient scale for which the numbers are compatible. Answers may vary. Sample:

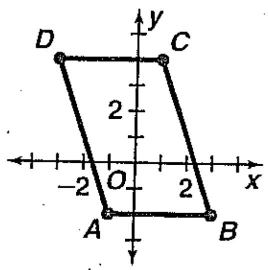
Scale 1 cm = 32 ft

52. The 3 in.-by-4 ft rectangle is really 3 in. by 48 in., so the ratio of its sides is $3 : 48$, or $1 : 16$. The 3 ft-by-4 yd rectangle is actually 1 yd by 4 yd, so the ratio of its sides is $1 : 4$. The ratios are not the same. Elaine did not convert units, and thought the ratios equaled 1. **53.** Answers may vary. Sample: ① $\frac{a}{b} = \frac{c}{d}$; $\frac{a+b}{b} = \frac{c+d}{d}$; $d(a+b) = b(c+d)$; $\frac{d(a+b)}{d(c+d)} = \frac{b(c+d)}{d(c+d)}$; $\frac{a+b}{c+d} = \frac{b}{d}$. ② By the Properties of Proportions, $\frac{b}{d} = \frac{a}{c}$. **54.** Answers may vary. Sample: ①: $\frac{a}{b} = \frac{c}{d}$; $\frac{a}{c} = \frac{b}{d}$; $\frac{a+c}{c} = \frac{b+d}{d}$; $d(a+c) = c(b+d)$; $\frac{d(a+c)}{d(b+d)} = \frac{c(b+d)}{d(b+d)}$; $\frac{a+c}{b+d} = \frac{c}{d}$ ② By the Properties of Proportions: $\frac{c}{d} = \frac{a}{b}$, so $\frac{a+b}{c+d} = \frac{a}{b}$. **55.** $\frac{a}{b} = \frac{c}{d}$; $\frac{a}{b} + 2 = \frac{c}{d} + 2$; $\frac{a}{b} + \frac{2b}{b} = \frac{c}{d} + \frac{2d}{d}$; $\frac{a+2b}{d} = \frac{c+2d}{d}$ **56.** Multiplying both sides of $\frac{a}{b} = \frac{c}{d}$ by $\frac{bd}{ac}$ results in $\frac{a}{b} \cdot \frac{bd}{ac} = \frac{c}{d} \cdot \frac{bd}{ac}$, so $\frac{d}{c} = \frac{b}{a}$. By the Symmetric Property of Equality, $\frac{b}{a} = \frac{d}{c}$. **57.** Multiplying both sides of $\frac{a}{b} = \frac{c}{d}$ by $\frac{b}{c}$ results in $\frac{a}{b} \cdot \frac{b}{c} = \frac{c}{d} \cdot \frac{b}{c}$, so $\frac{a}{c} = \frac{b}{d}$. **58.** $\frac{a}{b} = \frac{c}{d}$; $\frac{a}{b} + 1 = \frac{c}{d} + 1$; $\frac{a}{b} + \frac{b}{b} = \frac{c}{d} + \frac{d}{d}$; $\frac{a+b}{b} = \frac{c+d}{d}$ **59.** Solve for x: $\frac{x}{6} = \frac{x+10}{18}$; $18x = 6(x+10)$; $3x = x + 10$; $2x = 10$; $x = 5$. Solve for y: $\frac{x}{6} = \frac{4x}{y}$; $\frac{5}{6} = \frac{4(5)}{y}$; $5y = 6 \cdot 4(5)$; $y = 6 \cdot 4 = 24$. So, $x = 5$ and $y = 24$. **60.** Solve for y: $\frac{9}{y} = \frac{y}{25}$; $y^2 = 9(25)$; $y = \sqrt{9(25)} = 3(5) = 15$. Solve for x: $\frac{x}{5} = \frac{9}{y}$; $\frac{x}{5} = \frac{9}{15}$; $x = 5\left(\frac{9}{15}\right) = 3$. So, $x = 3$ and $y = 15$. **61.** Solve for x: $\frac{1}{x} = \frac{4}{x+9}$; $x + 9 = 4x$; $9 = 3x$; $x = 3$. Solve for y: $\frac{1}{x} = \frac{7}{y}$; $\frac{1}{3} = \frac{7}{y}$; $y = 3(7) = 21$. So, $x = 3$ and $y = 21$. **62.** $\frac{21}{x} = \frac{7}{3}$; $\frac{x}{21} = \frac{3}{7}$; $x = 21\left(\frac{3}{7}\right) = 9$. The answer is choice C. **63.** $\frac{4}{x-1} = \frac{1}{x}$; $4x = x - 1$; $3x = -1$; $x = -\frac{1}{3}$. The answer is choice G. **64.** $\frac{x}{x+6} = \frac{2}{3}$; $3x = 2(x+6)$; $3x = 2x + 12$; $x = 12$. The answer is choice D. **65.** $\frac{3}{8} = \frac{x+3}{9}$; $72\left(\frac{3}{8}\right) = 72\left(\frac{x+3}{9}\right)$; $27 = 8(x+3)$; $27 = 8x + 24$; $3 = 8x$; $x = \frac{3}{8}$. The answer is choice H. **66** [2] **a.** $\frac{2.75}{16} =$

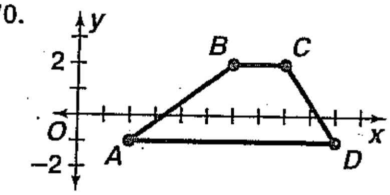
$\frac{23.2}{x}$ (OR equivalent proportion) **b.** about 135 km [1] incorrect proportion OR incorrect distance

67. 

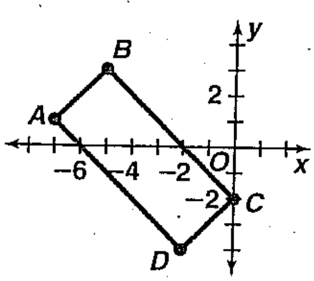
The traveler will wait at least 8 min if he or she arrives 0 to 12 min after the previous bus departs. $\frac{12}{60} = 60\%$ **68.** The probability is $A_{6\text{-in.}} : A_{12\text{-in.}} = \pi(6)^2 : \pi(12)^2 = 6^2 : 12^2 = 36 : 144 = 1 : 4 = \frac{1}{4}$, or 25%.

69. 

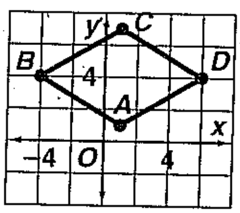
Opp. sides have the same slope, so opp. sides are ∥. The quad. is a ▱.

70. 

Exactly 2 sides are ∥, so the quad. is a trapezoid.

71. 

Adjacent sides are ⊥, so the quad. is a rectangle. Since rectangles are ▱s, it is also a ▱.

72. 

Since opp. sides are ∥, the quad. is a ▱. Since all sides are ≅, the ▱ is a rhombus. **73.** Since an isosc. △ has at least 2 ≅ sides and a scalene △ has no ≅ sides, the contradictory statements are I and III. **74.** By def. of complementary, Statement II indicates that $m\angle 1 + m\angle 2 = 90$, which contradicts Statement III. The contradictory statements are II and III. **75a.** To write the inverse, negate both the hyp. and conclusion: If an ∠ is not acute, then it does not measure between 0 and 90. **75b.** To write the contrapositive, switch the hyp. and conclusion of the inverse: If an ∠ does not measure between 0 and 90, then it is not acute. **76a.** To write the inverse, negate both the hyp. and conclusion: If two lines are not ∥, then they are not coplanar. **76b.** To write the contrapositive, switch the hyp. and conclusion of the inverse: If two lines are not coplanar, then they are not ∥. **77a.** To write the inverse, negate both the hyp. and conclusion: If two ∠s are not complementary, then the ∠s are not both acute. **77b.** To write the contrapositive, switch the hyp. and conclusion of the inverse: If two ∠s are not both acute, then they are not compl.

SOLVING QUADRATIC EQUATIONS *page 422*

1. $x^2 + 5x - 14 = 0$ is in standard form, so $x = \frac{-(5) \pm \sqrt{5^2 - 4(1)(-14)}}{2(1)} = \frac{-5 \pm \sqrt{25 + 56}}{2} = \frac{-5 \pm \sqrt{81}}{2} = \frac{-5 \pm 9}{2}$. Thus, $x = \frac{-5 + 9}{2} = 2$, or $x = \frac{-5 - 9}{2} = -7$. **2.** $4x^2 - 13x + 3 = 0$ is in standard form, so $x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(4)(3)}}{2(4)} = \frac{13 \pm \sqrt{169 - 48}}{8} = \frac{13 \pm \sqrt{121}}{8} = \frac{13 \pm 11}{8}$. Thus, $x = \frac{13 + 11}{8}$, which is 3, or $x = \frac{13 - 11}{8}$, which is $\frac{1}{4}$. **3.** $2x^2 + 7x + 3 = 0$ is in standard form, so $x = \frac{-7 \pm \sqrt{7^2 - 4(2)(3)}}{2(2)} = \frac{-7 \pm \sqrt{49 - 24}}{4} = \frac{-7 \pm \sqrt{25}}{4} = \frac{-7 \pm 5}{4}$. Thus, $x = \frac{-7 + 5}{4}$, which is $-\frac{1}{2}$, or $x = \frac{-7 - 5}{4}$, which is -3. **4.** $5x^2 + 2x - 2 = 0$ is in standard form, so $x = \frac{-2 \pm \sqrt{2^2 - 4(5)(-2)}}{2(5)} = \frac{-2 \pm \sqrt{4 + 40}}{10} = \frac{-2 \pm \sqrt{44}}{10}$. Thus, $x = \frac{-2 + \sqrt{44}}{10}$, which is about 0.46, or $x = \frac{-2 - \sqrt{44}}{10}$, which is about -0.21. **5.** Write $6x^2 + 10x = 5$ in standard form: $6x^2 + 10x - 5 = 0$. Then $x = \frac{-10 \pm \sqrt{10^2 - 4(6)(-5)}}{2(6)} = \frac{-10 \pm \sqrt{100 + 120}}{12} = \frac{-10 \pm \sqrt{220}}{12}$. So, $x = \frac{-10 + \sqrt{220}}{12}$, which is about 0.40, or $x = \frac{-10 - \sqrt{220}}{12}$, which is about -2.07. **6.** Write $1 = 2x^2 - 6x$ in standard form: $2x^2 - 6x - 1 = 0$. Then $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(-1)}}{2(2)} = \frac{6 \pm \sqrt{36 + 8}}{4} = \frac{6 \pm \sqrt{44}}{4}$. So, $x = \frac{6 + \sqrt{44}}{4}$, which is about 3.16, or $x = \frac{6 - \sqrt{44}}{4}$, which is about -0.16. **7.** Write $x^2 - 6x = 27$ in standard form: $x^2 - 6x - 27 = 0$. Then $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-27)}}{2(1)} = \frac{6 \pm \sqrt{36 + 108}}{2} = \frac{6 \pm \sqrt{144}}{2} = \frac{6 \pm 12}{2}$. So, $x = \frac{6 + 12}{2}$, which is 9, or $x = \frac{6 - 12}{2}$, which is -3. **8.** $2x^2 - 10x + 11 = 0$ is in standard form, so $x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(2)(11)}}{2(2)} = \frac{10 \pm \sqrt{100 - 88}}{4} = \frac{10 \pm \sqrt{12}}{4} = \frac{10 \pm 2\sqrt{3}}{4} = \frac{5 \pm \sqrt{3}}{2}$. So, $x = \frac{5 + \sqrt{3}}{2}$, which is about 3.37, or $x = \frac{5 - \sqrt{3}}{2}$, which is about 1.63. **9.** $8x^2 - 2x - 3 = 0$ is in standard form, so $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(8)(-3)}}{2(8)} = \frac{2 \pm \sqrt{4 + 96}}{16} = \frac{2 \pm \sqrt{100}}{16} = \frac{2 \pm 10}{16}$. Thus, $x = \frac{2 + 10}{16} = 0.75$, or $x = \frac{2 - 10}{16} = -0.5$.

8-2 Similar Polygons *pages 423–429*

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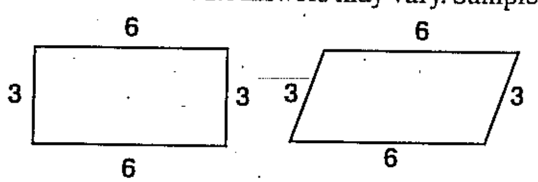
1. $\overline{AB} \cong \overline{HI}$; $\overline{BC} \cong \overline{IJ}$; $\overline{AC} \cong \overline{HJ}$ **2.** 6 **3.** 6 **4.** 3 **5.** 5

Check Understanding **1.** B corresponds to F, so $m\angle B = m\angle F$. $\overline{FG}$ corresponds to $\overline{BC}$, so the missing length is BC. **2.** It's given that all corresponding ∠s are ≅. Also, $\frac{12}{18} = \frac{14}{21} = \frac{16}{24}$ because each ratio is equivalent to $\frac{2}{3}$, so the sides are proportional. Because the $\angle$s are $\cong$ and the sides are proportional, the $\triangle$s are similar. **3.** $\frac{SR}{NM} = \frac{QR}{LM}$; $\frac{SR}{3.2} = \frac{6}{5}$; $SR = 3.2\left(\frac{6}{5}\right) = 3.84 \approx 3.8$ **4.** The height of the screen is shorter than its length, but the height of the photograph is longer than its length, so the orientation of the fields is opp. Compare ratios of the final image to the original image to find the smaller similarity ratio. Since $\frac{16}{3} > \frac{12}{5}$, the similarity ratio is $\frac{12}{5}$. Write and solve a proportion to find the width of the final image: $\frac{w}{3} = \frac{12}{5}$, so $w = \frac{36}{5}$, or 7.2 in. Then find the height of the final image: $\frac{h}{5} = \frac{12}{5}$, so $h = 12$, or 12 in. The photograph dimensions are 7.2 in. by 12 in. **5.** $\frac{n}{20} = \frac{1.618}{1}$; $n = 20(1.618) = 32.36$, or about 32.4 cm.

