Chapter 6 Quadrilaterals

DIAGNOSING READINESS page 286

1. Since corr. $\angle$s of $\parallel$ lines are $\cong$, then by def. of $\cong$, $x + 9 = 2x - 21; -x + 9 = -21; -x = -30; x = 30$. **2.** Since same-side int. $\angle$s of $\parallel$ lines are suppl., then $(3x - 14) + (2x - 16) = 180; 5x - 30 = 180; 5x = 210; x = 42$. **3.** Since vert. $\angle$s are $\cong$, the labeled values can represent corr. $\angle$s of $\parallel$ lines, which are also $\cong$. So, by def. of $\cong$, $5x = 176 - 3x; 8x = 176; x = 22$. **4.** Since alt. int. $\angle$s are $\cong$, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$. **5.** By the Triangle Angle-Sum Thm., $3x + 2x + 4x = 180; 9x = 180$, so $x = 20$. $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ if corr. $\angle$s are $\cong$. $m\angle A = 3x + 18 = 3(20) + 18 = 60 + 18 = 78$. $m\angle DCF = 4x = 4(20) = 80$. Since $78 \neq 80$, corr. $\angle$s are not $\cong$, so the lines are not $\parallel$. **6.** The lines are $\parallel$ if the same-side int. $\angle$s are suppl. Since vert. $\angle$s are $\cong$, $2x + 11 = 3x - 9; 20 = x$. $(3x - 9) + (6x + 9) = 9x = 9(20) = 180$. Since the same-side int. $\angle$s are suppl., the lines are parallel. **7.** The slopes are the same, so the lines are $\parallel$. **8.** Since the product of the slopes is -1, the lines are $\perp$. **9.** $2x - 3y = 1$ is equivalent to $y = \frac{2}{3}x - \frac{1}{3}$ and $3x - 2y = 8$ is equivalent to $y = \frac{3}{2}x - 4$. The slopes are different and their product is not -1, so they are *neither* $\parallel$ nor $\perp$. **10.** Since $\parallel$ lines have $\cong$ alt. int. $\angle$s, $\angle DAC \cong \angle BCA$, and $\angle BAC \cong \angle CDA$. By the Refl. Prop. of $\cong$, $\overline{AC} \cong \overline{CA}$, and the △s are $\cong$ by ASA. **11.** By the Refl. Prop. of $\cong$, the alt. is $\cong$ to itself, so the △s are $\cong$ by SAS. **12.** Vert. $\angle$s are $\cong$, so the △s are $\cong$ by AAS.

6-1 Classifying Quadrilaterals pages 288–293

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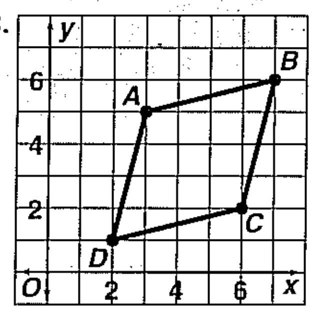
1. 10.8 **2.** 7.8 **3.** 12.7 **4.** $\frac{3}{4}$ **5.** $-\frac{8}{3}$ **6.** 1

Check Understanding 1a. $WXYZ$ is a quad. because it has 4 sides. It is a ▱ because both pairs of opp. sides are $\parallel$. It is a rhombus because all 4 sides are $\cong$. **1b.** Rhombus; a rhombus has all the characteristics of a quadrilateral, ▱, and a figure with all $\cong$ sides. **2.** Slope of $\overline{AB} = \frac{4 - 3}{2 - (-3)} = \frac{1}{5}$; slope of $\overline{BC} = \frac{-1 - 4}{3 - 2} = -5$; slope of $\overline{CD} = \frac{-2 - (-1)}{-2 - 3} = \frac{1}{5}$; slope of $\overline{DA} = \frac{3 - (-2)}{-3 - (-2)} = -5$. Since the slopes of opp. sides are $=$, then opp. sides are $\parallel$, so the quad. is a ▱. Since the product of adjacent slopes is -1, adjacent sides are $\perp$, so the ▱ is a rectangle. $AB = \sqrt{(-3 - 2)^2 + (3 - 4)^2} = \sqrt{25 + 1} = \sqrt{26}$; $BC = \sqrt{(3 - 2)^2 + (-1 - 4)^2} = \sqrt{1 + 25} = \sqrt{26}$; $CD = \sqrt{(3 - (-2))^2 + (-1 - (-2))^2} = \sqrt{25 + 1} = \sqrt{26}$; $AD = \sqrt{(-3 - (-2))^2 + (3 - (-2))^2} = \sqrt{1 + 25} = \sqrt{26}$. Since all sides are $=$, the rectangle is a square. **3.** Since it is a rhombus, all sides are $=$, so $5a + 4 = 3a + 8; 2a + 4 = 8; 2a = 4; a = 2$. Also, $4b - 2 = 3b + 2; b - 2 = 2; b = 4$. $LN = ST = NT = SL = 3b + 2 = 3(4) + 2 = 12 + 2 = 14$

