DIAGNOSING READINESS *page 346*

1. $3^2 = (3)(3) = 9$ **2.** $8^2 = (8)(8) = 64$ **3.** $12^2 = (12)(12) = 144$ **4.** $15^2 = (15)(15) = 225$ **5.** $\sqrt{16} = \sqrt{4^2} = 4$ **6.** $\sqrt{64} = \sqrt{8^2} = 8$ **7.** $\sqrt{100} = \sqrt{10^2} = 10$ **8.** $\sqrt{169} = \sqrt{13^2} = 13$ **9.** $x^2 = 36; x = \pm\sqrt{36} = \pm 6$ **10.** $a^2 = 104; a = \pm\sqrt{104} \approx \pm 10.2$ **11.** $x^2 - 48 = 0; x^2 = 48; x = \pm\sqrt{48} \approx \pm 6.9$ **12.** $b^2 - 65 = 0; b^2 = 65; b = \pm\sqrt{65} \approx \pm 8.1$ **13.** $\sqrt{8} = \sqrt{2^2 \cdot 2} = 2\sqrt{3}$ **14.** $\sqrt{27} = \sqrt{3^2 \cdot 3} = 3\sqrt{3}$ **15.** $\sqrt{48} = \sqrt{16 \cdot 3} = 4\sqrt{3}$ **16.** $6\sqrt{72} = 6\sqrt{36 \cdot 2} = 6 \cdot 6\sqrt{2} = 36\sqrt{2}$ **17.** The number of favorable outcomes is 3 red balls and the number of possible outcomes is $3 + 2$, or 5, so the probability is $\frac{3}{5}$. **18.** The number of favorable outcomes is 2 green balls and the number of possible outcomes is $3 + 2$, or 5, so the probability is $\frac{2}{5}$. **19.** There are no blue balls, so the number of existing favorable outcomes is 0 blue balls. The number of possible outcomes is $3 + 2$, or 5, so the probability is $\frac{0}{5}$, or 0. **20.** The number of favorable outcomes is 3 red and 2 green balls. The number of possible outcomes is $3 + 2$, or 5, so the probability is $\frac{3+2}{5}$, or 1. **21.** Nothing is known about the $\angle$ measures, but the sides are $\cong$, so it is a rhombus. **22.** Opp. $\measuredangle$s are $\cong$, so by Thm. 6-8, it is a ▱. **23.** The diagonals are $\perp$ and bisect each other, so it is a rhombus.

7-1 Areas of Parallelograms and Triangles *pages 348–354*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

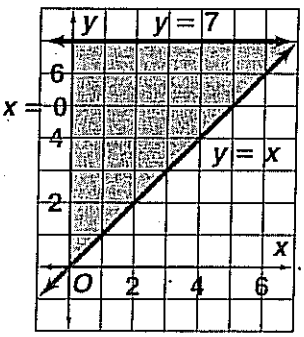
1. 25 cm^2 **2.** 28 $in.^2$ **3.** 11.5 m^2 **4.** $\frac{3}{2}$ ft^2 **5.** 6 $units^2$ **6.** 2 $units^2$ **7.** 8 $units^2$

Investigation The rectangle and ▱ have the same base length, height, and area. Their shapes are different.

Check Understanding **1.** $A = bh = (12)(9) = 108$; 108 m^2 **2.** If $\overline{HG}$ is the base, then $b = |-3 - 1| = 4$ and $h = |3 - (-2)| = 5$. $A = bh = (4)(5) = 20$; 20 $units^2$ **3.** $A = bh = (9)(15) = 135$. $A = bh$; $135 = (18)b$; $b = \frac{135}{18} = 7.5$; 7.5 cm **4.** $A = \frac{1}{2}bh = \frac{1}{2}(12)(5) = (6)(5) = 30$; 30 cm^2 **5.** $A_{\triangle} = \frac{1}{2}(2 \cdot 20)(6) = (20)(6) = 120$, or 120 ft^2; $A_{▱} = bh = (2 \cdot 20)(12) = (40)(12) = 480$, or 480 ft^2; $A_{\triangle} + A_{▱} = 120 + 480 = 600$, or 600 ft^2; $F = 0.004Av^2 = 0.004(600)(73)^2 = 0.004(600)(5329) = 12{,}789.600$, or 12,789 lb. Since $12{,}789.6 = 2(6394.8)$, the force is doubled.

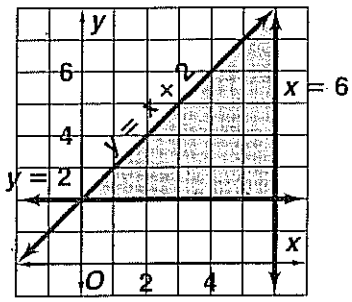
Exercises **1.** $A = bh = (20)(12) = 240$; 240 cm^2 **2.** $A = bh = (5.8)(3.5) = 20.3$; 20.3 m^2 **3.** $A = bh = (4.7)(5.7) = 26.79$; 26.79 $in.^2$ **4.** If $\overline{AB}$ is the base, then $b = |2 - 7| = 5$ and $h = |4 - 0| = 4$, so $A = bh = (5)(4) = 20$; 20 $units^2$. **5.** If $\overline{EF}$ is the base, then $b = |-4 - (-1)| = |-3| = 3$ and $h = |-3 - 0| = 3$, so $A = bh = (3)(3) = 9$; 9 $units^2$. **6.** If $\overline{IJ}$ is the base, then $b = |2 - 4| = |-2| = 2$ and $h = |2 - (-3)| = 5$, so $A = bh = (2)(5) = 10$; 10 $units^2$. **7.** If $\overline{NP}$ is the base, then $b = |-5 - 1| = 6$ and $h = |-6 - (-5)| = 1$, so $A = bh = (6)(1) = 6$; 6 $units^2$. **8.** $A = bh = (14)(8) = 112$; $A = bh = 10h = 112$; $h = 11.2$ **9.** $A = bh = (0.4)(0.3) = 0.12$; $A = bh = 0.5h = 0.12$; $h = 0.24$ **10.** $A = bh = (18)(12) = 216$; $A = bh = 13h = 216$; $h = 6\frac{8}{13}$ **11.** $A = \frac{1}{2}bh = \frac{1}{2}(4 + 3)(4) = \frac{1}{2}(7)(4) = 14$; 14 m^2 **12.** $A = \frac{1}{2}bh = \frac{1}{2}(4.5)(6) = 13.5$; 13.5 yd^2 **13.** $A = \frac{1}{2}bh = \frac{1}{2}(2)(3) = 3$; 3 ft^2 **14a.** Driving region area: $A = bh = (15)(50) = 750$; 750 ft^2. Parking space area: $A = bh = (10)(31 - 15) = (10)(16) = 160$. Total area $= 750 + 4(160) = 750 + 640 = 1390$; 1390 ft^2. **14b.** Compute the rectangular area of the entire lot and subtract the 2 triangular areas for the flowers. **14c.** Lot area $= bh = 31(50) = 1550$; 1550 ft^2. One flower area $= \frac{1}{2}bh = \frac{1}{2}(50 - 4 \cdot 10)(31 - 15) = \frac{1}{2}(50 - 40)(31 - 15) = 80$. The total area is $1550 - 2(80) = 1550 - 160 = 1390$; 1390 ft^2. The areas are =. **15.** $b = 5, h = 3; A = bh = (5)(3) = 15$; 15 $units^2$ **16.** $b = 4, h = 3; A = \frac{1}{2}bh = \frac{1}{2}(4)(3) = 6$; 6 $units^2$ **17.** $b = 4, h = 3, A = \frac{1}{2}bh = \frac{1}{2}(4)(3) = 6$; 6 $units^2$ **18.** $b = 4, h = 3; A = bh = (4)(3) = 12$; 12 $units^2$ **19.** $b = 9, h = 3; A = bh = (9)(3) = 27$; 27 $units^2$ **20.** $b = 3, h = 3; A = \frac{1}{2}bh = \frac{1}{2}(2)(3) = 3$; 3 $units^2$ **21.** $A_{ADJF} = A_{ABJF} + A_{BDJ} = (5)(3) + \frac{1}{2}(4)(3) = 15 + 6 = 21$; 21 $units^2$ **22.** $A = bh$; $24 = b(6)$; $b = 4$; 4 in. **23.** $A = \frac{1}{2}bh$; $98 = \frac{1}{2}x \cdot x$; $196 = x^2$; $x = \pm\sqrt{196} = \pm 14$, but since distance cannot be negative, $x = 14$; 14 cm. **24.** Let the measure of the base be $3a$ and the measure of the height be $2a$. Then $A = \frac{1}{2}bh$; $108 = \frac{1}{2}(3a)(2a)$; $108 = 3a^2$; $a^2 = 36$; $a = \pm 6$, but since distance is positive, $a = 6$.

25a.



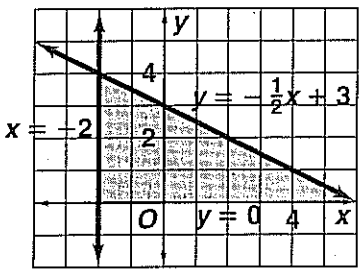
25b. $b = h = 7$; $A = \frac{1}{2}bh = \frac{1}{2}(7)(7) = 24.5$; 24.5 $units^2$

26a.



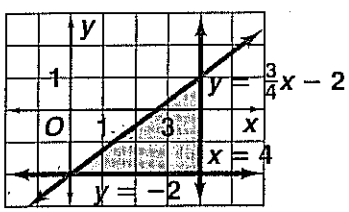
26b. $b = h = 6$; $A = \frac{1}{2}bh = \frac{1}{2}(6)(6) = 18$; 18 units^2

27a.



27b. $b = 8, h = 4$; $A = \frac{1}{2}bh = \frac{1}{2}(8)(4) = 16$; 16 units^2

28a.

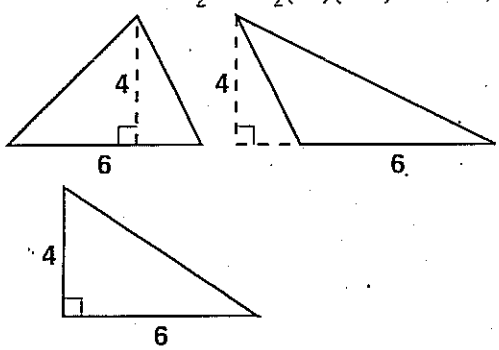


28b. $b = 4, h = 3$; $A = \frac{1}{2}bh = \frac{1}{2}(4)(3) = 6$; 6 units^2

29. The area does not change because the height, which is the distance between the two ∥ lines, and the base, AB, remain the same. **30.** The figure contains 7 whole squares and 2 half squares, so the area is $7 + 2\left(\frac{1}{2}\right) = 7 + 1 = 8$; 8 units^2.

31. The figure contains 2 half-regions of 1 by 3 units, or 3 units^2, plus 6 whole squares. So, the total area $= 3 + 6 = 9$; 9 units^2. **32.** The figure contains 6 whole squares and 4 half squares, so the total area $= 6 + 4\left(\frac{1}{2}\right) = 6 + 2 = 8$; 8 units^2. **33.** $A = \frac{1}{2}bh = \frac{1}{2}(60)(140) = 4200$; 4200 yd^2

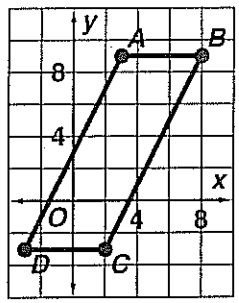
34.



Answers may vary. Sample: Choose the same base and height for each △ as was done in Exercise 29, and manipulate the position of the vertex not on the base.

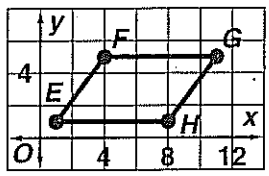
35a. The area of the entire figure is $6 \cdot 20$, or 120; 120 units^2. The area of the green ▱ is $bh = (4)(3) = 12$; 12 units^2. The area of the blue rectangle is $bh = (3)(4) = 12$; 12 units^2. The area of the red △ is $\frac{1}{2}bh = \frac{1}{2}(8)(3) = 12$; 12 units^2. The sum of the areas of the figures is 3(12), or 36; 36 units^2. The area of the grid region is $120 - 36 = 84$; 84 units^2. Since $84 > 36$, the fly is more likely to land on the blank grid. **35b.** Since each figure has an area of 12 units^2, the fly is equally likely to have landed on any one of them. So, no, the fly is not more likely to land on one figure than on another because the figures have the same area.

36.



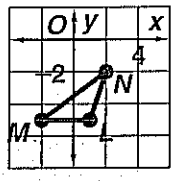
$b = CD = 5, h = 12$; $A = bh = (5)(12) = 60$; 60 units^2

37.



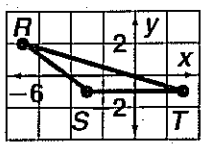
$b = EH = 7, h = 4$; $A = bh = (7)(4) = 28$; 28 units^2

38.



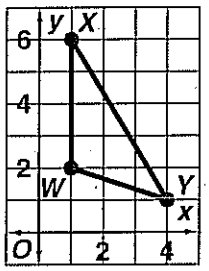
$b = ML = 3, h = 3$; $A = \frac{1}{2}bh = \frac{1}{2}(3)(3) = 4.5$; 4.5 units^2

39.



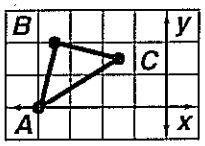
$b = ST = 6, h = 3$; $A = \frac{1}{2}bh = \frac{1}{2}(6)(3) = 9$; 9 units^2

40.



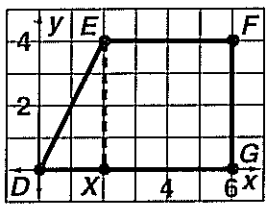
$b = XY = 4, h = 3$; $A = \frac{1}{2}bh = \frac{1}{2}(4)(3) = 6$; 6 units^2

41.



Slope of $\overline{BC} = \frac{4 - 3}{-7 - (-3)} = -\frac{1}{4}$; slope of $\overline{AB} = \frac{4 - 0}{-7 - (-8)} = \frac{4}{1} = 4$. Thus, $\overline{AB} \perp \overline{BC}$. $AB = \sqrt{(-8 - (-7))^2 + (0 - 4)^2} = \sqrt{(-1)^2 + (-4)^2} = \sqrt{1 + 16} = \sqrt{17}$; $BC = \sqrt{(-3 - (-7))^2 + (3 - 4)^2} = \sqrt{(4)^2 + (-1)^2} = \sqrt{17}$; $b = AB = \sqrt{17}, h = BC = \sqrt{17}$; $A = \frac{1}{2}\sqrt{17}\left(\sqrt{17}\right) = \frac{1}{2}(17) = 8.5$; 8.5 units^2

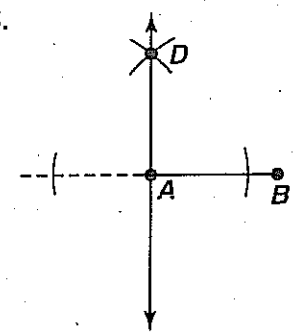
42.



Draw $\overline{EX}$ with X at (2, 0). Then $\overline{EX}$ divides the trapezoid into $\triangle EXD$ and rectangle $EFGX$. $A = A_{\triangle} + A_{\square} = \frac{1}{2}(2)(4) + (4)(4) = 4 + 16 = 20$; 20 units^2

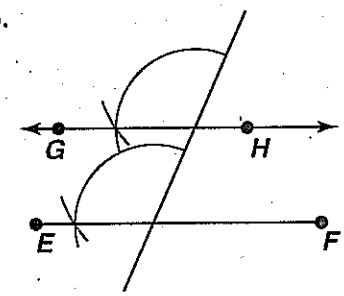
43. Draw $\overline{MX}$ with X at $(1, -2)$. Then $\overline{MX}$ divides the trapezoid into $\triangle MXN$ and rectangle $LMXK$.
$A = A_{\triangle} + A_{\square} = \frac{1}{2}(6)(8) + (8)(8) = 24 + 64 = 88$; 88 units^2 **44.** If the base of one $\triangle$ is x, then the base of the other $\triangle$ is $(25 - x)$, and the height for each is 25.
$A = A_{x-\triangle} + A_{(25-x)-\triangle} = \frac{1}{2}(x)(25) + \frac{1}{2}(25 - x)(25) = \frac{1}{2}(25x + 25^2 - 25x) = \frac{1}{2}(25^2) = \frac{1}{2}(625) = 312.5$; 312.5 ft^2
45. The trapezoid is a $\square$ and a $\triangle$. $A = A_{\square} + A_{\triangle} = (15)(21) + \frac{1}{2}(20)(21) = 315 + 210 = 525$; 525 cm^2
46. Extend the right vertical segment up to the 200-m segment to form a 60-m by 120-m rectangle and a $\triangle$ whose base is $(120 - 40)$ and height is $(200 - 60)$.
$A = A_{\square} + A_{\triangle} = (60)(120) + \frac{1}{2}(120 - 40)(200 - 60) = (60)(120) + \frac{1}{2}(80)(140) = 7200 + 5600 = 12{,}800$; 12,800 m^2
47. $a = 8, b = 9, c = 10, s = \frac{1}{2}(a + b + c) = \frac{1}{2}(8 + 9 + 10) = \frac{1}{2}(27) = 13.5$; $A = \sqrt{s(s - a)(s - b)(s - c)} = \sqrt{13.5(13.5 - 8)(13.5 - 9)(13.5 - 10)} = \sqrt{13.5(5.5)(4.5)(3.5)} = \sqrt{1169.4375} \approx 34$; 34 in.^{2}
48. $a = 15, b = 17, c = 21, s = \frac{1}{2}(a + b + c) = \frac{1}{2}(15 + 17 + 21) = 26.5$; $A = \sqrt{s(s - a)(s - b)(s - c)} = \sqrt{26.5(26.5 - 15)(26.5 - 17)(26.5 - 21)} = \sqrt{26.5(11.5)(9.5)(5.5)} = \sqrt{15{,}923.1875} \approx 126$; 126 m^2
49. $a = 6, b = 7, c = 11, s = \frac{1}{2}(a + b + c) = \frac{1}{2}(6 + 7 + 11) = 12$; $A = \sqrt{s(s - a)(s - b)(s - c)} = \sqrt{12(12 - 6)(12 - 7)(12 - 11)} = \sqrt{12(6)(5)(1)} = \sqrt{360} \approx 19$; 19 cm^2 **50.** $a = 10, b = 10.2, c = 11$, $s = \frac{1}{2}(a + b + c) = \frac{1}{2}(10 + 10.2 + 11) = 15.6$;
$A = \sqrt{s(s - a)(s - b)(s - c)} = \sqrt{15.6(15.6 - 10)(15.6 - 10.2)(15.6 - 11)} = \sqrt{15.6(5.6)(5.4)(4.6)} = \sqrt{2170.0224} \approx 47$; 47 ft^2
51a. $a = 9, b = 12, c = 15, s = \frac{1}{2}(a + b + c) = \frac{1}{2}(9 + 12 + 15) = 18$; $A = \sqrt{18(18 - 9)(18 - 12)(18 - 15)} = \sqrt{18(9)(6)(3)} = \sqrt{2916} = 54$; 54 in.^{2} **51b.** $A = \frac{1}{2}bh = \frac{1}{2}(12)(9) = 54$; 54 in.^{2} **52.** The legs are $\perp$ and are the two shortest sides, 10 in. and 24 in., so $b = 10$ and $h = 24$; $A = \frac{1}{2}bh = \frac{1}{2}(10)(24) = 120$; 120 in.^{2}. The answer is choice B. **53.** $b = CD = AB = 8, h = 8$; $A = bh = (8)(8) = 64$; 64 in.^{2}. The answer is choice G. **54.** Since the altitude is to the shorter side, $b = 176$ and $h = 290$; $A = bh = (176)(290) = 51{,}040$; 51,040 ft^2. The answer is choice A. **55.** If the perimeter is 60 m, then each side is $\frac{60}{3}$, or 20 m, so $b = 20$ and $h = 17.3$. $A = \frac{1}{2}bh = \frac{1}{2}(20)(17.3) = 173$; 173 m^2. The answer is choice F.
56 [2] **a.** It is the distance between $y = 2$ and $y = -3$, so $|2 - (-3)| = |5| = 5$. **b.** $A = bh = (5)(5) = 25$; 25 units^2 **57.** P is a units right and a units up from the origin: $P(a, a)$. **58.** P is $(a + c)$ units right and b units up from the origin: $P(a + c, b)$. **59.** The sum of the exterior $\measuredangle$s is 360, so each exterior $\angle$ measures $\frac{360}{5}$, or 72, so each interior $\angle$ measures $180 - 72$, or 108. **60.** From Exercise 59, $m\angle APE = 108$, so $m\angle E = 108$. $\triangle ENP$ is isosc., so $108 + 2m\angle EPN = 180$; $m\angle EPN = \frac{180 - 108}{2} = 36$. $m\angle APN = m\angle APE - m\angle EPN = 180 - 36 = 72$ **61.** By symmetry, $m\angle PAN = m\angle APN = 72$.
62. From Exercises 60 and 61, $m\angle APN = m\angle PAN = 72$, so $m\angle PNA = 180 - 2(72) = 180 - 144 = 36$.
63. From Exercise 59, $m\angle APE = 108$, so $m\angle E = 108$. $\triangle ENP$ is isosc., so $108 + 2m\angle EPN = 180$; $m\angle EPN = \frac{180 - 108}{2} = 36$. **64.** From Exercise 63, $m\angle EPN = m\angle ENP = 36$. Because $PENTA$ is regular and from Exercise 59 all the int. $\measuredangle$s measure 108, $m\angle ENT = 108$. From Exercise 62, $m\angle PNA = 36$. By $\angle$ addition, $m\angle ENT = m\angle ENP + m\angle PNA + m\angle ANT$; $108 = 36 + 36 + m\angle ANT$; $m\angle ANT = 108 - 72 = 36$.

65.



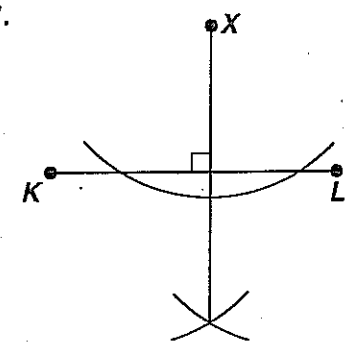
Extend $\overline{AB}$ away from B. With the compass pt. on A, swing arcs that intersect $\overleftrightarrow{AB}$ on each side of A. With the compass pt. on one of the intersections and open to more than half way, swing an arc about midway between A and B. Keeping the setting, place the compass pt. on the other arc intersection and swing an arc that intersects the previous arc. Label the intersection D. Draw $\overleftrightarrow{AD}$.

66.



Draw a transversal and construct $\cong$ corr. $\measuredangle$s.

67.



From X, swing an arc that intersects $\overline{KL}$ in 2 places. With the compass pt. on one of the arc intersections, swing an arc towards L that does not pass through X. Keep the same setting and swing an arc from the other arc intersection pt. so it intersects the previous arc. Draw a line, ray, or segment that includes the intersection of the two arcs and X.