Exercises **1.** D corresponds to H, so $\angle D \cong \angle JHY$. **2.** Y corresponds to R, so $\angle Y \cong \angle R$. **3.** T corresponds to X, so $\angle T \cong \angle JXY$. **4.** $\overline{DR}$ corresponds to $\overline{HY}$, so the missing side length is HY. **5.** $\overline{JX}$ corresponds to $\overline{JT}$, so the missing side length is JT. **6.** $\overline{DR}$ corresponds to $\overline{HY}$, so the missing side length is HY. **7.** They are similar if the corr. sides are proportional and the corr. $\angle$s are $\cong$. All the $\angle$s in both polygons measure 90, so corr. $\angle$s are $\cong$. However, since $\frac{20}{30} \neq \frac{36}{52}$, they are not similar. **8.** They are similar if the corr. sides are proportional and the corr. $\angle$s are $\cong$. The corr. $\angle$s are $\cong$ by the tick marks. Since $\frac{6}{8}$, $\frac{7.5}{10}$, and $\frac{12}{16}$ are all equivalent to $\frac{3}{4}$, the sides are proportional. Thus, the similarity ratio is $\frac{3}{4}$, and $QRST \sim XWYZ$. **9.** They are similar if the corr. sides are proportional and the corr. $\angle$s are $\cong$. All $\angle$s in both polygons are rt. $\angle$s, so corr. $\angle$s are $\cong$. Since $\frac{15}{25}$ and $\frac{18}{30}$ are both equivalent to $\frac{3}{5}$, the sides are proportional. The similarity ratio is $\frac{3}{5}$, and $JKLM \sim OPQN$. **10.** They are similar if the corr. sides are proportional and the corr. $\angle$s are $\cong$. Let x represent the measure of the 2 congruent $\angle$s in $ABCD$. Since the sum of the measures of the $\angle$s of a quad. is 360, then $2x + 2(110) = 360$, so $x = 70$. Thus, the corr. $\angle$s are $\cong$. Since all sides are $\cong$ in both quads., the ratio of their corr. sides is $\frac{8}{10}$, or $\frac{4}{5}$. So, the similarity ratio is $\frac{4}{5}$, and $ABCD \sim FGHE$. **11.** The polygons are similar if the corr. sides are proportional and corr. $\angle$s are $\cong$. Let n represent the measure of the 2 congruent $\angle$s in $PRST$. Since the sum of the measures of the $\angle$s of a quad. is 360, then $2n + 2(100) = 360$, so $n = 80$. Since $80 \neq 95$, the measures of the corr. $\angle$s are not $\cong$. **12.** The $\triangle$s are similar if the corr. sides are proportional and the corr. $\angle$s are $\cong$. The tick marks show that the corr. $\angle$s are $\cong$. Since $\frac{3.5}{2.5}$, $\frac{6.3}{4.5}$, and $\frac{8.4}{6.0}$ are each equivalent to $\frac{7}{5}$, the sides are proportional. Thus, the similarity ratio is $\frac{7}{5}$, and $\triangle ABC \sim \triangle FED$. **13.** $\frac{x}{8} = \frac{5}{10}$, so $x = 4$. $\frac{y}{6} = \frac{5}{10}$, so $y = 3$. **14.** $\frac{x}{10} = \frac{10}{5}$, so $x = 20$. $\frac{y}{35} = \frac{5}{10}$, so $y = 17.5$. $\frac{z}{15} = \frac{5}{10}$, so $z = 7.5$. **15.** $\frac{x}{12} = \frac{8}{6}$, so $x = 16$. $\frac{y}{6} = \frac{6}{8}$, so $y = 4.5$. $\frac{z}{10} = \frac{6}{8}$, so $z = 7.5$. **16.** $\frac{x}{4} = \frac{9}{6}$, so $x = 6$. $\frac{y}{12} = \frac{6}{9}$, so $y = 8$. $\frac{z}{15} = \frac{6}{9}$, so $z = 10$. **17.** The paper can be oriented either horizontally or vertically. Let the short side of the card corr. to the short side of the paper. Compare ratios of the final image to the original image to find the smaller similarity ratio. Since $\frac{8\frac{1}{2}}{3} > \frac{11}{5}$, use the similarity ratio $\frac{11}{5}$. Write and solve a proportion to find the width of the final image: $\frac{w}{3} = \frac{11}{5}$, so $w = \frac{33}{5}$, or 6.6 in. Then find the height of the final image: $\frac{h}{5} = \frac{11}{5}$, so $h = 11$, or 11 in. The enlargement will be 6.6 in. by 6 in. **18.** The orientation of length and width is the same for the map and the card. Let the short side of the map corr. to the short side of the paper. Compare ratios of the final image to the original image to find the lesser similarity ratio. Since $\frac{4}{9} > \frac{6}{15}$, use the similarity ratio $\frac{6}{15}$. Write and solve a proportion to find the width of the final image: $\frac{w}{9} = \frac{6}{15}$, so $w = 3.6$. Then find the length of the final image: $\frac{\ell}{15} = \frac{6}{15}$, so $\ell = 6$. The reduction will be 3.6 in. by 6 in. **19.** $\frac{s}{114} = \frac{1}{1.618}$, so $s = \frac{114}{1.618} \approx 70$, or about 70 mm. **20.** The shorter side can be 54 in. $\frac{\ell}{54} = 1.618$, so $\ell = 54(1.618) = 87.37$. The dimensions are 54 in. by 87.37 in. **21.** $\overline{DF}$ corr. to $\overline{HK}$, so the similarity ratio is $6:9$, or $2:3$. **22.** $\overline{HK}$ corr. to $\overline{DF}$, so the similarity ratio is $9:6$, or $3:2$. **23.** $m\angle F = 180 - (60 + 70) = 50$ **24.** $\angle F$ corr. to $\angle K$, and, from Exercise 23, $m\angle F = 50$, so $m\angle K = m\angle F = 50$. **25.** M and G are corr. vertices, so $m\angle M = m\angle G = 70$. **26.** $DF = 2$ and $HK = 3$, so $\frac{DF}{HK} = \frac{2}{3}$. **27.** From Exercise 22, the similarity ratio of $\triangle HKM$ to $\triangle DFG$ is $3:2$, or $\frac{3}{2}$. Also, $\overline{HM}$ corr. to $\overline{DG}$. Thus, $\frac{HM}{5} = \frac{3}{2}$, so $HM = \frac{15}{2}$, or 7.5 m. **28.** From Exercise 21, the similarity ratio of $\triangle DFG$ to $\triangle HKM$ is $2:3$, or $\frac{2}{3}$. Also, $\overline{GF}$ corr. to $\overline{MK}$. Thus, $\frac{GF}{8.4} = \frac{2}{3}$, so $GF = 5.6$, or 5.6 m. **29.** In $\cong$ polygons, all corr. $\angle$s are $\cong$ and all corr. sides are $\cong$. So, $\cong$ figures are similar because they have the same shape and their corr. sides all have a ratio of 1. **30a.** The similarity symbol is positioned over the equal symbol to create the congruence symbol, so the similarity symbol and equals symbol are used. **30b.** Answers may vary. $\cong$ figures are similar with $=$ ratios. **31.** 80 in. $= 6\frac{2}{3}$ ft and 16 in. $= 1\frac{1}{3}$ ft, so $\frac{x}{6\frac{2}{3}} = \frac{4}{4\frac{1}{3}}$; $\frac{3x}{20} = \frac{12}{4}$; $3x = 60$; $x = 20$. The length is 20 ft. **32.** $\triangle WLJ \sim \triangle QBV$, so $2x = 120$; $x = 60$. Then $\frac{x}{2} + 5 = \frac{60}{2} + 5 = 35$. By the Triangle Angle-Sum Thm., $y = 180 - (35 + 120) = 25$. So, $x = 60$ and $y = 25$. **33.** $\triangle PRQ \sim \triangle SRT$, so $\frac{x}{5.2} = \frac{3.5}{7}$; $x = 5.2\left(\frac{3.5}{7}\right) = 2.6$, or 2.6 cm. **34.** Use the least side of the first to the least side of the second. The ratio is $3:4$. **35.** Use the lesser side of the first to the lesser side of the second. The ratio is $9:3$, or $3:1$. **36.** The sides of the first $\triangle$ from least to greatest are 6, 8, 10. The sides of the second $\triangle$ from least to greatest are 3, 4, 5. Use the least side of the first to the least side of the second. The ratio is $6:3$, or $2:1$. **37.** The sides of the first polygon from least to greatest are 5, 7.5, 10, 17.5. The sides of the second polygon from least to

greatest are 10, 15, 20, 35. Use the least side of the first to the least side of the second. The ratio is 5 : 10, or 1 : 2. **38.** The sides of the first polygon from least to greatest are 6, 8, 10, 16. The sides of the second polygon from least to greatest are 4.5, 6, 7.5, 12. Use the greatest side of the first to the greatest side of the second. The ratio is 16 : 12, or 4 : 3. **39.** The sides of the first polygon from least to greatest are 4, 6, 8, 10. The sides of the second polygon from least to greatest are 6, 9, 12, 15. Use the least side of the first to the least side of the second. The ratio is 4 : 6, or 2 : 3. **40.** The ∠ measures stay the same. The sides are $\left(\frac{100 - 50}{100}\right)(4)$, or 2 cm each, and the ∠s are 60° and 120°. **41.** The ∠ measures stay the same. The sides are $\left(\frac{50}{100}\right)(4)$, or 2 cm each, and the ∠s are 60° and 120°. **42.** The ∠ measures stay the same. The sides are $\left(\frac{100 - 20}{100}\right)(4)$, or 3.2 cm each, and the ∠s are 60° and 120°. **43.** The ∠ measures stay the same. The sides are $\left(\frac{20}{100}\right)(4)$, or 0.8 cm each, and the ∠s are 60° and 120°. **44.** The ∠ measures stay the same. The sides are $\left(\frac{100 - 75}{100}\right)(4)$, or 1 cm each, and the ∠s are 60° and 120°. **45.** The ∠ measures stay the same. The sides are $\left(\frac{75}{100}\right)(4)$, or 3 cm each, and the ∠s are 60° and 120°. **46.** $\frac{10}{n} = \frac{1}{1.618}$; $n = 16.18$, or about 16.2 in. **47.** $\frac{n}{10} = \frac{1}{1.618}$; $n = \frac{10}{1.618}$, or about 6.2 in. **48.** The ratio of the long side to the short side of the old bill is $\frac{7.42}{3.13}$, or about 2.37 : 1. The ratio of the long side to the short side of the new bill is $\frac{6.14}{2.61}$, or about 2.35 : 1. Since the ratios are not =, the old and new bills are not similar rectangles. **49.** When ratios of corr. parts of one figure have the same similarity ratio to corr. parts of another figure, then the figures are similar. Explanations may vary. Sample: The ratios of radii, diameters, and circumferences of 2 circles are =, so all circles are similar to each other.
50. Either let the similarity ratio between the two figures be 2 : 1 or let the ratio between two sides of a ▱ have the ratio 2 : 1. Answers may vary. Sample:



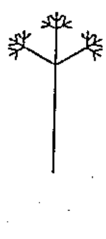
51a. ① Corr. sides of ~ polygons are proportional. ② Substitution ③ Cross-Product Property ④ Subtraction Property **51b.** $\sqrt{5} > 1$, so $\frac{1 - \sqrt{5}}{2}$ is negative. Length cannot be negative. **51c.** 1.6180
52a. $8 + 13 = 21; 13 + 21 = 34; 21 + 34 = 55; 34 + 55 = 89; 55 + 89 = 144; 89 + 144 = 233; 144 + 233 = 377$
52b. $\frac{8}{5} = 1.6; \frac{13}{8} = 1.625; \frac{21}{13} \approx 1.6154; \frac{34}{21} \approx 1.6190; \frac{55}{34} \approx 1.6176; \frac{89}{55} \approx 1.6182; \frac{144}{89} \approx 1.6180; \frac{233}{144} \approx 1.6181; \frac{377}{233} = 1.6180$ **52c.** The ratios hover above and below the golden ratio and, as the values increase, the ratios approach the golden ratio. **53.** A similarity ratio cannot be determined between the two figures since lengths in $\triangle HNV$ are not labeled. The answer is choice D. **54.** The ∠s are similar and N corr. to C, so the corr. ∠s are ≅. Thus, $m\angle N = m\angle C$. The answer is choice C. **55.** W corr. to H, so $m\angle W = m\angle H = 80$. Thus, $m\angle C = 180 - (80 + 38) = 62$. N corr. to C so, $m\angle N = m\angle C = 62$. Since $62 > 38$, the answer is choice A.
56 [2] **a.** $\frac{BC}{KL} = \frac{2}{3}$ and $BC = 8$, so $\frac{8}{KL} = \frac{2}{3}$. Thus, $KL = 12$, or 12 cm. **b.** $m\angle KLM = 38$ [1] incorrect length OR incorrect ∠ measure **57.** By the Cross-Product Property $9x = 7y$. **58.** By Properties of Proportions ③, $\frac{x}{y} = \frac{7}{9}$. **59.** By Properties of Proportions ④, $\frac{x + 7}{7} = \frac{y + 9}{9}$. **60.** By Thm. 6-5, the quad. is a ▱. **61.** Since it cannot be shown that one pair of sides is both ∥ and ≅, both pairs of opp. sides are ∥, or both pairs of opp. sides are ≅, the quad. is not necessarily a ▱. **62.** Since alt. int. ∠s are ≅, both pairs of opp. sides are ∥, so the quad. is a ▱. **63.** The base ∠s of $\triangle CEA$ are ≅, so $\triangle CEA$ is isosc. Since corr. ∠s of ∥ lines are ≅, the base ∠s of $\triangle FED$ are ≅, so $\triangle FED$ is isosc. Since alt. int. ∠s of ∥ lines are ≅, the base ∠s of $\triangle BCD$ are ≅, so $\triangle BCD$ is isosc. **64.** From Exercise 63, $\triangle BCD$ is isosc., so $\overline{CD} \cong \overline{BD}$. $AFDB$ is a ▱ since both pairs of opp. ∠s are ≅, so $\overline{BD} \cong \overline{FA}$. Thus, $\overline{CD} \cong \overline{BD} \cong \overline{FA}$. **65.** From Exercise 64, $FA = BD$, and from Exercise 63, $\triangle BCD$ is isosc., so $BD = CD = 3$. By the Transitive Prop., $AF = 3$. $AE = FE + FA = 5 + 3 = 8$ **66.** $2m\angle A + 42 = 180$, so $m\angle A = \frac{180 - 42}{2} = 69$.

CHECKPOINT QUIZ 1 *page 429*

1. model : table = 6 : 4(12) = 1 : 8 **2.** By Properties of Proportions ③, $\frac{a}{9} = \frac{b}{10}$. **3.** The short side to the short side ratio is $\frac{4}{6}$, and the long side to the long side ratio is $\frac{8}{10}$. Since $\frac{4}{6} \neq \frac{8}{10}$, the polygons are not similar. **4.** $\frac{y}{6} = \frac{18}{54}$; $y = 6\left(\frac{18}{54}\right) = \frac{18}{9} = 2$ **5.** $\frac{x - 2}{4} = \frac{5}{7}$; $x - 2 = \frac{20}{7}$; $x = \frac{20}{7} + 2 = \frac{20}{7} + \frac{14}{7} = \frac{34}{7}$ **6.** $\frac{\ell}{5 \text{ in.}} = \frac{5 \text{ ft}}{2 \text{ in.}}$; $\ell = \frac{25}{2}$, or 12.5 ft. **7.** Since A corr. to D, $m\angle A = m\angle D$. **8.** $\overline{BC}$ corr. to $\overline{BF}$, so the missing length is BF. **9.** The postcard is $\frac{1}{2}$ ft by $\frac{1}{3}$ ft. The similarity ratio of the final image to the original image is $3 : \frac{1}{2}$, or 6 : 1. So, $\frac{x}{4} = \frac{6}{1}$, or $x = 24$. The biggest possible enlargement is 3 ft by 2 ft. **10.** x and 10 are the lengths of the shortest sides. 8 and 24 are the lengths of the middle-length sides. So, $\frac{x}{10} = \frac{8}{24}$, resulting in $x = 3\frac{1}{3}$.

EXTENSION *pages 430–431*

1.



Every outward-pointing end receives 2 new branches.

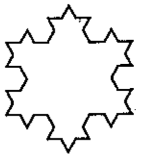
2.

Stage	0	1	2
Length	1	$\frac{4}{3}$	$\frac{16}{9}$

At each stage, the length is $\frac{4}{3}$ the length at the previous stage.

3. At each stage, the length is $\frac{4}{3}$ the length at the previous stage. At Stage 3, the length is $\frac{64}{27}$ units. At Stage 4, the length is $\frac{256}{81}$ units. **4a.** $\left(\frac{4}{3}\right)^n$, or $\frac{4^n}{3^n}$ **4b.** It increases infinitely.

5.



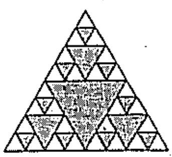
6. The snowflake began as an equilateral $\triangle$ and the sides are multiplied by the same number at each stage, so all sides are $=$. The snowflake remains equilateral for all stages.

7a.

Stage	Number of sides	Length of a side	Perimeter
0	1	1	3
1	12	$\frac{1}{3}$	4
2	48	$\frac{1}{9}$	$\frac{16}{3}$
3	192	$\frac{1}{27}$	$\frac{64}{9}$

At each stage, the number of sides changes by a factor of 4, the side length changes by a factor of $\frac{1}{3}$, and the perimeter changes by a factor of $\frac{4}{3}$. **7b.** The perimeter changes by a factor of $\frac{4}{3}$, so the perimeter at Stage 4 should be $\frac{4}{3}\left(\frac{64}{9}\right)$, or $\frac{256}{27}$. **7c.** Yes; the perimeter will continue to increase since it increases by a factor of $\frac{4}{3}$ at each stage. **8.** The snowflake goes beyond the original equilateral $\triangle$, so the area is increasing.

9.



8-3 Proving Triangles Similar pages 432–438

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1. SSS **2.** SAS **3.** ASA

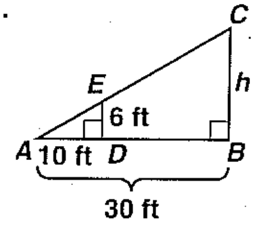
Investigation 1. The corr. sides have the same ratio, so the $\triangle$s are similar. **2.** If two $\angle$s of one $\triangle$ are congruent to two $\angle$s of another $\triangle$, then the $\triangle$s are *similar.*

Check Understanding 1. No; we don't know any of the side lengths. **2.** $\frac{AC}{EG} = \frac{CB}{GF} = \frac{AB}{EF} = \frac{3}{4}$, so the $\triangle$s are $\sim$ by the SSS $\sim$ Thm.; $\triangle ABC \sim \triangle EFG$. **3.** Since x is between the corr. $\angle$s, it corr. to length 6. Also, lengths 12 and 8 are measures of sides of corr. $\angle$s. Thus, $\frac{x}{6} = \frac{12}{8}$, so $x = 9$.
4. $\frac{x}{6} = \frac{9}{4}$; $x = \frac{54}{4} = 13.5$, or 13.5 ft.