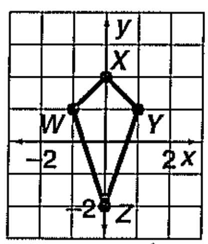
Exercises 1. Opp. sides are $\parallel$, so it's a ▱. The sides are $\perp$, so the ▱ is a rectangle. The sides are $=$, so the rect. is a square. **2.** Opp. sides are $\parallel$, so it is a ▱. **3.** Only one pair of opp. sides is $\parallel$, so it is a trapezoid. **4.** Opp. sides are $\parallel$, so it is a ▱. All sides are $=$, so the ▱ is a rhombus. **5.** Opp. sides are not $=$ but 2 pairs of adj. sides are $=$, so it is a kite. **6.** Only one pair of opp. sides is $\parallel$, so it is a trapezoid. The non-$\parallel$ sides are $=$, so it is an isosc. trap. **7.** Opp. sides are $\parallel$, so it is a ▱, and all sides are $=$, so the most precise name is rhombus. **8.** Opp. sides are $\parallel$, so it is a ▱. **9.** Opp. sides are $\parallel$, so it is a ▱, and all sides are $=$, so the most precise name is rhombus. **10.** Opp. sides are $\parallel$, so it is a ▱. All $\angle$s are rt. $\angle$s, so the most precise name is rectangle. **11.** Opp. sides are not $=$, but 2 pair of adj. sides are $=$, so it is a kite. **12.** Only one pair of opp. sides is $\parallel$, so it is a trapezoid. The non-$\parallel$ sides are $=$, so it is an isosc. trap.

13.



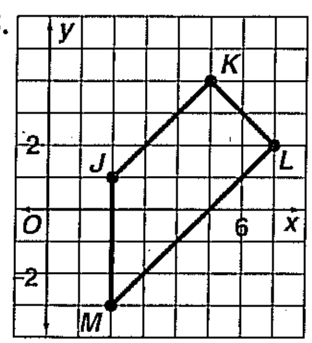
From the grid, the slope of $\overline{AB}$ = slope of $\overline{DC} = \frac{1}{4}$ and the slope of $\overline{AD}$ = slope of $\overline{BC} = 4$, so opp. sides are $\parallel$, thus it is a ▱. $AB = \sqrt{(7 - 3)^2 + (6 - 5)^2} = \sqrt{(4)^2 + (1)^2} = \sqrt{17}$; $BC = \sqrt{(7 - 6)^2 + (6 - 2)^2} = \sqrt{1 + 16} = \sqrt{17}$; $CD = \sqrt{(2 - 6)^2 + (1 - 2)^2} = \sqrt{16 + 1} = \sqrt{17}$; $AD = \sqrt{(2 - 3)^2 + (1 - 5)^2} = \sqrt{1 + 16} = \sqrt{17}$. All sides are $=$, so the ▱ is a rhombus.

14.

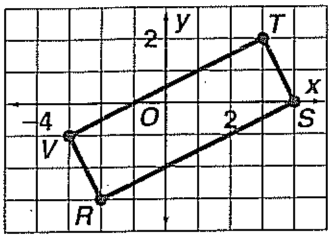


By examining the slopes from the grid, opp. sides are not $\parallel$, so it is not a ▱. No sides are $\parallel$, so it is not a trapezoid. $WX = YX = \sqrt{2}$ and $WZ = YZ = \sqrt{1^2 + 3^2} = \sqrt{10}$, indicating that two pairs of adj. sides are $=$, so the figure is a kite.

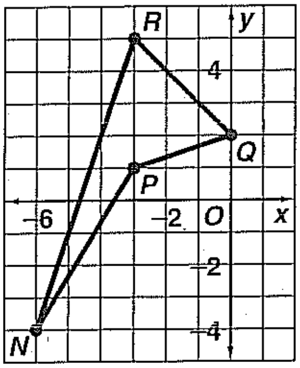
15.



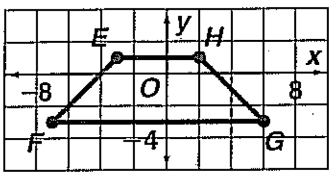
The slope of $\overline{JK} = 1 =$ the slope of $\overline{ML}$. $\overline{JM}$ is vertical and $\overline{KL}$ is not, so only one pair of sides is $\parallel$ and the non-$\parallel$ sides are not $=$ since $JM = 4$ and $KL \neq 4$. The figure is a trapezoid.

16. 