ALGEBRA 1 REVIEW *page 355*

1. $\sqrt{5} \cdot \sqrt{10} = \sqrt{5 \cdot 10} = \sqrt{5 \cdot 5 \cdot 2} = 5\sqrt{2}$ **2.** $\sqrt{243} = \sqrt{81 \cdot 3} = 9\sqrt{3}$ **3.** $\sqrt{128} \div \sqrt{2} = \frac{\sqrt{128}}{\sqrt{2}} = \sqrt{\frac{128}{2}} =$

$\sqrt{64} = 8$ **4.** $\sqrt{\frac{125}{4}} = \frac{\sqrt{125}}{\sqrt{4}} = \frac{\sqrt{25 \cdot 5}}{\sqrt{4}} = \frac{5\sqrt{5}}{2}$ **5.** $\sqrt{6} \cdot \sqrt{8} = \sqrt{6 \cdot 8} = \sqrt{3 \cdot 2 \cdot 2 \cdot 4} = \sqrt{3 \cdot 4 \cdot 4} = 4\sqrt{3}$ **6.** $\frac{\sqrt{36}}{\sqrt{3}} = \frac{\sqrt{36}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$ **7.** $\frac{\sqrt{144}}{\sqrt{2}} = \frac{\sqrt{144}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$ **8.** $\sqrt{3} \cdot \sqrt{12} = \sqrt{3 \cdot 12} = \sqrt{3 \cdot 3 \cdot 2 \cdot 2} = 3 \cdot 2 = 6$ **9.** $\sqrt{72} \div \sqrt{2} = \frac{\sqrt{72}}{\sqrt{2}} = \sqrt{\frac{72}{2}} = \sqrt{36} = 6$ **10.** $\sqrt{169} = \sqrt{13 \cdot 13} = 13$ **11.** $28 \div \sqrt{8} = \frac{28}{\sqrt{8}} = \frac{28}{\sqrt{4 \cdot 2}} = \frac{28}{2\sqrt{2}} = \frac{14}{\sqrt{2}} = \frac{14}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{14\sqrt{2}}{2} = 7\sqrt{2}$ **12.** $\sqrt{300} \div \sqrt{5} = \frac{\sqrt{300}}{\sqrt{5}} = \sqrt{\frac{300}{5}} = \sqrt{60} = \sqrt{4 \cdot 15} = 2\sqrt{15}$. **13.** $\sqrt{12} \cdot \sqrt{2} = \sqrt{6 \cdot 2} \cdot \sqrt{2} = \sqrt{6 \cdot 2 \cdot 2} = 2\sqrt{6}$ **14.** $\frac{\sqrt{24}}{\sqrt{3}} = \sqrt{\frac{24}{3}} = \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$ **15.** $\sqrt{\frac{75}{3}} = \sqrt{25} = 5$ **16.** $\sqrt{18} \cdot \sqrt{2} = \sqrt{9 \cdot 2} \cdot \sqrt{2} = 3\sqrt{2} \cdot \sqrt{2} = 3 \cdot 2 = 6$ **17.** $\sqrt{68} = \sqrt{4 \cdot 17} = 2\sqrt{17}$ **18.** $\sqrt{3} \cdot \sqrt{15} = \sqrt{3} \cdot \sqrt{3 \cdot 5} = \sqrt{3} \cdot \sqrt{3} \cdot \sqrt{5} = 3\sqrt{5}$ **19.** $\frac{\sqrt{20}}{\sqrt{5}} = \sqrt{\frac{20}{5}} = \sqrt{4} = 2$ **20.** $45 \div \sqrt{3} = \frac{45}{\sqrt{3}} = \frac{45}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{45\sqrt{3}}{3} = 15\sqrt{3}$ **21.** $\sqrt{\frac{25}{20}} = \sqrt{\frac{5 \cdot 5}{4 \cdot 5}} = \frac{\sqrt{5}}{\sqrt{4}} = \frac{\sqrt{5}}{2}$ **22.** $\sqrt{\frac{8}{28}} = \sqrt{\frac{2 \cdot 4}{7 \cdot 4}} = \sqrt{\frac{2}{7}} = \sqrt{\frac{2 \cdot 7}{7 \cdot 7}} = \frac{\sqrt{2 \cdot 7}}{\sqrt{7 \cdot 7}} = \frac{\sqrt{14}}{7}$ **23.** $\frac{\sqrt{6} \cdot \sqrt{3}}{\sqrt{9}} = \frac{\sqrt{6 \cdot 3}}{\sqrt{9}} = \frac{\sqrt{18}}{\sqrt{9}} = \sqrt{\frac{18}{9}} = \sqrt{2}$ **24.** $\frac{\sqrt{3} \cdot \sqrt{15}}{\sqrt{2}} = \frac{\sqrt{3} \cdot \sqrt{15}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{3 \cdot 15 \cdot 2}}{\sqrt{2 \cdot 2}} = \frac{\sqrt{3 \cdot 3 \cdot 5 \cdot 2}}{2} = \frac{3\sqrt{10}}{2}$

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1a. The combined areas of the 4 △s with the two smaller squares is = to the combined areas of the 4 △s with the largest square. **1b.** The area of one △ is $\frac{1}{2}ab$, so the area of 4 △s is $4 \cdot \frac{1}{2}ab$, or $2ab$. So, the area of 4 △s and the 2 smaller squares is $a^2 + b^2 + 2ab$, and the area of 4 △s and the largest square is $c^2 + 2ab$. **1c.** Since $a^2 + b^2 + 2ab = c^2 + 2ab$, then $a^2 + b^2 = c^2$, so the areas of the two smaller squares = the area of the larger square. **2.** The same relationship occurs. **3.** From Exercise 1c, $a^2 + b^2 = c^2$. **4.** yes **5.** The sum of the squares of the two legs of a right △ = the square of the length of the hypotenuse. **6.** No; for an obtuse △, $c^2 > a^2 + b^2$; for an acute △, $c^2 < a^2 + b^2$.

7-2 The Pythagorean Theorem and Its Converse *pages 357–364*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1. $3^2 + 4^2 = 5^2$ **2.** $5^2 + 12^2 = 13^2$ **3.** $6^2 + 8^2 = 10^2$ **4.** $4^2 + 4^2 = (4\sqrt{2})^2$

Check Understanding **1.** Since it's a rt. △, use the Pythagorean Thm.: $a^2 + 10^2 = 25^2$; $a^2 + 100 = 625$; $a^2 = 525$; $a = \pm\sqrt{525} = \pm\sqrt{25 \cdot 21} = \pm 5\sqrt{21}$. Since distance is positive, the missing side is $5\sqrt{21}$. Because the measures of the sides are not whole numbers, they do not form a Pythagorean triple. **2.** Use the Pythagorean Thm.: $a^2 + 6^2 = 12^2$; $a^2 + 36 = 144$; $a^2 = 108$; $a = \sqrt{108} = \sqrt{36 \cdot 3} = 6\sqrt{3}$. **3.** When paddling a boat, we think in whole-number distances. You want to know the nearest whole number value, which may not be apparent in a radical expression. **4.** $a^2 + b^2 = c^2$; $a^2 + 7^2 = \sqrt{53}^2$; $a^2 + 49 = 53$; $a^2 = 4$; $a = \sqrt{4} = 2$. So, $b = 7$ and $h = 2$: $A = \frac{1}{2}bh = \frac{1}{2}(7)(2) = 7$ cm^2. **5.** Substitute the greatest length for c in the Pythagorean Thm.: $a^2 + b^2 = c^2$; $16^2 + 48^2 \stackrel{?}{=} 50^2$; $256 + 2304 \stackrel{?}{=} 2500$; $2560 \neq 2500$. Since the Pythagorean Thm. is not satisfied, the △ is not a rt. △. **6.** $7^2 + 8^2 \stackrel{?}{=} 9^2$; $49 + 64 \stackrel{?}{=} 81$; $113 > 81$, $c^2 < a^2 + b^2$, so the △ is acute.

Exercises **1.** $6^2 + 8^2 = x^2$; $36 + 64 = x^2$; $100 = x^2$; $x = \pm\sqrt{100} = \pm 10$. Since distance is positive, $x = 10$. **2.** $24^2 + x^2 = 25^2$; $576 + x^2 = 625$; $x^2 = 49$; $x = \sqrt{49} = 7$ **3.** $16^2 + 30^2 = x^2$; $256 + 900 = x^2$; $x^2 = 1156$; $x = \sqrt{1156} = \sqrt{34 \cdot 34} = 34$ **4.** $x^2 + 16^2 = 20^2$; $x^2 + 256 = 400$; $x^2 = 144$; $x = \sqrt{144} = 12$ **5.** $x^2 + 72^2 = 97^2$; $x^2 + 5184 = 9409$; $x^2 = 4225$; $x = \sqrt{4225} = \sqrt{5^2 \cdot 13^2} = 5 \cdot 13 = 65$ **6.** $x^2 + 15^2 = 17^2$; $x^2 + 225 = 289$; $x^2 = 64$; $x = \sqrt{64} = 8$ **7.** $4^2 + 5^2 \stackrel{?}{=} 6^2$; $16 + 25 \stackrel{?}{=} 36$; $41 \neq 36$. Since $4^2 + 5^2 \neq 6^2$, then 4, 5, and 6 do not form a Pythagorean triple. **8.** $10^2 + 24^2 \stackrel{?}{=} 26^2$; $100 + 576 \stackrel{?}{=} 676$; $676 = 676$. Since $10^2 + 24^2 = 26^2$, then 10, 24, and 26 form a Pythagorean triple. **9.** $15^2 + 20^2 \stackrel{?}{=} 25^2$; $225 + 400 \stackrel{?}{=} 625$; $625 = 625$. Since $15^2 + 20^2 \stackrel{?}{=} 25^2$, then 15, 20, and 25 form a Pythagorean triple. **10.** $4^2 + 5^2 = x^2$; $16 + 25 = x^2$; $41 = x^2$; $x = \sqrt{41}$ **11.** $x^2 + 4^2 = 7^2$; $x^2 + 16 = 49$; $x^2 = 33$; $x = \sqrt{33}$ **12.** $x^2 + 15^2 = 18^2$; $x^2 + 225 = 324$; $x^2 = 99$; $x = \sqrt{99} = \sqrt{9(11)} = 3\sqrt{11}$ **13.** $10^2 + 16^2 = x^2$; $100 + 256 = x^2$; $356 = x^2$; $x = \sqrt{356} = \sqrt{4(89)} = 2\sqrt{89}$ **14.** $x^2 + x^2 = 6$; $2x^2 = 6$; $x^2 = 3$; $x = \sqrt{3}$ **15.** $5^2 + 5^2 = x^2$; $25 + 25 = x^2$; $50 = x^2$; $x = \sqrt{50} = \sqrt{25(2)} = 5\sqrt{2}$ **16.** The house is ⊥ to the ground, and the 15-ft ladder is the hypotenuse of a rt. △. Use the Pythagorean Thm.: $x^2 + 5^2 = 15^2$; $x^2 + 25 = 225$; $x^2 = 200$; $x = \sqrt{200} = \sqrt{100(2)} = 10\sqrt{2} \approx 14$; 14 ft. **17.** Since the playground is square, both legs measure x and the diagonal is 24 m. Use the Pythagorean Thm.: $x^2 + x^2 = 24^2$; $2x^2 = 576$; $x^2 = 288$; $x = \sqrt{288} \approx 17.0$; 17.0 m. **18.** Use the Pyth. Thm. to find the measure of the missing leg: $x^2 + 3^2 = 6^2$; $x^2 + 9 = 36$; $x^2 = 27$; $x = \sqrt{27} = \sqrt{9(3)} = 3\sqrt{3}$. So, $b = 3$ and $h = 3\sqrt{3}$; $A = \frac{1}{2}bh = \frac{1}{2}(3)(3\sqrt{3}) = \frac{9\sqrt{3}}{2}$. **19.** The alt. forms 2 rt. △s, so use the Pyth. Thm. with the measures of one of them to find the altitude: $h^2 + 6^2 = 8^2$; $h^2 + 36 = 64$; $h^2 = 28$; $h = \sqrt{28} = \sqrt{4(7)} = 2\sqrt{7}$. So, $b = 12$ and $h = 2\sqrt{7}$; $A = \frac{1}{2}bh = \frac{1}{2}(12)(2\sqrt{7}) = 12\sqrt{7}$; $12\sqrt{7}$ cm^2. **20.** The alt. forms 2 rt △s, so use the Pyth. Thm. with the measures of one of them to find the altitude: $h^2 + 5^2 = 10^2$; $h^2 + 25 = 100$; $h^2 = 75$; $h = \sqrt{75} = \sqrt{25(3)} = 5\sqrt{3}$. So, $h = 5\sqrt{3}$ and $b = 10$; $A = \frac{1}{2}bh = \frac{1}{2}(10)(5\sqrt{3}) = 25\sqrt{3}$; $25\sqrt{3}$ in. **21.** Use the Pyth. Thm.: $19^2 + 20^2 \stackrel{?}{=} 28^2$; $361 + 400 \stackrel{?}{=} 784$; $761 \neq 784$, so it is not a rt. △. **22.** Use the Pyth. Thm.: $8^2 + 24^2 \stackrel{?}{=} 25^2$; $64 + 576 \stackrel{?}{=} 625$; $640 \neq 625$, so it is not a rt. △. **23.** Use the Pyth. Thm.: $33^2 + 56^2 \stackrel{?}{=} 65^2$; $1089 + 3136 \stackrel{?}{=}$

4225; $4225 = 4225$, so the △ is a rt. △. **24.** Use the Pyth. Thm.: $a^2 + b^2 = c^2$; $(8)^2 + (15)^2 = (21)^2$; $64 + 225 \stackrel{?}{=} 441$; $289 < 441$. Since $c^2 > a^2 + b^2$, the △ is obtuse. **25.** Use the Pyth. Thm.: $a^2 + b^2 = c^2$; $(12)^2 + (16)^2 = (20)^2$; $144 + 256 \stackrel{?}{=} 400$; $400 = 400$. Since $c^2 = a^2 + b^2$, the △ is right. **26.** Use the Pyth. Thm.: $a^2 + b^2 = c^2$; $(4)^2 + (5)^2 = (6)^2$; $16 + 25 \stackrel{?}{=} 36$; $41 > 36$. Since $c^2 < a^2 + b^2$, the △ is acute. **27.** Use the Pyth. Thm.: $a^2 + b^2 = c^2$; $(16)^2 + (30)^2 = (34)^2$; $256 + 900 \stackrel{?}{=} 1156$; $1156 = 1156$. Since $c^2 = a^2 + b^2$, the △ is right. **28.** Use the Pyth. Thm.: $a^2 + b^2 = c^2$; $(0.3)^2 + (0.4)^2 = (0.6)^2$; $0.09 + 0.16 \stackrel{?}{=} 0.36$; $0.25 < 0.36$. Since $c^2 > a^2 + b^2$, the △ is obtuse. **29.** Use the Pyth. Thm.: $a^2 + b^2 = c^2$; $(11)^2 + (12)^2 = (15)^2$; $121 + 144 \stackrel{?}{=} 225$; $265 > 225$. Since $c^2 < a^2 + b^2$, the △ is acute. **30.** Use the Pyth. Thm.: $a^2 + b^2 = c^2$; $(\sqrt{3})^2 + (2)^2 \stackrel{?}{=} (3)^2$; $3 + 4 \stackrel{?}{=} 9$; $7 < 9$. Since $c^2 > a^2 + b^2$, the △ is obtuse. **31.** Use the Pyth. Thm.: $c^2 > a^2 + b^2 = c^2$; $(18)^2 + (80)^2 = (82)^2$; $324 + 6400 \stackrel{?}{=} 6724$; $6724 = 6724$. Since $c^2 = a^2 + b^2$, the △ is right. **32.** Use the Pyth. Thm.: $a^2 + b^2 = c^2$; $(20)^2 + (21)^2 = (28)^2$; $400 + 441 \stackrel{?}{=} 784$; $841 > 784$. Since $c^2 < a^2 + b^2$, the △ is acute. **33.** Use the Pyth. Thm.: $a^2 + b^2 = c^2$; $(12)^2 + (23)^2 = (31)^2$; $144 + 529 \stackrel{?}{=} 961$; $673 < 961$. Since $c^2 > a^2 + b^2$, the △ is obtuse. **34.** Since 30, 40, and 50 are $3 \cdot 10$, $4 \cdot 10$, and $5 \cdot 10$, it is a multiple of a 3-4-5 rt. △. Thus it is a rt. △. **35.** $\sqrt{7} < \sqrt{11} < 4$. Use the Pyth. Thm.: $a^2 + b^2 = c^2$; $(\sqrt{7})^2 + (\sqrt{11})^2 = (4)^2$; $7 + 11 \stackrel{?}{=} 16$; $18 > 16$. Since $c^2 < a^2 + b^2$, the △ is acute. **36.** The alt. forms 2 rt ◬, so use the Pyth. Thm. with the measures of one of them to find the altitude: $x^2 + 24^2 = 26^2$; $x^2 + 576 = 676$; $x^2 = 100$; $x = \sqrt{100} = 10$. **37.** The alt. forms 2 rt ◬, so use the Pyth. Thm. with the measures of one of them to find the altitude: $h^2 + 4^2 = (4\sqrt{5})^2$; $h^2 + 16 = 16(5)$; $h^2 = 16(5) - 16 = 64$; $h = \sqrt{64} = 8$. Use the Pyth. Thm. to find the value of x: $h^2 + 16^2 = x^2$; $8^2 + 16^2 = x^2$; $64 + 256 = x^2$; $x^2 = 320$; $x = \sqrt{320} = \sqrt{64(5)} = 8\sqrt{5}$. **38.** The alt. forms 2 rt ◬, so use the Pyth. Thm. with the measures of one of them to find the altitude: $x^2 + 1^2 = 3^2$; $x^2 + 1 = 9$; $x^2 = 8$; $x = \sqrt{8} = \sqrt{4(2)} = 2\sqrt{2}$. **39.** $3 + 4 + 5 = 12$, so have three people hold the rope 3 units, 4 units, and 5 units apart in the shape of a △. **40.** The alt. forms 2 rt. ◬, so use the Pyth. Thm. with the measures of one of them to find the altitude: $h^2 + 3^2 = (3\sqrt{2})^2$; $h^2 + 9 = 9(2)$; $h^2 = 9$; $h = \sqrt{9} = 3$. Since one leg is 3 and the hyp. is 5 for the rt. △ to the right of the alt, it is a 3-4-5 right △. The missing side is 4, so the entire base length is $4 + 3$, or 7 in. So, $h = 3$ and $b = 7$: $A = \frac{1}{2}bh = \frac{1}{2}(7)(3) = 10.5$; 10.5 in.^{2}. **41.** The alt. divides that half of the ▱ into 2 rt. ◬. The sides of the △ on the left are 2 times those of a 3-4-5 rt. △, so its missing side is 6 ft. The △ on the right is the 8-15-17 triple, so its missing side is 15 ft. So, $b = 6 + 15$, or 21 ft and $h = 8$ ft. $A = bh = (21)(8) = 168$; 168 ft^2 **42.** The alt. divides the △ into 2 rt. ◬. The one on the left is a 3-4-5 rt. △, so its missing side is 4 m, which is also the height. Use the Pyth. Thm. to find the missing part of the base: $x^2 + 4^2 = (4\sqrt{2})^2$; $x^2 + 16 = 16(2)$; $x^2 = 16$; $x = \sqrt{16} = 4$, or 4 m. So, $b = 4 + 3 = 7$ m and $h = 4$ m: $A = \frac{1}{2}bh = \frac{1}{2}(7)(4) = 14$; 14 m^2. **43.** The alt. divides that half of the ▱ into 2 rt. ◬. Use the Pyth. Thm. to find the missing base from the △ on the left: $x^2 + 4^2 = (2\sqrt{5})^2$; $x^2 + 16 = 4(5)$; $x^2 + 16 = 20$; $x^2 = 4$; $x = \sqrt{4} = 2$. Use the Pyth. Thm. to find the missing base from the △ on the right: $y^2 + 4^2 = (2\sqrt{13})^2$; $y^2 + 16 = 4(13)$; $y^2 + 16 = 52$; $y^2 = 36$; $y = \sqrt{36} = 6$. So, $b = 2 + 6 = 8$ in. and $h = 4$ in: $A = bh = (8)(4) = 32$; 32 in.^{2}. **44.** The diagonal of the square is 6 in. and it is also the hyp. of an isosc. rt. △. Use the Pyth. Thm.: $x^2 + x^2 = 6^2$; $2x^2 = 36$; $x^2 = 18$; $x = \sqrt{18} \approx 4.2$. **45.** It is a rectangle if $\triangle RST$ is a rt. △. Use the Pyth. Thm.: $7^2 + 24^2 \stackrel{?}{=} 25^2$; $49 + 576 \stackrel{?}{=} 625$; $625 = 625$, so $\triangle RST$ is a rt. △ with legs $\overline{RS}$ and $\overline{ST}$. Thus, $\angle RST$ is a rt. ∠. It can be shown that all other ∠s of the quadrilateral are also right. Therefore, $RSTW$ is a rectangle. **46a.** Since $\overline{PR}$ is horizontal, subtracting the y-values creates 0, so the distance is the positive value of the difference in x-values: $PR = |x_2 - x_1|$. Since $\overline{QR}$ is vertical, subtracting the x-values creates 0, so the distance is the positive value of the difference in y-values: $QR = |y_2 - y_1|$. **46b.** $PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$ **46c.** $PQ = \pm\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, but since distance is positive, $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. **47.** Answers may vary. Sample: Using 2 segments of length 1, construct the hyp. of the right △ formed by these segments. Using the hyp. found as one leg and a segment of length 1 as the other leg, construct the hyp. of the △ formed by those legs. Repeat until there are $(n - 1)$ ◬ in all. The hyp. of the final △ will have length $\sqrt{n}$. **48.** Assume that the given measures are the legs and use the Pyth. Thm.: $20^2 + 21^2 = c^2$; $400 + 441 = 841 = c^2$; $c = \sqrt{841} = 29$. **49.** Assume that the given measures are the legs and use the Pyth. Thm.: $14^2 + 48^2 = c^2$; $196 + 2304 = 2500 = c^2$; $c = \sqrt{2500} = 50$. **50.** Assume that the given measures are the legs and use the Pyth. Thm.: $13^2 + 85^2 = c^2$; $169 + 7225 = 7394 = c^2$; $c = \sqrt{7394} \approx 85.988$, which is not a whole number. Assume that 85 is the hyp. and use the Pyth. Thm.: $a^2 + 13^2 = 85^2$; $a^2 + 169 = 7225$; $a^2 = 7056$; $a = \sqrt{7056} = 84$. **51.** Assume that 12 and 37 are the legs and use the Pyth. Thm.: $12^2 + 37^2 = c^2$; $144 + 1369 = c^2$; $1513 = c^2$; $c = \sqrt{1513} \approx 38.897$, which is not a whole number. Assume that 37 is the hyp. and use the Pyth. Thm.: $a^2 + 12^2 = 37^2$; $a^2 + 144 = 1369$; $a^2 = 1225$; $a^2 = \sqrt{1225} = 35$. **52a.** $|4 - 5| < j < |4 + 5|$; $1 < j < 9$; $c^2 < (4)^2 + (5)^2$, so $j < \sqrt{16 + 25}$ which is about 6.4. Answers may vary. Sample: $j = 6$ **52b.** $|4 - 5| < k < |4 + 5|$; $1 < k < 9$; $c^2 > (4)^2 + (5)^2$, so $k > \sqrt{41}$, which is about 6.4. Answers may vary. Sample: $k = 7$ **53a.** $|2 - 4| < j < |2 + 4|$; $2 < j < 6$; $c^2 < (2)^2 + (4)^2$, so $j < \sqrt{4 + 16}$, which is about 4.47. Answers may vary. Sample: Since $c \geq 4$, let $j = 4$. **53b.** $|2 - 4| < k < |2 + 4|$; $2 < k < 6$; $c^2 > (2)^2 + (4)^2$, so $k > \sqrt{20}$, which is about 4.47. Answers may vary. Sample: $k = 5$ **54a.** $|6 - 9| < j < |6 + 9|$; $3 < j < 15$; $9^2 < (6)^2 + (j)^2$, so $j > \sqrt{45}$, which is about 6.7. Answers may vary. Sample: $j = 8$ **54b.** $|6 - 9| < k < |6 + 9|$;

$3 < k < 15; c^2 > (6)^2 + (9)^2$, so $k > \sqrt{6^2 + 9^2}$, which is about 10.817. Answers may vary. Sample: $k = 11$

55a. $|5 - 10| < j < |5 + 10|; 5 < j < 15; c^2 < (5)^2 + (10)^2$, so $j < \sqrt{5^2 + 10^2}$, which is about 11.18. Answers may vary. Sample: $j = 11$ **55b.** $|5 - 10| < k < |5 + 10|$; $5 < k < 15; c^2 > (5)^2 + (10)^2$, so $k > \sqrt{5^2 + 10^2}$, which is about 11.18. Answers may vary. Sample: $k = 12$

56a. $|6 - 7| < j < |6 + 7|; 1 < j < 13; c^2 < (6)^2 + (7)^2$, so $j < \sqrt{6^2 + 7^2}$, which is about 9.22. Answers may vary. Sample: $j = 8$ **56b.** $|6 - 7| < k < |6 + 7|; 1 < k < 13$; $c^2 > (6)^2 + (7)^2$, so $k > \sqrt{6^2 + 7^2}$, which is about 9.22. Answers may vary. Sample: $k = 10$ **57a.** $|9 - 12| < j < |9 + 12|; 3 < j < 21; c^2 < (9)^2 + (12)^2$, so $j < \sqrt{9^2 + 12^2}$, which is 15. Answers may vary. Sample: $j = 14$

57b. $|9 - 12| < k < |9 + 12|; 3 < k < 21; c^2 > (9)^2 + (12)^2$, so $k > \sqrt{9^2 + 12^2}$, which is 15. Answers may vary. Sample: $k = 19$ **58a.** $|8 - 17| < j < |8 + 17|; 9 < j < 25$; $c^2 < (8)^2 + (17)^2$, so $j < \sqrt{8^2 + 17^2}$, which is about 18.79. Answers may vary. Sample: $j = 18$ **58b.** $|8 - 17| < k < |8 + 17|; 9 < k < 25; c^2 > (8)^2 + (17)^2$, so $k > \sqrt{8^2 + 17^2}$, which is about 18.79. Answers may vary. Sample: $k = 19$

59a. $|9 - 40| < j < |9 + 40|; 31 < j < 49; c^2 < (9)^2 + (40)^2$, so $j < \sqrt{9^2 + 40^2}$, which is 41. Answers may vary. Sample: $j = 39$ **59b.** $|9 - 40| < k < |9 + 40|$; $31 < k < 49; c^2 > (9)^2 + (40)^2$, so $k > \sqrt{9^2 + 40^2}$, which is 41. Answers may vary. Sample: $k = 42$ **60a.** Each side of the square is c, so its area is c^2. **60b.** The area of the 4 △s is $4 \cdot \frac{1}{2}bh = 2bh = 2(b)(a)$, or $2ab$. The area of the small square is $(b - a)^2$. The total area of the 4 △s and small square is $2ab + (b - a)^2$. **60c.** $2ab + (b - a)^2 = 2ab + b^2 - 2ab + a^2 = a^2 + b^2$, so $c^2 = a^2 + b^2$. **61.** The hyp. = the radius + 600 km = 6370 + 600 = 6970 km, so $x^2 + 6370^2 = 6970^2; x^2 + 40{,}576{,}900 = 48{,}580{,}900$; $x^2 = 8{,}004{,}041; x = \sqrt{8{,}004{,}041} \approx 2830$; 2830 km.