Exercises 1. The sides of $\triangle ABC$ from least to greatest are 2, 3, 4. The sides of $\triangle FED$ from least to greatest are 3, 4.5, 6. The three ratios are $\frac{2}{3}$, $\frac{3}{4.5} = \frac{6}{9} = \frac{2}{3}$, and $\frac{4}{6} = \frac{2}{3}$. The ratios of the three sides are $=$, so $\triangle ABC \sim \triangle FED$ by the SSS $\sim$ Thm. **2.** No; not enough information is provided to use AA, SAS, or SSS. **3.** For Exercise 1, all ratios are equivalent to $\frac{2}{3}$, so the similarity ratio is $\frac{2}{3}$ for the side lengths of $\triangle ABC$ to the side lengths of $\triangle FED$. It is not possible to find a similarity ratio for the $\triangle$s in Exercise 2 because the $\triangle$s are not necessarily similar. **4.** Since $\overline{FG} \parallel \overline{JK}$ with 2 transversals, 2 pairs of alt. int. $\angle$s are $\cong$. Thus, $\triangle FHG \sim \triangle KHJ$ by the AA $\sim$ Post. **5.** Since $\frac{6}{3} \neq \frac{10}{4}$, the $\triangle$s are not similar. **6.** Since $\frac{20}{20+25} \neq \frac{25}{25+30}$ $\left(\frac{20}{45} \neq \frac{25}{55}\right)$, the $\triangle$s are not similar. **7.** Yes; $\triangle APJ \sim \triangle ABC$. Since $\frac{3}{8} = \frac{2}{5\frac{1}{3}}$, then $\frac{AP}{AB} = \frac{PJ}{BC} = \frac{AJ}{AC}$, so the $\triangle$s are similar by the SSS $\sim$ Thm. Or since $\frac{3}{8} = \frac{2}{5\frac{1}{3}}$, then $\frac{AP}{BA} = \frac{JA}{CA}$ and $\angle A \cong \angle A$, so the $\triangle$s are similar by the SAS $\sim$ Thm. **8.** Yes; $\triangle NMP \sim \triangle NQR$. Since $\frac{8}{18} = \frac{8}{18}$ and $\angle N \cong \angle N$, the $\triangle$s are similar by the SAS $\sim$ Thm. **9.** List the lengths of the sides for $\triangle JKL$ from least to greatest: 32, 45, 60. List the lengths of the sides for $\triangle PRQ$ from least to greatest: 22, 30, 40. Then write the corr. ratios to see if they are $=$. $\frac{32}{22} \neq \frac{45}{30}$, so the $\triangle$s are not similar. **10.** Two $\angle$s are congruent, so the $\triangle$s are similar by the AA $\sim$ Post. $\frac{x}{5} = \frac{4.5}{3}$; $x = 7.5$ **11.** Since same-side int. $\angle$s are suppl., the length x is that of a midsegment which is $\parallel$ to the base of the $\triangle$. Then 2 transversals create 2 pairs of $\cong$ corr. $\angle$s, so the $\triangle$s are similar by the AA $\sim$ Post. A midsegment is half the length of the base it's $\parallel$ to, so $x = \frac{5}{2}$, or 2.5.
12. Vert. $\angle$s are $\cong$, creating 2 pairs of $\cong$ $\angle$s, so the $\triangle$s are similar by the AA $\sim$ Post. $\frac{x}{22} = \frac{14}{24}$, so $x = 12\frac{5}{6}$. **13.** The $\parallel$ lines and 2 transversals create 2 pairs of $\cong$ corr. $\angle$s, so the $\triangle$s are similar by the AA $\sim$ Post. $\frac{x}{9} = \frac{2+6}{6}$; $x = 9\left(\frac{8}{6}\right) = 12$
14. The parallel lines and two transversals create 2 pairs of $\cong$ corr. $\angle$s, so the $\triangle$s are similar by the AA $\sim$ Post. $\frac{x+4}{4} = \frac{15}{5}$; $x + 4 = 4(3)$; $x = 12 - 4 = 8$ **15.** The parallel lines and two transversals create 2 pairs of $\cong$ corr. $\angle$s, so the $\triangle$s are similar by the AA $\sim$ Post. $\frac{x}{x+7.5} = \frac{12}{18}$; $18x = 12(x + 7.5)$; $18x = 12x + 90$; $6x = 90$; $x = 15$ **16.** $\frac{6}{9} = \frac{4}{6}$ and the included vert. $\angle$s are $\cong$, so the $\triangle$s are similar by the SAS $\sim$ Thm. $\frac{x}{8} = \frac{6}{4}$; $4x = 48$; $x = 12$, or 12 m. **17.** The two $\triangle$s share an $\angle$ that is $\cong$ to itself by the Refl. Prop., so the $\triangle$s are similar by the AA $\sim$ Post. $\frac{x}{110} = \frac{2(90)}{90}$; $90x = 110(2)(90)$; $x = 110(2) = 220$ **18.** Two $\angle$s are $\cong$, so the $\triangle$s are similar by the AA $\sim$ Post. $\frac{x}{5.25} = \frac{15}{5}$; $5x = 5.25(15)$; $x = 5.25(3) = 15.75$, or 15 ft 9 in. **19.** Since vert. $\angle$s are $\cong$, the $\triangle$s are similar by the AA $\sim$ Post. $\frac{x}{15} = \frac{120}{20}$; $20x = 15(120)$; $x = 15(6) = 90$, or 90 ft. **20.** Answers may vary. Hanalore Krause can measure her shadow and use similar $\triangle$s to find the length of the shadow of the proposed building. **21.** $\frac{s}{265} = \frac{1}{1.75}$; $s = \frac{265}{1.75} \approx 151$, or 151 m. **22a.** Since exactly 2 sides are $\parallel$, $RSTW$ is a trapezoid. **22b.** Since the $\parallel$ lines are intersected by 2

transversals, 2 pairs of alt. int. ∡ are ≅, creating 2 pairs of ≅ ∡ in $\triangle RSZ$ and $\triangle TWZ$. Thus, $\triangle RSZ \sim \triangle TWZ$ by the AA ~ Post. **23a.** No; an isosc. △ may be acute, right, or obtuse, so the corr. ∡ may not be ≅. **23b.** Yes; every isosc. rt. △ is a 45°-45°-90° △. Therefore, by the AA ~ Thm. they are all ~. **24.** Yes; since the vert. ∡ are ≅ and since $\frac{3}{6} = \frac{2}{4}$, $\triangle GMK \sim \triangle SMP$ by the SAS ~ Thm. **25.** Yes; since the vert. ∡ are ≅ and since $\frac{2}{4} = \frac{3}{6}$, $\triangle AWV \sim \triangle AST$ by the SAS ~ Thm. **26.** Yes; since $\frac{3}{4.5} = \frac{4}{6}$, $\triangle XYZ \sim \triangle MNK$ by the SSS ~ Thm. **27.** No; since the ∥ lines are intersected by just one transversal, only one pair of ∡ can be shown ≅, so the ∆ are not necessarily similar. **28.** $\frac{t}{15} = \frac{3}{1}$; $t = 15(3) = 45$, or 45 ft. **29.** Check students' work. **30.** Choose to compare the sides between marked ≅ ∡. The ratio is 4.5 : 3, or 3 : 2. **31.** The ratio is (3 + 3) : 3, or 2 : 1. **32.** The ratio is 24 : 14, or 12 : 7. **33.** The ratio is (6 + 2) : 6, or 4 : 3. **34.** The ratio is 15 : 5, or 3 : 1. **35.** The ratio is 18 : 12, or 3 : 2. **36.** The ratio is 6 : 4, or 3 : 2. **37.** The ratio is (90 + 90) : 90, or 2 : 1. **38.** The ratio is 15 : 5, or 3 : 1. **39.** The ratio is 120 : 20, or 6 : 1. **40.** Check students' work. Constructions can be drawn according to these instructions: Draw $\triangle ABC$. Construct $\angle A \cong \angle R$. Construct $\overline{RS}$ such that $RS = 3AB$, and $\overline{RT}$ such that $RT = 3AC$. Connect points S and T. **41a.** Add the lengths of the outside edges. The perimeter of the △ on the left is 29 + 29 + 40 = 98, or 98 m. The perimeter of the △ on the right is 37 + 37 + 24 = 98, or 98 m. **41b.** The area of the △ on the left is $\frac{1}{2}(40)(21) = 420$, or 420 m^2. The area of the △ on the right is $\frac{1}{2}(24)(35) = 420$, or 420 m^2. **41c.** No; the ∆ given are a counterexample to this conjecture. Since $\frac{29}{37} \neq \frac{40}{24}$, the ∆ are not similar because the sides are not in proportion. **42.** ① $\ell_1 \parallel \ell_2$, $\overline{EF} \perp \overline{AF}$, $\overline{BC} \perp \overline{AF}$ (Given) ② $\angle EFD$ and $\angle BCA$ are rt. ∡. (Def. of ⊥) ③ $\angle EFT \cong \angle BCA$ (All rt. ∡ are ≅.) ④ $\angle BAC \cong \angle EDF$ (If ∥ lines, then corr. ∡ are ≅.) ⑤ $\triangle ABC \sim \triangle EDF$ (AA ~ Post.) ⑥ $\frac{BC}{EF} = \frac{AC}{DF}$ (Def. of similar) ⑦ $\frac{BC}{AC} = \frac{EF}{DF}$ (Prop. of Proportions-2) **43.** ① $\frac{BC}{AC} = \frac{EF}{DF}$, $\overline{EF} \perp \overline{AF}$, $\overline{BC} \perp \overline{AF}$ (Given) ② $\angle ACB$ and $\angle DFE$ are rt. ∡. (Def. of ⊥) ③ $\angle ACB \cong \angle DFE$ (All rt. ∡ are ≅.) ④ $\triangle ABC \sim \triangle DEF$ (SAS ~ Thm.) ⑤ $\angle BAC \cong \angle EDF$ (Def. of similar) ⑥ $\ell_1 \parallel \ell_2$ (If corr. ∡ are ≅, then ∥ lines.) **44.** ① $RT \cdot TQ = MT \cdot TS$ (Given) ② $\frac{MT}{TQ} = \frac{RT}{TS}$ (Prop. of Proportions) ③ $\angle RTM \cong \angle STQ$ (Vert. ∡ are ≅.) ④ $\triangle RTM \sim \triangle STQ$ (SAS ~ Thm.) **45.** Since 8 : (8 + 2) = 12 : (12 + 3) because they both simplify to 4 : 5, and since they both share the included $\angle A$ which is congruent to itself by the Refl. Prop. of ≅, $\triangle ABC \sim \triangle ANK$ by the SAS ~ Thm. The answer is choice C. **46.** Since $\overline{AB} \parallel \overline{GL}$ and 2 transversals intersect the ∥ lines, 2 pairs of alt. int. ∡ are ≅. Thus, $\triangle ABC \sim \triangle GLC$ by the AA ~ Post. The answer is choice I. **47.** [2] **a.** All corr. ∡ of similar ∆ are ≅, so $\angle Q \cong \angle X$. Subtract $m\angle V$ and $m\angle L$ from 180 to get $m\angle Q$, which is = to $m\angle X$. **b.** 52 [1] incorrect explanation OR incorrect ∠ measure

48. [4] **a.**



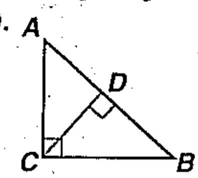
$\triangle ABC \sim \triangle ADE$; AA ~ Post. **b.** $\frac{6}{10} = \frac{h}{30}$ OR equivalent proportion; 18 ft [3] appropriate methods and appropriate diagram but incorrect solution [2] correct diagram and similarity statement OR correct proportion [1] correct height, no work shown **49.** T corr. to E, so $\angle T \cong \angle E$. **50.** D corr. to P, so $\angle D \cong \angle P$. **51.** A corr. to Y, so $\angle A \cong \angle Y$. **52.** $TRAP$ is represented in the numerator and $EZYD$ is represented in the denominator, so $\overline{RA}$ corr. to $\overline{ZY}$. The missing length is ZY. **53.** $TRAP$ is represented in the left ratio and $EZYD$ is represented in the right ratio, so $\overline{TR}$ corr. to $\overline{EZ}$. The missing length is EZ. **54.** $EZYD$ is represented in the numerator and $TRAP$ is represented in the denominator, so $\overline{AR}$ corr. to $\overline{YZ}$. The missing length is YZ. **55.** The y-coordinate W is c, and the y-coordinate of Z is $-c$. W and Z must also have the same x-coordinate. Sample: $W(-b, c)$, $Z(-b, -c)$ **56.** The y-coordinate of W is c, and the y-coordinate of Z is 0. WZ must have the opposite slope of its opposite side, so the slope of WZ is $\frac{c}{a - b}$. Sample: $W(-b, c)$, $Z(-a, 0)$ **57.** $|9 - 15| < x < |9 + 15|$; $|-6| < x\ |24|$; $6 < x < 24$

8-4 Similarity in Right Triangles pages 439–444

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1. 6 **2.** $\frac{14}{3}$ **3.** $\frac{24}{5}$ **4.** 39 **5.** 2 **6.** $\frac{8}{3}$ **7.** $\frac{40}{9}$ **8.** 18

9.



Check students' work. Sample: $\triangle ADC$ and $\triangle BCD$

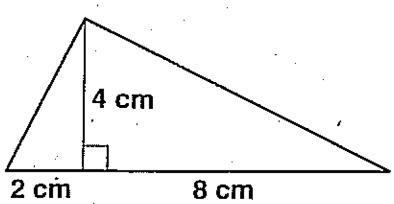
Investigation **1.** ∠4 and ∠7 **2.** ∠6 and ∠8 **3.** ∠5 and ∠9 **4.** The ∆ are ~. **5.** $\overline{RS}$ is the short leg and $\angle RST$ is the rt. ∠, so $\triangle RST \sim \triangle RWS \sim \triangle SWT$.

Check Understanding **1.** $\frac{15}{x} = \frac{x}{20}$; $x^2 = (15)(20) = (3)(5)(2)(2)(5)$; $x = \sqrt{(3)(5)(2)(2)(5)} = (5)(2)\sqrt{3} = 10\sqrt{3}$ **2.** Solve for x: $\frac{4}{x} = \frac{x}{4 + 12}$; $x^2 = 4(4 + 12) = 4(16)$; $x = \sqrt{4(16)} = 2(4) = 8$. Solve for y: $\frac{4}{y} = \frac{y}{12}$; $y^2 = 4(12)$; $y = \sqrt{4(12)} = \sqrt{4(4)(3)} = 4\sqrt{3}$. **3.** Use the Pyth. Thm.: $CD^2 + 180^2 = 300^2$; $CD^2 + 32{,}400 = 90{,}000$; $CD^2 = 57{,}600$; $CD = 240$, or 240 m.