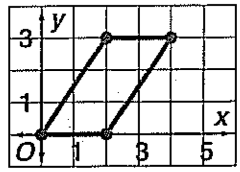
The slope of $\overline{VR}$ = the slope of $\overline{TS} = -2$. The slope of $\overline{TV}$ = the slope of $\overline{SR} = \frac{3}{6} = \frac{1}{2}$. Both pairs of opp. sides are ∥, so the figure is a ▱. The product of the slopes of adj. sides is −1, so all 4 ∠s are right ∠s. Since all sides are not =, the ▱ is not a square. The ▱ is a rectangle.

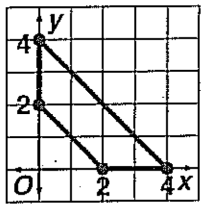
17. 

The slope of $\overline{NP} = \frac{5}{3}$. The slope of $\overline{PQ} = \frac{1}{3}$. The slope of $\overline{QR} = \frac{-3}{3} = -1$. The slope of $\overline{RN} = \frac{-9}{3} = -3$. None of the slopes are the same, so no sides are ∥. The figure is not a ▱.
$NP = \sqrt{(-6+3)^2 + (-4-1)^2} = \sqrt{34}$;
$PQ = \sqrt{(0+3)^2 + (2-1)^2} = \sqrt{10}$;
$QR = \sqrt{(0+3)^2 + (2-5)^2} = \sqrt{18}$;
$RN = \sqrt{(-6+3)^2 + (-4-5)^2} = \sqrt{90}$. Since no sides are =, the figure is a quadrilateral.

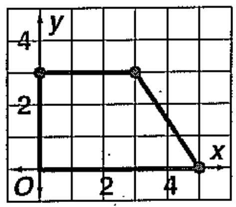
18. 

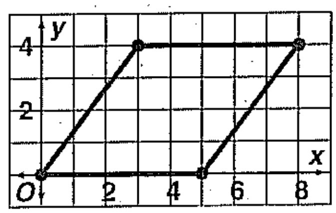
The slope of $\overline{EH}$ = the slope of $\overline{FG} = 0$, so $EFGH$ is a trapezoid. $EF = HG = \sqrt{32}$, or $4\sqrt{2}$, so the trapezoid is isosc. **19.** $AB = BC = 13$, so $x + 2 = 13$; $x = 11$. $CD = x + 12 = 11 + 12 = 23$, so $AD = 23 = y - 6$; $y = 29$. **20.** From the tick marks, $x + 0.5 = 2x - 3.5$; $x = 4$. In a kite, the other pair of sides are also =: $2y - 2.8 = y + 2$; $y = 4.8$. $x + 0.5 = 4 + 0.5 = 4.5$. $2x - 3.5 = 2(4) - 3.5 = 4.5$. $2y - 2.8 = 2(4.8) - 2.8 = 6.8$. $y + 2 = 4.8 + 2 = 6.8$ **21.** From the tick marks, $2y - 5 = y + 1$, so $y = 6$. $y + 1 = 6 + 1 = 7$. $2y - 5 = 2(6) - 5 = 7$. In a kite, the other pair of sides are also =: $3x - 4 = x$; $x = 2$. $x = 3x - 4 = 2$ **22.** $2x + 2 = 4$; $x = 1$. $3x + 1 = 3(1) + 1 = 4$. $11x - 2 = 11(1) - 2 = 9$. So, the sides are 4, 4, 4, 9. **23.** In a rhombus, all sides are =, so $5x = 4x + 3 = 3y = 15$. $x = 3$ and $y = 5$ and all sides are 15. **24.** In a square, all sides are =, so $2y - 5 = y - 1$; $y = 4$. All sides are $y - 1 = 4 - 1 = 3$. $2x - 7 = 3$, so $x = 5$. **25.** Since the trap. is isosc., $DE = GF$, so $a - 4 = 11$; $a = 15$. $EF = a = 15$. $DE = FG = 11$. $DG = 2a + 2 = 2(15) + 2 = 32$. Since ∥ lines create same-side int. ∠s that are suppl., and since the suppl. of one of 2 ≅ ∠s is suppl. of the other, $(4c - 20) + c = 180$; $5c - 20 = 200$; $5c = 200$; $c = 40$. $m\angle D = m\angle G = c = 40$. $m\angle E = m\angle F = 4c - 20 = 4(40) - 20 = 140$ **26.** In a rhombus, all sides are =, so $2r - 4 = r + 1$, so $r = 5$. So, all sides are $r + 1 = 5 + 1 = 6$. Since opp. sides are ∥ and since same-side int. ∠s are suppl., $x + (2x + 6) = 180$; $3x = 174$; $x = 58$. $m\angle H = m\angle J = x = 58$. $m\angle K = m\angle I = 180 - 58 = 122$

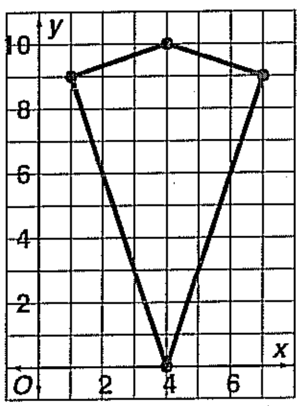
27. rectangle formed by horizontal lines in blue sky abutting edge of picture and smokestack, square window in center, trapezoid shed to left of silos
28. Check students' work.
29. Answers may vary. Sample: Draw a ▱ with no right ∠s and no = adjacent sides.