62. The perimeter is 2 + 1 + 1 + 1 + 1 + 1 + the length of the diagonal. The diagonal is the hyp. of a △ with legs of 3 and 4, so the third leg must be 5. The total perimeter is 2 + 1 + 1 + 1 + 1 + 1 + 5, or 12; 12 cm. **63.** The perimeter is 1 + 2 + the hyp. of a 3-4-5 △ + the hyp. of a △ having legs of 2 and 4. The hyp. of the 3-4-5 △ is 5. The hyp. of the other △ is $\sqrt{2^2 + 4^2} = \sqrt{4 + 16} = \sqrt{20} \approx 4.5$. So, the total perimeter to the nearest tenth is $1 + 2 + 5 + 4.5 = 12.5$; 12.5 cm. **64.** The perimeter is the sum of 8 hyp. each having legs of 1 and 2 cm. $P = 8(\sqrt{1^2 + 2^2}) = 8\sqrt{1 + 4} = 8\sqrt{5} \approx 17.9$; 17.9 cm

65a. Answers may vary. Sample: $n = 6; 2n = 2(6) = 12$; $n^2 - 1 = 6^2 - 1 = 36 - 1 = 35$ **65b.** $12^2 + 35^2 \stackrel{?}{=} 37^2$; $144 + 1225 \stackrel{?}{=} 1369; 1369 = 1369$, so they form a Pyth. triple. **66a.** $\triangle ABC$ is a 3-4-5 rt. △, so the hyp. is 5 in.

66b. $\triangle ABD$ is a rt. △, so use the Pyth. Thm.: $d_1^2 + 2^2 = d_2^2; 5^2 + 2^2 = d_2^2; 25 + 4 = d_2^2; d_2 = \sqrt{25 + 4} = \sqrt{29}$.

66c. Substitute $AC^2 + BC^2$ for d_1^2 in $d_1^2 + BD^2 = d_2^2$: $AC^2 + BC^2 + BD^2 = d_2^2$, so $d_2 = \sqrt{AC^2 + BC^2 + BD^2}$, or $d_2 = \sqrt{BD^2 + AC^2 + BC^2}$.

66d. The longest fishing pole would be the length of d_2: $\sqrt{BD^2 + AC^2 + BC^2} = \sqrt{18^2 + 24^2 + 16^2} = \sqrt{324 + 576 + 256} = \sqrt{1156} = \sqrt{34^2} = 34$; 34 in.

67. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = \sqrt{(1 - 0)^2 + (2 - 0)^2 + (3 - 0)^2} = \sqrt{(1)^2 + (2)^2 + (3)^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$

68. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = \sqrt{(-3 - 0)^2 + (4 - 0)^2 + (-6 - 0)^2} = \sqrt{(-3)^2 + (4)^2 + (-6)^2} = \sqrt{9 + 16 + 36} = \sqrt{61}$

69. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = \sqrt{(2 - (-1))^2 + (1 - 3)^2 + (7 - 5)^2} = \sqrt{(3)^2 + (-2)^2 + (2)^2} = \sqrt{9 + 4 + 4} = \sqrt{17}$

70. Draw rt. $\triangle FDE$ with legs $\overline{DE}$ of length a and $\overline{EF}$ of length b and hyp. of length x. Then $a^2 + b^2 = x^2$, by the Pyth. Thm. We are given $\triangle ABC$ with sides of length a, b, c and $a^2 + b^2 = c^2$. By subst., $c^2 = x^2$, so $c = x$. Since all side lengths of $\triangle ABC$ and $\triangle FDE$ are the same, $\triangle ABC \cong \triangle FDE$ by SSS. $\angle C \cong \angle E$ by CPCTC, so $m\angle C = 90$. Therefore, $\triangle ABC$ is a right △. **71.** Use the Pyth. Thm.: $17^2 + 20^2 = c^2; 289 + 400 = c^2; 689 = c^2; c = \sqrt{689} \approx 26.2$. **72.** Use the Pyth. Thm.: $a^2 + 16^2 = 34^2; a^2 + 256 = 1156; d^2 = 900; d = \sqrt{900} = 30$. **73.** Assume 40 and 41 are legs and use the Pyth. Thm.: $40^2 + 41^2 = c^2; c^2 = 1600 + 1681 = 3281$, so $c = \sqrt{3281}$, which is not a whole number. Assume 41 is the hypotenuse and use the Pyth. Thm.: $a^2 + 40^2 = 41^2; a^2 + 1600 = 1681; d^2 = 81; d = \sqrt{81} = 9$. So, the third side is a leg that measures 9.

74. The measure of the third side a is between $|20 - 30|$ and $|20 + 30|$, or $10 < c < 50$. Since 20 and 30 are the shorter measures, then c is the longest measure. Then $c^2 > 20^2 + 30^2; c^2 > 400 + 900; c^2 > 1300; c > \sqrt{1300}$, which is about 36.0555. The least whole number for c is 37. **75.** Use the Pyth. Thm.: $10^2 + 10^2 = c^2; 100 + 100 = c^2; 200 = c^2; c = \sqrt{200}$, which is about 14.1 cm. To answer the question, grid 14.1. **76.** Since the △ is isosc, then both legs must measure 12 cm. Since they are ⊥, $b = 12$ and $h = 12$. $A = \frac{1}{2}bh = \frac{1}{2}(12)(12) = 72$ cm^2

77. Since the legs of an isosc. rt. △ are ⊥ and ≅, $b = h = x$. $A = \frac{1}{2}bh; 112.5 = \frac{1}{2}x^2; 225 = x^2; x = \sqrt{225} = 15$; 15 ft

78. By the Angle Bisector Thm., $RS = TS$, so $2x + 19 = 7x - 16; -5x + 19 = -16; -5x = -35; x = 7$. $RS = 2x + 19 = 2(7) + 19 = 14 + 19 = 33$ **79.** By the Angle Bisector Thm., $RS = TS$, so $2(7y - 11) = 5y + 5$; $14y - 22 = 5y + 5; 9y - 22 = 5; 9y = 27; y = 3$. $RS = 2(7y - 11) = 2(7)(3) - 11) = 2(21 - 11) = 2(10) = 20$

80. Since $\triangle PQR \cong \triangle STV$, then $\angle P \cong \angle S$ by CPCTC. So, $4w + 5 = 6w - 15; -2w + 5 = -15; -2w = -20; w = 10$. **81.** $\overline{RQ} \cong \overline{VT}$ by CPCTC. So, $10y - 6 = 5y + 9$; $5y - 6 = 9; 5y = 15; y = 3$. **82.** $\angle T \cong \angle Q$ by CPCTC. So, $2x - 40 = x + 10; x - 40 = 10; x = 50$. **83.** $\overline{PR} \cong \overline{SV}$ by CPCTC. So, $2z + 3 = 4z - 11; -2z + 3 = -11$; $-2z = -14; z = 7$.

a. The altitude divides $\triangle ABC$ into 2 rt. $\triangle$s. First use the Pyth. Thm. to find the length of the altitude. Then use the Pyth. Thm. with the length of the altitude to find BC: $\text{alt}^2 + 2^2 = 6^2$; $\text{alt}^2 + 4 = 36$; $\text{alt}^2 = 32$; so the altitude $= \sqrt{32} = \sqrt{16(2)} = 4\sqrt{2}$. $\text{alt}^2 + 8^2 = BC^2$; $(4\sqrt{2})^2 + 64 = BC^2$; $16(2) + 64 = BC^2$; $32 + 64 = BC^2$; $96 = BC^2$; $BC = \sqrt{96} \approx 9.8$. The perimeter to the nearest tenth is $9.8 + 6 + (2 + 8) = 25.8$. **b.** $h = 4\sqrt{2}$ and $b = 2 + 8 = 10$. $A = \frac{1}{2}bh = \frac{1}{2}(10)(4\sqrt{2}) = 20\sqrt{2} \approx 28.3$

7-3 Special Right Triangles pages 366–372

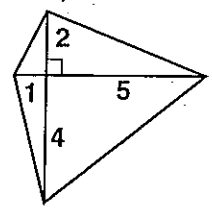
Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM*.
1. 45, 45, 90 **2.** 30, 60, 90 **3.** 45, 45, 90

Check Understanding **1.** The hyp. in a 45°-45°-90° $\triangle$ $= \sqrt{2} \cdot \text{leg} = \sqrt{2} \cdot 5\sqrt{3} = 5\sqrt{6}$. **2.** The length of a leg of a 45°-45°-90° $\triangle$ is the hyp. $\div \sqrt{2} = 10 \div \sqrt{2} = \frac{10}{\sqrt{2}} = \frac{10\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$. **3.** Multiply the leg by $\sqrt{2}$: $100\sqrt{2} \approx 141$; 141 ft. **4.** The shorter leg is half the hyp.: $12 \div 2 = 6$. The longer leg is $\sqrt{3}$ times the shorter leg: $6\sqrt{3}$. **5.** The longer leg is $\sqrt{3}$ times the shorter leg: $\sqrt{6} \cdot \sqrt{3} = \sqrt{18} = \sqrt{9(2)} = 3\sqrt{2}$. The hyp. is twice the shorter leg: $2\sqrt{6}$. **6a.** Drop an altitude from a vertex to the opp. side of the rhombus to create a 30°-60°-90° $\triangle$. The alt. is opp. the 30° angle, so it is half the hyp.: $10 \div 2 = 5$. $A = bh = (10)(5) = 50$; 50 in.^{2} **6b.** The shortest side is half the hyp.: $10 \div 2 = 5$; 5 in. The other side is $\sqrt{3}$ times the shortest side: $5\sqrt{3}$, or $5\sqrt{3}$ in. The diagonals of the rhombus divide it into 4 $\cong$ $\triangle$s. So, the total area is $4\left(\frac{1}{2}bh\right) = 2bh = 2(5)(5\sqrt{3}) = 50\sqrt{3}$; $50\sqrt{3}$ in.^{2}.

Exercises **1.** Both legs are $=$, so $x = 8$. The hyp. is $\sqrt{2}$ times the leg, so $y = 8\sqrt{2}$. **2.** Both legs are $=$, so $x = \sqrt{2}$. The hyp. is $\sqrt{2}$ times the leg, so $y = \sqrt{2} \cdot \sqrt{2} = 2$. **3.** The hyp. is $\sqrt{2}$ times the leg, so $y = 60\sqrt{2}$. **4.** In a 45°-45°-90° $\triangle$, the legs are both $=$ to the hyp. $\div \sqrt{2}$, so $x = y = \frac{15\sqrt{2}}{\sqrt{2}} = 15$. **5.** A leg is the hyp. $\div \sqrt{2}$, so $x = \frac{8}{\sqrt{2}} = \frac{8\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$. **6.** A leg is the hyp. $\div \sqrt{2}$, so $a = \frac{9\sqrt{2}}{2} = 9$. **7.** A leg is the hyp. $\div \sqrt{2}$, so $x = \frac{\sqrt{6}}{\sqrt{2}} = \sqrt{\frac{6}{2}} = \sqrt{3}$. **8.** A leg is the hyp. $\div \sqrt{2}$, so $b = \frac{12}{\sqrt{2}} = \frac{12\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$. **9.** The $\triangle$ is an isosc. rt. $\triangle$, so it is a 45°-45°-90° $\triangle$. The hyp. is $\sqrt{2}$ times a leg, so $y = \sqrt{2} \cdot \sqrt{5} = \sqrt{10}$. **10.** The diagonal of a square divides it into two 45°-45°-90° $\triangle$s. So, the side of the plate is a leg that is the hyp. $\div \sqrt{2}$. So, each side is $20 \div \sqrt{2}$, or $\frac{20}{\sqrt{2}} = \frac{20\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{20\sqrt{2}}{2} = 10\sqrt{2} \approx 14.1$; 14.1 cm. **11.** The distance between 2 tips of the blades is the hyp. of a 45°-45°-90° $\triangle$, so the length of the blade is $36 \div \sqrt{2}$, or $\frac{36}{\sqrt{2}} = \frac{36\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{36\sqrt{2}}{2} = 18\sqrt{2}$; $18\sqrt{2}$ ft. **12.** The side opp. the 30° $\angle$ is half the hyp., so $x = 40 \div 2 = 20$. The side opp. the 60° $\angle$ is $\sqrt{3}$ times the shorter leg, so $y = 20\sqrt{3}$. **13.** The side opp. the 30° $\angle$ is half the hyp., so $x = 2\sqrt{3} \div 2 = \sqrt{3}$. The side opp. the 60° $\angle$ is $\sqrt{3}$ times the shorter leg, so $y = \sqrt{3} \cdot \sqrt{3} = 3$. **14.** The side opp. the 30° $\angle$ is half the hyp., so $x = 10 \div 2 = 5$. The side opp. the 60° $\angle$ is $\sqrt{3}$ times the shorter leg, so $y = 5\sqrt{3}$. **15.** The hyp. is twice the side opp. the 30° $\angle$, so $x = 2(12) = 24$. The leg opp. the 60° $\angle$ is $\sqrt{3}$ times the shorter leg, so $y = 12\sqrt{3}$. **16.** The side opp. the 30° $\angle$ is the longer leg $\div \sqrt{3}$, so $y = \frac{2\sqrt{3}}{\sqrt{3}} = 2$. The hyp. is twice the shorter leg, so $x = 2(2) = 4$. **17.** The hyp. is twice the shorter leg, so $x = 2(2\sqrt{3}) = 4\sqrt{3}$. The longer leg is $\sqrt{3}$ times the shorter leg, so $y = \sqrt{3}(2\sqrt{3}) = 2 \cdot 3 = 6$. **18.** The shorter leg is the longer leg length $\div \sqrt{3}$, so $x = \frac{9\sqrt{3}}{\sqrt{3}} = 9$. The hyp. is twice the shorter leg, so $y = 2(9) = 18$. **19.** The shorter leg is the longer leg $\div \sqrt{3}$, so $x = \frac{9}{\sqrt{3}} = \frac{9\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{9\sqrt{3}}{3} = 3\sqrt{3}$. **20.** The shorter leg is the length of the longer leg $\div \sqrt{3}$, so $x = \frac{15}{\sqrt{3}} = \frac{15\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{15\sqrt{3}}{3} = 5\sqrt{3}$. The hyp. is twice the shorter leg, so $y = 2\left(5\sqrt{3}\right) = 10\sqrt{3}$. **21.** The alt. of an equilateral $\triangle$ divides the $\triangle$ into two 30°-60°-90° $\triangle$s. The side opp. the 30° $\angle$ is 5 cm, so the longer leg is $5\sqrt{3}$ cm. $A = \frac{1}{2}bh = \frac{1}{2}(10)\left(5\sqrt{3}\right) = 25\sqrt{3} \approx 43.3$. The area is about 43.3 cm^2. **22.** The altitude creates a 30-60°-90° $\triangle$ with a 5-cm hyp. The shorter leg is 2.5 cm, so the longer leg is $2.5\sqrt{3}$ cm. $A = bh = (5)\left(2.5\sqrt{3}\right) \approx 21.7$. The area is about 21.7 cm^2. **23.** The altitude creates a 45°-45°-90° $\triangle$ with a 12-m hyp. and the alt. is a leg. The leg measures $\frac{12}{\sqrt{2}} = \frac{12\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$. $A = bh = (12)(6\sqrt{2}) \approx 101.8$. The area is about 101.8 m^2. **24.** The leg measuring a is of a 45°-45°-90° $\triangle$, so it is $\frac{7\sqrt{2}}{\sqrt{2}}$, or 7. Also, $c = a = 7$. In the 30°-60°-90° $\triangle$, b is the hyp., which is twice a, or $2(7) = 14$. Also, d is the measure of the longer leg, so it is $7\sqrt{3}$. So, $a = 7$, $b = 14$, $c = 7$, and $d = 7\sqrt{3}$. **25.** $4\sqrt{3}$ is the length of the hyp. of a 30°-60°-90° $\triangle$, so $c = \frac{4\sqrt{3}}{2}$, or $2\sqrt{3}$. Also, $a = 2\sqrt{3}\left(\sqrt{3}\right) = 2(3) = 6$. The other rt. $\triangle$ is a 45°-45°-90° $\triangle$, so $d = a = 6$, and $b = a\sqrt{2}$, or $6\sqrt{2}$. So, $a = 6$, $b = 6\sqrt{2}$, $c = 2\sqrt{3}$, and $d = 6$. **26.** 10 is the hyp. length of a 30°-60°-90° $\triangle$, so $d = 10 \div 2$, or 5, and $b = 5\sqrt{3}$. In the other rt. $\triangle$, b is the short leg, so $a = 2b = 2\left(5\sqrt{3}\right) = 10\sqrt{3}$, and $c = b\sqrt{3} = \left(5\sqrt{3}\right)\sqrt{3} = 5(3) = 15$. So, $a = 10\sqrt{3}$, $b = 5\sqrt{3}$, $c = 15$, and $d = 5$. **27.** Draw an altitude to create a $6 \times 2\sqrt{3}$ rectangle and a 30°-60°-90° $\triangle$. The alt. is $2\sqrt{3}$ and is the longer leg of the $\triangle$. The short leg of the $\triangle$ is 2. So, $b = 6 - 2$, or 4. Then $a = 2(2)$, or 4. So, $a = 4$ and $b = 4$. **28.** Draw an altitude to create a $4 \times a$ rectangle and a 45°-45°-90° $\triangle$. Then a is the length of a leg of the $\triangle$, so $a = \frac{3\sqrt{2}}{\sqrt{2}}$, or 3. Both legs of the $\triangle$ are 3, so $b = 4 + 3 = 7$. So, $a = 3$ and $b = 7$. **29.** Draw an altitude to create a 6×8 rectangle and a 45°-45°-90° $\triangle$. One leg, the alt., is