Exercises **1.** $\frac{4}{x} = \frac{x}{9}$; $x^2 = (4)(9)$; $x = \sqrt{(4)(9)} = (2)(3) = 6$ **2.** $\frac{4}{x} = \frac{x}{10}$; $x^2 = (4)(10)$; $x = \sqrt{(4)(10)} = 2\sqrt{10}$ **3.** $\frac{4}{x} = \frac{x}{12}$; $x^2 = (4)(12) = (4)(4)(3)$; $x = \sqrt{(4)(4)(3)} = 4\sqrt{3}$ **4.** $\frac{3}{x} =$

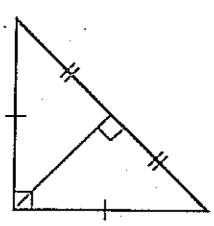
$\frac{x}{48}$; $x^2 = (3)(48) = (3)(3)(4)(4)$; $x = \sqrt{(3)(3)(4)(4)} = (3)(4) = 12$ **5.** $\frac{7}{x} = \frac{x}{56}$; $x^2 = (7)(56) = (7)(7)(2)(2)(2)$; $x = \sqrt{(7)(7)(2)(2)(2)} = (7)(2)\sqrt{2} = 14\sqrt{2}$ **6.** $\frac{5}{x} = \frac{x}{125}$; $x^2 = (5)(125) = (5)(5)(5)(5)$; $x = (5)(5) = 25$ **7.** $\frac{9}{x} = \frac{x}{24}$; $x^2 = (9)(24) = (3)(3)(2)(2)(6)$; $x = (3)(2)\sqrt{6} = 6\sqrt{6}$ **8.** $\frac{7}{x} = \frac{x}{9}$; $x^2 = (7)(9)$; $x = 3\sqrt{7}$ **9.** By Cor. 1 to Thm. 8-3, $\frac{r}{h} = \frac{h}{s}$, so the missing length is s. **10.** By Cor. 2 to Thm. 8-3, $\frac{c}{a} = \frac{a}{r}$, so the missing length is r. **11.** By Cor. 2 to Thm. 8-3, $\frac{c}{b} = \frac{b}{s}$, so the missing length is c. **12.** By Cor. 2 to Thm. 8-3, $\frac{r}{a} = \frac{a}{c}$, so the missing length in both places is a. **13.** By Cor. 1 to Thm. 8-3, $\frac{r}{h} = \frac{h}{s}$, so the missing length is h. **14.** By Cor. 2 to Thm. 8-3, $\frac{s}{b} = \frac{b}{c}$, so the missing length is b. **15.** $\frac{x}{6} = \frac{6}{4}$; $x = \frac{(6)(6)}{4} = 9$ **16.** The hyp. is divided into 2 parts measuring 40 and $50 - 40$, or 10, so $\frac{10}{x} = \frac{x}{40}$; $x^2 = (10)(40) = 400 = 20^2$; $x = 20$. **17.** $\frac{4}{x} = \frac{x}{4 + 21}$; $x^2 = 4(4 + 21) = 4(25)$; $x = 2(5) = 10$ **18.** $\frac{9}{x} = \frac{x}{9 + 3}$; $x^2 = (9)(9 + 3) = (9)(12) = (9)(4)(3)$; $x = (3)(2)\sqrt{3} = 6\sqrt{3}$ **19.** $\frac{16}{x} = \frac{x}{9}$; $x^2 = (16)(9)$; $x = (4)(3) = 12$ **20.** $\frac{144}{x} = \frac{x}{25}$; $x^2 = (144)(25) = (12^2)(5^2)$; $x = (12)(5) = 60$ **21a.** Use the Pyth. Thm. to find the distance from Alba to Blare: $c = \sqrt{30^2 + 40^2} = 50$. Let n represent the distance from Blare to the service station. Then $\frac{n}{30} = \frac{30}{50}$, so $n = \frac{900}{50}$, or 18 mi. **21b.** The distance from Alba to the service station is $50 - 18$, or 32 mi. $\frac{32}{x} = \frac{x}{18}$; $x^2 = (32)(18) = (16)(2)(2)(9) = (4^2)(2^2)(3^2)$; $x = (4)(2)(3) = 24$, or 24 mi. **22.** $\overline{JK}$ is the longer leg and $\overline{KL}$ is the shorter leg in $\triangle JKL$, so $\triangle JKL \sim \triangle KNL \sim \triangle JNK$. **23a.** The altitude is the geometric mean of the parts of the hyp., so $\frac{2}{h} = \frac{h}{8}$; $h^2 = (2)(8) = 16$; $h = 4$, or 4 cm.

23b.



23c. Answers may vary. Sample: Draw a 10-cm segment 2 cm from one endpoint, and construct a $\perp$ and measure a length 4 cm on it. Connect endpoints to form a $\triangle$.

24a.

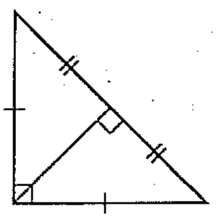


24b. They are =. Explanations may vary. Sample: The altitude and the hyp. segments are = legs of two isosc. ▲s. **25.** The hyp. is vertical, so the altitude is horizontal. The y-value is 6, since it intersects the hyp. at (4, 6). $AD = 4$ and $BD = 9$, so CD is the geometric mean of 4 and 9. $\frac{4}{CD} = \frac{CD}{9}$; $CD^2 = (4)(9)$; $CD = (2)(3) = 6$. The x-value for C can be 6 units right or 6 units left of $D(4, 6)$. So, $C = (4 + 6, 6) = (10, 6)$, or $C = (4 - 6, 6) = (-2, 6)$. **26.** $\frac{3}{x} = \frac{x}{16}$; $x^2 = (16)(3)$; $x = 4\sqrt{3}$ **27.** $\frac{4}{x} = \frac{x}{49}$; $x^2 = (4)(49)$; $x = (2)(7) = 14$ **28.** $\frac{\sqrt{8}}{x} = \frac{x}{\sqrt{2}}$; $x^2 = \sqrt{2} \cdot \sqrt{8} = \sqrt{16} = 4$; $x = \sqrt{4} = 2$ **29.** $\frac{\sqrt{28}}{x} = \frac{x}{\sqrt{7}}$; $x^2 = \sqrt{7} \cdot \sqrt{28} = \sqrt{196} = 14$; $x = \sqrt{14}$ **30.** $\frac{\frac{1}{2}}{x} = \frac{x}{2}$; $x^2 = \left(\frac{1}{2}\right)(2) = 1$; $x = 1$ **31.** $\frac{5}{x} = \frac{x}{1.25}$; $x^2 = (5)(1.25) = 6.25$; $x = \sqrt{6.25} = 2.5$ **32.** $\frac{1}{x} = \frac{x}{1000}$; $x^2 = 1000 = (100)(10)$; $x = 10\sqrt{10}$ **33.** $\frac{11}{x} = \frac{x}{1331}$; $x^2 = (11)(1331) = (11)(11)(11)(11)$; $x = (11)(11) = 121$ **34.** Solve for x: $\frac{9}{x} = \frac{x}{9 + 7}$; $x^2 = 9(9 + 7) = 9(16)$; $x = 3(4) = 12$. Solve for y: $\frac{9}{y} = \frac{y}{7}$; $y^2 = 9(7)$; $y = 3\sqrt{7}$. Solve for z: $\frac{7}{z} = \frac{z}{7 + 9}$; $z^2 = (7 + 9)(7) = (16)(7)$; $z = 4\sqrt{7}$. **35.** Solve for x: $\frac{30 - 6}{x} = \frac{x}{30}$; $x^2 = (30 - 6)(30) = (24)(30) = 720$; $x = \sqrt{720} = \sqrt{144 \cdot 5} = 12\sqrt{5}$. Solve for y: $\frac{30 - 6}{y} = \frac{y}{6}$; $y^2 = (30 - 6)(6) = (24)(6) = 144$; $y = 12$. Solve for z: $\frac{6}{z} = \frac{z}{30}$; $z^2 = (6)(30) = (6)(6)(5)$; $z = 6\sqrt{5}$. **36.** Solve for x: $\frac{x}{6} = \frac{6}{9}$; $x = \frac{36}{9} = 4$. Solve for y: $\frac{x}{y} = \frac{y}{x + 9}$; $\frac{4}{y} = \frac{y}{4 + 9}$; $y^2 = 4(4 + 9) = 4(13)$; $y = 2\sqrt{13}$. Solve for z: $\frac{9}{z} = \frac{z}{9 + x}$; $\frac{9}{z} = \frac{z}{9 + 4}$; $z^2 = (9)(9 + 4) = (9)(13)$; $z = 3\sqrt{13}$. **37.** If the shorter segment on the hyp. is x, then the longer one is $2x$, so 8 is the geometric mean of x and $2x$. $\frac{x}{8} = \frac{8}{2x}$; $2x^2 = (8)(8)$; $x^2 = \frac{(8)(8)}{2}$; $x = \frac{8}{\sqrt{2}} = 4\sqrt{2}$. The total length of the hyp. is $x + 2x$, or $3x$, so the hyp. is $3(4\sqrt{2})$, or $12\sqrt{2}$. **38a.** Given **38b.** Corollary 2 of Thm. 8-3 **38c.** Cross-Product Prop. **38d.** Addition Prop. of Equality **38e.** Dist. Prop. **38f.** Segment Addition Post. **38g.** Substitution **39.** Let the height of the totem pole be $(x + 2)$ m. Then 3 is the geometric mean of x and 2. $\frac{2}{3} = \frac{3}{x}$; $2x = 9$; $x = 4.5$, so the height is about $4.5 + 2$, or about 6.5 m. **40.** The alt. to the hyp. creates 2 new 30°-60°-90° ▲s, and the hyp. segment closest to the 10-cm leg is 5 cm. But the entire hyp. of the original $\triangle$ is 20 cm, so the other hyp. segment is $(20 - 5)$, or 15 cm. Thus, h is the geometric mean of 15 and 5. $\frac{15}{h} = \frac{h}{5}$; $h^2 = (5)(15) = (5)(5)(3)$; $h = 5\sqrt{3}$, or $5\sqrt{3}$ cm. **41.** Solve for h: Since the 2 legs are 3 and 4, $h = 5$. Solve for h_1: $\frac{h_1}{\ell_1} = \frac{\ell_1}{h}$; $\frac{h_1}{3} = \frac{3}{5}$; $h_1 = \frac{9}{5}$. Solve for h_2: $h_2 = h - h_1 = 5 - \frac{9}{5} = \frac{25 - 9}{5} = \frac{16}{5}$. Solve for a: $\frac{h_1}{a} = \frac{a}{h_2}$; $\frac{\frac{9}{5}}{a} = \frac{a}{\frac{16}{5}}$; $a^2 = \left(\frac{9}{5}\right)\left(\frac{16}{5}\right)$; $a = \frac{(3)(4)}{5} = \frac{12}{5}$.

42. Solve for a: $\frac{h_1}{a} = \frac{a}{h_2}$; $\frac{4}{a} = \frac{a}{9}$; $a^2 = (4)(9)$; $a = (2)(3) = 6$. Solve for h: $h = h_1 + h_2 = 4 + 9 = 13$. Use the Pyth. Thm. to solve for ℓ_1: $\ell_1 = \sqrt{a^2 + (h_1)^2} = \sqrt{6^2 + 4^2} = \sqrt{52} = 2\sqrt{13}$. Use the Pyth. Thm. to solve for ℓ_2: $\ell_2 = \sqrt{a^2 + (h_2)^2} = \sqrt{6^2 + 9^2} = \sqrt{117} = 3\sqrt{13}$. **43.** Solve for h_2: $\frac{h_1}{a} = \frac{a}{h_2}$; $\frac{6}{6} = \frac{6}{h_2}$; $h_2 = 6$. Solve for h: $h = h_1 + h_2 = 6 + 6 = 12$. The ▲s are isosc. rt. ▲s. Solve for ℓ_1 and ℓ_2: The largest $\triangle$ has a hyp. of 12, so $\ell_1 = \ell_2 = \frac{12}{\sqrt{2}} = 6\sqrt{2}$. **44.** Use the Pyth. Thm. to solve for h_1: $h_1 = \sqrt{(\ell_1)^2 - a^2} = \sqrt{5^2 - 4^2} = 3$. Solve for h_2: $\frac{h_2}{a} = \frac{a}{h_1}$;

$\frac{h_2}{4} = \frac{4}{3}; h_2 = \frac{16}{3}$. Solve for h: $h = h_1 + h_2 = 3 + \frac{16}{3} = \frac{9 + 16}{3} = \frac{25}{3}$. Use the Pyth. Thm. to solve for ℓ_2: $\ell_2 = \sqrt{a^2 + (h_2)^2} = \sqrt{4^2 + \left(\frac{16}{3}\right)^2} = \sqrt{6 + \frac{256}{9}} = \sqrt{\frac{9 \cdot 16}{9} + \frac{256}{9}} = \sqrt{\frac{144 + 256}{9}} = \sqrt{\frac{400}{9}} = \frac{20}{3}$. **45.** Use the Pyth. Thm. to solve for ℓ_1: $\ell_1 = \sqrt{h^2 - (\ell_2)^2} = \sqrt{13^2 - 12^2} = 5$. Solve for h_1: $\frac{h_1}{\ell_1} = \frac{\ell_1}{h}; \frac{h_1}{5} = \frac{5}{13}; h_1 = \frac{25}{13}$. Solve for h_2: $h_2 = h - h_1 = 13 - \frac{25}{13} = \frac{169 - 25}{13} = \frac{144}{13}$. Solve for a: $\frac{h_1}{a} = \frac{a}{h_2}; \frac{\frac{25}{13}}{a} = \frac{a}{\frac{144}{13}}; a^2 = \left(\frac{25}{13}\right)\left(\frac{144}{13}\right); a = \frac{(5)(12)}{13} = \frac{60}{13}$. **46.** Use the Pyth. Thm. to solve for a: $a = \sqrt{(\ell_1)^2 - (h_1)^2} = \sqrt{4^2 - 3^2} = \sqrt{16 - 9} = \sqrt{7}$. Solve for h_2: $\frac{h_1}{a} = \frac{a}{h_2}; \frac{3}{\sqrt{7}} = \frac{\sqrt{7}}{h_2}; 3h_2 = 7; h_2 = \frac{7}{3}. h = h_1 + h_2 = 3 + \frac{7}{3} = \frac{16}{3}$. Use the Pyth. Thm. to solve for $\ell_2 = \ell_2 = \sqrt{h^2 - (\ell_1)^2} = \sqrt{\left(\frac{16}{3}\right)^2 - 4^2} = \sqrt{\frac{256}{9} - 16} = \sqrt{\frac{256}{9} - \frac{16 \cdot 9}{9}} = \sqrt{\frac{256}{9} - \frac{144}{9}} = \sqrt{\frac{112}{9}} = \sqrt{\frac{16 \cdot 7}{9}} = \frac{4\sqrt{7}}{3}$.
47. Use the Pyth. Thm. to solve for ℓ_1: $\ell_1 = \sqrt{a^2 + (h_1)^2} = \sqrt{8^2 + 16^2} = \sqrt{64 + 256} = \sqrt{320} = \sqrt{64(5)} = 8\sqrt{5}$. Solve for h_2: $\frac{h_2}{a} = \frac{a}{h_1}; \frac{h_2}{8} = \frac{8}{16}; h_2 = 4$. Solve for h; $h = h_1 + h_2 = 16 + 4 = 20$. Use the Pyth. Thm. to solve for ℓ_2: $\ell_2 = \sqrt{h^2 - (\ell_1)^2} = \sqrt{20^2 - (8\sqrt{5})^2} = \sqrt{400 - (64)(5)} = \sqrt{80} = \sqrt{16(5)} = 4\sqrt{5}$. **48.** Solve for h_2: $\frac{h_2}{\ell_2} = \frac{\ell_2}{h_2 + h_1}; \frac{h_2}{6\sqrt{3}} = \frac{6\sqrt{3}}{h_2 + 3}; h_2(h_2 + 3) = 36(3); h_2^2 + 3h_2 = 108; h_2^2 + 3h_2 - 108 = 0; h_2 = \frac{-3 \pm \sqrt{3^2 - 4(1)(-108)}}{2(1)} = \frac{-3 \pm \sqrt{9 + 432}}{2} = \frac{-3 \pm \sqrt{441}}{2} = \frac{-3 \pm 21}{2}; h_2 = 9$. Solve for h: $h = h_1 + h_2 = 3 + 9 = 12$. Solve for ℓ_1: The lengths follow the pattern of a 30°-60°-90° △, so $\ell_1 = \frac{1}{2}h = \frac{1}{2}(12) = 6$. Solve for a: The lengths follow the pattern of a 30°-60°-90° △, so $a = h_1\sqrt{3} = 3\sqrt{3}$.

49a.