30. Answers may vary. Sample: Draw an isosc. right △. The midsegment connecting the legs and hypotenuse form the ∥ sides of the trapezoid.


31. Impossible; since one pair of opp. sides is ∥, the same-side int. ∠s are suppl. If one of those ∠s measures 90, the other also measures 90. So, a trapezoid cannot have only one right ∠.

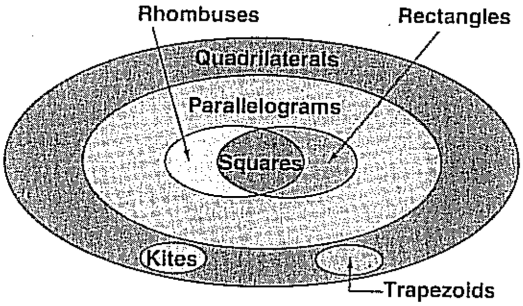
32. Answers may vary. Sample: Draw a rectangle and elongate two sides so they no longer form rt. ∠s with the ∥ sides.


33. Answers may vary. Sample: Draw a ▱ with no right ∠s and all sides =.


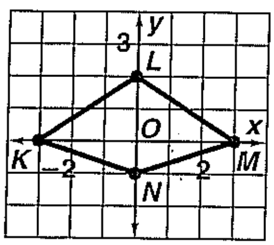
34. Answers may vary. Sample: Draw 2 rt. ∠s that share a hypotenuse so that pairs of = sides are adjacent.


35. A rhombus has 4 ≅ sides, while a kite has 2 pairs of adj. sides ≅, but no opp. sides are ≅. Opp. sides of a rhombus are ∥, while opp. sides of a kite are not ∥.

36.



Rhombuses and rectangles must be inside the ▱ region, while trapezoids must be outside the ▱ region.
37. True; a square has all the properties of a rectangle and a rhombus, so a square is both a rectangle and a rhombus. **38.** False; a ▱ has 2 pairs of ∥ sides, and a trapezoid only has one pair of ∥ sides. **39.** False; a rhombus has ≅ opp. sides, and a kite does not have ≅ opp. sides. **40.** True; a square has all the properties of a ▱, so some ▱s are squares. **41.** False; counterexamples include a kite and a trapezoid. So, not every quadrilateral is a ▱. **42.** False; a rhombus does not require right ∠s. A square, however, does have all the properties of a rhombus, so all squares are rhombuses. **43.** Rhombus; all 4 sides are ≅ because they come from the same cut. **44.** Check students' work. **45.** Check students' work. **46.** Check students' sketches. Possible answers: isosc. trapezoids, trapezoids having only 2 ≅ adj. sides **47.** Check students' sketches. Possible answers: ▱, rectangle, rhombus, square **48.** Check students' sketches. Possible answers: rectangle, square **49.** Check students' sketches. Possible answers: trapezoid, isosc. trapezoid, kite, square, rhombus **50.** A trapezoid has only one pair of ∥ sides. A ▱ has 2 pairs of ∥ sides. **51.** ▱, rectangle, kite **52.** ▱, rhombus **53.** ▱, rhombus, square **54.** kite, ▱, rhombus **55a.** The fourth point can be any point on the y-axis except (0, 0) where it would become an isosc. △, (0, 2) where it shares a point with L, and (0, −2) where it becomes a rhombus. Answers may vary. Sample:



55b. The y-axis is the ⊥ bis. of $\overline{KM}$, so if N is on $\overline{KM}$, then $NM = NK$. So, for the points N mentioned in part ⓐ, $KL = LM$ and $KN = NM$, but $KL \neq KN$. **56.** If the missing side is ∥ to the opp. side, then it is a ▱ that is a rectangle. If the missing side is not ∥ to the opp. side, then it is a trapezoid. **57.** If each of the 2 missing sides is ∥ to its opp. side, then it is a ▱. If opp. sides are also =, then it is a rhombus. If only one pair of opp. sides is ∥, then it is a trapezoid. If the non-∥ sides of the trapezoid are =, then it is an isosc. trap. If no opp. sides are ∥, but the missing sides are =, then it is a kite. **58.** If both missing sides are ∥ to their opp. sides, then it is a ▱. If all sides are =, then it is a rhombus. If only one pair of opp. sides is ∥, it is a trapezoid. If the trapezoid's non-∥ sides are ≅, it is an isosc. trap. **59.** If both pairs of opp. sides are ∥, then the figure is a ▱ that is a rectangle. If the sides are also all =, then the figure is a square, which is also a rhombus. If only one pair of opp. sides is ∥, then the figure is a trapezoid. If no pair of opp. sides are =, but if two pairs of adj. sides are =, then the figure is a kite. **60.** All squares are rhombuses, so choice A is true. All squares are ▱s, so choice B is sometimes true and choice D is always true. Trapezoids are never ▱s, so the answer is choice C. **61.** Since a rectangle is a ▱, choices F and G are always true. Sides of a rectangle form right ∠s, so choice H is always true. Only rectangles that are squares have all sides ≅, so the answer is choice I.
62. By def. of rhombus, the answer is choice C.
63. Since a square and a rectangle each have 4 right ∠s and since right ∠s are ≅, eliminate choices F and G. A square has more restrictions than a rectangle that all ▱s do not have. The answer is choice H. **64.** [2] The slope of $\overline{AB}$ is $-\frac{3}{2}$. The slope of $\overline{BC}$ is 1. Since the product of the slopes is not −1, $\overline{AB}$ and $\overline{BC}$ are not ⊥. Since there is one ∠ that is not a right ∠, and since a rectangle requires 4 right ∠s, the figure could not be a rectangle. [1] incorrect slope OR failure to recognize the information provided by the slopes **65.** $3 + 6 > 8$, $3 + 8 > 6$, and $8 + 6 > 3$, so since the sum of the lengths of any 2 sides is greater than the third side, 8, 6, and 3 could be the lengths of a △. **66.** $7 + 20 > 5$, $20 + 5 > 7$, but $5 + 7$ is not > 20. So, 5, 20, and 7 cannot be the measures of a △. **67.** $8 + 5 > 3$, $8 + 3 > 5$, but $3 + 5$ is not > 8. So, 3, 5, and 8 are not the measures of the sides of a △. **68.** Since $\overline{MN} \cong \overline{SR}$, $SR = MN = 28$ mm. **69.** Since $\overline{VT} \cong \overline{PQ}$, $VT = PQ = 16$ mm. **70.** Since $\overline{ST} \cong \overline{MQ}$, then $ST = MQ = 12$ mm. **71.** Since $\angle Q \cong \angle T$, $m\angle Q = m\angle T = 130$. The sum of the interior ∠s of a quadrilateral is 360, so $m\angle S = m\angle M = 360 - (130 + 90 + 58) = 360 - 278 = 82$. **72.** $\angle V \cong \angle P$, so $m\angle V = m\angle P = 90$. **73.** $\angle R \cong \angle N$, so $m\angle R = m\angle N = 58$. **74.** The slope will be −3 and the y-intercept is 4, so the equation is $y = -3x + 4$.

6-2 Properties of Parallelograms *pages 294–301*

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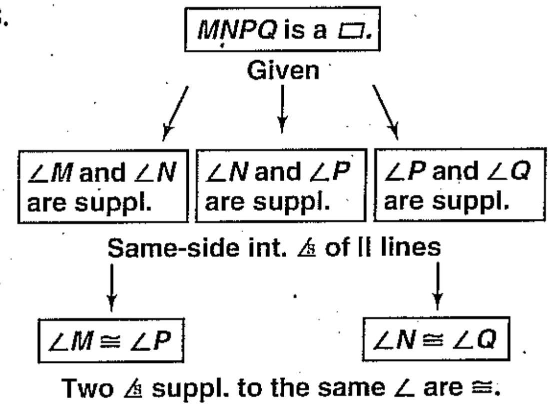
1. ASA **2a.** $\angle HGE$ **2b.** $\angle GHE$ **2c.** $\angle HEG$ **2d.** $\overline{GH}$ **2e.** $\overline{HE}$ **2f.** $\overline{EG}$ **3.** They are ∥.

Check Understanding **1.** Yes; by the Converse of the Same-Side Int. Angles Thm., both pairs of opp. sides are ∥. **2.** Since opp. ∠s of a ▱ are ≅, $6y + 4 = 3y + 37$; $3y = 33$; $y = 11$; $m\angle E = 6y + 4 = 6(11) + 4 = 66 + 4 = 70 = m\angle G$; $m\angle F = 180 - 70 = 110 = m\angle H$. **3.** Since the diag. of a ▱ bis. each other, $a = b + 2$ and $b + 10 = 2a - 8$. Substitute $b + 2$ for a in the second equation: $b + 10 = 2(b + 2) - 8$; $b + 10 = 2b + 4 - 8$; $b + 10 = 2b - 4$; $b = 14$. $a = b + 2 = 14 + 2 = 16$ **4.** From Thm. 6-4, $HG = GF = FE = 2.5$. From the diagram, $EH = HG + GF + FE = 2.5 + 2.5 + 2.5 = 7.5$.