6, so the other leg must also be 6. Thus, $a = 8 + 6 = 14$. Also, $b = 6\sqrt{2}$. So, $a = 14$ and $b = 6\sqrt{2}$. **30.** Rika; the shortest side of a △ must be opp. the smallest ∠ and the longest side is opp. the largest ∠. The smallest ∠ of the △ is 30°. Sandra marked the shorter leg as opposite the 60° ∠. **31.** Answers may vary. Sample: A ramp up to a door is 12 ft long. It has an incline of 30°. How high off the ground is the door? The distance from the ground to the door is opp. the 30° ∠, so it is half of 12 ft, which is 6 ft. **32a.** The side of the barn is opp. the 60° ∠, so the distance from the vertex of that ∠ to the base of the barn is $\frac{24}{\sqrt{3}} = \frac{24\sqrt{3}}{\sqrt{3}\sqrt{3}} = 8\sqrt{3}$. The length of the belt is the hyp., so its length is $2(8\sqrt{3}) = 16\sqrt{3} \approx 28$. The belt length is about 28 ft. **32b.** Use $d = rt$: $28 = 100t$; $t = 0.28$. It takes about 0.28 min. **33a.** Each leg of the 45°-45°-90° △ is $\frac{6}{\sqrt{2}}$, or $3\sqrt{2}$ m. So, the side opp. the 30° ∠ is $3\sqrt{2}$ m. Thus, the hyp. is $2(3\sqrt{2})$, or $6\sqrt{2}$ m. The longer brace is about 8.5 m. **33b.** The side opp. the 60° ∠ is $3\sqrt{2}(\sqrt{3})$, or $3\sqrt{6}$ m. So, the difference is $3\sqrt{6} - 3\sqrt{2} \approx 3.1$. The difference is about 3.1 m. **34.** The base and height are each one of the legs, and the legs measure $\frac{14\sqrt{2}}{\sqrt{2}}$, or 14 m. $A = \frac{1}{2}bh = \frac{1}{2}(14)(14) = 98$. The area is 98 m^2. **35.** The alt. is the longer leg of a 30°-60°-90° △, so the shorter side is $\frac{8\sqrt{3}}{\sqrt{3}}$, or 8 cm. The base is 2(8), or 16 cm. $A = \frac{1}{2}bh = \frac{1}{2}(16)(8\sqrt{3}) = 64\sqrt{3} \approx 110.9$. The area is about 110.9 cm^2. **36.** Each △ in the square is a 45°-45°-90° △, so each leg measures $\frac{24}{\sqrt{2}}$, or $12\sqrt{2}$ ft. $A = s^2 = (12\sqrt{2})^2 = 12^2(\sqrt{2})^2 = 144(2) = 288$. The area is 288 ft^2. **37.** An altitude creates a 45°-45°-90° △ having a 4-yd hyp. The height is $\frac{4}{\sqrt{2}}$ which is $2\sqrt{2}$. $A = bh = 4(2\sqrt{2}) \approx 11.3$. The area is about 11.3 yd^2. **38.** The altitude creates a 30°-60°-90° △ having a hyp. of 6 m. The shorter leg is half the hyp., or 3 m. The height is $3\sqrt{3}$ m. $A = bh = (3)(3\sqrt{3}) = 9\sqrt{3} \approx 15.6$. The area is about 15.6 m. **39.** The shorter leg is half the hyp., or $3\sqrt{3}$. The longer leg is $\sqrt{3}$ times the shorter leg; $3\sqrt{3}(\sqrt{3}) = 3(3) = 9$. $A = \frac{1}{2}bh = \frac{1}{2}(3\sqrt{3})(9) \approx 23.4$. The area is about 23.4 units^2. **40a.** The hyp. of a 45°-45°-90° △ having legs of 1 unit is $\sqrt{2}$. Use the Pyth. Thm.: $1^2 + (\sqrt{2})^2 = d^2$; $1 + 2 = d^2$; $d = \sqrt{3}$. So, d is $\sqrt{3}$ units. **40b.** The hyp. of a 45°-45°-90° △ having legs of 2 units is $2\sqrt{2}$. Use the Pyth. Thm.: $2^2 + (2\sqrt{2})^2 = d^2$; $2^2 + 2^2(\sqrt{2})^2 = d^2$; $4 + 4(2) = d^2$; $4 + 8 = d^2$; $d = \sqrt{12} = 2\sqrt{3}$. So, d is $2\sqrt{3}$ units. **40c.** The pattern is $1\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}$, for sides of 1, 2, and 3, respectively. So, the diagonal of a cube that is s units per side is $s\sqrt{3}$ units long. **41a.** Each ∠ of an equilateral △ is 60°. So, the altitude is opp. the 60° ∠ in a 30°-60°-90° △. Half the base of the equilateral △ is $\frac{1}{\sqrt{3}}$, or $\frac{\sqrt{3}}{3}$, so the entire base is $\frac{2\sqrt{3}}{3}$. $A = \frac{1}{2}bh = \frac{1}{2}\left(\frac{2\sqrt{3}}{3}\right)(1) = \frac{\sqrt{3}}{3}$. The area is $\frac{\sqrt{3}}{3}$ units^2. **41b.** If the altitude is h, then half the base is $\frac{h\sqrt{3}}{3}$, so the total base is $\frac{2h\sqrt{3}}{3}$. $A = \frac{1}{2}bh = \frac{1}{2}\left(\frac{2h\sqrt{3}}{3}\right)h = \frac{h^2\sqrt{3}}{3}$. The formula is $A = \frac{h^2\sqrt{3}}{3}$. **41c.** $A = \frac{h^2\sqrt{3}}{3} = \frac{6^2\sqrt{3}}{3} = \frac{36\sqrt{3}}{3} = 12\sqrt{3}$. The area is $12\sqrt{3}$ units^2. **42.** For Column A: The diagonal of a square forms a 45°-45°-90° △. The legs are 3 and the diagonal is $3\sqrt{2}$. For Column B: Each leg is $\frac{3}{\sqrt{2}}$, or $\frac{3\sqrt{2}}{2}$ units long. Since $3\sqrt{2} > \frac{3\sqrt{2}}{2}$, the answer is choice A. **43.** For Column A: The shorter leg is half the hyp. so it is 2. For Column B: The shorter leg is $\frac{\sqrt{3}}{\sqrt{3}}$ or 1, so the hyp. is 2. Since 2 = 2, the answer is choice C. **44.** There are no measures involving length, so the answer is choice D. **45.** The diagonal is the hyp. of a 45°-45°-90° △ having legs of 4, so it is $4\sqrt{2}$ units long. The answer is choice D. **46** [2] **a.** Let a leg measure x. Then $A = \frac{1}{2}x^2 = 16$ and $x = 4\sqrt{2}$. **b.** The hypotenuse is $\sqrt{2} \cdot$ leg, so $4\sqrt{2}(\sqrt{2}) = 4(2) = 8$. [1] incorrect calculation OR no explanation **47.** The alt. divides the △ into 2 ≅ △s, and the base of each is 8 cm. Use the Pyth. Thm.: $a^2 + 8^2 = 20^2$; $a^2 + 64 = 400$; $a^2 = 336$; $a = \sqrt{336} = \sqrt{16 \cdot 21} = 4\sqrt{21}$. The altitude is $4\sqrt{21}$ cm. **48.** From Exercise 47, the height is $4\sqrt{21}$ cm. $A = \frac{1}{2}bh = \frac{1}{2}(16)(4\sqrt{21}) = 32\sqrt{21}$. The area is $32\sqrt{21}$ cm^2.

49. no



50. Since a pair of opp. ∠s are rt ∠s, they are =, so the figure could be a rectangle or a kite. Since opp. sides are also =, it cannot be a kite. It must be a rectangle, which is a ▱. **51.** No; the figure could be an isosc. trapezoid. **52.** Two ∠s and a nonincluded side are ≅, so the △s are ≅ by AAS. **53.** No sides are ≅, so the △s could be different sizes. It is not possible to conclude that the △s are ≅. **54.** Two sides and a nonincluded ∠ are ≅. Since there is no SSA post. or thm., it is not possible to conclude that the △s are ≅. **55.** Two ∠s and an included side are ≅, so the △s are ≅ by ASA.

CHECKPOINT QUIZ 1 *page 372*

1. $A = \frac{1}{2}bh = \frac{1}{2}(21)(8) = 84$. The area is 84 in.^{2}. **2.** $A = bh = (8)(14) = 112$. The area is 112 cm^2. **3.** To find the height, use the Pyth. Thm.: $h^2 + 6^2 = 10^2$; $h^2 + 36 = 100$; $h^2 = 64$; $h = \sqrt{64} = 8$. The base is 6 + 6, or 12. $A = \frac{1}{2}bh = \frac{1}{2}(12)(8) = 48$. The area is 48 m^2. **4.** The measures are 3 times those of the special 3-4-5 △, so the missing side is 3(4), or 12. **5.** For x: The △ is an isosc. △, so $x = 10$. For y: The △ is a 45°-45°-90° △, so the hyp. is $10\sqrt{2}$. So, $x = 10$ and $y = 10\sqrt{2}$. **6.** The △ is a 30°-60°-90° △, so the sides in order from shortest to longest have the ratio $a : a\sqrt{3} : 2a$. Since $a = 12$, $x = 12\sqrt{3}$ and $y = 24$. **7.** The longest side is 9, so $9^2 \stackrel{?}{=} 7^2 + 8^2$; $81 \stackrel{?}{=} 49 + 64$; $81 < 113$, so the △ is acute. **8.** The longest side is 39, so $39^2 \stackrel{?}{=} 15^2 + 36^2$; $1521 \stackrel{?}{=} 225 + 1296$; $1521 = 1521$, so the △ is right. **9.** The longest side is 16, so $16^2 \stackrel{?}{=} 10^2 + 12^2$; $256 \stackrel{?}{=} 100 + 144$; $256 > 244$, so the △ is obtuse. **10.** The diagonal is the hyp. of a 45°-45°-90° △, so each side measures $\frac{40}{\sqrt{2}}$, or $20\sqrt{2}$. Each side of the square is the length of a leg of the rt. △, so they measure about 28.3 cm.

7-4 Areas of Trapezoids, Rhombuses, and Kites pages 373–379

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

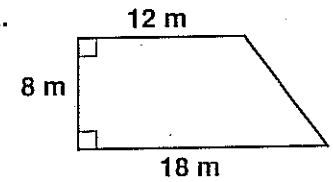
1. $A = bh$ or $A = \ell w$ **2.** $A = \frac{1}{2}bh$ **3.** 9 units^2 **4.** 7 units^2 **5.** 13.5 units^2

Investigation 1a. The length of the ▱ base is the sum of the two lengths of the trapezoid bases: $b_1 + b_2$. **1b.** The area of a trapezoid is the product of the base and height: $h(b_1 + b_2)$. **2.** Since 2 ≅ trapezoids create the ▱, the area of each trapezoid is half the area of the ▱. **3.** $A = \frac{1}{2}h(b_1 + b_2)$

Check Understanding 1. $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(7)(12 + 15) = \frac{1}{2}(7)(27) = 94.5$. The area is 94.5 cm^2. **2.** The △ is isosc., so $h = 2$. $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(2)(7 + 5) = 12$; 12 m^2 **3.** $A = \frac{1}{2}d_1d_2 = \frac{1}{2}(12)(9) = 54$; 54 in.^{2} **4.** $9^2 + 12^2 = 15^2$

Exercises 1. $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(16)(21 + 38) = 472$; 472 in.^{2} **2.** $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(8.5)(24.3 + 9.7) = 144.5$; 144.5 cm^2 **3.** $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(9)(18 + 6) = 108$; 108 ft^2 **4.** $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(309)(205 + 511) = \frac{1}{2}(309)(716) = 110{,}622$; about 110,622 mi^2 **5.** $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(10)(12 + 18) = 150$; 150 cm^2 **6.** $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}\left(\frac{1}{3}\right)(2 + 3) = \frac{5}{6}$; $\frac{5}{6}$ ft^2 **7.** $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(111)(342 + 438) = \frac{1}{2}(111)(780) = 43{,}290$; about 43,290 mi^2 **8.** The altitude forms a 3-4-5 △, so the altitude is 4 ft. One base is 6 ft and the other base is 6 ft + 3 ft, or 9 ft. $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(4)(6 + 9) = 30$; 30 ft^2 **9.** The △ formed by the altitude has measures twice those in the 3-4-5 special △, so the missing measure is 2(3), or 6. So one base is 6 and the other base is 6 + 6, or 12. $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(8)(6 + 12) = 72$; 72 m^2 **10.** One base is 8 m and the other is (8 + 5), or 13 m. The alt. forms a special 5-12-13 △, so the altitude is 12 m. $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(12)(8 + 13) = 126$; 126 m^2 **11.** The alt. forms a 30°-60°-90° △ whose sides are in the ratio $a : a\sqrt{3} : 2a$. Since $2a = 8$, then $a = 4$, so the height is $4\sqrt{3}$. One side is 15 ft and the other side is (15 − 4), or 11 ft. $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}\left(4\sqrt{3}\right)(15 + 11) = \frac{1}{2}\left(4\sqrt{3}\right)(26) = 52\sqrt{3}$; $52\sqrt{3}$ ft^2 **12.** The altitude forms an isosc. rt. △ whose leg is 15, so the missing leg length is 15 in. One base is 13 in. The other base is (9 + 13 + 15), or 37 in. $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(15)(37 + 13) = 375$; 375 in.^{2} **13.** The altitudes form 2 isosc. rt. △ and a rectangle between them. Since the hyp. is $8\sqrt{2}$, the legs on each △ are 8 m. One base is 8 m. The other base is (8 + 8 + 8), or 24 m. The height is 8 m. $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(8)(8 + 24) = 128$; 128 m^2 **14.** $A = \frac{1}{2}d_1d_2 = \frac{1}{2}(2 + 8)(8 + 8) = \frac{1}{2}(10)(16) = 80$; 80 in.^{2} **15.** $A = \frac{1}{2}d_1d_2 = \frac{1}{2}(2 + 4)(3 + 3) = \frac{1}{2}(6)(6) = 18$; 18 m^2 **16.** $A = \frac{1}{2}d_1d_2 = \frac{1}{2}(6)(4 + 4) = 24$; 24 ft^2 **17.** $A = \frac{1}{2}d_1d_2 = \frac{1}{2}(7)(16) = 56$; 56 ft^2 **18.** Since the diagonals of a rhombus bisect each other, the diagonals are 40 ft and 60 ft. $A = \frac{1}{2}d_1d_2 = \frac{1}{2}(40)(60) = 1200$; 1200 ft^2 **19.** Since all sides of a rhombus are ≅ and its diags. are ⊥, the diagonals divide the rhombus into four triangles whose measures are twice those in the special 3-4-5 △. Thus, the missing side length of the △ is 6 in. The diags. of a rhombus bis. each other, so the diagonals are 12 in. and 16 in. $A = \frac{1}{2}d_1d_2 = \frac{1}{2}(12)(16) = 96$; 96 in.^{2} **20.** The missing diagonal creates four 3-4-5 triangles, of which the missing length is 4 m. So, the missing diag. is 8 m. $A = \frac{1}{2}d_1d_2 = \frac{1}{2}(8)(6) = 24$ m^2 **21.** $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(4)(4 + 6) = 20$; 20 in.^{2}

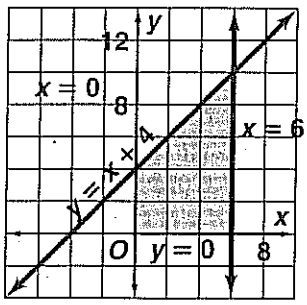
22a.



22b. An altitude creates an 8 × 12 rectangle and a rt. △ having a base of (18 − 12), or 6 m and a height of 8 m. The measures are twice that of the 3-4-5 special rt. △, so the missing side is 2(5), or 10 m. $P = 10 + 12 + 8 + 18 = 48$; 48 m **22c.** $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(8)(12 + 18) = 120$; 120 m^2 **23.** Check students' work. **24.** $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(3)(4 + 2) = 9$; 9 cm^2 **25.** $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(3)(8 + 5) = \frac{1}{2}(3)(13) = 19.5$; 19.5 cm^2 **26.** Use the Pyth. Thm. to find the height: $h^2 + 1^2 = 3^2$; $h^2 + 1 = 9$; $h^2 = 8$; $h = \sqrt{8}$; $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}\left(\sqrt{8}\right)(3 + (1 + 4)) = \frac{1}{2}\left(\sqrt{8}\right)(8) \approx 11.3$; 11.3 cm^2. **27.** The altitude is opp. the 30° ∠, so it is half the hyp., which is 4 ft. The side opp. the 60° ∠ is $4\sqrt{3}$ ft, so the longer base is $(4\sqrt{3} + 9)$ ft and the shorter base is 9 ft. $A = \frac{1}{2}h\left(b_1 + b_2\right) = \frac{1}{2}(4)\left(9 + 4\sqrt{3} + 9\right) = 2\left(18 + 4\sqrt{3}\right) \approx 49.9$; 49.9 ft^2 **28.** An altitude creates a rectangle and an isosc. rt. △ whose hyp. is 1.7 m. So, each leg of the △ is $\frac{1.7}{\sqrt{2}}$ m. Thus, the height is $\frac{1.7}{\sqrt{2}}$ m. $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}\left(\frac{1.7}{\sqrt{2}}\right)(0.9 + 2.1) = \frac{1}{2}\left(\frac{1.7}{\sqrt{2}}\right)(3) \approx 1.8$; 1.8 m^2 **29.** $QS = 6$ and $RT = 6$, so $A = \frac{1}{2}d_1d_2 = \frac{1}{2}(6)(6) = 18$; 18 units^2. **30.** Using the distance formula, $TR = \sqrt{5^2 + 5^2} = 5\sqrt{2}$. $\triangle QTS$ is an isosc. rt. △ whose legs are each 3 units, so $QS = 3\sqrt{2}$ units. $A = \frac{1}{2}d_1d_2 = \frac{1}{2}\left(5\sqrt{2}\right)\left(3\sqrt{2}\right) = \frac{1}{2}(5)(3)\left(\sqrt{2} \cdot \sqrt{2}\right) = \frac{1}{2}(5)(3)(2) = 15$; 15 units^2 **31.** For diagonal $\overline{RT}$: $R = (-2, 2)$ and $T = (1, -1)$, so $RT = \sqrt{(1 - (-2))^2 + (-1 - 2)^2} = \sqrt{(3)^2 + (-3)^2} = 3\sqrt{2}$. For diagonal $\overline{QS}$: $Q = (-3, -2)$ and $S = (2, 3)$, so $QS = \sqrt{(2 - (-3))^2 + (3 - (-2))^2} = \sqrt{(5)^2 + (5)^2} = 5\sqrt{2}$. $A = \frac{1}{2}d_1d_2 = \frac{1}{2}\left(3\sqrt{2}\right)\left(5\sqrt{2}\right) = \frac{1}{2}(3)(5)(2) = 15$; 15 units^2 **32.** The quad. is a trapezoid with horizontal bases $\overline{RS}$ and

$\overline{QT}$. The height, RQ, is 5, $RS = 5$, and $QT = 7$. $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(5)(5 + 7) = 30$; 30 units^2 **33.** The diagonals create two isosc. rt. △s whose hyp. is $9\sqrt{2}$, so each leg must be 9. One diagonal is $(9 + 6)$, or 15 m. The other diagonal is $(9 + 9)$, or 18 m. $A = \frac{1}{2}d_1d_2 = \frac{1}{2}(15)(18) = 135$; 135 m^2

34a.



34b. The lines intersect to form a trapezoid. **34c.** From the graph, one base is 4 units, and the other base is 10 units. The height is 6 units. $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(6)(4 + 10) = 42$; 42 units^2

35. The diagonals of a rhombus bis. each other, so one diag. measures 6 cm. The diagonals of a rhombus are also ⊥, so they divide this rhombus into 4 isosc. rt. △s. Thus, the other diagonal is also 6 cm. $A = \frac{1}{2}(6)(6) = 18$; 18 cm^2 **36.** The diagonals of a rhombus are ⊥ bis. of each other so they form four 30°-60°-90° △s whose shorter leg is 4 m. The longer leg is $4\sqrt{3}$. Thus, the diagonals are 8 m and $8\sqrt{3}$ m. $A = \frac{1}{2}(8)(8\sqrt{3}) = 32\sqrt{3}$; $32\sqrt{3}$ m^2 **37.** The diags. of a rhombus are ⊥ bis. of each other, so one diagonal is 16 in. The 8-in. segment is opp. the 60° ∠, so the leg opp. the 30° ∠ is $8 \div \sqrt{3}$, which is $\frac{8\sqrt{3}}{3}$. Thus, the other diag. is $\frac{16\sqrt{3}}{3}$ in. $A = \frac{1}{2}(16)(\frac{16\sqrt{3}}{3}) = \frac{128\sqrt{3}}{3}$; $\frac{128\sqrt{3}}{3}$ in.^{2} **38a.** The height of both △s is the same, h. The formula for the area of the △ having base b_1 is $A = \frac{1}{2}b_1h$. The formula for the area of the △ having base b_2 is $A = \frac{1}{2}b_2h$. **38b.** The area of the trapezoid is the sum of the areas of the two △s, so $A = \frac{1}{2}b_1h + \frac{1}{2}b_2h = \frac{1}{2}h(b_1 + b_2)$. **39.** If the shorter base is $2x$, then the longer base is $2(2x)$, or $4x$. Then $h = \frac{2x + 4x}{2}$, or $h = 3x$. Since $A = \frac{1}{2}h(b_1 + b_2)$, then $324 = \frac{1}{2}(3x)(2x + 4x)$; $324 = \frac{1}{2}(3x)(6x)$; $324 = 9x^2$; $x^2 = 36$; $x = 6$. Thus, $b_1 = 2x = 2(6) = 12$, $b_2 = 4x = 4(6) = 24$, and $h = 3x = 3(6) = 18$. So, $b_1 = 12$ cm, $b_2 = 24$ cm, and $h = 18$ cm. **40.** At both $x = 1$ and $x = -1$, $y = 0.25(1)^2 = 0.25$. There are 2 ≅ △s whose base is 1 and whose height is 0.25 m and the sum of their areas is $2(\frac{1}{2})(1)(0.25)$, or 0.25 m^2. There are also 2 ≅ trapezoids having a height of 1 m, one base of 0.25 m, and a second base of $0.25(-2)^2$, or 1 m. So, the sum of the trapezoid areas is $2(\frac{1}{2})(1)(0.25 + 1)$, or 1.25 m^2. The total area of the 2 trapezoids and 2 △s is $(1.25 + 0.25)$, or 1.5 m^2. **41.** An altitude from B to $\overline{DC}$ forms a 30°-60°-90° △ whose hyp. is 20 in. Thus, the height is 10 in. and the longer leg is $10\sqrt{3}$ in. An altitude from D to $\overline{AB}$ is also 10 in. and it creates an isosc. rt. △ having 10-in. legs. The 2 altitudes create a 10-in. by $(15 - 10)$-in., or 10-in. by 5-in. rectangle. Thus, $AB = 15$ in., $DC = (5 + 10\sqrt{3})$ in., and $h = 10$ in. $A = \frac{1}{2}(10)(15 + 5 + 10\sqrt{3}) = \frac{1}{2}(10)(20 + 10\sqrt{3}) = 100 + 50\sqrt{3}$; about 186.6 in.^{2} **42.** $A = \frac{1}{2}d_1d_2$; $120 = \frac{1}{2}(20)d_2$; $120 = 10d_2$; $d_2 = 12$; the answer is choice A. **43.** $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(6)(10 + 16) = 78$; the answer is choice H. **44.** The formula for the area of a kite and rhombus is identical. Since the numbers for the diagonals are also the same, the areas are the same. The answer is choice C. **45.** The area of a rhombus is greatest if it is a square. Its area can be minute if the rhombus is "stretched" so one diagonal is near 10 and the other is near 0. Since there is no way of determining the area of the rhombus, the answer is choice D. **46.** For Column A: $A = \frac{1}{2}(8)(10) = 40$; for Column B: $A = \frac{1}{2}(9)(9) = 40.5$; since $40 < 40.5$, the answer is choice B.

47. [2] **a.**

b. In the rt. △ shown, the other leg is 6. Since there are 2 ≅ triangles, the longer base is $x + 12$ and the shorter base is x. $A = \frac{1}{2}h(b_1 + b_2)$; $160 = \frac{1}{2}(8)(x + x + 12)$ and $x = 14$. Bases are 14 cm and 26 cm. [1] correct answer without explanation OR correct explanation and calculation error **48.** In an isosc. rt. △, the ratio of the sides is $a : a : a\sqrt{2}$, so $a = 50$. Thus, $A = \frac{1}{2}(50)(50) = 1250$; 1250 in.^{2}. **49.** The diag. of a square divides it into 2 isosc. rt. △s. The sides are in the ratio of $a : a : a\sqrt{2}$. Since $a\sqrt{2} = 10$, $a = 5\sqrt{2}$. So, the length of a side is $5\sqrt{2}$ units. **50.** $A = s^2$, so $s = \sqrt{20}$. The diagonal of a square divides it into 2 isosc. rt. △s, and the hyp. is the length of a leg times $\sqrt{2}$. Thus, the diagonal is $\sqrt{20} \cdot \sqrt{2}$, or $\sqrt{40}$, which is about 6.3. The diagonal is about 6.3 cm long. **51.** The incenter is the point of concurrency of the ∠ bisectors of a △. The ∠ bisectors are always between the sides of the ∠, so the incenter will *always* be inside the △. **52.** The orthocenter is the intersection of the lines containing the altitudes of a △. It is also the center of the circle that is circumscribed about the △. Since the center of the circumscribed circle is sometimes inside, sometimes outside, and sometimes on the △, the orthocenter *sometimes* lies outside the △. **53.** The centroid is the point of concurrency of the medians of the △. It is also the center of mass of the △. The center of mass must always lie inside the △, so the centroid can *never* lie on the △. **54.** The sum of the measures of the exterior ∠s of a polygon is 360, so each ext. ∠ of a regular 9-gon is $\frac{360}{9}$, or 40°. The interior ∠ is its supplement, so its measure is $180 - 40$, or 140°.

7-5 Areas of Regular Polygons pages 380–385

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1. $25\sqrt{3}$ cm^2 **2.** 50 ft^2 **3.** $\frac{100\sqrt{3}}{3}$ m^2 **4.** 24 in. **5.** $16\sqrt{3}$ cm

Check Understanding **1.** For $m\angle 1$: An octagon has

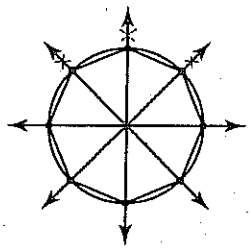
8 sides, so $m\angle 1 = \frac{360}{8}$, or 45°. For $m\angle 2$: The apothem bis. the vertex ∠, so $m\angle 2 = \frac{45}{2}$, or 22.5°. For $m\angle 3$: The acute ◭ of a rt. △ are compl., so $m\angle 3 = 90 - 22.5$, or 67.5°. **2.** $a = 8; p = 5(11.6) = 58; A = \frac{1}{2}ap = \frac{1}{2}(8)(58) = 232$; the area is 232 cm². **3.** The radii divide the hexagon into six equilateral and equiangular ◬. The apothem divides the △ into two 30°-60°-90° ◬. Since each side of the hexagon is 16 ft, the side opp. the 30° ∠ is 8 ft. So, the apothem is $8\sqrt{3}$ ft. The perimeter is 6(16), or 96 ft; $A = \frac{1}{2}ap = \frac{1}{2}(8\sqrt{3})(96) = 384\sqrt{3}$. The area is $384\sqrt{3}$ ft². **4.** Each side of the large △ is 4 times the length of the small △. So, $a = 4(6.35)$, or 35.4, and $p = 4(38.1\sqrt{3})$, or $152.4\sqrt{3}$; $A = \frac{1}{2}ap = \frac{1}{2}(25.4)(152.4\sqrt{3}) \approx 3352$; the area is about 3352 in.².