Draw a right △ with an alt. to the hyp. Given: rt. $\triangle ABC$ with alt. $\overline{CD}$; prove: $AC \cdot BC = AB \cdot CD$. **49b.** Yes; area $\triangle ABC = \frac{1}{2}AC \cdot BC$, so $AC \cdot BC = 2 \times$ area $\triangle ABC$. Also, area $\triangle ABC = \frac{1}{2}AB \cdot CD$, so $AB \cdot CD = 2 \times \triangle ABC$. Since both products are equal to 2 × area $\triangle ABC$, $AC \cdot BC = AB \cdot CD$.
50. $\frac{x}{x + 3} = \frac{x + 3}{12}; (x + 3)^2 = 12x; x^2 + 6x + 9 = 12x; x^2 - 6x + 9 = 0; (x - 3)(x - 3) = 0; x = 3$ **51.** $\frac{x}{x + 2} = \frac{x + 2}{x + 5}; x(x + 5) = (x + 2)(x + 2); x^2 + 5x = x^2 + 4x + 4; 5x = 4x + 4; x = 4$ **52.** $\frac{8}{2x + 1} = \frac{2x + 1}{x + 8}; 8(x + 8) = (2x + 1)(2x + 1); 8x + 64 = 4x^2 + 4x + 1; 0 = 4x^2 - 4x - 63; x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(-63)}}{2(4)} = \frac{4 \pm \sqrt{16 + 1008}}{8} = \frac{4 \pm \sqrt{1024}}{8} = \frac{4 \pm 32}{8}; x = \frac{36}{8} = 4.5$
53. Let the hyp. segment closest to the side of length 3 be x. Then $\frac{x}{3} = \frac{3}{5}$, so $x = \frac{9}{5}$. The other hyp. segment is $5 - \frac{9}{5}$, or $\frac{16}{5}$. Now the altitude a to the hyp. is the geometric mean of $\frac{9}{5}$ and $\frac{16}{5}$; $\frac{\frac{9}{5}}{a} = \frac{a}{\frac{16}{5}}; a^2 = \frac{(9)(16)}{25}; a = \frac{(3)(4)}{5} = \frac{12}{5}$. **54.** Let the hyp. segment closest to the side of length 5 be x. Then $\frac{x}{5} = \frac{5}{13}$, so $x = \frac{25}{13}$. The other hyp. segment is $13 - \frac{25}{13}$, or $\frac{144}{13}$. Now the altitude a to the hyp. is the geometric mean of $\frac{25}{13}$ and $\frac{144}{13}$; $\frac{\frac{25}{13}}{a} = \frac{a}{\frac{144}{13}}; a^2 = \frac{(25)(144)}{169}; a = \frac{(5)(12)}{13} = \frac{60}{13}$. **55.** Let the hyp. segment closest to the side of length 8 be x. Then $\frac{x}{8} = \frac{8}{17}$, so $x = \frac{64}{17}$. The other hyp. segment is $17 - \frac{64}{17}$, or $\frac{225}{17}$. Now the altitude a to the hyp. is the geometric mean of $\frac{64}{17}$ and $\frac{225}{17}$; $\frac{\frac{64}{17}}{a} = \frac{a}{\frac{225}{17}}; a^2 = \frac{(64)(225)}{289}; a = \frac{(8)(15)}{17} = \frac{120}{17}$. **56.** Let the hyp. segment closest to the side of length 20 be x. Then $\frac{x}{20} = \frac{20}{29}$, so $x = \frac{400}{29}$. The other hyp. segment is $29 - \frac{400}{29}$, or $\frac{441}{29}$. Now the altitude a to the hyp. is the geometric mean of $\frac{400}{29}$ and $\frac{441}{29}$; $\frac{\frac{400}{29}}{a} = \frac{a}{\frac{441}{29}}; a^2 = \frac{(400)(441)}{841}; a = \frac{(20)(21)}{29} = \frac{420}{29}$. **57.** $\frac{12}{m} = \frac{m}{18}; m^2 = (12)(18) = 216 = 36(6)$; $m = 6\sqrt{6}$. The answer is choice D. **58.** $\frac{2}{m} = \frac{m}{36}; m^2 = 36(2); m = 6\sqrt{2}$. The answer is choice G. **59.** $\frac{16}{m} = \frac{m}{16 + 9}$; $m^2 = 16(16 + 9) = 16(25); m = 4(5) = 20$. The answer is choice C. **60.** $\frac{5}{a} = \frac{a}{15}; a^2 = (5)(15) = 75 = 25(3); a = 5\sqrt{3}$. The answer is choice H. **61** [2] **a.** Solve for x by making a proportion from similar rt. △s. The proportion is $\frac{x}{13} = \frac{9}{x}$. Now cross multiply and solve for x. **b.** $x^2 = 9(13)$, so $x = 3\sqrt{13}$. [1] incorrect proportion OR incorrect x-value **62a.** Since vert. ∠s are ≅, 2 pairs of corr. ∠s are ≅, so $\triangle RNM \sim \triangle PNQ$ by the AA ~ Post. **62b.** AA ~ Post. **63a.** $\frac{PR}{AC} = \frac{14}{21} = \frac{2}{3}; \frac{RQ}{CB} = \frac{10}{15} = \frac{2}{3}; \frac{PQ}{AB} = \frac{12}{18} = \frac{2}{3}$, so $\triangle PRQ \sim \triangle ACB$ by the SSS ~ Thm.
63b. SSS ~ Thm. **64a.** Since $\frac{10}{16} \neq \frac{12}{20}$, $\triangle FGE$ is not similar to $\triangle FHD$, and since $\frac{10}{20} \neq \frac{12}{16}$, $\triangle FGE$ is not similar to $\triangle FDH$. So, the △s cannot be proved similar by the given information. **65.** The area of the ▱ is $bh = (10)(6) = 60$, so $8h = 60; h = 7.5$. **66.** The area of the ▱ is $bh = (15)(8) = 120$, so $b(12) = 120; b = 10$.
67. Because the diags. of a ▱ bis. each other, $2x = y + 2$ and $y = x + 3$. Substitute $(x + 3)$ for y in $2x = y + 2$: $2x = (x + 3) + 2; 2x = x + 5; x = 5$. Substitute 5 for x in $y = x + 3$: $y = 5 + 3 = 8$. **68.** The diags. of a ▱ bis. each other, so $4x = 3y - 3$ and $y + 6 = 2x + 3$. Solve the second equation for y: $y = 2x - 3$. Substitute $2x - 3$ for y in $4x = 3y - 3$: $4x = 3(2x - 3) - 3; 4x =$

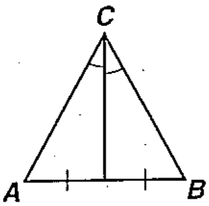
$6x - 9 - 3$; $4x = 6x - 12$; $-2x = -12$; $x = 6$. Substitute 6 for x in $y + 6 = 2x + 3$: $y + 6 = 2(6) + 3$; $y + 6 = 12 + 3$; $y = 12 + 3 - 6 = 9$. **69.** Opp. sides of a ▱ are ≅, so $5x = 4y - 1$ and $2x + 3 = y + 5$. Solve the second equation for y: $y = 2x - 2$. Substitute $2x - 2$ for y in $5x = 4y - 1$: $5x = 4(2x - 2) - 1$; $5x = 8x - 8 - 1$; $5x = 8x - 9$; $-3x = -9$; $x = 3$. Substitute 3 for x in $2x + 3 = y + 5$: $2(3) + 3 = y + 5$; $6 + 3 = y + 5$; $9 = y + 5$; $y = 4$.

TECHNOLOGY — *page 445*

1. It divides the sides into proportional segments. **2.** The segments are proportional to the lengths of the other sides of the △. **3.** The corr. segment ratios are =. **4.** The corr. segment ratios are =.

8-5 Proportion in Triangles — *pages 446–452*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*
1. 28 cm **2.** $3\frac{3}{7}$ mm **3.** 9.8 in. **4.** 11.25 ft
Check Understanding **1.** $\frac{x + 1.5}{5} = \frac{x}{2.5}$; $2.5(x + 1.5) = 5x$; $2.5x + 3.75 = 5x$; $3.75 = 2.5x$; $x = 1.5$ **2.** Solve for x: $\frac{x}{30} = \frac{15}{26}$; $x = \frac{(30)(15)}{26} = \frac{450}{26} = \frac{225}{13}$. Solve for y: $\frac{y}{16.5} = \frac{26}{15}$; $y = \frac{(16.5)(26)}{15} = 28.6$. **3.** $\frac{y}{8} = \frac{3.6}{5}$; $y = \frac{(8)(3.6)}{5} = 5.76$
Exercises **1.** $\frac{x}{10} = \frac{6}{8}$; $x = \frac{60}{8} = 7.5$ **2.** $\frac{x + 4}{2x} = \frac{9}{12}$; $12(x + 4) = 9(2x)$; $4(x + 4) = 3(2x)$; $4x + 16 = 6x$; $16 = 2x$; $x = 8$ **3.** $\frac{x}{2} = \frac{13 - x}{3}$; $3x = 2(13 - x)$; $3x = 26 - 2x$; $5x = 26$; $x = 5.2$ **4.** b corr. to e, so a corr. to d. The missing length is d. **5.** e corr. to b, so f corr. to c. The missing length is c. **6.** f corr. to c, so e corr. to b. The missing length is b. **7.** $b + c$ corr. to $e + f$, so a corr. to d. The missing length is d. **8.** $\frac{x}{5} = \frac{6}{4}$; $x = \frac{30}{4} = 7.5$ **9.** $\frac{x}{4} = \frac{5}{6}$; $x = \frac{20}{6} = 3\frac{1}{3}$ **10.** $\frac{x}{24} = \frac{8}{8 + 12}$; $x = \frac{(24)(8)}{20} = 9.6$ **11.** $\frac{x}{12} = \frac{5}{10}$; $x = 6$ **12.** $\frac{x}{3} = \frac{8}{5}$; $x = 4.8$ **13.** $\frac{x}{20} = \frac{14}{8}$; $x = 35$
14. $\frac{x}{6 - x} = \frac{6}{4}$; $4x = 36 - 6x$; $10x = 36$; $x = 3.6$
15. $\frac{x}{10 - x} = \frac{8}{6}$; $6x = 80 - 8x$; $14x = 80$; $x = \frac{80}{14} = \frac{40}{7}$
16. $\frac{x}{9} = \frac{8}{6}$; $x = 12$ **17.** JR corr. to RS, so KJ corr. to KS. The missing length is KS. **18.** KJ corr. to KS, so JP corr. to SQ. The missing length is SQ. **19.** QL corr. to SQ, so PM corr. to JP. The missing length is JP. **20.** TQ corr. to PT, so KQ corr. to KP. The missing length is KP. **21.** LW corr. to MW, so KL corr. to KM. The missing length is KM. **22.** KQ corr. to KP, so LQ corr. to MP. The missing length is MP. **23.** KS corr. to SQ, so KJ corr. to JP. The missing length is JP. **24.** KM corr. to MW, so KL corr. to LW. The missing length is LW.
25. $\frac{x}{500} = \frac{425}{380}$; $x = \frac{500 \cdot 425}{380} \approx 559$, or about 559 ft.
26. $\frac{x}{600} = \frac{425}{380}$; $x = \frac{600 \cdot 425}{380} \approx 671$, or about 671 ft.
27. The hyp. is $\sqrt{5^2 + 12^2} = 13$, so if one hyp. segment is x, the other is $(13 - x)$. Thus, $\frac{x}{5} = \frac{13 - x}{12}$; $12x = 5(13 - x)$; $12x = 65 - 5x$; $17x = 65$; $x \approx 3.8$ and $13 - x \approx 9.2$, so the segment lengths are about 3.8 cm and 9.2 cm. **28.** If the shorter side is a and the longer side is b, then $a = \frac{2b}{3}$, $b - a < 15$, and $b + a > 15$. Answers may vary. Sample: 9 cm and 13.5 cm **29.** Use the perimeter to find an alternative expression for y: $x + y + 12 + 8 = 50$; $y = 30 - x$. $\frac{x}{12} = \frac{y}{8}$; $\frac{x}{12} = \frac{30 - x}{8}$; $8x = 12(30 - x)$; $8x = 360 - 12x$; $20x = 360$; $x = 18$, or 18 m. Solve for y: $y = 30 - x = 30 - 18 = 12$, or 12 m.
30a.

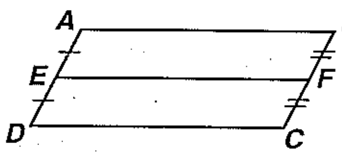


30b. Let $2x$ represent the length of the bisected side. Then x represents each of its segments. So, $\frac{AB}{x} = \frac{BC}{x}$ by the Triangle-Angle-Bisector Thm. Multiplying both sides by x results in $AB = BC$, so, by def. of isosc. △, △ABC is isosc. **31.** $\frac{4x}{5x} = \frac{4x + 8}{6x - 10}$; $4x(6x - 10) = 5x(4x + 8)$; $24x^2 - 40x = 20x^2 + 40x$; $4x^2 - 40x = 40x$; $4x^2 - 80x = 0$; $4x(x - 20) = 0$, so $x = 0$ or $x = 20$. If $x = 0$, the distances between the ∥ lines would be 0, which is impossible since the lines would then be the same line. So, $x = 20$. **32.** $\frac{7x}{10x - 4} = \frac{5x}{6x}$; $42x^2 = 50x^2 - 20x$; $-8x^2 + 20x = 0$; $-4x(2x - 5) = 0$, so $x = 0$ or $x = 2.5$. If $x = 0$, the △ lengths would be 0, which is impossible. So, $x = 2.5$. **33.** $\frac{x}{x + 6} = \frac{x - 3}{x + 1}$; $x(x + 1) = (x + 6)(x - 3)$; $x^2 + x = x^2 + 3x - 18$; $x = 3x - 18$; $-2x = -18$; $x = 9$ **34a.** $\frac{AB}{BC}$ **34b.** $\frac{WX}{XY}$ **34c.** $\frac{AB}{BC} = \frac{WX}{XY}$ **35.** Measure $\overline{AC}$, $\overline{CE}$, and $\overline{BD}$. Use the Side-Splitter Thm. Write the proportion $\frac{AC}{CE} = \frac{AB}{BD}$ and solve for AB. **36.** If the 7.5-cm side is nearest the 5-cm segment, then $\frac{x}{7.5} = \frac{3}{5}$, so $x = 4.5$, or 4.5 cm. If the 7.5-cm side is nearest the 3-cm segment, then $\frac{y}{7.5} = \frac{5}{3}$, so $y = 12.5$, or 12.5 cm. **37.** $\frac{GE}{3} = \frac{4}{2}$; $GE = 6$ **38.** $\frac{HC}{5} = \frac{2}{4}$, so $BC = 2.5$. **39.** From Exercise 37, $GE = 6$. $\frac{AB}{3} = \frac{2 + 4}{4}$, so $AB = 4.5$. $EA = 4 + 2 = 6$, $AB = 4.5$, and $EG = 6 + 3 = 9$. So, the perimeter is $EA + AB + EG = 6 + 4.5 + 9 = 19.5$. **40.** Solve for h: The △ is in the 3-4-5 family of right ⚠s, so $h = 10$. Solve for h_1: $\frac{h_1}{10 - h_1} = \frac{\ell_1}{\ell_2}$; $\frac{h_1}{10 - h_1} = \frac{6}{8}$; $8h_1 = 6(10 - h_1)$; $8h_1 = 60 - 6h_1$; $14h_1 = 60$; $h_1 = \frac{60}{14} \approx 4.3$. Solve for h_2: $h_2 = 10 - h_1 \approx 10 - 4.3 = 5.7$. **41.** Solve for h: $h = h_1 + h_2 = 4 + 9 = 13$. Use the Pyth. Thm. to solve for h: $h^2 = 5.3^2 + 11.9^2 = 28.09 + 141.61 = 169.7$; $h = \sqrt{169.7} \approx 12.9$. Solve for h_1: $\frac{h_1}{\ell_1} = \frac{\ell_2}{h}$; $\frac{h_1}{5.3} = \frac{5.3}{\sqrt{169.7}}$; $h_1 = \frac{28.09}{\sqrt{169.7}} \approx 2.2$. Solve for h_2: $\frac{h_2}{\ell_2} = \frac{\ell_2}{h}$; $\frac{h_2}{11.9} = \frac{11.9}{\sqrt{169.7}}$; $h_2 = \frac{141.61}{\sqrt{169.7}} \approx 10.9$. **42.** $h = h_1 + h_2 = 3.8 + 9.2 = 13$. Solve for ℓ_1: $\frac{h_1}{\ell_1} = \frac{\ell_1}{h}$; $\frac{3.8}{\ell_1} = \frac{\ell_1}{13}$; $\ell_1^2 = 49.4$; $\ell_1 = \sqrt{49.4} \approx 7.0$. Solve for ℓ_2: $\frac{h_2}{\ell_2} = \frac{\ell_2}{h}$; $\frac{9.2}{\ell_2} = \frac{\ell_2}{13}$; $\ell_2^2 \approx$

119.6; $\ell_2 = \sqrt{119.6} \approx 10.9$. **43.** By segment subtraction, $h_2 = h - h_1 = h - 5$. By the Triangle-Angle-Bisector Thm., $\frac{\ell_2}{\ell_1} = \frac{h_2}{h_1}$, so $\ell_2 = \frac{\ell_1 h_2}{h_1} = \frac{(5\sqrt{2})(h-5)}{5} = \sqrt{2}(h-5)$. Use the Pyth. Thm. to solve for h: $\ell_1^2 + \ell_2^2 = h^2$; $(5\sqrt{2})^2 + (\sqrt{2}(h-5))^2 = h^2$; $50 + 2(h^2 - 10h + 25) = h^2$; $h^2 - 20h + 100 = 0$; $(h-10)^2 = 0$; $h = 10$. Solve for h_2: $h_2 = h - 5 = 10 - 5 = 5$. Solve for ℓ_2: $\frac{\ell_2}{\ell_1} = \frac{h_2}{h_1}$; $\frac{\ell_2}{5\sqrt{2}} = \frac{5}{5}$; $\ell_2 = 5\sqrt{2} \approx 7.1$. **44.** Use the Pyth. Thm. to solve for h: $h = \sqrt{15^2 + 8^2} = \sqrt{225 + 64} = \sqrt{289} = 17$. Use the Triangle-Angle-Bisector Thm. to solve for h_1: $\frac{h_1}{h_2} = \frac{\ell_1}{\ell_2}$; $\frac{h_1}{17 - h_1} = \frac{15}{8}$; $8h_1 = 15(17 - h_1)$; $8h_1 = 255 - 15h_1$; $23h_1 = 255$; $h_1 = \frac{255}{23} \approx 11.1$. Solve for h_2: $h_2 = 17 - h_1 \approx 17 - 11.1 = 5.9$. **45.** Solve for h_2: $h_2 = h - h_1 = 8 - 4 = 4$. Since the ∠-bisector also bisects the hyp., the △ is an isosc. rt. △. Thus, $\ell_1 = \ell_2 = \frac{h}{\sqrt{2}} = \frac{8}{\sqrt{2}} = 4\sqrt{2} \approx 5.7$.