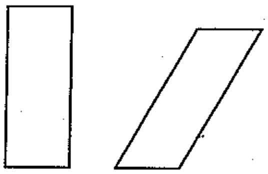
Exercises **1.** Since opp. sides of a ▱ are ∥, same-side int. ∠s are suppl.: $x + 53 = 180$; $x = 127$. **2.** Since opp. sides of a ▱ are $\parallel$, same-side int. ∡ are suppl.: $x + 113 = 180$; $x = 67$. **3.** Since opp. sides of a ▱ are $\parallel$, same-side int. ∡ are suppl.: $x + 104 = 180$; $x = 76$. **4.** Since opp. sides of a ▱ are $\parallel$, same-side int. ∡ are suppl.: $x + 56 = 180$; $x = 124$. **5.** Since opp. sides of a ▱ are $\parallel$, same-side int. ∡ are suppl.: $x + 80 = 180$; $x = 100$. **6.** Since opp. sides of a ▱ are $\parallel$, same-side int. ∡ are suppl.: $x + 62 = 180$; $x = 118$. **7.** Since opp. sides of a ▱ are $\cong$, $4x + 5 = 8$; $4x = 3$, $x = \frac{3}{4}$. **8.** Since opp. sides of a ▱ are $\cong$, $3x + 2 = 14$; $3x = 12$; $x = 4$. **9.** Since opp. sides of a ▱ are $\cong$, $7x - 2 = 5x + 6$; $2x - 2 = 6$; $2x = 8$; $x = 4$. **10.** Since opp. sides of a ▱ are $\cong$, $2x + 4 = 10$, so $x = 3$. $GM = FE = 10$ and $FG = EM = 7x - 1 = 7(3) - 1 = 20$ **11.** Since opp. sides of a ▱ are $\cong$, $x - 3.5 = 18.5$, so $x = 22$. $AB = DC = x + 1.6 = 22 + 1.6 = 23.6$ and $BC = AD = 18.5$ **12.** Since opp. ∡ of a ▱ are $\cong$, $6a + 10 = 130$; $6a = 120$; $a = 20$. **13.** Since opp. ∡ of a ▱ are $\cong$, $3a + 50 = 104$; $3a = 54$; $a = 18$. **14.** Since opp. ∡ of a ▱ are $\cong$, $4a - 4 = 2a + 30$; $2a - 4 = 30$; $2a = 34$; $a = 17$. **15.** Since opp. ∡ of a ▱ are $\cong$, $4a - 12 = 3a$; $a = 12$. $m\angle Q = m\angle S = 3a = 3(12) = 36$. $m\angle R = m\angle P = 180 - 36 = 144$ **16.** Since opp. ∡ of a ▱ are $\cong$, $20a + 30 = 17a + 48$; $3a + 30 = 48$; $a + 10 = 16$; $a = 6$. $m\angle J = m\angle H = 5a = 5(6) = 30$. $m\angle I = m\angle K = 180 - 30 = 150$ **17.** Since the diagonals of a ▱ bis. each other, $2x = y + 4$ and $y = x + 2$, so substitute $x + 2$ for y in the first equation: $2x = (x + 2) + 4$; $x = 6$. $y = x + 2 = 6 + 2 = 8$ **18.** Since diagonals of a ▱ bis. each other, $2x = y + 3$ and $y = x + 2$, so substitute $x + 2$ for y in the first equation: $2x = (x + 2) + 3$; $x = 5$. $y = x + 2 = 4 + 2 = 7$ **19.** Since the diagonals of a ▱ bis. each other, $2y = 3x - 1$ and $y = x + 3$, so substitute $x + 3$ for y in the first equation: $2(x + 3) = 3x - 1$; $2x + 6 = 3x - 1$; $x = 7$. $y = x + 3 = 7 + 3 = 10$ **20.** Since the diagonals of a ▱ bis. each other, $3x = 2y$ and $2x = y + 3$, so $y = 2x - 3$. Substitute $2x - 3$ for y in the first equation: $3x = 2(2x - 3)$; $3x = 4x - 6$; $x = 6$. $y = 2x - 3 = 2(6) - 3 = 9$ **21.** The diagonals of a ▱ bis. each other, so $6y = 8x$ and $2y = 2x + 2$, so $y = x + 1$. Substitute $x + 1$ for y in the first equation: $6(x + 1) = 8x$; $6x + 6 = 8x$; $x = 3$. $y = x + 1 = 3 + 1 = 4$ **22.** By Thm. 6-4, $ED = EF = 12$. $FD = EF + ED = 12 + 12 = 24$ **23.** Pick 4 equally spaced lines on the paper. Place the paper so that the first button is on the first line and the last button is on the fourth line. Draw a line between the first and last buttons. The remaining buttons should be placed where the drawn line crosses the middle 2 $\parallel$ lines on the paper. **24.** By Thm. 6-4, $ZU = XT = 3$. **25.** By Thm. 6-4, $XZ = XT = 3$. **26.** By Thm. 6-4, $UZ = XZ = XT = 3$, so $XU = XR + ZU = 3 + 3 = 6$. **27.** By Thm. 6-4, $UZ = XZ = XT = 3$, so $TZ = XR + XT = 3 + 3 = 6$. **28.** By Thm. 6-4, $UZ = XZ = XT = 3$, so $TU = TX + XR + ZU = 3 + 3 + 3 = 9$. **29.** By Thm. 6-4, $XV = WY = 2.25$. **30.** By Thm., 6-4, $YX = WY = 2.25$. **31.** By Thm., 6-4, $YX = XV = WY = 2.25$, so $YV = YX + XV = 2.25 + 2.25 = 4.5$. **32.** By Thm. 6-4, $YX = XV = WY = 2.25$, so $WX = YX + WY = 2.25 + 2.25 = 4.5$. **33.** By Thm. 6-4, $YX = XV = WY = 2.25$, so $WV = XV + YX + WY = 2.25 + 2.25 + 2.25 = 6.75$. **34.** $CD = AB = BC - 5$ and $AD = BC$. $P = AB + BC + CD + AD = (BC - 5) + BC + (BC - 5) + BC = 48$; $4BC - 10 = 48$; $4BC = 58$; $BC = 14.5$. So, $AB = CD = BC - 5 = 14.5 - 5 = 9.5$, or 9.5 in., and $AD = BC = 14.5$, or 14.5 in. **35.** $BC = AD = 2AB + 7$ and $CD = AB$. $P = AB + BC + CD + AD = 92$; $AB + (2AB + 7) + AB + (2AB + 7) = 6AB + 14 = 92$; $6AB = 78$; $CD = AB = 13$, or 13 cm, and $AD = BC = 2AB + 7 = 2(13) + 7 = 33$, or 33 cm. **36a.** Opp. sides of a ▱ are $\parallel$: $\overline{DC}$. **36b.** Opp. sides of a ▱ are $\parallel$: $\overline{AD}$. **36c.** If lines are $\parallel$, then alt int. ∡ are $\cong$: $\cong$. **36d.** Reflexive **36e.** 2 ∡ and an included side are $\cong$: ASA. **36f.** Corr. parts of $\cong$ △ are $\cong$; CPCTC. **37a.** Given **37b.** Opp. sides of a ▱ are $\parallel$, by def. of ▱. **37c.** If 2 lines are $\parallel$, then alt. int. ∡ are $\cong$. **37d.** If 2 lines are $\parallel$, then alt. int. ∡ are $\cong$. **37e.** Reflexive Prop. of $\cong$ **37f.** 2 ∡ and an included side are $\cong$: ASA. **37g.** 2 ∡ and an included side are $\cong$: ASA. **37h.** Corr. parts of $\cong$ △ are $\cong$: CPCTC. **37i.** CPCTC