Exercises **1.** $m\angle 1 = \frac{360}{3} = 120; m\angle 2 = \frac{1}{2}m\angle 1 = \frac{1}{2}(120) = 60; m\angle 3 = 90 - 60 = 30$ **2.** $m\angle 4 = \frac{360}{4} = 90$; $m\angle 5 = \frac{1}{2}m\angle 4 = \frac{1}{2}(90) = 45; m\angle 6 = 90 - 45 = 45$ **3.** $m\angle 7 = \frac{360}{6} = 60; m\angle 8 = \frac{1}{2}m\angle 7 = \frac{1}{2}(60) = 30; m\angle 9 = 90 - 30 = 60$ **4.** $p = 5(35.3) = 176.5; A = \frac{1}{2}ap = \frac{1}{2}(24.3)(176.5) = 2144.475$; the area is 2144.475 cm². **5.** $p = 7(28) = 196; A = \frac{1}{2}ap = \frac{1}{2}(29.1)(196) = 2851.8$; the area is 2851.8 ft². **6.** $p = 8(50) = 400; A = \frac{1}{2}ap = \frac{1}{2}(60.4)(400) = 12{,}080$; the area is 12,080 in.². **7.** $p = 9(20) = 180; A = \frac{1}{2}ap = \frac{1}{2}(27.5)(180) = 2475$; the area is 2475 in.². **8.** $p = 10(12.3) = 123; A = \frac{1}{2}ap = \frac{1}{2}(19)(123) = 1168.5$; the area is 1168.5 m². **9.** $p = 12(14) = 168; A = \frac{1}{2}ap = \frac{1}{2}(26.1)(168) = 2192.4$; the area is 2192.4 cm². **10.** There are 6 ◭ whose vertex is the center of the circle, so each measures $\frac{360}{6}$, or 60°. So, the △ formed by two consecutive radii is an equilateral △. The apothem divides the △ into two 30°-60°-90° ◬. The side opp. the 30° ∠ is 9 ft, so the apothem is $9\sqrt{3}$ ft. The perimeter is 6(18), or 108 ft. Then $A = \frac{1}{2}ap = \frac{1}{2}(9\sqrt{3})(108) \approx 841.8$; the area is about 841.8 ft². **11.** Each ∠ formed by radii at the center of the circle is $\frac{360}{3}$, or 120°. The apothem divides that ∠ into two 60° ◭, forming a 30°-60°-90° △. The side opp. the 60° ∠ is 4 in., so the apothem is $4\sqrt{3}$ in. The perimeter is 3(8), or 24 in. Then $A = \frac{1}{2}ap = \frac{1}{2}(4\sqrt{3})(24) \approx 83.1$; the area is 83.1 in.². **12.** There are 6 ◭ whose vertex is the center of the circle, so each measures $\frac{360}{6}$, or 50°. So, the △ formed by two consecutive radii is an equilateral △. The apothem divides the △ into two 30°-60°-90° ◬. The side opp. the 30° ∠ is 3 m, so the apothem is $3\sqrt{3}$ m. The perimeter is 6(6), or 36 m. Then $A = \frac{1}{2}ap = \frac{1}{2}(3\sqrt{3})(36) \approx 93.5$; the area is about 93.5 m². **13.** There are 4 sides, so 2 consecutive radii form an ∠ measuring $\frac{360}{4}$, or 90°. Each leg measures 6 cm, so the hyp. is $6\sqrt{2}$ cm. Then $A = s^2 = (6\sqrt{2})^2 = 36(2) = 72$; the area is 72 cm². **14.** The apothem bis. an ∠ measuring $\frac{360}{6}$, or 60, forming a 30°-60°-90° △. The side opp. the 60° ∠ is $8\sqrt{3}$, so the shorter leg is 8. Each side of the hexagon is 2(8), or 16, so the perimeter is 6(16), or 96. Then $A = \frac{1}{2}ap = \frac{1}{2}(8\sqrt{3})(96) = 384\sqrt{3}$; the area is $384\sqrt{3}$ in.². **15.** The radius divides the ∠ into two 30° ◭. The apothem forms a 30°-60°-90° △ with the radius, and its hyp. is 20 ft. The apothem is half the hyp., or 10 ft. The longer leg is $10\sqrt{3}$, which is half the side of the △. So, the perimeter of the △ is $3(2)(10\sqrt{3})$, or $60\sqrt{3}$ ft. Then $A = \frac{1}{2}ap = \frac{1}{2}(10)(60\sqrt{3}) = 300\sqrt{3}$; the area is $300\sqrt{3}$ ft². **16.** Two consecutive radii create an ∠ measuring $\frac{360}{6}$, or 60°. So, the △ they form is an equilateral △ that is $6\sqrt{3}$ m on each side. The perimeter is $6(6\sqrt{3})$, or $36\sqrt{3}$ m. The apothem divides the △ into two 30°-60°-90° ◬. The shorter leg is $\frac{6\sqrt{3}}{2}$, or $3\sqrt{3}$ m. The longer leg, which is the apothem, is $\sqrt{3}$ times the shorter leg. The apothem is $(3\sqrt{3})(\sqrt{3}) = 3(3) = 9$. So, $A = \frac{1}{2}ap = \frac{1}{2}(9)(36\sqrt{3}) = 162\sqrt{3}$; the area is $162\sqrt{3}$ m². **17.** Each ∠ of the equilateral △ measures 60°. The radius bis. each ∠ into two 30° ◭. Also, the radius is the hyp. of a 30°-60°-90° △, so it is twice the short leg. Thus, the radius is 2(5), or 10 m. The longer leg is $5\sqrt{3}$, which is half the side of the △. The perimeter of the △ is $3(2)(5\sqrt{3})$, or $30\sqrt{3}$ m. So, $A = \frac{1}{2}ap = \frac{1}{2}(5)(30\sqrt{3}) = 75\sqrt{3}$; the area is $75\sqrt{3}$ m². **18.** The radius is 4 in. and is also $\frac{2}{3}$ the height of the △, so the height is $\frac{3}{2}(4)$, or 6 in. So, the apothem is 6 − 4, or 2 in. Half the side of the △ is $2\sqrt{3}$, so the perimeter is $3(2)(2\sqrt{3}) = 12\sqrt{3}$. Then $A = \frac{1}{2}ap = \frac{1}{2}(2)(12\sqrt{3}) = 12\sqrt{3}$; the area is $12\sqrt{3}$ in.

19a. $\frac{360}{5} = 72$ **19b.** $90 - \frac{72}{2} = 90 - 36 = 54$ **20a.** $\frac{360}{8} = 45$ **20b.** $90 - \frac{45}{2} = 90 - 22.5 = 67.5$ **21a.** $\frac{360}{9} = 40$ **21b.** $90 - \frac{40}{2} = 90 - 20 = 70$ **22a.** $\frac{360}{12} = 30$ **22b.** $90 - \frac{30}{2} = 90 - 15 = 75$ **23.** $p = 8(8) = 64$; $A = \frac{1}{2}ap = \frac{1}{2}(9.7)(64) = 310.4$ ft² **24a.** A regular 3-sided polygon is an equilateral △. So, if the height is $3x$, then the radius is $2x$ and the apothem is x. Each side of the △ is then $2x\sqrt{3}$, so the perimeter is $3(2x\sqrt{3})$, or $6x\sqrt{3}$. Thus, $36 = \frac{1}{2}(x)(6x\sqrt{3})$; $36 = 3x^2\sqrt{3}$; $x^2 = \frac{36}{3\sqrt{3}} = 4\sqrt{3}$, so $x = \sqrt{4\sqrt{3}}$, or $2\sqrt[4]{3}$ and $2x\sqrt{3} = 2(2\sqrt[4]{3})\sqrt{3} \approx 9.1$. One side measures about 9.1 in. **24b.** A regular quadrilateral is a square, so $36 = s^2$, and $s = 6$. One side measures 6 in. **24c.** The radii of a regular hexagon divide it into 6 equilateral ◬. If a radius measures $2x$, then each side measures $2x$ and the apothem measures $x\sqrt{3}$. The perimeter measures $6(2x)$, or $12x$. Thus, $36 = \frac{1}{2}(x\sqrt{3})(12x)$; $36 = 6x^2\sqrt{3}$; $x^2 = 2\sqrt{3}$, so $x = \sqrt{2\sqrt{3}}$. Each side measures $2x = 2\sqrt{2\sqrt{3}} \approx 3.7$. One side measures 3.7 in. **24d.** Answers may vary. Sample: About 4 in.; the length of a side of a pentagon should be between 3.7 in. and 6 in. **25.** $m\angle 1 = \frac{360}{10} = 36$; $m\angle 2 = \frac{1}{2}m\angle 1 = 18; m\angle 3 = 90 - m\angle 2 = 90 - 18 = 72$ **26.** In a rt. △, the hyp. is the longest side. The apothem is one leg of a rt. △ and the radius is the hypotenuse. **27.** The altitude bis. the base of the △, creating two

6.5-cm segments. It also creates two 30°-60°-90° △, so the height, which is the longer leg, is $6.5\sqrt{3}$ cm. Thus, $A = \frac{1}{2}bh = \frac{1}{2}(13)(6.5\sqrt{3}) \approx 73$. The area is about 73 cm^2. **28.** If the radius is 10, then the apothem is 5, and the perimeter is $3(2)(5\sqrt{3})$, or $30\sqrt{3}$. Thus, $A = \frac{1}{2}(5)(30\sqrt{3}) \approx 130$. The area is about 130 in.^{2}. **29.** If the radius is 4.6, then the apothem is 2.3, and the perimeter is $3(2)(2.3\sqrt{3})$, or $13.8\sqrt{3}$. Thus, $A = \frac{1}{2}(2.3)(13.8\sqrt{3}) \approx 27$. The area is about 27 m^2. **30.** If the radius is 8.9, then the apothem is 4.45, and the perimeter is $3(2)(4.45\sqrt{3})$, or $26.7\sqrt{3}$. So, $A = \frac{1}{2}(4.45)(26.7\sqrt{3}) \approx 103$. The area is about 103 ft^2. **31.** If the radius is 13, then the apothem is 6.5, and the perimeter is $3(2)(6.5\sqrt{3})$, or $39\sqrt{3}$. So, $A = \frac{1}{2}(6.5)(39\sqrt{3}) \approx 220$. The area is about 220 cm^2.

32a–c. regular octagon



32d. The measure of the ∡ created by the radii at the center of the circle should be $\frac{360}{6}$, or 60°. Construct an equilateral △ whose sides are each the length of a radius. **33.** If the perimeter is 120 m, then each side is $\frac{120}{6}$, or 20 m. The radii divide the hexagon into 6 equilateral △. The apothem is the height of a △. Since the short leg is $\frac{20}{2}$, or 10 m, the apothem is $10\sqrt{3}$ m. So, $A = \frac{1}{2}(10\sqrt{3})(120) = 600\sqrt{3}$. The area is $600\sqrt{3}$ m^2. **34.** Check students' work. **35.** The regular quad. is a square whose diameter is 2(8), or 16 cm. So, each leg is a side that measures $\frac{16}{\sqrt{2}}$, or $8\sqrt{2}$ cm. Hence, $A = s^2 = (8\sqrt{2})^2 = 64(2) = 128$. The area is 128 cm^2. **36.** The radii divide the hexagon into 6 equilateral △. The apothem is the height of a △. Since the shorter leg is $\frac{4}{2}$, or 2 cm, the apothem is $2\sqrt{3}$. The perimeter of the hexagon is 6(4), or 24 cm. Hence, $A = \frac{1}{2}(2\sqrt{3})(24) = 24\sqrt{3}$. The area is $24\sqrt{3} \approx 41.6$ cm^2. **37.** The apothem and radius create a 30°-60°-90° △. The apothem is $10\sqrt{3}$, half a side of the △ is $(10\sqrt{3})\sqrt{3}$, or 30, so the perimeter is 3(2)(30), or 180 m. Thus, $A = \frac{1}{2}(10\sqrt{3})(180) = 900\sqrt{3}$. The area is $900\sqrt{3} \approx 1558.8$ m^2. **38.** The polygon is a square whose diameter is twice the radius, or $10\sqrt{2}$ ft. Each side of the square is a leg of an isosc. rt. △, so they each measure $\frac{10\sqrt{2}}{\sqrt{2}}$, or 10 ft. Then $A = s^2 = 10^2 = 100$. The area is 100 ft^2. **39.** The altitude bis. the base of the △ into two 4-in. segments. The altitude also bis. the 60° vertex ∠, creating a 30°-60°-90° △. Since the short leg is 4 in., the longer leg is $4\sqrt{3}$ in. The longer leg is also the △ height. So, $A = \frac{1}{2}bh = \frac{1}{2}(8)(4\sqrt{3}) = 16\sqrt{3}$. The area is $16\sqrt{3} \approx 27.7$ in.^{2}. **40.** The radii of a hexagon divide it into 6 equilateral △. Each side of the △ is $3\sqrt{3}$ m long, so each side of the hexagon is also $3\sqrt{3}$ m long. The perimeter is $6(3\sqrt{3})$, or $18\sqrt{3}$ m. The apothem creates a 30°-60°-90° △ with the radius, so the short leg is $\frac{3\sqrt{3}}{2}$ and the apothem is $\frac{3\sqrt{3}\sqrt{3}}{2}$, or 4.5 m. Hence $A = \frac{1}{2}(4.5)(18\sqrt{3}) = \frac{81\sqrt{3}}{2}$. The area is $\frac{81\sqrt{3}}{2} \approx 70.1$ m^2.

41a. $b = s; h = \frac{\sqrt{3}}{2}s; A = \frac{1}{2}bh = \frac{1}{2}(s)(\frac{\sqrt{3}}{2}s) = \frac{1}{4}s^2\sqrt{3}$

41b. apothem $= \frac{s}{2} \div \sqrt{3} = \frac{s}{2\sqrt{3}} = \frac{s\sqrt{3}}{2\sqrt{3}\sqrt{3}} = \frac{s\sqrt{3}}{6}; p = 3s;$ $A = \frac{1}{2}(\frac{s\sqrt{3}}{6})(3s) = \frac{1}{4}s^2\sqrt{3}$ **42.** The apothem is ⊥ to a side of the pentagon. The right △ are formed with the radii of the pentagon. So, the △ are ≅ by HL. Therefore the ∡ from the center are ≅ by CPCTC, and the apothem bisects the vertex ∠. **43.** For regular n-gon $ABCDE\ldots$, let P be the intersection of the bisectors of $\angle ABC$ and $\angle BCD$. Show that $\overrightarrow{DP}$ is the bisector of $\angle CDE$. Proof: By the Angle Add. Post., $m\angle BCP + m\angle DCP = m\angle BCD$. Also, $m\angle BCP = m\angle DCP$ by def. of bis. Then $2m\angle BCP = m\angle BCD$. Since all ∡ of a regular polygon are =, $m\angle CDE = 2m\angle BCP$. Also, $m\angle CDE = m\angle CDP + m\angle EDP$ by the Angle Add. Post. Then (*) $2m\angle BCP = m\angle CDP + m\angle EDP$ by substitution. To continue with the (*) equation, prove $m\angle BCP = m\angle CDP$, using ≅ △. Since all ∡ of a regular polygon are ≅, and since $\overline{BP}$ and $\overline{CP}$ are ∠ bisectors, the bisectors create 4 ≅ ∡ by division, so $\angle ABP \cong \angle PBC \cong \angle BCP \cong \angle DCP$. Since all sides of a regular polygon are ≅, $\overline{BC} \cong \overline{CD}$, so $\triangle BPC \cong \triangle CPD$ by ASA. By CPCTC, $\angle BCP \cong \angle CDP$, so $m\angle BCP = m\angle CDP$. Substitute $m\angle CDP$ for $m\angle BCP$ in (*) equation to get $2m\angle CDP = m\angle CDP + m\angle EDP$. Then $m\angle CDP \cong m\angle EDP$ by subtraction. Thus, $\overrightarrow{DP}$ bisects $\angle CDE$, so P is the point of concurrency. **44a.** Since $OV_1 = 4$, then $OV_2 = 4$. So, $4 = \sqrt{x^2 + y^2}; 4 = \sqrt{x^2 + x^2}; 4 = \sqrt{2x^2}; 4 = x\sqrt{2};$ $x = \frac{4}{\sqrt{2}} \approx 2.8$; since $x = y$, the coordinates are (2.8, 2.8). **44b.** $b = OV_1 = 4$ and $h \approx 2.8$, so $A = \frac{1}{2}bh \approx \frac{1}{2}(4)(2.8) = 5.6$. The area is 5.6 units^2. **44c.** There are 8 △, so the total area is about 8(5.6), or about 45 units^2. **45a.** Use the formula for the area of the square to find each side of the △: $10 = s^2$, so $s = \sqrt{10}$. Half the base of the △ is $\frac{\sqrt{10}}{2}$, so the height is $\frac{\sqrt{10}}{2} \cdot \sqrt{3}$, or $\frac{\sqrt{30}}{2}$; $A = \frac{1}{2}bh = \frac{1}{2}(\sqrt{10})(\frac{\sqrt{30}}{2}) = \frac{1}{4}\sqrt{300} = \frac{10\sqrt{3}}{4} = \frac{5\sqrt{3}}{2}$; the area is $\frac{5\sqrt{3}}{2}$ cm^2. **45b.** $\frac{\frac{5\sqrt{3}}{2}}{10} = \frac{\frac{5\sqrt{3}}{2}}{\frac{10}{1}} = \frac{\frac{5\sqrt{3}}{2} \cdot 2}{\frac{10}{1} \cdot 2} = \frac{5\sqrt{3}}{20} = \frac{\sqrt{3}}{4}$

46. $A = \frac{1}{2}(25.1)(182) = 2284.1$; the answer is choice B. **47.** $1235.2 = \frac{1}{2}(19.3)p; 2470.4 = 19.3p; p = 128$; the answer is choice F. **48.** The radii divide the hexagon into 6 ≅ △. The radius forms a rt. △ with the apothem and is the hyp., so the short leg is 5 ÷ 2, or 2.5 ft and the longer leg is $2.5\sqrt{3}$. So, the apothem is $2.5\sqrt{3}$ and the perimeter is 6(5), or 30. The area is $\frac{1}{2}(2.5\sqrt{3})(30) \approx 65$. The answer is choice B. **49** [2] **a.** Divide the decagon into 10 ≅ △. Consider one △ with hyp. of 35.6 and leg 11. The apothem can be found using the Pyth. Thm., so $(35.6)^2 - 11^2 = 1146.36$ and leg ≈ 33.9; 33.9 in. **b.** $A = \frac{1}{2}ap \approx \frac{1}{2}(33.9)(220) = 3729$; the area is about 3729 in.^{2}. **50** [4] **a.** Area of $\triangle BCG = \frac{1}{2}bh = \frac{1}{2}(8\sqrt{3})(12) = 48\sqrt{3}$. **b.** Area of $ABCDEF = 6 \cdot 48\sqrt{3} = 288\sqrt{3}$. **c.** Find the

area of one △ and mult. it by 6, or use the formula for the area of a reg. polygon. [3] appropriate methods, but with one computational error [2] incorrect formulas OR no explanation [1] incorrect calculations, correct explanation **51.** $A = \frac{1}{2}d_1d_2 = \frac{1}{2}(8)(11.5) = 46$; the area is 46 m^2. **52.** $A = \frac{1}{2}d_1d_2$; $150 = \frac{1}{2}(10)d_2$; $d_2 = 30$; the length is 30 in. **53.** $A = \frac{1}{2}h(b_1 + b_2)$; $42 = \frac{1}{2}(7)(4 + b_2)$; $12 = 4 + b_2$; $b_2 = 8$; the base is 8 m. **54.** Look for △s that have the given segments as sides: $\triangle DAB \cong \triangle CBA$. **55.** Look for △s that have the given segments as sides: $\triangle ACG \cong \triangle BDF$. **56.** Look for △s that have the given ∠s as parts: $\triangle DFA \cong \triangle CGB$. **57a.** $r = \frac{d}{2} = \frac{3}{2} = 1.5$; $A = \pi r^2 \approx (3.14)(1.5)^2 = (3.14)(2.25) \approx 7.1$; the area is about 7.1 mi^2. **57b.** $200 \approx (3.14)r^2$; $r^2 \approx 63.694$; $r \approx 8$; the radius is about 8 mi.

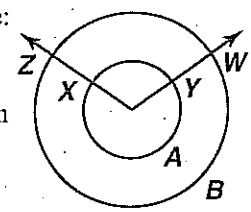
7-6 Circles and Arcs pages 386–393

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM*.
1. 14 cm **2.** 3.2 m **3.** 5 ft **4.** 2.5 in. **5.** 32 **6.** 137 **7.** 180 **8.** 76

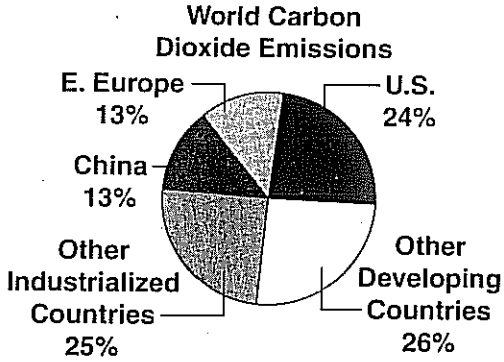
Check Understanding **1a.** number of hours spent doing an activity **1b.** Each section represents the average of the 3600 participants' answers. **2.** Answers may vary. Samples: $\widehat{CEA}, \widehat{DAE}, \widehat{ACD}, \widehat{EDC}$ **3.** $m\angle COD = m\widehat{CD} = 58$; $m\widehat{CDA} = m\angle COA = 180$; $m\widehat{AD} = 360 - (m\angle AOC + m\widehat{CD}) = 360 - (180 + 58) = 360 - 238 = 122$; $m\widehat{BAD} = 360 - (32 + 58) = 360 - 90 = 270$ **4.** $C = \pi d = \pi(22) \approx 69$; 100 ft = 1200 in.; $1200 \div 69 \approx 17$; the wheel makes a little more than 17 full revolutions. **5.** The measure of a semicircle is 180, so $\frac{180}{360} \cdot 2\pi r = \frac{1}{2}(2\pi)(1.3) = 1.3\pi$; the length is 1.3π m.