46. The ratio of the legs of a 30°-60°-90° △ is $\sqrt{3} : 1$. Let the right-∠ bisector divide the hyp. into segments x and y. Then, by the Triangle-Angle-Bis. Thm., $\frac{x}{y} = \sqrt{3}$, so $x = \sqrt{3}y$. **47a.** Given **47b.** The 4th Property of Proportions **47c.** Segment Addition Post. **47d.** Reflexive Prop. of ≅ **47e.** Two sides of an included ∠ are proportional: SAS ~ Thm. **47f.** Corr. ∡ of ~ ▵ are ≅. **47g.** $\overline{QX}$ is a transversal to $\overline{RS}$ and $\overline{XY}$, creating corr. ∡ 1 and 2: If corr. ∡ are ≅, then the lines are ∥. **48.** Yes; since $\frac{6}{10} = \frac{9}{15}$, the segments are ∥ by the Corollary to the Side-Splitter Thm. **49.** No; $\frac{28}{12} = \frac{7}{3}$ and $\frac{24}{10} = \frac{12}{5}$, so $\frac{28}{12} \neq \frac{24}{10}$. **50.** Yes; since $\frac{15}{12} = \frac{20}{16}$, the segments are ∥ by the Corollary to the Side-Splitter Thm. **51a.** Base the def. on that of a midsegment of a △ and a midsegment of a trapezoid: A midsegment of a ▱ connects the midpts. of 2 opp. sides.

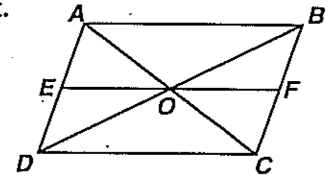
51b.



Given: ▱$ABCD$ with $\overline{EF}$ connecting the midpts. of $\overline{AD}$ and $\overline{BC}$; prove: $\overline{AB} \parallel \overline{EF}$ and $\overline{EF} \parallel \overline{CD}$.

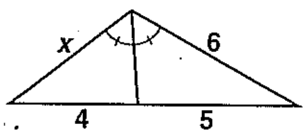
① ▱$ABCD$ (Given) ② $\overline{AD} \parallel \overline{BC}$, or $\overline{AE} \parallel \overline{BF}$ and $\overline{ED} \parallel \overline{FC}$ (Def. of ▱) ③ $\overline{AD} \cong \overline{BC}$ (Opp. sides of a ▱ are ≅.) ④ E and F are midpts. of $\overline{AD}$ and $\overline{BC}$. (Given) ⑤ $\overline{AE} \cong \overline{ED}$ and $\overline{BF} \cong \overline{FC}$ (Def. of midpt.) ⑥ $AE + ED = BF + FC$ (Subst.) ⑦ $AE = BF$ (Subtr.) ⑧ $ABFE$ is a ▱. (If one pair of opp. sides is ≅ and ∥, the quad. is a ▱.) ⑨ $ED = FC$ (Subtr.) ⑩ $EFCD$ is a ▱. (If one pair of opp. sides is ≅ and ∥, the quad. is a ▱.) ⑪ $\overline{AB} \parallel \overline{EF}$ and $\overline{EF} \parallel \overline{CD}$ (Opp. sides of a ▱ are ∥.)

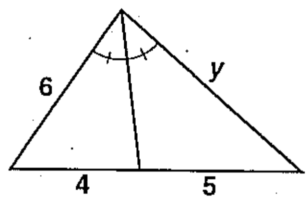
51c.



Given: ▱ $ABCD$ with midsegment $\overline{EF}$ and the diagonals intersecting at O; prove: $\overline{EF}$ bis. $\overline{AC}$ and $\overline{BD}$. Let $\overline{BD}$ intersect $\overline{EF}$ at X. Since $\overline{EF}$ is a midsegment of the ▱, it bis. $\overline{BC}$, so $BF = FC$ and $BC = 2BF$. By the midsegment theorem proved in Exercise 51b, $\overline{EF} \parallel \overline{DC}$. So, $\angle BXF \cong \angle BDC$ and $\angle BFX \cong \angle BCD$. Thus, $\triangle BFX \sim$ to $\triangle BCD$ by the AA ~ Post., and the similarity ratio is 1 : 2. Then $\frac{BX}{BD} = \frac{1}{2}$, or $2BX = BD$. Since $BD = DX + BX$, then $2BX = DX + BX$, so $BX = DX$. Since the diagonals of a ▱ bis. each other, points X and O are the same point, so both diagonals are bis. by the midsegment. **52.** $\frac{x}{2x + 10} = \frac{12}{30}$; $30x = 12(2x + 10)$; $30x = 24x + 120$; $6x = 120$; $x = 20$. The answer is choice D. **53.** Use the Pyth. Thm. to find the hyp.: $h = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25$. Let x represent the short segment on the hyp. Then $25 - x$ represents the other segment. $\frac{h}{7} = \frac{25 - h}{24}$; $24h = 175 - 7h$; $31h = 175$; $h = \frac{175}{31} \approx 5.6$. The answer is choice F. **54.** [2] $\frac{n + 1}{28} = \frac{20}{35}$; $35n + 35 = 560$; $n = 15$ [1] correct proportion solved incorrectly **55.** [4]

a.





b. $\frac{x}{4} = \frac{6}{5}$; $x = 4.8$, or 4.8 cm. $\frac{6}{4} = \frac{y}{5}$; $y = 7.5$, or 7.5 cm.

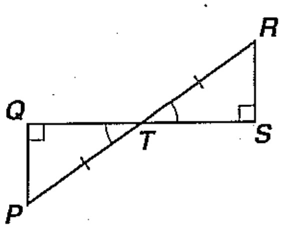
[3] correct drawings and proportions with one computational error [2] one possibility drawn and done correctly OR two correct drawings [1] one correct drawing OR one correct proportion **56.** $\frac{\text{long leg}}{\text{short leg}} = \frac{n}{h} = \frac{h}{m}$. The missing length is m. **57.** $\frac{\text{short leg}}{\text{hyp.}} = \frac{m}{b} = \frac{b}{c}$. The missing length is m. **58.** $\frac{\text{long leg}}{\text{hyp.}} = \frac{n}{a} = \frac{a}{c}$. The missing length is c. **59.** $\frac{\text{long leg}}{\text{short leg}} = \frac{m}{h} = \frac{h}{n}$. The missing length is h. **60.** Solve for x: In a 30°-60°-90° △, the hyp. is twice the shorter leg, so $x = 2(12) = 24$. Solve for y: In a 30°-60°-90° △, the longer leg is $\sqrt{3}$ times the length of the shorter leg, so $y = 12\sqrt{3}$. **61.** Solve for x: In a 30°-60°-90° △, the shorter leg is half the hyp., so $x = 9$. Solve for y: In a 30°-60°-90° △, the longer leg is $\sqrt{3}$ times the shorter leg, so $y = 9\sqrt{3}$. **62.** Solve for x: In a 30°-60°-90° △, the shorter leg is $\frac{1}{\sqrt{3}}$ times the longer leg, so $x = 15\sqrt{3}\left(\frac{1}{\sqrt{3}}\right) = 15$. Solve for y: In a 30°-60°-90° △, the hyp. is twice the shorter leg, so $y = 2x = 2(15) = 30$. **63.** In a rectangle, the diags. are =, so $5x + 8 = x + 32$; $4x = 24$; $x = 6$. $RT = SV = x + 32 = 6 + 32 = 38$ **64.** In a rectangle, the diags. are =, so $42 - x = 9x - 8$; $-10x = -50$; $x = 5$. $RT = SV = 9x - 8 = 9(5) - 8 = 45 - 8 = 37$ **65.** In a rectangle, the diags. are =, so $8x - 4 = 6x + 9$; $2x = 13$; $x = 6.5$. $RT = SV = 6x + 9 = 6(6.5) + 9 = 39 + 9 = 48$ **66.** In a rectangle, the diags. are =, so $3x + 5 = 5x + 4$; $-2x = -1$; $x = 0.5$. $RT = SV = 5x + 4 = 5(0.5) + 4 = 2.5 + 4 = 6.5$

CHECKPOINT QUIZ 2 *page 452*

1. Since 2 pairs of ∠s are ≅, $\triangle ABC \sim \triangle XYZ$ by the AA ~ Post. **2.** Two sides are proportional and the included ∠s are ≅, so $\triangle WST \sim \triangle HJG$ by the SAS ~ Thm. **3.** Solve for w; $\frac{w}{3} = \frac{3}{2}$, so $w = 4.5$. Use the Pyth. Thm. to solve for x: $x = \sqrt{w^2 + 3^2} = \sqrt{4.5^2 + 3^2} = \sqrt{20.25 + 9} = \frac{3\sqrt{13}}{2} \approx 5.41$. **4.** $\frac{m}{16} = \frac{6}{8}$, so $m = 12$. **5.** $\frac{x+5}{10} = \frac{10}{5}$; $x + 5 = 20$; $x = 15$ **6.** $\frac{x}{12} = \frac{5}{8}$; $x = \frac{60}{8} = 7.5$ **7.** $\frac{5}{y} = \frac{y}{16}$; $y^2 = 16(5)$; $y = 4\sqrt{5}$ **8.** $\frac{y}{6} = \frac{3}{5}$; $y = \frac{18}{5} = 3.6$ **9.** $\frac{x}{28} = \frac{10}{16}$; $x = \frac{280}{16} = 17.5$ **10.** $\frac{x}{55} = \frac{63}{45}$; $x = \frac{(55)(63)}{45} = \frac{(55)(63)}{(5)(9)} = (11)(7) = 77$

READING MATH *page 453*

Check Understanding Given: $\overline{PQ} \perp \overline{QS}$, $\overline{RS} \perp \overline{QS}$, T is the midpt. of $\overline{PR}$; prove: $\triangle PQT \cong \triangle RST$.



① $\overline{PQ} \perp \overline{QS}$, $\overline{RS} \perp \overline{QS}$ (Given) ② $\angle Q$ and $\angle S$ are rt. ∠s. (Def. of ⊥; from step 1) ③ $\angle Q \cong \angle S$ (All rt. ∠s are ≅; from step 2.) ④ $\angle QTP \cong \angle STR$ (Vert. ∠s are ≅.) ⑤ T is the midpt. of $\overline{PR}$ (Given) ⑥ $\overline{PT} \cong \overline{RT}$ (Def. of midpt.; from step 5) ⑦ $\triangle PQT \cong \triangle RST$ (AAS; from steps 3, 4, and 6)

8-6 Perimeters and Areas of Similar Figures *pages 454–459*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1. 28 in.; 49 in.^{2} **2.** 24 m; 32 m^2 **3.** 24 cm; 24 cm^2 **4.** 8 cm; 3 cm^2 **5.** 16 cm; 12 cm^2 **6.** 24 cm; 27 cm^2

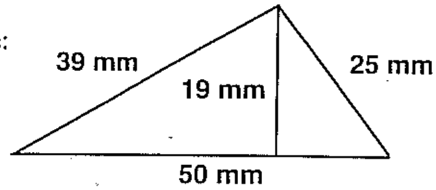
Investigation **1.** Check students' work. **2.** Check students' work. **3.** The ratio for perimeters is the same, but the ratio for areas is the similarity ratio squared.

Check Understanding **1a.** The ratio of the perimeters is the same as the similarity ratio, so it is also 5 : 7. **1b.** The ratio of the areas is the square of the similarity ratio, so it is $5^2 : 7^2$, or 25 : 49. **2.** The ratio of the areas is $\left(\frac{3}{4}\right)^2$, or $\frac{9}{16}$. Solve for the area of the smaller ▱: $\frac{A}{96} = \frac{9}{16}$; $A = \frac{96(9)}{16} = 54$, or 54 in.^{2}. **3.** The cost of glass is more closely associated with the amount of glass than with the perimeter of the glass, so use the area ratio of $3^2 : 5^2$, or 9 : 25. Solve for the dollar amount: $\frac{2.50}{d} = \frac{9}{25}$; $9d = 62.5$; $d \approx 6.94$, so the cost should be \$6.94. **4.** $\sqrt{1875} : \sqrt{135} = \sqrt{625(3)} : \sqrt{9(15)} = 25\sqrt{3} : 3\sqrt{15} = 25\sqrt{3}\sqrt{15} : 3\sqrt{15}\sqrt{15} = 25\sqrt{45} : 3(15) = 25\sqrt{9(5)} : 45 = 75\sqrt{5} : 45 = 5\sqrt{5} : 3$