38.



39. Since alt. int. ∡ of $\parallel$ lines are $\cong$, $m\angle 1 = 38$. By the Triangle Angle-Sum Thm., $m\angle 2 = 180 - (110 + 38) = 32$. Since opp. ∡ of a ▱ are $\cong$, $m\angle 3 = 110$. **40.** Since same-side int. ∡ of $\parallel$ lines are suppl., $m\angle 1 + 71 + 28 = 180$, so $m\angle 1 = 180 - (71 + 28) = 81$. Since alt. int. ∡ of $\parallel$ lines are $\cong$, $m\angle 2 = 28$ and $m\angle 3 = 71$. **41.** $m\angle 1 + 85 = 180$, so $m\angle 1 = 95$. By the Triangle Ext. Angle Thm., $m\angle 2 + 48 = 85$, so $m\angle 2 = 37$. Since alt. int. ∡ of $\parallel$ lines are $\cong$, $m\angle 3 = m\angle 2 = 37$. **42.** Only if the horizontal lines are $\parallel$ can Brian assume that $QS = SV$. Since they are not marked as $\parallel$, the horizontal lines may not be $\parallel$. **43.** Since same-side int. ∡ of $\parallel$ lines are suppl., if one $\angle$ is x and the other is $9x$, then $x + 9x = 180$, so $x = 18$. One $\angle$ is 18 and the other is 9(18), or 162. **44.** Since same-side int. ∡ of $\parallel$ lines are suppl., $2x + x = 180$, so $x = 60$. **45.** Since same-side int. ∡ of $\parallel$ lines are suppl., $y + 3y = 180$, so $y = 45$. Since opp. ∡ of a ▱ are $\cong$, $y = 3x$; $45 = 3x$; $x = 15$. **46.** Since same-side int. ∡ of $\parallel$ lines are suppl., $x + 71 = 180$, so $x = 109$. Since opp. ∡ of a ▱ are $\cong$, $y + 21 = x$, so $y + 21 = 109$ and $y = 88$. Since opp. ∡ of a ▱ are $\cong$, $z - 5 = 71$, so $z = 76$. **47.** Since opp. ∡ of a ▱ are $\cong$, $2x - 1 = x + 24$; $x = 25$. Since same-side int. ∡ of $\parallel$ lines are suppl., $(y + 16) + (x + 24) = 180$, so $y + 16 + 25 + 24 = 180$; $y = 115$. **48.** Since opp. sides of a ▱ are $\cong$, $x = y$ and $x + y = 3y - 6$. Substitute x for y in the second equation: $x + x = 3x - 6$; $-x = -6$; $x = 6$. $x = y = 6$ **49.** Since opp. sides of a ▱ are $\cong$, $2x - 12 = x - 2$, so $x = 10$. Also, $y = 8 - y$;