Exercises **1.** 7% of 360 = 0.07(360) ≈ 25 **2.** 8% of 360 = 0.08(360) ≈ 29 **3.** 9% of 360 = 0.09(360) ≈ 32 **4.** 6% of 360 = 0.06(360) ≈ 22 **5.** 7% of 360 = 0.07(360) ≈ 25 **6.** 16% of 360 = 0.16(360) ≈ 58 **7.** 9% of 360 = 0.09(360) ≈ 32 **8.** 38% of 360 = 0.38(360) ≈ 137 **9.** Any arc smaller than a semicircle is a minor arc. Answers may vary. Samples: $\widehat{BF}, \widehat{DF}, \widehat{ED}$ **10.** Any arc greater than a semicircle is a major arc. Answers may vary. Samples: $\widehat{CFD}, \widehat{DBF}, \widehat{FEB}$ **11.** A semicircle is a half circle. Look for arcs whose endpts. are the endpts. of a diameter. Answers may vary. Samples: $\widehat{CBF}, \widehat{CEF}, \widehat{FCE}, \widehat{BFE}$ **12.** Adjacent arcs do not overlap and share the same endpt. Answers may vary. Sample: $\widehat{FE}$ and $\widehat{ED}$ **13.** An acute central ∠ is an ∠ measuring between 0 and 90 and whose vertex is the center of a circle. Answers may vary. Sample: $\angle FOE$ **14.** Any pair of vertical ∠s is ≅. Look for intersecting diameters. Answers may vary. Samples: $\angle BOF$ and $\angle EOC$; $\angle FOE$ and $\angle BOC$ **15.** The measure of an arc is = to the measure of its central ∠. Since $m\angle TPC = 128$, then $m\widehat{TC} = 128$. **16.** Since $m\angle TPD = 180$, then $m\widehat{TBD} = 180$. **17.** $m\widehat{BTC} = m\widehat{BT} + m\widehat{TC} = m\angle BPT + m\angle TPC = 90 + 128 = 218$ **18.** $m\widehat{TCB} = 360 - m\widehat{TB} = 360 - m\angle TPB = 360 - 90 = 270$ **19.** $m\widehat{CD} = 360 - m\widehat{CBD} = 360 - (m\widehat{CT} + m\widehat{TB} + m\widehat{BD}) = 360 - (m\angle CPT + m\angle TPB + m\angle BPD) = 360 - (128 + 90 + 90) = 360 - 308 = 52$ **20.** $m\widehat{CBD} = m\widehat{CT} + m\widehat{TB} + m\widehat{BD} = m\angle CPT + m\angle TPB + m\angle BPD = 128 + 90 + 90 = 308$ **21.** $m\widehat{TCD} = m\angle TPD = 180$ **22.** $m\widehat{DB} = m\angle DPB = 90$ **23.** $m\widehat{TDC} = 360 - m\widehat{TC} = 360 - m\angle TPC = 360 - 128 = 232$ **24.** $m\widehat{TB} = m\angle TPB = 90$ **25.** $m\widehat{BC} = 360 - m\widehat{BTC} = 360 - (m\widehat{BT} + m\widehat{TC}) = 360 - (m\angle BPT + m\angle TPC) = 360 - (90 + 128) = 360 - 218 = 142$ **26.** $m\widehat{BCD} = 360 - m\widehat{BD} = 360 - m\angle BPD = 360 - 90 = 270$ **27.** $C = \pi d = 20\pi$; the circumference is 20π cm. **28.** $C = 2\pi r = 2\pi(3) = 6\pi$; the circumference is 6π ft. **29.** $C = 2\pi r = 2\pi(4.2) = 8.4\pi$; the circumference is 8.4π m. **30.** $C = \pi d = 14\pi$; the circumference is 14π in. **31.** $C = 2\pi r = 2\pi\left(\frac{1}{2}\right) = \pi$; the circumference is π m. **32.** $C = 2\pi r = 2\pi(29) = 58\pi$; the circumference is 58π cm. **33.** The distance each bicycle travels is the circumference of its wheel. For the 26-in. wheel, $C = \pi d = 26\pi$. For the 18-in. wheel, $C = 18\pi$. The difference is $26\pi - 18\pi$, which is 8π, or about 25 in. **34.** $\frac{45}{360} \cdot 2\pi 14 = \frac{7\pi}{2}$; the arc length is $\frac{7\pi}{2}$ cm. **35.** The arc measure is 180 − 60, or 120; $\frac{120}{260} \cdot \pi(24) = 8\pi$; the arc length is 8π ft. **36.** The arc measure is 360 − 90, or 270; $\frac{270}{360} \cdot 2\pi(18) = 27\pi$; the arc length is 27π m. **37.** The arc measure is 360 − 30, or 330; $\frac{330}{360} \cdot \pi(36) = 33\pi$; the arc length is 35π in. **38.** The arc measure is 180; $\frac{180}{360} \cdot \pi(23) = \frac{23\pi}{2}$; the arc length is $\frac{23\pi}{2}$ m. **39.** Since vert. ∠s are ≅, the measure of the central ∠ to the arc is 25, so the arc measure is 25; $\frac{25}{360} \cdot 2\pi(9) = \frac{5\pi}{4}$; the arc length is $\frac{5\pi}{4}$ m.

40. Drawings may vary. Sample: The measures are =, but $m\widehat{ZW} > m\widehat{XY}$ because the radius of circle B is greater than the radius of circle A. So, the lengths are not ≅.



41.



Find the measure of the central ∡ by multiplying 360 by the percent for each category. **42.** $m\angle EOF = m\angle JOH = m\widehat{JH} = 70$ **43.** $m\widehat{EJH} = m\angle EOH = 180$ **44.** $m\widehat{FH} = 180 - m\widehat{JH} = 180 - 70 = 110$ **45.** Since $2m\angle FOG + m\angle JOH = 180$, then $m\angle FOG = \frac{1}{2}(180 - m\angle JOH) = \frac{1}{2}(180 - m\widehat{JH}) = \frac{1}{2}(180 - 70) = 55$. **46.** From Exercise 45, $m\angle FOG = 55$, so $m\angle GOH = 55$; $m\widehat{JEG} = 360 - m\widehat{GOJ} = 360 - (m\widehat{GH} + m\widehat{HJ}) = 360 - (m\angle GOH + m\widehat{HJ}) = 360 - (55 + 70) = 360 - 125 = 235$ **47.** $m\widehat{HFJ} = 360 - m\widehat{JH} = 360 - 70 = 290$ **48.** Check students' work. **49a.** A full circle rotation of the minute hand takes 60 minutes, so 1 minute is $\frac{1}{60}$ of 360, or 6 degrees. **49b.** From Exercise 49a, 1 minute is 6 degrees, so 5 min. is 5(6), or 30 degrees. **49c.** The minute hand moves $\frac{1}{3}$ circle in 20 minutes, so $\frac{1}{3}$ or 360 is 120 degrees. **50a.** The numbers on a clock represent an hour and they are located at 5-min. intervals. From Exercise 49b, they are 30°. So, the hour hand moves 30° per hour and $\frac{1}{60}$ of 30 per minute; $\frac{30}{60} = \frac{1}{2}$; the hour hand moves $\frac{1}{2}°$ per min. **50b.** From Exercise 50a, the hour hand moves $\frac{1}{2}°$ per minute, so in 5 min. it moves $5\left(\frac{1}{2}\right)$, or 2.5°. **50c.** From Exercise 50a, the hour hand moves $\frac{1}{2}°$ per min., so in 20 min. it moves $20\left(\frac{1}{2}\right)$, or 10°. **51.** The minute hand makes $\frac{2640}{360}$, or $7\frac{1}{3}$ rotations between 12:00 (0°) and 7:20, so at 7:20 the minute hand is $\frac{1}{3}$ of 360, or 120° clockwise from vertical. From Exercise 50a, the hour hand moves 30° per hour, so in $7\frac{1}{3}$, or $\frac{22}{3}$ h, it moves $30\left(\frac{22}{3}\right)$, or 220° from vertical. The ∠ formed by the two hands is the difference: 220 − 120 = 100; the ∠ formed measures 100. **52.** $c + (4c - 10) = 180$; $5c - 10 = 180$; $5c = 190$; $c = 38$ **53.** $(x + 40) + (2x + 60) + (3x + 20) = 360$; $6x + 120 = 360$; $x + 20 = 60$; $x = 40$ **54.** The central ∠ formed by East St. and Maple St. is 40°. The arc measure is the same as the sum of the related ∠ measures, so it is 90 + 90 + 40, or 220°. **55.** $C = \pi d$; $100\pi = \pi d$; $d = 100$; the diameter is 100 in. **56.** $C = 2\pi r$; $100\pi = 2\pi r$; $r = 50$; the radius is 50 in. **57.** $\frac{120}{360} \cdot 100\pi = \frac{100\pi}{3}$; the arc length is $\frac{100\pi}{3}$ in. **58.** Let a represent the radius of circle A and let b represent the radius of circle B. The length of a 60° arc in circle A is $\frac{60}{360} \cdot 2\pi a$, or $\frac{\pi a}{3}$, and the length of a 45° arc in circle B is $\frac{45}{360} \cdot 2\pi b$, or $\frac{\pi b}{4}$. Then, $\frac{\pi a}{3} = \frac{\pi b}{4}$, so $4a = 3b$. So, $b = \frac{4a}{3}$. The ratio of $a : b$ is $a : \frac{4a}{3} = 1 : \frac{4}{3} = 3 : 4$. **59.** The shorter arc length is $\frac{180}{360} \cdot 20\pi$, or 10π ft. The longer arc is $\frac{180}{360} \cdot (20 + 2 \cdot 3)\pi$, or 13π ft. There are eleven 3-ft. segments for a total of 33 ft. The total amount of wrought iron is $10\pi + 13\pi + 33$, or about 105 ft. **60a.** There are $\frac{360}{7.2}$, or 50, 7.2° central ∡ in a full circle. So, the circumference is 50(500), or 25,000 mi. **60b.** $\frac{24{,}902}{25{,}000} \approx 0.99608 \approx 1$, so the estimate seems quite accurate. **61.** The midpt. of the diameter is the center of the circle, so its coordinates are $\left(\frac{1+4}{2}, \frac{3+7}{2}\right)$, or (2.5, 5). **62.** The length of the diameter is $\sqrt{(4-1)^2 + (7-3)^2} = \sqrt{9 + 16} = 5$; $C = \pi d = 5\pi$; the circumference is 5π units. **63.** The measure of the arc is 180 + 45, or 225, so the length is $\frac{225}{360} \cdot 2\pi(4.1)$, or 5.125π; 5.125π ft. **64.** The arc measure is 180 − 50 = 130, so its length is $\frac{130}{360} \cdot \pi(7.2)$, or 2.6π; 2.6π in. **65.** The arc measure is 180 − 90, or 90, so the arc length is $\frac{90}{360} \cdot 2\pi(6)$, or 3π; 3π m. **66.** $2\pi(10) - 2\pi(3) = 14\pi \approx 44$; the point 10 cm from the center travels about 44.0 cm farther.

67. Outside; a point on the outside travels farther in the same amount of time, so it goes faster. **68.** $\frac{120}{360} \cdot \pi d = 6\pi$; $\frac{\pi d}{3} = 6\pi$; $\pi d = 18\pi$; $d = 18$; the diameter is 18 cm.

69. The diameter is $\sqrt{(3-3)^2 + (7-(-1))^2} = \sqrt{0 + (8)^2} = 8$; $\frac{180}{360} \cdot 8\pi \approx 12.6$; the arc length is about 12.6 units. **70a.** Arcs sharing the same central ∠ have the same measure. Answers may vary. Sample: $\widehat{BD}$ and $\widehat{FE}$. **70b.** Since $m\widehat{FE} = 70$, then $m\angle FOE = 70$. So, $m\angle AOB = 180 - 70 = 110$. By the Angle Add. Post., $2x + 110 = 180$, so $2x = 70$, and $x = 35$.

71. Assumptions may vary. Sample: Assume that the inside quad. contains $\frac{1}{4}$ of each circle and that the circles are ≅. Then the perimeter of the shaded figure is the circumference of 1 circle. The outer square is 4 in., which is 2 diameters long. Thus, one diameter measures 2 in., so $C = \pi(2) = 2\pi$. The perimeter of the shaded figure is 2π in. **72.** The outside perimeter consists of 2 semicircles having a diameter of (40 + 2(10)) yd and 2 segments 100 yd long. The 2 semicircles combined make 1 whole circle, so its circumference is πd, or 60π yd. The total perimeter is $60\pi + 2(100)$, or about 388.5 yd.

73. $\frac{60}{360} \cdot 2\pi(12) = 4\pi$, so the answer is choice B.

74. $\frac{240}{360} \cdot 2\pi x = 16\pi$; $\frac{4\pi}{3}x = 16\pi$; $x = 16\pi\left(\frac{3}{4\pi}\right) = 12$; the answer is choice G. **75.** [2] Each side of the path consists of a quarter circumference of a 4-m-radius circle and a quarter circumference of a 6-m-radius circle. The length of both sides is $2\left(\frac{1}{4} \cdot 2\pi(6) + \frac{1}{4} \cdot 2\pi(4)\right) = \frac{1}{2}(2\pi \cdot 6) + \frac{1}{2}(2\pi \cdot 4) = 6\pi + 4\pi = 10\pi \approx 31.4$; she needs about 31.4 m. **76.** Since the polygon is a 12-gon, $m\angle 1 = \frac{360}{12} = 30$. Then $m\angle 2 = \frac{1}{2}m\angle 1 = \frac{1}{2}(30) = 15$. Since ∠3 and ∠2 are complements, $m\angle 3 = 90 - m\angle 2 = 90 - 15 = 75$. Since ∠4 is an exterior ∠, its measure is $\frac{360}{12}$, or 30. **77.** The apothem bis. a side. Use the Pyth. Thm.: $5^2 + a^2 = 19.3^2$; $25 + a^2 = 372.49$; $a^2 = 347.49$; $a \approx 18.6$. The apothem is about 18.6 mm. **78.** The perimeter is 12(10), or 120 mm. From Exercise 77, the apothem is about 18.6 mm. $A = \frac{1}{2}ap \approx \frac{1}{2}(18.6)(120) = 1116$. The area is about 1116 mm^2. **79.** No; the figure could be an

isosc. trapezoid. To ensure that it's a ▱, both pairs of opp. sides must be ≅, or both pairs of opp. sides must be ∥, or at least one pair of opp. sides must be both ∥ and ≅. **80.** Yes; by Thm. 6-5, if the diags. bis. each other, then the quad. is a ▱. **81.** Yes; by Thm. 6-6, if one pair of opp. sides is both ≅ and ∥, then the quad. is a ▱. **82.** By Thm. 3-1, two nonvertical parallel lines *always* have the same slope. **83.** By Thm. 3-4, two perpendicular lines *never* have slopes that are reciprocals.

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1. Since the end result has meters in the numerator, the answer is choice B. **2.** Since the end result has feet in the numerator, the answer is choice A. **3.** Since the end result has yards in the numerator, the answer is choice B. **4.** Since the end result has inches in the numerator, the answer is choice A. **5.** Since the end result has kilometers in the numerator, the answer is choice B.

6. $4\text{m} \cdot \frac{100\text{ cm}}{\text{m}} = 400\text{ cm}$; the answer is 400.

7. $360\text{ in.} \cdot \frac{1\text{ yd}}{36\text{ in.}} = 10\text{ yd}$; the answer is 10.

8. $9\text{ mm} \cdot \frac{1\text{ cm}}{10\text{ mm}} = 0.9\text{ cm}$; the answer is 0.9.

9. $17\text{ yd} \cdot \frac{3\text{ ft}}{\text{yd}} = 51\text{ ft}$; the answer is 51.

10. $2.5\text{ ft} \cdot \frac{12\text{ in.}}{\text{ft}} = 30\text{ in.}$; the answer is 30.

11. $35\text{m} \cdot \frac{1\text{ km}}{1000\text{ m}} = 0.035\text{ km}$; the answer is 0.035.

12. $2\text{ yd} \cdot \frac{36\text{ in.}}{\text{yd}} = 72\text{ in.}$; the answer is 72.

13. $23\text{ cm} \cdot \frac{10\text{ mm}}{\text{cm}} = 230\text{ mm}$; the answer is 230.

14. $2\text{ ft} \cdot \text{ft} \cdot \frac{12\text{ in.}}{\text{ft}} \cdot \frac{12\text{ in.}}{\text{ft}} = 288\text{ in.}^2$; the answer is 288.

15. $500\text{ mm} \cdot \text{mm} \cdot \frac{1\text{ cm}}{10\text{ mm}} \cdot \frac{1\text{ cm}}{10\text{ mm}} = 5\text{ cm}^2$; the answer is 5.

16. $840\text{ in.} \cdot \text{in.} \cdot \frac{1\text{ ft}}{12\text{ in.}} \cdot \frac{1\text{ ft}}{12\text{ in.}} = 5\frac{5}{6}\text{ ft}^2$; the answer is $5\frac{5}{6}$.

17. $3\text{ km} \cdot \text{km} \cdot \frac{100{,}000\text{ cm}}{\text{km}} \cdot \frac{100{,}000\text{ cm}}{\text{km}} = 30{,}000{,}000{,}000\text{ cm}^2$; the answer is 30,000,000,000.

18. $7\text{ m} \cdot \text{m} \cdot \frac{1\text{ km}}{1000\text{ m}} \cdot \frac{1\text{ km}}{1000\text{ m}} = 0.000007\text{ km}^2$; the answer is 0.000007.

19. $360\text{ in.} \cdot \text{in.} \cdot \frac{1\text{ yd}}{36\text{ in.}} \cdot \frac{1\text{ yd}}{36\text{ in.}} = \frac{5}{18}\text{ yd}^2$; the answer is $\frac{5}{18}$.

20. $900\text{ cm} \cdot \text{cm} \cdot \text{cm} \cdot \frac{1\text{ m}}{100\text{ cm}} \cdot \frac{1\text{ m}}{100\text{ cm}} \cdot \frac{1\text{ m}}{100\text{ cm}} = 0.0009\text{ m}^3$; the answer is 0.0009.

21. $4\text{ yd} \cdot \text{yd} \cdot \text{yd} \cdot \frac{3\text{ ft}}{\text{yd}} \cdot \frac{3\text{ ft}}{\text{yd}} \cdot \frac{3\text{ ft}}{\text{yd}} = 36\text{ ft}^3$; the answer is 108.

22. $75\text{ m} \cdot \text{m} \cdot \frac{1\text{ km}}{1000\text{ m}} \cdot \frac{1\text{ km}}{1000\text{ m}} = 0.000075\text{ km}^2$; the answer is 0.000075.

23. $250\text{ in.} \cdot \text{in.} \cdot \frac{1\text{ yd}}{36\text{ in.}} \cdot \frac{1\text{ yd}}{36\text{ in.}} = \frac{125}{648}\text{ yd}^2$; the answer is $\frac{125}{648}$.

24. $3\text{ km} \cdot \text{km} \cdot \text{km} \cdot \frac{1000\text{ m}}{\text{km}} \cdot \frac{1000\text{ m}}{\text{km}} \cdot \frac{1000\text{ m}}{\text{km}} = 3{,}000{,}000{,}000\text{ m}^3$; the answer is 3,000,000,000.

25. $2\text{ km} \cdot \text{km} \cdot \frac{1{,}000{,}000\text{ mm}}{\text{km}} \cdot \frac{1{,}000{,}000\text{ mm}}{\text{km}} = 2{,}000{,}000{,}000{,}000\text{ mm}^2$; the answer is 2,000,000,000,000.

26. $60\text{ in.} \cdot \text{in.} \cdot \frac{1\text{ yd}}{36\text{ in.}} \cdot \frac{1\text{ yd}}{36\text{ in.}} = \frac{5}{108}\text{ yd}^2$; the answer is $\frac{5}{108}$.

7-7 Areas of Circles and Sectors *pages 395–400*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1. 4.5 cm **2.** 16 ft **3.** 12π in. **4.** 6π m

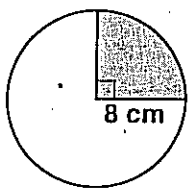
Investigation **1.** They are equal. **2.** There are 2 sides of the figure of length b, so $b \approx \frac{1}{2}C$. **3.** Since $C = 2\pi r$, and since $b \approx \frac{1}{2}C$, then $b \approx \frac{1}{2}(2\pi r) = \pi r$. **4.** $A = bh \approx (\pi r)(r) = \pi r^2$

Check Understanding **1.** The radius of the 14-in. pizza is 7 in., and the radius of the 12-in. pizza is 6 in.; $A_{14\text{-in.}} - A_{12\text{-in.}} = \pi(7)^2 - \pi(6)^2 = 49\pi - 36\pi = 13\pi \approx 41$; the difference is about 41 in.^{2} **2.** $\frac{208}{360} \cdot \pi(10)^2 = \frac{520\pi}{9} \approx 181.5\text{ cm}^2$ **3.** For the area of △: $h = 6\sqrt{3}$ and $b = 12$, so $A = \frac{1}{2}(12)(6\sqrt{3}) = 36\sqrt{3}$; for the area of the segment: $\frac{60}{360} \cdot \pi(12)^2 - 36\sqrt{3} = 24\pi - 36\sqrt{3} \approx 13.0$; the area of the segment is about 13.0 cm^2.

Exercises **1.** $r = \frac{d}{2} = \frac{6}{2} = 3$; $A = \pi(3)^2 = 9\pi$; the area is $9\pi\text{ m}^2$. **2.** The radius is 5.5 cm, so $A = \pi(5.5)^2 = 30.25\pi$. The area is $30.25\pi\text{ cm}^2$. **3.** The radius is 0.85 ft, so $A = \pi(0.85)^2 = 0.7225\pi$. The area is $0.7225\pi\text{ ft}^2$. **4.** The radius is $\frac{1}{3}$ in., so $A = \pi\left(\frac{1}{3}\right)^2 = \frac{\pi}{9}$. The area is $\frac{\pi}{9}$ in.^{2}. **5.** Subtract the areas of the two circles. $A_{300} - A_{250} = \pi(300)^2 - \pi(250)^2 = 90{,}000\pi - 62{,}500\pi = 27{,}500\pi \approx 86{,}394$; the area of the 300-ft-radius field is about 86,394 ft^2 more. **6.** The radii of the 8-ft diameter circle and the 6-ft diameter circle are 4 ft and 3 ft, respectively. $A_8 - A_6 = \pi(4)^2 - \pi(3)^2 = 16\pi - 9\pi = 7\pi \approx 22$; the area of the 8-ft diameter circle is about 22 ft^2 more. **7.** $\frac{45}{360} \cdot \pi(18)^2 = 40.5\pi$; the area is $40.5\pi\text{ yd}^2$. **8.** $\frac{90}{360} \cdot \pi(16)^2 = 64\pi$; the area is $64\pi\text{ cm}^2$. **9.** The radius is 13 m; $\frac{180 - 120}{360} \cdot \pi(13)^2 = \frac{169\pi}{6}$; the area is $\frac{169\pi}{6}\text{ m}^2$. **10.** $\frac{12}{360} \cdot \pi(12)^2 = 12\pi$; the area is 12π in.^{2}. **11.** $\frac{360 - 90}{360} \cdot \pi(4)^2 = 12\pi$; the area is $12\pi\text{ ft}^2$. **12.** The radius is 8 cm. $\frac{360 - 45}{360} \cdot \pi(8)^2 = 56\pi$; the area is $56\pi\text{ cm}^2$. **13.** $\frac{90}{360} \cdot \pi(5)^2 = \frac{25\pi}{4}$; the area is $\frac{25\pi}{4}\text{ m}^2$. **14.** $\frac{15}{360} \cdot \pi(6)^2 = \frac{3\pi}{2}$; the area is $\frac{3\pi}{2}\text{ ft}^2$. **15.** The radius is 8 in. $\frac{135}{360} \cdot \pi(8)^2 = 24\pi$; the area is 24π in.^{2}. **16.** The radius is 7.5 cm. $\frac{180}{360} \cdot \pi(7.5)^2 = 28.125\pi$; the area is $28.125\pi\text{ cm}^2$. **17.** The central ∠ is 120°, so the height of the △ is 3 cm, and its base is $6\sqrt{3}$. The area of the △ is $\frac{1}{2}(6\sqrt{3})(3) = 9\sqrt{3}$. The area of the segment is the area of the sector minus the area of the △: $\frac{120}{360} \cdot \pi(6)^2 - 9\sqrt{3} = 12\pi - 9\sqrt{3} \approx 22.1$; the area is about 22.1 cm^2. **18.** The central ∠ is 90, so the height and base of the △ are both 8 ft. The area of the △ is $\frac{1}{2}(8)(8) = 32$. The area of the segment is the area of the sector minus the area of the △: $\frac{90}{360} \cdot \pi(8)^2 - 32 = 16\pi - 32 \approx 18.3$. The area of the segment is about 18.3 ft^2. **19.** The △ is an equiangular △,