Exercises **1.** The similarity ratio is 2 : 4, or 1 : 2. The ratio of the perimeters is 1 : 2. The ratio of the areas is $1^2 : 2^2$, or 1 : 4. **2.** The similarity ratio is 8 : 6, or 4 : 3. The ratio of the perimeters is 4 : 3. The ratio of the areas is $4^2 : 3^2$, or 16 : 9. **3.** The similarity ratio is 14 : 21, or 2 : 3. The ratio of the perimeters is 2 : 3. The ratio of the areas is $2^2 : 3^2$, or 4 : 9. **4.** The similarity ratio is 15 : 25, or 3 : 5. The ratio of the perimeters is 3 : 5. The ratio of the areas is $3^2 : 5^2$, or 9 : 25. **5.** The similarity ratio is 6 : 3, or 2 : 1. The ratio of the areas is $2^2 : 1^2$, or 4 : 1. $\frac{A}{6} = \frac{4}{1}$; $x = 24$. The area is 24 in.^{2}. **6.** The similarity ratio is 12 : 18, or 2 : 3. The ratio of the areas is $2^2 : 3^2$, or 4 : 9. $\frac{A}{121} = \frac{4}{9}$; $A = \frac{121(4)}{9} \approx 54$. The area is about 54 m^2. **7.** The similarity ratio is 12 : 16, or 3 : 4. The area ratio is $3^2 : 4^2$, or 9 : 16. $\frac{A}{105} = \frac{9}{16}$; $A = \frac{(105)9}{16} \approx 59$. The area is about 59 ft^2. **8.** The similarity ratio is 13 : 5, so the area ratio is $13^2 : 5^2$, or 169 : 25. $\frac{A}{65} = \frac{169}{25}$; $A = \frac{65(169)}{25} \approx 439$. The area is about 439 m^2. **9.** Since $\frac{9}{12} = \frac{12}{16}$, the floors are similar. The similarity ratio is 9 : 12, or 3 : 4, so the area ratio is $3^2 : 4^2$, or 9 : 16. $\frac{216}{x} = \frac{9}{16}$; $9x = 16(216)$; $x = 384$. The cost for the other floor would be \$384. **10.** The similarity ratio is 1 : 4, so the area ratio is 1 : 16. You should pay 16 times as much as \$2.95, or \$47.20. **11.** The area ratio is 4 : 16, so the similarity ratio and perimeter ratio will each be $\sqrt{4} : \sqrt{16}$, which is 2 : 4, or 1 : 2. **12.** The area ratio is 75 : 12 = 25 : 4, so the similarity ratio and perimeter ratio will each be 5 : 2. **13.** The area ratio is 49 : 9, so the similarity ratio and perimeter ratio will each be 7 : 3. **14.** The area ratio is 18 : 32, which is 9 : 16, so the similarity ratio and perimeter ratio will each be 3 : 4. **15.** The area ratio is $16\sqrt{3} : \sqrt{3}$, or 16 : 1, so the similarity ratio and perimeter ratio will each be 4 : 1. **16.** The area ratio is $2\pi : 200\pi$, or 1 : 100, so the similarity ratio and perimeter ratio will each be 1 : 10. **17.** The perimeter ratio is the same as the similarity ratio, so it is 3 : 1. The area ratio is the square of the similarity ratio, so it is 9 : 1. **18.** The perimeter ratio is the same as the similarity ratio, so it is 2 : 5. The area ratio is the square of the similarity ratio, so it is 4 : 25. **19.** The perimeter ratio is the same as the similarity ratio, so it is 2 : 3. The area ratio is the square of the similarity ratio, so it is 4 : 9. **20.** The perimeter ratio is the same as the similarity ratio, so it is 7 : 4. The area ratio is the square of the similarity ratio, so it is 49 : 16. **21.** The perimeter ratio is the same as the similarity ratio, so it is 6 : 1. The area ratio is the square of the similarity ratio, so it is 36 : 1. **22.** The similarity ratio is 4 : 1, so the area ratio is 16 : 1. $\frac{A}{50} = \frac{16}{1}$; $A = 50(16) = 800$. The area is 800 cm^2. **23.** While the ratio of lengths is 2 : 1, the ratio of areas is 4 : 1. **24.** The similarity ratio is 3 : 1, so the area ratio is 9 : 1. $\frac{2.7}{x} = \frac{9}{1}$; $9x = 2.7$; $x = 0.3$. The area is 0.3 cm^2. **25.** The similarity ratio is 15 : 5, or 3 : 1, so the area ratio is 9 : 1. $\frac{A}{28} = \frac{9}{1}$; $A = 28(9) = 252$. The area is 252 m^2. **26.** The area of the larger △ is 48 cm^2, so the

area ratio is 3 : 48, or 1 : 16. The similarity ratio is the square root of the area ratio, so it is 1 : 4. Solve for x: $\frac{x}{8} = \frac{1}{4}$; $x = 2$, or 2 cm. Solve for y: $\frac{y}{12} = \frac{1}{4}$; $y = 3$, or 3 cm.
27. The area of the larger $\triangle$ is 48 cm^2, so the area ratio is 6 : 48, or 1 : 8. The similarity ratio is the square root of the area ratio, so it is $\sqrt{2}$: 4. Solve for x: $\frac{x}{8} = \frac{\sqrt{2}}{4}$; $x = 2\sqrt{2}$, or $2\sqrt{2}$ cm. Solve for y: $\frac{y}{12} = \frac{\sqrt{2}}{4}$; $y = 3\sqrt{2}$, or $3\sqrt{2}$ cm. **28.** The area of the larger $\triangle$ is 48 cm^2, so the area ratio is 12 : 48, or 1 : 4. The similarity ratio is the square root of the area ratio, so it is 1 : 2. Solve for x: $\frac{x}{8} = \frac{1}{2}$; $x = 4$, or 4 cm. Solve for y: $\frac{y}{12} = \frac{1}{2}$; $y = 6$, or 6 cm.
29. The area of the larger $\triangle$ is 48 cm^2, so the area ratio is 16 : 48, or 1 : 3. The similarity ratio is the square root of the area ratio, so it is $\sqrt{3}$: 3. Solve for x: $\frac{x}{8} = \frac{\sqrt{3}}{3}$; $x = \frac{8\sqrt{3}}{3}$, or $\frac{8\sqrt{3}}{3}$ cm. Solve for y: $\frac{y}{12} = \frac{\sqrt{3}}{3}$; $y = 4\sqrt{3}$, or $4\sqrt{3}$ cm.
30. The area of the larger $\triangle$ is 48 cm^2, so the area ratio is 24 : 48, or 1 : 2. The similarity ratio is the square root of the area ratio, so it is $\sqrt{2}$: 2. Solve for x: $\frac{x}{8} = \frac{\sqrt{2}}{2}$; $x = 4\sqrt{2}$, or $4\sqrt{2}$ cm. Solve for y: $\frac{y}{12} = \frac{\sqrt{2}}{2}$; $y = 6\sqrt{2}$, or $6\sqrt{2}$ cm. **31.** The area of the larger $\triangle$ is 48 cm^2, so the area ratio is 48 : 48, or 1 : 1. The similarity ratio is the square root of the area ratio, so it is 1 : 1. So, $x = 8$ and $y = 12$. **32.** The area ratio is 27 : 48, or 9 : 16, so the similarity ratio is 3 : 4. Solve for the corr. side in the smaller rectangle: $\frac{x}{16} = \frac{3}{4}$; $x = 12$, or 12 in. The other side in the smaller rectangle is $\frac{27}{12}$, or 2.25 in. So, the dimensions of the smaller rectangle are 2.25 in. by 12 in. Solve for the other side in the larger rectangle: $\frac{48}{16} = 3$, so the dimensions of the larger rectangle are 3 in. by 16 in.
33a. Check students' work. **33b.** Check students' work. **33c.** Write a proportion using compatible numbers with the area ratio. Estimates may vary. Sample: 205 m^2
34. The similarity ratio of small to large is 8 : 32, or 1 : 4, so the perimeter ratio is also 1 : 4. **35a.** The similarity ratio is $5x : 2x$, or 5 : 2, so the perimeter ratio is 5 : 2, or $\frac{5}{2}$.
35b. The similarity ratio is 5 : 2, so the area ratio is $5^2 : 2^2$, which is 25 : 4, or $\frac{25}{4}$. **36a.** The similarity ratio is 8 : 3, so the perimeter ratio is 8 : 3, or $\frac{8}{3}$. **36b.** The similarity ratio is 8 : 3, so the area ratio is $8^2 : 3^2$, which is 64 : 9, or $\frac{64}{9}$. **37a.** The similarity ratio is 2 : 1, so the perimeter ratio is also 2 : 1, or $\frac{2}{1}$. **37b.** The similarity ratio is 2 : 1, so the area ratio is $2^2 : 1^2$, which is 4 : 1 or $\frac{4}{1}$. **38.** Writing a ratio using the number of students in each case provides the similarity ratio. The playground is measured in terms of area, so the similarity ratio should be squared for the area. Answers may vary. Sample: The proposed playground is more than adequate. The number of students has approximately doubled. The proposed playground would be four times larger than the original playground.

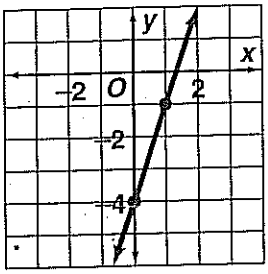
39a. Answers may vary. Sample:



39b. Answers may vary. Sample: For the sample $\triangle$ shown in part (a), the perimeter is 39 + 25 + 50, or 114 mm. Its area is $\frac{1}{2}(50)(19)$, or 475 mm^2. **39c.** The similarity ratio for the sample $\triangle$ is 200 : 50. Solve for the perimeter: $\frac{P}{114} = \frac{200}{50}$, so $P = 456$, or 456 yd. Solve for the area: $\frac{A}{475} = \frac{40{,}000}{2500}$, so $A = 7600$, or 7600 yd^2.
40a. The radii of a regular hexagon divide it into 6 equilateral $\triangle$s whose height is $\sqrt{3}$ and base is 2. The total area is $6\left(\frac{1}{2}\right)(2)\left(\sqrt{3}\right)$, or $6\sqrt{3}$ cm^2. **40b.** Find the area of the 6-cm hexagon: The similarity ratio is 6 : 2, or 3 : 1, so the area ratio is 9 : 1. $\frac{A}{6\sqrt{3}} = \frac{9}{1}$, so $A = 54\sqrt{3}$, or $54\sqrt{3}$ cm^2. Find the area of the 3-cm hexagon: The similarity ratio is 3 : 2, so the area ratio is 9 : 4. $\frac{A}{6\sqrt{3}} = \frac{9}{4}$, so $A = 13.5\sqrt{3}$, or $13.5\sqrt{3}$ cm^2. Find the area of the 8-cm hexagon: The similarity ratio is 8 : 2, or 4 : 1, so the area ratio is 16 : 1. $\frac{A}{6\sqrt{3}} = \frac{16}{1}$, so $A = 96\sqrt{3}$, or $96\sqrt{3}$ cm^2.
41. Similar rectangles having = perimeters have a similarity ratio of 1 : 1, so they are always congruent. So, two similar rectangles with the same perimeter are *always* congruent. **42.** If the rectangles are congruent, then they are also similar. However, a 1-by-8 rect. and a 2-by-4 rect. have the same area, but they are not similar. So, two rectangles with the same area are *sometimes* similar. **43.** If the rectangles were congruent, then both measures would be the same. If they were similar but not congruent, and had a similarity ratio of a : 1, then the perimeter of one would be a times the perimeter of the other and the area of one would be a^2 times the area of the other. So, two rectangles with the same area and different perimeters are *never* similar. **44.** If the similar figures are congruent, then they have the same area. If they are not congruent but are similar, then the areas are in the ratio of the square of their similarity ratio. So, similar figures *sometimes* have the same area.
45. The similarity ratio is 3 : 5, so the area ratio is 9 : 25. $\frac{A}{81} = \frac{25}{9}$, so $A = 225$. **46.** The area ratio is 9 : 16, so the similarity ratio is 3 : 4. The perimeter ratio and the similarity ratio are the same, so $\frac{p}{900} = \frac{3}{4}$; $p = 675$.
47. The similarity ratio is the same as the perimeter ratio, so it is 1 : 3. So, the area ratio is 1 : 9. $\frac{a}{486} = \frac{1}{9}$, so $a = 54$. **48.** The similarity ratio is 192 : 528, or 4 : 11, so the area ratio is 16 : 121. $\frac{2288}{a} = \frac{16}{121}$, so $a = 17303$. **49.** The area ratio is 3267 : 9075, or 9 : 25, so the similarity ratio is 3 : 5. The perimeter ratio is also 3 : 5. $\frac{p}{270.5} = \frac{3}{5}$, so $p = 162.3$. **50.** Solve for x: $\frac{x}{60} = \frac{37.5}{50}$; $x = 45$.

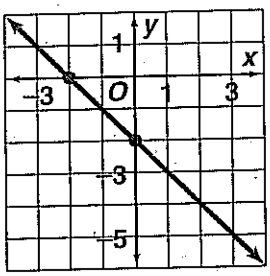
Solve for y: $\frac{y}{37.5} = \frac{129}{x}; \frac{y}{37.5} = \frac{129}{45}; y = 107.5$. **51.** If the 8-cm side is adjacent to the 6-cm segment, then $\frac{m}{4} = \frac{8}{6}$, so $m = 5\frac{1}{3}$, or $5\frac{1}{3}$ cm. If the 8-cm side is adjacent to the 4-cm segment, then $\frac{n}{6} = \frac{8}{4}$, so $n = 12$, or 12 cm. **52.** The diag. creates two isosc. rt. △s, so each leg is $\frac{10}{\sqrt{2}}$ cm long. The area of the square is $\left(\frac{10}{\sqrt{2}}\right)^2$, or 50 cm^2. **53.** The perimeter is 5(20), or 100 units. $A = \frac{1}{2}ap = \frac{1}{2}(13.8)(100) = 690$, or 690 units^2. **54.** The perimeter is 8(10), or 80 units. $A = \frac{1}{2}ap = \frac{1}{2}(12)(80) = 480$, or 480 units^2. **55.** The perimeter is 12(2), or 24 units. $A = \frac{1}{2}ap = \frac{1}{2}(3.7)(24) = 44.4$, or 44.4 units^2.

56.



Slope-intercept form is $y = mx + b$. $6x - 2y = 8$; $-2y = -6x + 8$; $y = 3x - 4$

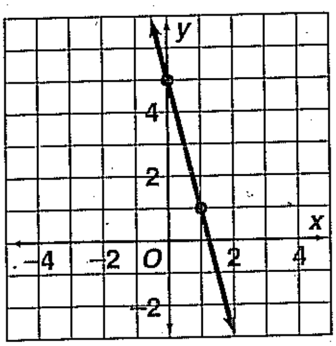
57.



Slope-intercept form is $y = mx + b$. $x + y = -2$; $y = -x - 2$

58. Slope-intercept form is $y = mx + b$. $3x = 4y$; $4y = 3x$; $y = \frac{3}{4}x$

59.



Slope-intercept form is $y = mx + b$. $2x + \frac{1}{2}y = \frac{5}{2}$; $4x + y = 5$; $y = -4x + 5$

60. Equation forms may vary. Sample: Substitute −3 for m in $y = mx + b$: $y = -3x + b$. Then substitute 1 for x and −2 for y in $y = -3x + b$ and solve for b: $-2 = -3(1) + b$; $-2 = -3 + b$; $b = 1$. So, the equation is $y = -3x + 1$. **61.** Equation forms may vary. Sample: The slope is $\frac{-1-7}{0-3} = \frac{8}{3}$. Substitute $\frac{8}{3}$ for m, 0 for x_1, and −1 for y_1 in $(y - y_1) = m(x - x_1)$: $y - (-1) = \frac{8}{3}(x - 0)$; $y + 1 = \frac{8}{3}x$.

TEST-TAKING STRATEGIES — *page 460*

1. For choice A, $\frac{1}{12} \stackrel{?}{=} \frac{1-3}{8}; \frac{1}{12} \neq \frac{-2}{8}$, so A is not the answer. For choice B, $\frac{3}{12} \stackrel{?}{=} \frac{3-3}{8}; \frac{1}{4} \neq 0$, so B is not the answer. For choice C, $\frac{6}{12} \stackrel{?}{=} \frac{6-3}{8}; \frac{1}{2} \neq \frac{3}{8}$, so C is not the answer. Since $\frac{9}{12} = \frac{9-3}{8}$, the answer is choice D. **2.** Test 20.5 for a: $\frac{8}{13} \stackrel{?}{=} \frac{20}{20.5 + 12}; \frac{8}{13} \stackrel{?}{=} \frac{20}{32.5}; \frac{8}{13} = \frac{40}{65}$. The answer is choice F. **3.** $\frac{4}{x} = \frac{x}{9}$; $x^2 = (4)(9)$; $x = (2)(3) = 6$. The answer is choice B. **4.** $\frac{y+4}{4} = \frac{2y+5}{6}$; $6(y + 4) = 4(2y + 5)$; $3(y + 4) = 2(2y + 5)$; $3y + 12 = 4y + 10$; $-y = -2$; $y = 2$. The answer is choice F. **5.** $AF = 20 - 10 = 10$, so $\overline{CF}$ is a midsegment. Thus, $AC = CD = 4$. The answer is choice B.

CHAPTER REVIEW — *pages 461–463*

1. By def. of similar, two polygons are *similar* if corresponding △s are congruent and corresponding sides are proportional. **2.** The *Cross-Product Property* states that the product of the extremes is equal to the product of the means. **3.** A *golden rectangle* is a rectangle that can be divided into a square and a rectangle that is similar to the original rectangle. **4.** The ratio of the lengths of corresponding sides of two similar figures is the *similarity ratio*. **5.** A *proportion* is a statement that two ratios are equal. **6.** Finding distances using similar △s is called *indirect measurement*. **7.** The length and width of a golden rectangle are in the *golden ratio*.

8. 6 ft is the same as 6(12), or 72 inches, so the ratio is $1\frac{1}{2} : 72 = 2\left(1\frac{1}{2}\right) : 2(72) = 3 : 144 = 1 : 48$. **9.** 3 ft 6 in. is the same as 36 + 6, or 42 in., so the ratio is $1\frac{3}{4} : 42 = 7 : 168 = 1 : 24$. **10.** By the Cross-Product Prop., the statement is true. **11.** By Properties of Proportions (2), and because the cross product equation is equivalent to the original proportion, the statement is true. **12.** Since this is not a cross product for the original statement, the statement is false. **13.** By Properties of Proportion (3), and because the cross-product equation is equivalent to the original proportion, the statement is true. **14.** $\angle M \cong \angle R$, $\angle N \cong \angle S$, $\angle P \cong \angle T$; $\frac{MN}{RS} = \frac{MP}{RT} = \frac{NP}{ST}$ **15.** $\frac{24}{n} = \frac{1}{1.618}$; $n \approx 39$ in. or $\frac{n}{24} = \frac{1}{1.618}$; $n = \frac{24}{1.618}$; $n \approx 15$ in. **16.** Choose corr. sides that have known measures. $FR : DA = 4 : 6 = 2 : 3$ **17.** Choose both shortest sides in the △s to write the similarity ratio. $10 : 4 = 5 : 2$ **18.** $\frac{x}{6} = \frac{3}{2}$; $x = 9$ **19.** Solve for x: Both x and 8 are measures of sides of corr. ≅ △s, so the other two legs also corr. $\frac{x}{8} = \frac{9}{6}$; $x = 12$. Solve for y: The hyp. of the small △ is 10. So, $\frac{y}{10} = \frac{9}{6}$; $y = 15$.

20. Two sides are proportionate and the included △s are congruent, so $\triangle XYZ \sim \triangle JKL$ by the SAS ~ Thm. **21.** From least to greatest, the ratios of the sides in $\triangle QRS$ are 8 : 12 : 17 and the ratios of the sides in $\triangle TUV$ are 4 : 5 : 8.5, or 8 : 10 : 17. The ratios of the sides are not the same, so the corr. sides are not proportional. Thus, the △s are not similar. **22.** One pair of acute △s is congruent and their right △s are congruent, so the △s are similar by the AA ~ Post. **23.** $\frac{t}{18} = \frac{1.5}{2}$, so $t = 13.5$, or 13.5 ft. **24.** Solve for x: $\frac{9}{x} = \frac{x}{9 + 16}$; $x^2 = 9(9 + 16) = (9)(25)$; $x = (3)(5) = 15$. Solve for y: $\frac{9}{y} = \frac{y}{16}$; $y^2 = (9)(16)$; $y = (3)(4) = 12$. Solve for z: $\frac{16}{z} = \frac{z}{9 + 16}$; $z^2 = (16)(9 + 16) = (16)(25)$; $z = (4)(5) = 20$. **25.** Solve for x: $\frac{14-8}{x} = \frac{x}{14}$; $x^2 = (14-8)(14) = (6)(14) = 84$; $x = \sqrt{84} = 2\sqrt{21}$. Solve for y: $\frac{14-8}{y} = \frac{y}{8}$; $y^2 = (6)(8) = 48$; $y = \sqrt{48} = 4\sqrt{3}$. Solve for z: $\frac{8}{z} = \frac{z}{14}$; $z^2 = (8)(14) = 112$; $z = \sqrt{112} = 4\sqrt{7}$. **26.** Use the 30°-60°-90° relation to solve for x: $x = 2\sqrt{3}$. Solve for y: $\frac{y}{x} = \frac{x}{2}$; $\frac{y}{2\sqrt{3}} = \frac{2\sqrt{3}}{2}$; $y = \frac{(2\sqrt{3})^2}{2} = \frac{4(3)}{2} = 6$. Solve for z: $\frac{y}{z} = \frac{z}{y+2}$; $\frac{6}{z} = \frac{z}{6+2}$; $z^2 = 6(6+2) = 48$; $z = \sqrt{48} = 4\sqrt{3}$. **27.** $\frac{x}{15} = \frac{7}{14}$; $x = 7.5$ **28.** $\frac{x}{11} = \frac{7}{14}$; $x = 5.5$ **29.** $\frac{x}{15} = \frac{40}{16}$; $x = 37.5$ **30.** The similarity ratio is $8:12$, or $2:3$, so the area ratio is $2^2:3^2$, or $4:9$. **31.** The similarity ratio is $6:4$, or $3:2$, so the area ratio is $3^2:2^2$, or $9:4$. **32.** The similarity ratio is $3:6$, or $1:2$, so the area ratio is $1^2:2^2$, or $1:4$. **33.** The similarity ratio is $\sqrt{8}:\sqrt{25}$, or $2\sqrt{2}:5$, so the perimeter ratio is $2\sqrt{2}:5$. **34.** The similarity ratio is $3:7$, so the area ratio is $3^2:7^2$, or $9:49$. $\frac{36}{a} = \frac{9}{49}$; $9a = 36(49)$; $a = 4(49) = 196$, or 196 cm^2.

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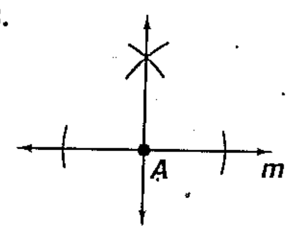
1. $\frac{4}{5} = \frac{x}{20}$; $x = \frac{(20)4}{5} = 16$ **2.** $\frac{6}{x} = \frac{10}{7}$; $10x = 42$; $x = 4.2$ **3.** $\frac{x}{3} = \frac{8}{12}$; $x = \frac{24}{12} = 2$ **4.** Solve for x: The $x°$ $\angle$ corr. to the 42° $\angle$, so $x = 42$. Solve for y: The sum of the $\angle$s of a quad. is 360, so $y = 360 - (90 + 90 + 42) = 360 - 222 = 138$. Solve for z: $\frac{z}{12} = \frac{6}{8}$; $z = \frac{12 \cdot 6}{8} = 9$. **5.** $\frac{x}{2} = \frac{8}{4}$; $x = 4$ **6.** Solve for x: The $x°$ $\angle$ corr. to the 63° $\angle$, so $x = 63$. Solve for y: $\frac{y}{4} = \frac{16}{8}$; $y = 8$. **7.** Order the lengths from least to greatest and then write the ratios of corr. sides to determine if they are =: 8, 10, 10 and 12, 15, 15, so the ratios are $\frac{8}{12}, \frac{10}{15}, \frac{10}{15}$. Since all ratios $= \frac{2}{3}$, $\triangle PRQ \sim \triangle TWV$ by the SSS $\sim$ Thm. **8.** The labeled sides are on each side of a pair of $\cong$ $\angle$s. The ratio of the shorter sides is $\frac{12}{14}$, or $\frac{6}{7}$. The ratio of the longer sides is $\frac{16}{18}$, or $\frac{8}{9}$. Since $\frac{6}{7} \neq \frac{8}{9}$, the corr. sides are not in proportion, so the $\triangle$s are not similar. **9.** 2 pairs of $\angle$s are $\cong$, so $\triangle ABC \sim \triangle FDE$ by the AA $\sim$ Post. **10.** By the Triangle-Angle-Bis. Thm., $\frac{AD}{DC} = \frac{AB}{BC}$. **11.** Use the corollaries to Thm. 8-3 to write relationships of the sides. Answers may vary. Samples: $\frac{AD}{AC} = \frac{AC}{AB}$, $\frac{BD}{CB} = \frac{CB}{BA}$, $\frac{AD}{CD} = \frac{CD}{DB}$ **12.** $\frac{p}{9} = \frac{1}{1.5}$; $p = \frac{9}{1.5} = 6$, or 6 m **13.** $\frac{x}{3} = \frac{9}{2}$; $x = 13.5$, or 13.5 m **14.** $\frac{10}{x} = \frac{x}{15}$; $x^2 = (10)(15) = 150$; $x = \sqrt{150} = \sqrt{25(6)} = 5\sqrt{6}$ **15.** $\frac{4}{x} = \frac{x}{9}$; $x^2 = (4)(9)$; $x = (2)(3) = 6$ **16.** $\frac{6}{x} = \frac{x}{12}$; $x^2 = (6)(12) = 72 = (36)(2)$; $x = 6\sqrt{2}$ **17.** Check students' work. **18.** $\frac{6}{10} = \frac{10}{6 + (x - 6)}$; $\frac{6}{10} = \frac{10}{x}$; $\frac{x}{10} = \frac{10}{6}$; $x = \frac{100}{6} = 16\frac{2}{3}$ **19.** $\frac{x}{20} = \frac{6}{12}$; $x = \frac{20 \cdot 6}{12} = 10$ **20.** $\frac{x}{10} = \frac{6}{11}$; $x = \frac{60}{11} = 5\frac{5}{11}$ **21.** $\frac{8}{12} = \frac{12}{x+8}$; $8(x + 8) = 144$; $x + 8 = 18$; $x = 10$ **22.** Answers may vary. Sample: To measure the height of a tree on a sunny day, measure the length of the shadow it casts. Then measure the length of the shadow that you cast. Since you already know your height, you can use $\sim$ $\triangle$s to write and solve a proportion. **23.** The similarity ratio is $7:8$, so the area ratio is $7^2:8^2$, or $49:64$. **24.** The similarity ratio is $12:8$, or $3:2$, so the area ratio is $3^2:2^2$, or $9:4$.

STANDARDIZED TEST PREP — page 465

1. The center of the circle is at the circumcenter, which is where the $\perp$ bis. of the sides of the $\triangle$ intersect. In a rt. isosc. $\triangle$, they intersect at the midpt. of the hyp. In this $\triangle$, that point is $(-1, -1)$. The answer is choice D. **2.** By def. of both square and rectangle, each has 4 rt. $\angle$s, so eliminate choices F and H. A square is a rhombus, so a rhombus sometimes has 4 rt. $\angle$s. Eliminate choice I. The trapezoid remains. If it had 4 rt. $\angle$s, then both pairs of opp. sides would be $\parallel$, which contradicts the def. of trapezoid. The answer is choice G. **3.** The hyp. is 15 and the leg is 10. Use the Pyth. Thm.: $x = \sqrt{15^2 - 10^2} = \sqrt{225 - 100} = \sqrt{125} \approx 11$. The answer is choice D. **4.** Same-side-int. $\angle$s of $\parallel$ lines are suppl., so $(2x + 10) + (5x - 5) = 180$; $7x + 5 = 180$; $7x = 175$; $x = 25$. The answer is choice I. **5.** They are all written in $y = mx + b$ form in which m is the slope. Eliminate choices A and B, since they don't have slope 3. Substitute 2 for x and 5 for y in the remaining 2 equations to see which equation holds true for $(2, 5)$. $5 \neq 3(2) + 1$, but $5 = 3(2) - 1$. The answer is choice D. **6.** The diagonals of a kite are $\perp$ and the sum of the $\angle$s of a $\triangle$ is 180, so $x + 22 + 90 = 180$; $x = 68$. The answer is choice I. **7.** $\frac{x}{5} = \frac{3}{4}$; $x = \frac{15}{4} = 3.75$. The answer is choice A. **8.** No two sides are =, so it can't be isosc. $\angle B$ is opp. the longest side, so it is the largest $\angle$. Since $7^2 + 24^2 = 25^2$, the $\triangle$ is a right $\triangle$. The answer is choice G. **9.** Since both pairs of opp. sides are =, the quad. is a ▱. In a ▱ the diags. bis. each other, so $AE = EC$. The answer is choice C. **10.** No $\angle$ measures are given and the diagram offers no way to compare the lengths of the sides of $\triangle ADC$, so a comparison of its $\angle$s is not possible. The answer is choice D. **11.** Since both pairs of opp. sides are =, the quad. is a ▱. In a ▱, both pairs of opp. sides are $\parallel$, so all pairs of alt. int. $\angle$s are $\cong$. Thus, the $\angle$s are =. The answer is choice C. **12.** The area of a rhombus is $\frac{1}{2}$ the prod. of its diags.: $A = \frac{1}{2}(20)(14.2) = 142$. The area is 142 cm^2. (To answer the question, grid 142.)

13.



[2] $\perp$ line is constructed accurately. [1] line m with pt. A drawn and with 2 arcs marked on m at equal distances from A

14a. Only one pair of corr. sides is $\cong$ and one pair of vert. $\angle$s are $\cong$. Two more pairs of sides or one more pair of $\angle$s must be shown $\cong$ before the $\triangle$s can be proved $\cong$. The answer is no. **14b.** To prove by SAS, the included $\angle$ pair would need to be shown $\cong$. To prove by SSS, the third pair of sides would need to be shown $\cong$. The answer is $\angle BCD \cong \angle EDC$ or $\overline{BD} \cong \overline{EC}$. **14c.** By the Refl. Prop. of $\cong$, $\overline{AF} \cong \overline{FA}$. **14d.** Since 3 pairs of sides are $\cong$, the answer is SSS. [4] all 4 answers correct [3] any 3 answers correct [2] any 2 answers correct [1] any 1 answer correct

REAL-WORLD SNAPSHOTS *pages 466–467*

Activity 1: For the KTHI-TV tower, the hyp. is $3963 + \frac{2063}{5280}$, or about 3963.39 mi., so $x \approx \sqrt{3963.4^2 - 3963^2} \approx 55.65$, or 55.7 mi. For the CN Tower, the hyp. is $3963 + \frac{1815}{5280}$, or about 3963.34 mi., so $x \approx \sqrt{3963.34^2 - 3963^2} \approx 52.2$, or about 52.2 mi. For the Empire State Building, the hyp. is $3963 + \frac{1250}{5280}$, or about 3963.24 mi., so $x \approx \sqrt{3963.24^2 - 3963^2} \approx 43.3$, or about 43.3 mi. For the Chrysler Building, the hyp. is $3963 + \frac{1046}{5280}$, or about 3963.20 mi., so $x \approx \sqrt{3963.2^2 - 3963^2} \approx 39.6$, or about 39.6 mi. For the 4-story town house, the hyp. is $3963 + \frac{66}{5280}$, or about 3963.01 mi., so $x \approx \sqrt{3963.01^2 - 3963^2} \approx 10.0$, or about 10.0 mi. For the Eiffel Tower, the hyp. is $3963 + \frac{1052}{5280}$, or about 3963.20 mi., so $x \approx \sqrt{3963.2^2 - 3963^2} \approx 39.7$, or 39.7 mi. For the Bank of China Building, the hyp. is $3963 + \frac{1033}{5280}$, or about 3963.20 mi., so $x \approx \sqrt{3963.2^2 - 3963^2} \approx 39.4$, or about 39.4 mi. For the 1 Canada Square Building, the hyp. is $3963 + \frac{797}{5280}$, or about 3963.15 mi., so $x \approx \sqrt{3963.15^2 - 3963^2} \approx 34.6$, or about 34.6 mi. For the Saturn V rocket, the hyp. is $3963 + \frac{364}{5280}$, or about 3963.07 mi., so $x = \sqrt{3963.07^2 - 3963^2} \approx 23.4$, or about 23.4 mi. For St. Peter's Basilica, the hyp. is $3963 + \frac{451}{5280}$, or about 3963.09 mi, so $x \approx \sqrt{3963.09^2 - 3963^2} \approx 26.0$, or about 26.0 mi. For the Great Pyramid at Giza, the hyp. is $3963 + \frac{481}{5280}$, or about 3963.09 mi, so $x \approx \sqrt{3963.09^2 - 3963^2} \approx 26.87$, or about 26.9 mi. For the Cologne Cathedral, the hyp. is $3963 + \frac{513}{5280}$, or about 3963.10 mi, so $x \approx \sqrt{3963.1^2 - 3963^2} \approx 27.75$, or about 27.8 mi. For the Leaning Tower in Pisa, the hyp. is about $3963 + \frac{185}{5280}$, or about 3963.035 mi, so $x \approx \sqrt{3963.035^2 - 3963^2} \approx 16.66$, or about 16.7 mi.

Activity 2: **a.** The longest side of the △ is the side that includes the tree, since the tree is taller than the lookout point on the ship. The hyp. is $3963 + \frac{50}{5280}$, or about 3964.0095 mi. The long leg of the rt. △ is $3963 + \frac{40}{5280}$, or about 3963.0076 mi. So, $x \approx \sqrt{3963.0095^2 - 3963.0076^2} \approx 15.011$, or about 15 mi. **b.** The hyp. is $3963 + \frac{50}{5280}$, or about 3964.0095 mi. The long leg of the rt. △ is $3963 + \frac{15}{5280}$, or about 3963.0028 mi. So, $x \approx \sqrt{3963.0095^2 - 3963.0028^2} \approx 6.126$, or about 6 mi. **c.** 15 mi > 6 mi, or you can see about 2.5 times as far if you are 25 ft higher.