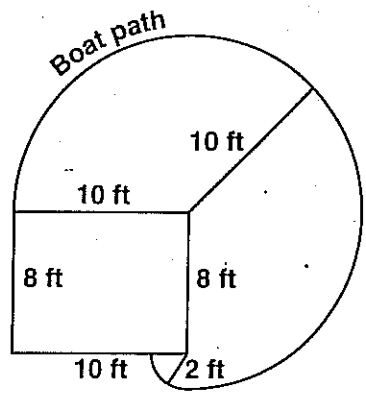
so the height of the $\triangle$ is $3\sqrt{3}$ and the base is 6. The area of the $\triangle$ is $\frac{1}{2}(6)(3\sqrt{3}) = 9\sqrt{3}$. The area of the segment is the area of the sector minus the area of the $\triangle$: $\frac{60}{360} \cdot \pi(6)^2 - 9\sqrt{3} = 6\pi - 9\sqrt{3} \approx 3.3$. The area of the segment is about 3.3 m^2. **20.** The area of the sector $= \frac{60}{360} \cdot \pi(15)^2 = \frac{225\pi}{6}$; the area of the $\triangle = \frac{225\sqrt{3}}{4}$; the area of the segment $= \frac{225\pi}{6} - \frac{225\sqrt{3}}{4} \approx 20.4$; 20.4 m^2. **21.** The area of the sector $= \frac{120}{360} \cdot \pi(14)^2 = \frac{196\pi}{3}$; the area of the $\triangle = 49\sqrt{3}$; the area of the segment $= \frac{196\pi}{3} - 49\sqrt{3} \approx 120.4$; 120.4 cm^2. **22.** The area of the shaded region $= A_{circle} - A_{segment} = A_{circle} - (A_{sector} - A_{\triangle}) = A_{circle} + A_{\triangle} - A_{sector} = \pi(18)^2 + \frac{1}{2}(18)(18) - \frac{90}{360} \cdot \pi(18)^2 = 324\pi + 162 - 81\pi = 243\pi + 162$; the area is $(243\pi + 162)$ ft^2. **23.** The area of the shaded region $= A_{circle} - A_{segment} = A_{circle} - (A_{sector} - A_{\triangle}) = A_{circle} + A_{\triangle} - A_{sector} = \pi(9)^2 + \frac{1}{2}(4.5)(2 \cdot 4.5\sqrt{3}) - \frac{120}{360} \cdot \pi(9)^2 = 81\pi + 20.25\sqrt{3} - 27\pi = 54\pi + 20.25\sqrt{3}$; the area is $(54\pi + 20.25\sqrt{3})$ cm^2. **24.** The area of the shaded region $= A_{circle} - A_{segment} = A_{circle} - (A_{sector} - A_{\triangle}) = A_{circle} + A_{\triangle} - A_{sector} = \pi(12)^2 + \frac{1}{2}(12)(6\sqrt{3}) - \frac{60}{360} \cdot \pi(12)^2 = 144\pi + 36\sqrt{3} - 24\pi = 120\pi + 36\sqrt{3}$; the area is $(120\pi + 36\sqrt{3})$ m^2. **25.** $A_{square} - A_{circle} = 2^2 - \pi(1)^2 = 4 - \pi$; the area is $(4 - \pi)$ ft^2. **26.** Two half circles = 1 whole circle, so $A_{square} - A_{circle} = 8^2 - \pi(4)^2 = 64 - 16\pi$; the area is $(64 - 16\pi)$ ft^2. **27.** Four quarter circles = 1 whole circle, so $A_{square} - A_{circle} = (2 \cdot 14)^2 - \pi(14)^2 = 784 - 196\pi$; the area is $(784 - 196\pi)$ $in.^2$. **28.** Use the Pyth. Thm. to find the radius of the circle: $r^2 + 80^2 = 100^2$; $r^2 = 10{,}000 - 6400$; $r^2 = 3600$; $r = 60$; $A = \pi(60)^2 = 3600\pi \approx 11{,}310$; the area is about 11,310 ft^2. **29.** The base area of a piece from the top is $\frac{1}{8}\pi(8)^2$, or 8π $in.^2$. The base area of a lower outside piece is $\frac{1}{12}\pi(13)^2 - \frac{1}{12}\pi(8)^2 = \frac{1}{12}\pi(13^2 - 8^2) = \frac{1}{12}\pi(105) = 8.75\pi$. Since $8.75\pi > 8\pi$, the lower outside piece is larger. **30.** The area of a circle having radius 12 in. is 144π. The area of a circle having radius 4 in. is 16π. Since $\frac{144\pi}{16\pi} = 9$, exactly 9 circles with radius 4 in. will have the same total area as a circle with radius 12 in. **31.** The radius is 10 in., so $\frac{1}{20}\pi(10)^2 = 5\pi \approx 15.7$; the area is about 15.7 $in.^2$. **32.** $\frac{90}{360} \cdot \pi r^2 = 36\pi$; $r^2 = 144$; $r = 12$; the radius is 12 in.

33.



Answers may vary. Sample: Choose a measure for the arc and use it to solve for r, or choose a measure for r and use it to solve for the arc measure. **34a.** Answers may vary. Sample: Subtract the minor arc segment area from the area of the circle, or add the areas of the major sector and the $\triangle$ formed. **34b.** $A_{sector} - A_{\triangle} = \frac{90}{360} \cdot \pi(10)^2 - \frac{1}{2}(10)(10) = 25\pi - 50$; $A_{circle} - A_{segment} = A_{circle} - (A_{sector} - A_{\triangle}) = A_{circle} - A_{sector} + A_{\triangle} = \pi(10)^2 - 25\pi + 50 = 75\pi + 50$ **35.** The shaded region consists of 6 segments, and each of the central $\measuredangle$s is 60°, so the related $\triangle$s are equiangular; $6\left(\frac{60}{360} \cdot \pi(7)^2 - \frac{1}{2}(7)(3.5\sqrt{3})\right) = 6\left(\frac{49}{6}\pi - 12.25\sqrt{3}\right) = 49\pi - 73.5\sqrt{3}$; the area is $(49\pi - 73.5\sqrt{3})$ m^2. **36.** The area of the shaded region is found by subtracting the area of 2 segments from the area of the square. The area of the 2 segments is $2\left(\frac{90}{360} \cdot \pi(10)^2 - \frac{1}{2}(10)(10)\right) = 2(25\pi - 50)$, or $(50\pi - 100)$ m^2. The area of the square is 10^2, or 100 m^2. The area of the shaded region is $100 - (50\pi - 100)$, or $(200 - 50\pi)$ m^2. **37.** Analyze the area of the shaded region by drawing a vertical radius in the largest semicircle. To its right is a shaded semicircle the same size as the unshaded semicircle on its left side. Thus, the area of the shaded region is $\frac{1}{4}$ the area of the largest semicircle whose radius is 4 m. $A = \frac{1}{4}\pi(4)^2; = 4\pi$; the area is 4π m^2. **38.** Let x represent the length of the radius of the circle. Then, the area of the circle is πx^2, $AB = 2x$, and $PR = 2x$. Since $PR = 2x$, $PQ = \frac{2x}{\sqrt{2}}$, or $x\sqrt{2}$, and so the area of $PQRS$ is $2x^2$. The area of the yellow region is $\pi x^2 - 2x^2$, or $(\pi - 2)x^2$. The area of the blue region is $(2x)^2 - 2x^2 = 4x^2 - 2x^2 = 2x^2$. Since $(\pi - 2) < 2$, the blue region is smaller than the yellow region. **39.** The region is twice the area of a segment. Call the intersection points of the two circles A and B and the intersection of $\overline{TU}$ and $\overline{AB}$, call X. Then $AU = TA = TB = BU = TU = 10$, so $\triangle ATU$ and $\triangle BTU$ are equilateral $\triangle$s. Thus, $UX = 5$ and $AX = 5\sqrt{3}$, so $AB = 10\sqrt{3}$. The area of $\triangle AUB = \frac{1}{2}(10\sqrt{3})(5) = 25\sqrt{3}$. So, the total area of the region contained in both circles is $2\left(\frac{120}{360} \cdot \pi(10)^2 - 25\sqrt{3}\right) = 2\left(\frac{100\pi}{3} - 25\sqrt{3}\right) = \frac{200\pi}{3} - 50\sqrt{3}$; the area is $\left(\frac{200\pi}{3} - 50\sqrt{3}\right)$ units^2.

40a.



40b. Find the area of $\frac{3}{4}$ of a circle of radius 10 and add $\frac{1}{4}$ of a circle of radius 2.

40c. $\frac{3}{4}(\pi 10^2) + \frac{1}{4}(\pi 2^2) = \frac{3}{4}(100\pi) + \frac{1}{4}(4\pi) = 75\pi + \pi = 76\pi \approx 239$; the area is about 239 ft^2. **41.** $\frac{10}{360} \cdot 72\pi = 2\pi$; the answer is choice A. **42.** A sector of 90° is $\frac{1}{4}$ the area of the circle, so the area of the entire circle is 4 times the area of the sector, or 4π. The answer is choice G. **43.** [2] The area of the rectangle is $(2)(12) = 24$; the area of the semicircles $= 3\left(\frac{1}{2}\pi \cdot 2^2\right) = 3(2\pi) = 6\pi$;

the area of the shaded region $= 24 - 6\pi \approx 5.2$; the area is about 5.2 m^2. **44.** $\frac{180 - 60}{360} \cdot \pi(30) = 10\pi$; the length is 10π cm. **45.** $\frac{72}{360} \cdot 2\pi(5) = 2\pi$; the length is 2π m. **46.** $\frac{180 - 40}{360} \cdot 2\pi(36) = 28\pi$; the area is 28π in. **47.** Let n be the length of each of the 3 ≅ sides. Then the fourth side is $n + 4$. Add the sides: $3n + n + 4 = 49$; $4n + 4 = 49$; $4n = 45$; $n = 11.25$ and $n + 4 = 15.25$; the four sides are 11.25 in., 11.25 in., 11.25 in., and 15.25 in. **48.** Show that $\triangle BCE \cong \triangle ACD$ by ASA. Then use $\overline{CE} \cong \overline{CD}$ and segment subtraction to show that $\overline{BD} \cong \overline{AE}$. Now since vertical ∠s are ≅, then $\angle BFD \cong \angle AFE$, so $\triangle BDF \cong \triangle AEF$ by ASA.

CHECKPOINT QUIZ 2 *page 400*

1. $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(9)(12 + 18) = 135$; the area is 135 $in.^2$. **2.** $A = \frac{1}{2}d_1d_2 = \frac{1}{2}(13)(9) = 58.5$; the area is 58.5 m^2. **3.** The radii of the hexagon form 6 equilateral △s. So, the side of the hexagon is $2 \cdot \frac{6}{\sqrt{3}}$, or $4\sqrt{3}$. The perimeter is $6(4\sqrt{3}) = 24\sqrt{3}$. So, $A = \frac{1}{2}(6)(24\sqrt{3})$, or $72\sqrt{3}$; the area is $72\sqrt{3}$ $in.^2$. **4.** The apothem is $\frac{1}{3}$ the height, so the height is 9 ft. Then each side is $2 \cdot \frac{9}{\sqrt{3}}$, or $6\sqrt{3}$ ft. The area is $\frac{1}{2}(6\sqrt{3})(9)$, or $27\sqrt{3}$ ft^2. **5.** The diagonal of the square is 8 yd, so each side is $\frac{8}{\sqrt{2}}$ yd. The area of the square is $\left(\frac{8}{\sqrt{2}}\right)^2 = \frac{64}{2}$, or 32 yd^2. **6.** The diameter is 20 in. so the radius is 10 in. The area of the circle is $\pi(10)^2$, or 100π $in.^2$. **7.** $\frac{120}{360} \cdot \pi(9)^2 = 27\pi$; the area is 27π m^2. **8.** The area of the △ is $\frac{1}{2}(8)(8)$, or 32 cm^2. The area of the segment $= \frac{90}{360} \cdot \pi(8)^2 - 32$, or $(16\pi - 32)$ cm^2. **9.** $C = 2\pi(5) = 10\pi \approx 31.4$; the circumference is about 31.4 m. **10.** $\frac{45}{360} \cdot 2\pi(18) = \frac{9\pi}{2}$; the length is $\frac{9\pi}{2}$.

TECHNOLOGY *page 401*

1. As the number of sides increases, the perimeter and circumference become closer and so do the polygon area and circle area, so each ratio will approach 1. **2a.** yes **2b.** The ratio of perimeter to circumference approaches 1 faster than the area ratios. **3.** Use the formula for circumference of a circle to estimate the perimeter: $C = 2\pi(10) = 20\pi \approx 63$; the perimeter is about 63 cm. Use the formula for area of a circle to estimate the area of the polygon: $A = \pi(10)^2 = 100\pi \approx 314$; the area is about 314 cm^2.

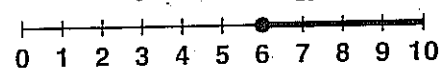
7-8 Geometric Probability *pages 402–407*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1. $\frac{1}{3}$ **2.** $\frac{1}{2}$ **3.** 1 **4.** $\frac{1}{4}$ **5.** $\frac{1}{6}$ **6.** $\frac{1}{2}$ **7.** $\frac{1}{3}$ **8.** $\frac{1}{2}$

Check Understanding **1.** There are an infinite number of points on $\overline{AB}$, which is 10 units long, and an infinite number of points on $\overline{CD}$, which is 4 units long. The probability is $\frac{4}{10}$, or $\frac{2}{5}$. **2.** $\frac{10}{25} = \frac{2}{5}$ **3a.** $\frac{\pi(2)^2}{144} = \frac{4\pi}{144} \approx 0.087$; it becomes about 8.7%, which is about 4 times greater than 2.2%. **3b.** $\frac{\pi(3)^2}{144} = \frac{9\pi}{144} \approx 0.196$; it becomes about 19.6%, which is about 9 times greater than 2.2%. **4.** Yes; 1.4% is 1.4 out of 100, so theoretically you should win 1.4 times out of 100.

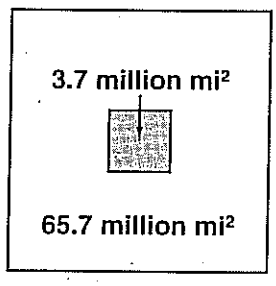
Exercises **1.** There are an infinite number of points on $\overline{AK}$ which is 10 units long and an infinite number of points on $\overline{CH}$ which is 5 units long, so the probability is $\frac{5}{10}$ or $\frac{1}{2}$. **2.** There are an infinite number of points on $\overline{AK}$ which is 10 units long, and an infinite number of points on $\overline{FG}$ which is 1 unit long, so the probability is $\frac{1}{10}$. **3.** There are an infinite number of points on $\overline{AK}$ which is 10 units long, and an infinite number of points on $\overline{DJ}$ which is 6 units long, so the probability is $\frac{6}{10}$, or $\frac{3}{5}$. **4.** There are an infinite number of points on $\overline{AK}$ which is 10 units long, and an infinite number of points on $\overline{EI}$ which is 4 units long, so the probability is $\frac{4}{10}$, or $\frac{2}{5}$. **5.** There are an infinite number of points on the entire segment $\overline{AK}$ which is 10 units long and an infinite number of points on the desired segment $\overline{AK}$, so the probability is $\frac{10}{10}$, or 1. **6.** Since 5 and 9 are both less than 20, the letters are in the order of A, M, N, B, or its reverse. So, $AM + MN + NB = 20$; $5 + MN + 9 = 20$; $MN = 6$. The probability is $\frac{6}{20}$, or $\frac{3}{10}$.

7. 

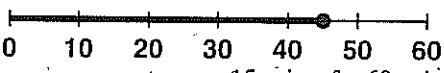
The total length of time is 10 minutes and the desired length of time is 4 minutes, so the probability is $\frac{4}{10}$, which is $\frac{2}{5}$, or 40%. **8.** The total number of seconds for the 3 colors is $30 + 5 + 25$, or 60 s. Green is the desired color at 30 s, so the probability of seeing green is $\frac{30}{60}$, which is $\frac{1}{2}$, or 50%. **9.** The total number of seconds is $20 + 5 + 50$, or 75 s. Green is the desired color at 20 s, so the probability of seeing green is $\frac{20}{75}$, which is $\frac{4}{15}$, or about 27%. **10.** The total number of seconds is $40 + 5 + 25$, or 70 s. Green is the desired color at 40 s, so the probability is $\frac{40}{70}$, which is $\frac{4}{7}$, or about 57%. **11.** The total number of seconds is $25 + 5 + 45$, or 75 s. Green is the desired color at 25 s, so the probability is $\frac{25}{75}$, which is $\frac{1}{3}$ or about 33%. **12.** The total number of seconds is $35 + 8 + 32$, or 75 s. Green is the desired color at 35 s, so the probability is $\frac{35}{75}$, which is $\frac{7}{15}$, or about 47%. **13.** The total number of seconds is $50 + 4 + 26$, or 80 s. Green is the desired color at 50 s, so the probability is $\frac{50}{80}$, which is $\frac{5}{8}$, or 62.5%. **14.** The total number of minutes in the time span is 30. The bridge is raised for 5 min. So, the probability of finding the bridge raised is $\frac{5}{30}$, or $\frac{1}{6}$. **15.** The △ is divided into 4 ≅ △s, and ≅ polygons have = areas. Exactly 1 is shaded, so the probability of hitting the shaded △ is $\frac{1}{4}$, or 25%. **16.** Let x represent the radius of the small circle. Then $2x$ represents the radius of the entire circle. The area of the smaller △ is π

x^2, and the area of the entire circle is $\pi(2x)^2$, or $4x^2\pi$. So, the probability is $\frac{x^2\pi}{4x^2\pi}$, which is $\frac{1}{4}$, or 25%. **17.** The radii divide the pentagon into 5 ≅ △ which have = areas. Two of the △ are shaded, so the probability is $\frac{2}{5}$, or 40%. **18.** Since a 120° sector is not shaded, the sector that is shaded has a central ∠ of 360 − 120, or 240°. So, the part of the circle that is shaded is $\frac{240}{360}$, which is $\frac{2}{3}$, or about 67%. Thus, the probability is $\frac{2}{3}$, or about 67%. **19.** There are 4 ≅ semicircles, which is equivalent to 2 circles each having diameter s, the length of the side of the square. The total area is the area of the 2 circles and the area of the square: $(2)\pi\left(\frac{s}{2}\right)^2 + s^2$, or $\frac{s^2\pi + 2s^2}{2}$. The shaded area is the area of the 2 circles, which is $\frac{s^2\pi}{2}$. So, the ratio of the shaded area to the total area is $\frac{\frac{s^2\pi}{2}}{\frac{s^2\pi + 2s^2}{2}} = \frac{s^2\pi}{s^2\pi + 2s^2} = \frac{s^2\pi}{s^2(\pi + 2)} = \frac{\pi}{\pi + 2} \approx$ or about 61%. So, the probability is $\frac{\pi}{\pi + 2}$, or about 61%. **20.** If s is the side of the square, then the area of the board is s^2. The combination of two ≅ semicircles has the same area as one circle. The diameter is s, so the radius is $\frac{s}{2}$. The shaded area is the area of the square minus the area of one circle, which is $s^2 - \pi\left(\frac{s}{2}\right)^2$, or $\frac{s^2(4 - \pi)}{4}$. The ratio of the shaded area to the entire area is $\frac{\frac{s^2(4 - \pi)}{4}}{s^2} = \frac{4 - \pi}{4} \approx 21\%$. So, the probability is $\frac{4 - \pi}{4}$, or about 21%. **21.** The radius of the entire target is 5(12.2), or 61 cm, so the area of the entire target is $\pi 61^2$, or 3721π cm^2. The radius of the yellow circle is 12.2 cm, so the area of the yellow circle is $\pi(12.2)^2$, or 148.84π cm^2. The ratio of the yellow circle to the entire target is $\frac{148.84\pi}{3721\pi} = 4\%$. So, the probability of hitting the yellow zone is 4%. **22.** The probability is $\frac{MN}{BZ}$, so $\frac{MN}{20} = 0.3$. Thus, $MN = 6$. **23.** Let the side of the square be $2n$, so the area of the square is $4n^2$. The radius of the circle is n, so the area of the circle is $n^2\pi$. The ratio of the circle to the square is $\frac{n^2\pi}{4n^2}$, or $\frac{\pi}{4}$, so the probability of hitting the circle is $\frac{\pi}{4}$. **24.** Let the side of the square be $4n$, so the area of the square is $16n^2$. The radius of one circle is n, so the area of the 4 circles is $4n^2\pi$. The ratio of the circle to the square is $\frac{4n^2\pi}{16n^2}$, or $\frac{\pi}{4}$, so the probability of hitting a circle is $\frac{\pi}{4}$. **25.** Let the side of the square be $6n$, so the area of the square is $36n^2$. The radius of each circle is n, so the area of the 9 circles is $9n^2\pi$. The ratio of the circle to the square is $\frac{9n^2\pi}{36n^2}$, or $\frac{\pi}{4}$, so the probability of hitting a circle is $\frac{\pi}{4}$. **26.** The desired target is a circle having a radius of $\sqrt{10}$ in., so its area is 10π. The radius of the square is 10 in., so one side of the square is $10\sqrt{2}$ in. The area of the entire square target is $(10\sqrt{2})^2$, or 200 in.^{2}. The probability of hitting the desired target is $\frac{10\pi}{200}$ or about 16%. **27a.** The probability of hitting the target is 1.4%, so, out of 100 tosses, the quarter would award a prize 100(0.014), or 1.4 times. Out of 1000 tosses, the quarter would award a prize 1000(0.014), or 14 prizes. **27b.** The value of 1000 quarters is 1000(0.25), or \$250. They would lose 14(10), or \$140 for the 14 prizes. So, the profit would be 250 − 140, or \$110. **28.** The total interval is 20 + 5, or 25 min. The time when the wait for the next bus is greater than 10 min. is immediately following the departure of a bus until 15 min. after the departure, or 15 min. The probability is $\frac{15}{25}$, which is $\frac{3}{5}$, or 60%. **29.** Each cycle is 60 s. The probability of facing a red light is 60%. So, $\frac{r}{60} = 0.6$; $r = 36$; the light is red for 36 s.

30. $\frac{3.7}{65.7} \approx 56\%$

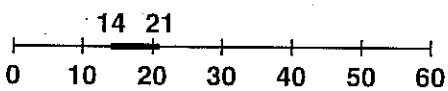


31a.



If it starts after 45 min, you cannot erase 15 min of a 60-min tape.

31b.



The rehearsal was taped from the 21st to the 29th minute on the tape, so to erase the entire amount, the erasure could be anywhere from 29 − 15, or the 14th min, to the 21st min. The probability is $\frac{21 - 14}{45}$, which is $\frac{7}{45}$, or about 16%. **32.** $AK = 10$, so the probability is $\frac{8 - 2}{10}$, or $\frac{3}{5}$. **33.** $7 \le x \le 10$; $AK = 10$, so the probability is $\frac{10 - 7}{10}$, or $\frac{3}{10}$. **34.** $2x \le 9$; $x \le 4.5$; $0 \le x \le 4.5$; $AK = 10$, so the probability is $\frac{4.5 - 0}{10}$, or $\frac{9}{20}$. **35.** $\frac{1}{2}x - 5 \ge 0$; $\frac{1}{2}x \ge 5$; $x \ge 10$; $x = 10$, which is only one point out of an infinite number of points on $\overline{AK}$. The probability is 0. **36.** $2 \le 4x \le 3$; $\frac{1}{2} \le x \le \frac{3}{4}$; $AK = 10$, so the probability is $\frac{\frac{3}{4} - \frac{1}{2}}{10}$, or $\frac{1}{40}$. **37.** $0 \le \frac{1}{3}x + 1 \le 5$; $-1 \le \frac{1}{3}x \le 4$; $-3 \le x \le 12$; this range exceeds the range of $\overline{AK}$, so the probability is 1. **38.** If $x - 6 \ge 0$, or $x \ge 6$, then $x - 6 \le 1.5$; $x \le 7.5$; therefore, $6 \le x \le 7.5$. If $x - 6 \le 0$, or $x \le 6$, then $-(x - 6) \le 1.5$; $x - 6 \ge -1.5$; $x \ge 4.5$; therefore, $4.5 \le x \le 6$. It follows that $4.5 \le x \le 7.5$. $AK = 10$, so the probability is $\frac{7.5 - 4.5}{10} = \frac{3}{10}$. **39.** $\sqrt{2} \le \pi x \le \sqrt{10}$; $\frac{\sqrt{2}}{\pi} \le x \le \frac{\sqrt{10}}{\pi}$; $AK = 10$, so the probability is $\frac{\frac{\sqrt{10} - \sqrt{2}}{\pi}}{10}$, which is $\frac{\sqrt{10} - \sqrt{2}}{10\pi}$, or about 0.06. **40.** Assume that the center of the ball is heading for the target whose diameter is 40 cm, so its radius is 20 cm. The area of background is $\pi(20)^2$, or 400π cm^2. The desired part of the target, however, can be hit squarely by the center of the ball or be nicked by just half the ball to be effective. So, the radius of the ball increases the effective radius of

the red portion by 3.6 cm. So, the radius of the desired target is 10 + 3.6, or 13.6 cm. The area of the desired target is $\pi(13.6)^2$, or 184.96π cm^2. The probability is $\frac{184.96\pi}{400\pi}$, or about 46%. **41.** Assume that the center of the ball is heading for the square target whose sides are 40 cm, so the area of the target is 40^2, or 1600 cm^2. The desired part of the target, however, can be hit squarely by the center of the ball or be nicked by just half the ball to be effective. So, the radius of the ball increases the effective side of the circle by 3.6 cm. So, the radius of the desired target is 10 + 3.6, or 13.6 cm. The area of the desired target is $\pi(13.6)^2$, or 184.96π cm^2. The probability is $\frac{184.96\pi}{1600}$, or about 36%. **42.** Assume that, as long as the center of the ball hits the target, the subject will get dunked. The diameter of the target is 40 cm, so its radius is 20 cm. The area of the background is $\pi(20)^2$. The strike zone of the target is 20 cm by 20 cm. Since the radius of the ball is 3.6 cm, height of the target is enhanced to 2(3.6) + 20, or 27.2 cm. Likewise, the width is enhanced to 27.2 cm. Since the ball is round, however, so are the corners of the enhanced strike zone. Each corner is $\frac{1}{4}$ the area of the circle of the ball, which is $\frac{1}{4} \cdot 3.6^2\pi$, or 3.24π cm^2. The total area of the strike zone is the sum of the original square plus the area of the four 3.6 cm-by-20 cm rectangles in the enhanced region plus 4 times the area of the quarter circle of the ball: $20^2 + 4(3.6)(20) + 4 \cdot \frac{1}{4} \cdot 3.6^2\pi = 400 + 288 + 12.96\pi = 688 + 12.96\pi$. The percent is the area of the strike zone divided by the area of the target: $\frac{688 + 12.96\pi}{400\pi} \approx 58\%$. The percent is about 58%. **43.** Assume that the center of the ball is heading for the square target whose side is 40 cm, so the area of background is 40^2, or 1600 cm^2. The desired part of the target, however, can be hit squarely by the center of the ball or be nicked by just half the ball to be effective. So, the radius of the ball increases the effective side of the desired target by 3.6 cm at both ends of a side. So, the side of the desired target is 20 + 7.2, or 27.2 cm. The area of the desired target is 27.2^2, or 739.84 cm^2. The probability is $\frac{739.84}{1600}$, or about 46%. **44a.** $3 + 3 > 4$ and $3 + 4 > 3$, so, since the sum of the lengths of every 2 straws is greater than the length of the third straw, they can form a △. **44b.** $4 + 5 > 1$, $5 + 1 > 4$, but $4 + 1$ is not greater than 5, so, since the sum of the lengths of every 2 straws is not greater than the length of the third straw, they cannot form a △. **44c.** From parts ⓐ and ⓑ, Kimi can cut from 1 in. and 5 in. from one end of the 6-in. straw. The range is 4 in. and the length of the entire straw is 6 in., so the probability is $\frac{4}{6}$, or $\frac{2}{3}$. **45a.** Check students' work. **45b.** Check students' work. **46a.** The radius of the target is 1 m, or 100 cm. The probability is 0.2, so $\frac{r^2\pi}{100^2\pi} = 0.2$; $r^2 = 0.2(10{,}000)$; $r \approx 46$; the radius is about 46 cm. **46b.** The radius of the target is 1 m, or 100 cm. The probability is 0.4, so $\frac{r^2\pi}{100^2\pi} = 0.4$; $r^2 = 0.4(10{,}000)$; $r \approx 63$; the radius is about 63 cm. **46c.** The radius of the target is 1 m, or 100 cm. The probability is 0.5, so $\frac{r^2\pi}{100^2\pi} = 0.5$; $r^2 = 0.5(10{,}000)$; $r \approx 71$; the radius is about 71 cm. **46d.** The radius of the target is 1 m, or 100 cm. The probability is 0.6, so $\frac{r^2\pi}{100^2\pi} = 0.6$; $r^2 = 0.6(10{,}000)$; $r \approx 77$; the radius is about 77 cm. **46e.** The radius of the target is 1 m, or 100 cm. The probability is 0.8, so $\frac{r^2\pi}{100^2\pi} = 0.8$; $r^2 = 0.8(10{,}000)$; $r \approx 89$; the radius is about 89 cm. **46f.** The radius of the target is 1 m, or 100 cm. The probability is 1, so $\frac{r^2\pi}{100^2\pi} = 1$; $r^2 = 1(10{,}000)$; $r = 100$; the radius is 100 cm. **47.** The diameter of the entire target is 14 cm, so the radius is 7 cm. The area of the target is $\pi(7)^2$, or 49π cm^2. The diameter of the region having blue at the perimeter is 1 + 2 + 2, or 5 cm, so the area of the blue-red-yellow region is $\pi 5^2$, or 25π cm^2. The area of the gray region is $A_{\text{target}} - A_{\text{blue-red-yellow}} = 49\pi - 25\pi = 24\pi$; the area of the gray region is 24π cm^2. So, the probability is $\frac{24\pi}{49\pi}$, or about 49%. The blue and red regions combined is $A_{\text{blue-red-yellow}} - A_{\text{yellow}} = 25\pi - \pi = 24\pi$, so since the area of the blue-red region is the same as the gray region, the probability is the same. **48.** The radius of each circle is 1 m, so the area of each circle is π. There are 4 circles, so their total area is 4π m^2. The area of the square is 4^2, or 16 m^2. The area of the shaded region is $A_{\text{square}} - A_{\text{4 circles}} = 16 - 4\pi$. So, the probability is $\frac{16 - 4\pi}{16}$, or about 21%. The answer is choice A. **49.** From Exercies 48, the probability of not hitting a circle is 21%, so the probability of hitting a circle is 100% − 21%, or 79%. The answer is choice I. **50.** [2] $\frac{\text{area of circle}}{\text{area of triangle}} = \frac{\pi(1)^2}{3\sqrt{3}} \approx 0.6 = 60\%$ [1] no work shown OR correct explanation and a computational error **51.** [4] **a.** $\frac{1}{3}(\text{area of circle}) = \frac{1}{3}(\pi \cdot 28^2) = \frac{784\pi}{3}$; the area of the sector is $\frac{784\pi}{3}$ m^2. **b.** Area of △ $= \frac{1}{2}bh = \frac{1}{2}(28\sqrt{3})(14) = 196\sqrt{3}$, so area of segment $= \frac{784\pi}{3} - 196\sqrt{3} \approx 481.5$; the area of the segment is about 481.5 m^2. The shaded area is the area of the 120° sector minus the area of the △. [3] one computational error OR incorrect explanation [2] one computational error and incorrect explanation [1] one correct answer OR a correct explanation **52.** $C = 2\pi r$; $20\pi = 2\pi r$; $r = 10$; $A = \pi r^2 = \pi(10)^2 = 100\pi$; the area is 100π ft^2.

53. $\frac{30}{360} \cdot \pi(12)^2 = 12\pi$; the area is 12π cm^2.

54. A semicircle is a half-circle. If the diameter is 20 ft, then the radius is 10 ft. The area is $\frac{1}{2}\pi r^2 = \frac{1}{2}\pi 10^2 = 50\pi$. The area is 50π ft^2. **55.** Consecutive ∠s in a ▱ are suppl., so $4x + x = 180$; $5x = 180$; $x = 36$. Also, $y + x = 180$; $y + 36 = 180$; $y = 144$; so $x = 36$ and $y = 144$. **56.** Consecutive ∠s in a ▱ are suppl, so $x + 3x = 180$; $4x = 180$; $x = 45$. Also $x + (y + x) = 180$; $45 + (y + 45) = 180$; $y + 90 = 180$; $y = 90$; so $x = 45$ and $y = 90$. **57a.** $D = \left(\frac{1 + 5}{2}, \frac{-4 + 6}{2}\right) = (3, 1)$; $E = \left(\frac{-3 + 5}{2}, \frac{2 + 6}{2}\right) = (1, 4)$ **57b.** Slope of $\overline{DE} = \frac{1 - 4}{3 - 1} = -\frac{3}{2}$; slope of $\overline{AC} =$

$\frac{-4-2}{1-(-3)} = -\frac{3}{2}$. **57c.** $\overline{DE} \parallel \overline{AC}$ because they have the same slope. **57d.** $DE = \sqrt{(3-1)^2 + (1-4)^2} = \sqrt{4+9} = \sqrt{13}$, and $AC = \sqrt{(-3-1)^2 + (2-(-4))^2} = \sqrt{16+36} = \sqrt{52} = \sqrt{4 \cdot 13} = 2\sqrt{13}$ **57e.** $DE = \frac{1}{2}AC$ because $\sqrt{13} = \frac{1}{2} \cdot 2\sqrt{13}$.

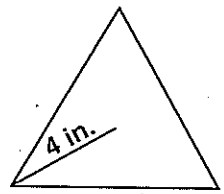
TEST-TAKING STRATEGIES *page 408*

1. Since the perimeter is 20 cm and one side is 0.1 cm, then its opp. side must also be 0.1 cm. Thus, the sum of the remaining 2 sides is $20 - 2(0.1)$, or 19.8 cm. So, each of the two remaining sides is $\frac{19.8}{2}$, or 9.9. The area of a rectangle with sides of 0.1 cm and 9.9 cm is 0.99 cm^2. **2.** Since $\overline{AC}$ is the longest side and is opp. $\angle B$, then the largest $\angle$ is $\angle B$, so I is true. Since $\sqrt{1}^2 + \sqrt{2}^2 = \sqrt{3}^2$, the $\triangle$ is a right $\triangle$. Since the side lengths are not in the ratio of $1 : \sqrt{3} : 2$, then III is false. The answer is choice C. **3.** For option I: The perimeter of the square is 4(2), or 8 cm and the perimeter of the $\triangle$ is $3\left(2 \cdot \frac{3}{\sqrt{3}}\right)$, or $6\sqrt{3}$ cm. Since $8 \leq 6\sqrt{3}$, option 1 is false. So, the answer is choice H. **4.** Option I is true by Thm. 3-10. Since the expression for determining the sum of the measures of the interior angles of an n-gon is $180(n-2)$, option II is true. Option 3 is sometimes true, since not all rhombuses are squares. The answer is choice B.

CHAPTER REVIEW *pages 409–411*

1. An altitude is the distance from a vertex to the opp. side of a $\triangle$, and the base is what that opp. side is called. So, you can use any side as the *base* of a $\triangle$. **2.** By def. of sector, a *sector* of a circle is a region bounded by two radii and the intercepted arc. **3.** By def. of diameter, a segment that contains the center of a circle and has both endpoints on the circle is the *diameter* of a circle. **4.** By def. of apothem, in a regular polygon, the perpendicular distance from the center to a side is the *apothem* of the ▱. **5.** By def. of adjacent arcs, two arcs of a circle with exactly one point in common are *adjacent arcs*. **6.** $A = \frac{1}{2}bh = \frac{1}{2}(4)(5) = 10$; the area is 10 m^2. **7.** $A = bh = (9)(10) = 90$; the area is 90 $in.^2$. **8.** $A = \frac{1}{2}bh = \frac{1}{2}(11)(6) = 33$; the area is 33 ft^2. **9.** Use the 30°-60°-90° $\triangle$ relationship to determine the height of the trapezoid. $h\ 6\sqrt{3}$; $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(6\sqrt{3})(11 + (15 + 6)) = \frac{1}{2}(6\sqrt{3})(32) = 96\sqrt{3}$; the area is $96\sqrt{3}$ mm^2. **10.** The diagonals create four right ◬, each side having twice the measure of a 3-4-5 $\triangle$. So, the missing diagonal lengths are 6 ft and 6 ft. The diagonal lengths are 12 ft and 16 ft, so $A = \frac{1}{2}d_1d_2 = \frac{1}{2}(12)(16) = 96$; the area is 96 ft^2. **11.** Since the diagonals are $\perp$, $A = \frac{1}{2}d_1d_2 = \frac{1}{2}(10+8)(6.5+6.5) = \frac{1}{2}(18)(13) = 117$; the area is 117 cm^2.

12.



The radius is $\frac{2}{3}h$, so $h = 6$. The base then is $2\left(\frac{6}{\sqrt{3}}\right)$, or $4\sqrt{3}$, so $A = \frac{1}{2}bh = \frac{1}{2}(4\sqrt{3})(6) \approx 20.8$; the area is about 20.8 cm^2. **13.** If the radius of the square is 8 mm, then the diagonal is 16 mm. Each side then is $\frac{16}{\sqrt{2}}$, so $A = s^2 = \left(\frac{16}{\sqrt{2}}\right)^2 = \frac{256}{2} = 128$; the area is 128 mm^2. **14.** The radii of a hexagon divide it into 6 equiangular ◬. The apothem divides a $\triangle$ into two 30°-60°-90° ◬, of which the side of the equiangular $\triangle$ is the hyp. of the 30°-60°-90° $\triangle$. So, the apothem is $\frac{7\sqrt{3}}{2}$ and the perimeter is 6(7), or 42. The area of the hexagon is $\frac{1}{2}ap = \frac{1}{2}\left(\frac{7\sqrt{3}}{2}\right)(42) \approx 127.3$; the area is about 127.3 cm^2. **15.** Since the ratio of one leg to the hyp. is 12 : 20, or 3 : 5, it is a 3-4-5 $\triangle$ with each side mult. by 4. So, the missing side is 4(4), or 16. **16.** Use the Pyth. Thm.: $14^2 + 16^2 = x^2$; $196 + 256 = x^2$; $x^2 = 452$; $x = \pm\sqrt{452} = \pm\sqrt{4 \cdot 113} = \pm 2\sqrt{113}$; since area is positive, the area is $2\sqrt{113}$. **17.** Use the Pyth. Thm.: $8^2 + 15^2 = x^2$; $64 + 225 = x^2$; $x^2 = \sqrt{289} = 17$. **18.** In a 30°-60°-90° $\triangle$, the sides are in the ratio of $1 : \sqrt{3} : 2$, so since 9 is opp. the smallest $\angle$, $x = 9\sqrt{3}$ and $y = 2(9) = 18$. **19.** The $\triangle$ is an isosc. rt. $\triangle$, so $x = 12\sqrt{2}$. **20.** In a 30°-60°-90° $\triangle$, the sides are in the ratio of $1 : \sqrt{3} : 2$, so since 20 is opp. the second largest $\angle$, $x = \frac{20}{\sqrt{3}} = \frac{20\sqrt{3}}{3}$. Then $y = 2\left(\frac{20\sqrt{3}}{3}\right) = \frac{40\sqrt{3}}{3}$. **21.** $m\angle APD = 180 - 90 - 60 = 30$ **22.** $m\widehat{AC} = 180 - m\angle APB = 180 - 60 = 120$ **23.** $m\widehat{ABD} = m\angle APB + 180 + m\angle CPD = 60 + 180 + 90 = 330$ **24.** $m\angle CPA = 360 - (60 + 180) = 120$ **25.** $\frac{110}{360} \cdot 2\pi(4) = \frac{22}{9}\pi$; the length is $\frac{22}{9}\pi$ in. **26.** $\frac{360 - (180 + 120)}{360} \cdot 2\pi(3) = \pi$; the length is π mm. **27.** $\frac{90}{360} \cdot \pi(8)^2 - \frac{1}{2}(8)(8) = \frac{5760\pi}{360} - 32 \approx 18.3$; the area is about 18.3 m^2. **28.** The $\angle$ bisector of the vertex $\angle$ of 120° in the isosc. $\triangle$ creates a 30°-60°-90° $\triangle$, so its height is 3 and the other leg is $3\sqrt{3}$. The entire length of that side of the larger $\triangle$ is $2(3\sqrt{3})$, or $6\sqrt{3}$ cm. $\frac{1}{2}A_{circle} - A_{\triangle} = \frac{1}{2}\pi(6)^2 - \frac{1}{2}(6\sqrt{3})(3) = 18\pi - 9\sqrt{3} \approx 41.0$; the area is about 41.0 cm^2. **29.** The shaded area is half the area of the $\triangle$, so the probability is $\frac{1}{2}$, or 50%. **30.** The shaded area is $\frac{1}{4} + \frac{1}{2} \cdot \frac{1}{4}$ or $\frac{3}{8}$, so the probability of hitting the shaded area is $\frac{3}{8}$, or 37.5%. **31.** $\frac{60}{360}$, or $\frac{1}{6}$ of the circle is shaded, so the probability is $\frac{1}{6}$, or about 16.7%.

CHAPTER TEST *page 412*

1. $A = \frac{1}{2}bh = \frac{1}{2}(13)(12) = 78$; the area is 78 ft^2. **2.** The radius of the square is 6 mm, so the diameter is 12 mm and divides the square into 2 isosc. rt. ◬. Each side of the square, then, is $\frac{12}{\sqrt{2}}$, so $A = s^2 = \left(\frac{12}{\sqrt{2}}\right)^2 = \frac{144}{2} = 72$, so the area is 72 mm^2. **3.** The hyp. of the 30°-60°-90° $\triangle$ is 9 m., so the height of the ▱ is $4.5\sqrt{3}$ m. The area, then, is $bh = (8)(4.5\sqrt{3}) \approx 62.4$; the area is about 62.4 m^2.

4. $A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(3)(3 + 6) = 13.5$; the area is 13.5 in.^{2}. **5.** The radii of a hexagon divide it into 6 equiangular ◬. The apothem divides a △ into two 30°-60°-90° ◬ whose hyp. is 4 ft. So, the apothem is $2\sqrt{3}$. The perimeter is 2(2)(6), or 24 ft. Thus, $A = \frac{1}{2}ap = \frac{1}{2}(2\sqrt{3})(24) \approx 41.6$; the area is about 41.6 ft^2. **6.** The perimeter is 6(6), or 36 cm, so $A = \frac{1}{2}ap = \frac{1}{2}(7.2)(36) = 129.6$. The area is 129.6 cm^2. **7.** Use the Pyth. Thm.: $7^2 + 11^2 = x^2$; $49 + 121 = x^2$; $x^2 = 170$; $x = \sqrt{170}$. **8.** Use the Pyth. Thm.: $x^2 + 13^2 = 15^2$; $x^2 + 169 = 225$; $x^2 = 56$; $x = \sqrt{56} = \sqrt{4 \cdot 14} = 2\sqrt{14}$. **9.** In an isosc. rt. △, the side lengths are in the ratio of $1 : 1 : \sqrt{2}$, so $x = y = \frac{11}{\sqrt{2}} = \frac{11\sqrt{2}}{2}$.
10. In a 30°-60°-90° △, the side lengths are in the ratio of $1 : \sqrt{3} : 2$, so $x = \frac{12}{\sqrt{3}} = 4\sqrt{3}$ and $y = 2(4\sqrt{3}) = 8\sqrt{3}$.
11. $12^2 \stackrel{?}{=} 9^2 + 10^2$; $144 \stackrel{?}{=} 81 + 100$; $144 < 181$, so the △ is acute. **12.** $17^2 \stackrel{?}{=} 8^2 + 15^2$; $289 \stackrel{?}{=} 64 + 225$; $289 = 289$, so the △ is right. **13.** $10^2 \stackrel{?}{=} 5^2 + 6^2$; $100 \stackrel{?}{=} 25 + 36$; $100 > 61$, so the △ is obtuse. **14.** The ratio of the length of the longer leg to the length of the shorter leg is $\sqrt{3} : 1$. The ratio of the length of the hypotenuse to the length of the shorter leg is $2 : 1$. Find the length of the shorter leg first, then use it to find the remaining lengths.
15. $m\angle BPC = 90 - m\angle APB = 90 - 50 = 40$
16. $m\widehat{AB} = m\angle APB = 50$ **17.** $m\widehat{ADC} = 360 - m\widehat{CA} = 360 - m\angle CPA = 360 - 90 = 270$ **18.** $m\widehat{ADB} = 360 - m\widehat{AB} = 360 - m\angle APB = 360 - 50 = 310$
19. $\frac{120}{360} \cdot 2\pi(5) = \frac{10}{3}\pi$; the length is $\frac{10}{3}\pi$ in.
20. $\frac{90}{360} \cdot 2\pi(3) = 1.5\pi$; the length is 1.5π cm. **21.** The central ∠ of the shaded region measures 180 − 80, or 100, so $A = \frac{100}{360} \cdot \pi(6)^2 = 10\pi$, or about 31.42 m^2.
22. The ∠ bis. of the 120° vertex ∠ of the isosc. △ creates two 30°-60°-90° ◬ whose hyp. is 7 ft. The height then is 3.5 ft, and the longest side of the isosc. △ is $2(3.5\sqrt{3})$ or $7\sqrt{3}$ ft. So, $A = \frac{120}{360} \cdot \pi(7)^2 - \frac{1}{2}(7\sqrt{3})(3.5) = \frac{5880}{360}\pi - 12.25\sqrt{3}$, or about 30.10 ft^2. **23.** Check students' work. **24.** $A = A_{\triangle} + A_{\text{semicircle}} = \frac{1}{2}(12)(12) + \frac{1}{2}\pi(6)^2 = 72 + 18\pi$; the area is $(72 + 18\pi)$ cm^2. **25.** $A = A_{▱} - A_{\text{circle}} = (6 + 3)(6) - \pi(3)^2 = 54 - 9\pi$; the area is $(54 - 9\pi)$ m^2. **26.** The gates go down 3 times per hour for 3 minutes each time for a total of $3 \cdot 3$, or 9 min each hour. Since 60 min are in an hour, the probability is $\frac{9}{60}$, or $\frac{3}{20}$.

STANDARDIZED TEST PREP *page 413*

1. From the last sentence in the first paragraph, the answer is choice B. **2.** Solve the equation wire length = $QP + QR + RS$ for QP. The answer is choice F.
3. Solve the equation $AB = TP + CD + VS$ for TP, using the fact that opp. sides of a rectangle are ≅. The answer is choice C. **4.** Though $\overline{QT}$ is a leg of a rt. △, nothing can be determined about the measures of either of the other two ∠s of $\triangle PQT$. The answer is choice H.
5. Subtract QT from CT, which must be done to solve for CQ in choice D. The answer is choice D. **6.** By symmetry, since $ACTP$ has 4 rt. ∠s, then $DBSV$ also has 4 rt. ∠s, so the quadrilateral is a rectangle. **7.** Mark two level points on the wall where you want the top corners of the picture. The wire is divided into 3 sections that are 10 in., 14 in., and 10 in. long. The △ formed is a 6-8-10 rt. △. The horizontal distance from the corner of the picture is 8 in. and the vertical distance is 9 − 6, or 3 in. From the two points, measure 8 in. towards each other and 3 in. down. **8.** Mark two level points on the wall where you want the widest part of the mirror. The wire is divided into 3 sections that are 9 in., 10 in. and 9 in. long. The △ formed is a rt. △ whose hyp. is 9 and horizontal leg is $\frac{22 - 10}{2}$, or 6 in. By the Pyth. Thm., the distance up from each point is about 6.7 in. So, from the points, measure 6 in. towards each other and 6.7 in. up.