$2y = 8; y = 4$. **50.** Since diagonals of a ▱ bis. each other, $2x - 5 = x + 7; x - 5 = 7; x = 12$. Also, $6y + 1 = 4y + 9$; $2y + 1 = 9; 2y = 8; y = 4$. **51.** Since the diagonals of a ▱ bis. each other, $AE = EC$, so $3x + y = 2x + y; x = 0$. $AC = AE + EC$; $4x + 10 = (3x + y) + (2x + y)$; $4(0) + 10 = 3(0) + y + 2(0) + y; 10 = 2y; y = 5$
52. Since the diagonals of a ▱ bis. each other, $2x = 3y$ and $x + 8 = 2y + 5$, so $x = 2y - 3$. Substitute $2y - 3$ for x in the first equation: $2(2y - 3) = 3y; 4y - 6 = 3y$; $y = 6. x = 2y - 3 = 2(6) - 3 = 9$ **53.** The opp. ∠s in a ▱ are ≅, so they have = measures. Consecutive ∠s are suppl., so the sum of the given ∠ with a consecutive ∠ is 180. One of the other ∠s will have the same measure as the given ∠. The other 2 ∠s will be suppl. to the given ∠. **54a.** Draw a ▱ and then "squish" it for the second one. Answers may vary. Sample:



54b. No; the corr. sides may be ≅, but the ∠s may not be. **55a.** Given **55b.** Both pairs of opp. sides of a quad. are ≅, by def. of ▱. **55c.** Opp. sides of a ▱ are ≅. **55d.** Trans. Prop. of ≅ **55e.** If 2 lines are ∥ to the same line, then they are ∥ to each other. **55f.** If 2 lines are ∥, then the corr. ∠s are ≅. **55g.** Trans. Prop. of ≅ **55h.** 2 ∠s and a nonincluded side are ≅: AAS. **55i.** CPCTC **56.** Answers may vary. Sample: ① $LENS$ and $NBTH$ are ▱s. (Given) ② $\angle ELS \cong \angle ENS$ and $\angle GTH \cong \angle GNH$ (Opp. ∠s of a ▱ are ≅.) ③ $\angle ENS \cong \angle GNH$ (Vert. ∠s are ≅.) ④ $\angle ELS \cong \angle GTH$ (Trans. Prop. of ≅) **57.** Answers may vary. Sample: In ▱s $LENS$ and $NGTH$, $\overline{GT} \parallel \overline{EH}$ and $\overline{EH} \parallel \overline{LS}$ by the def. of ▱. Therefore, $\overline{LS} \parallel \overline{GT}$ because if 2 lines are ∥ to the same line, then they are ∥ to each other. **58.** Answers may vary. Sample: ① $LENS$ and $NGTH$ are ▱s. (Given) ② $\angle GTH \cong \angle GNH$ (Opp. ∠s of a ▱ are ≅.) ③ $\angle ENS \cong \angle GNH$ (Vert. ∠s are ≅.) ④ $\overline{EL} \parallel \overline{NS}$ (Opp. sides of a ▱ are ≅.) ⑤ $\angle LEN$ is suppl. to $\angle ENS$. (Same-side int. ∠s of ∥ lines are suppl.) ⑥ $\angle LEN \cong \angle GTH$ (Trans. Prop. of ≅) ⑦ $\angle E$ is suppl. to $\angle T$. (Suppl. of ≅ ∠s are suppl.) **59.** Answers may vary. Sample: In ▱$RSTW$ and ▱$XYTZ$, $\angle R \cong \angle T$ and $\angle X \cong \angle T$ because opp. ∠s of a ▱ are ≅. Then, $\angle R \cong \angle X$ by the Trans. Prop. of ≅. **60.** In ▱$RSTW$ and ▱$XYTZ$, $\overline{XY} \parallel \overline{TW}$ and $\overline{RS} \parallel \overline{TW}$ by the def. of a ▱. Then $\overline{XY} \parallel \overline{RS}$ because if 2 lines are ∥ to the same line, then they are ∥ to each other. **61.** $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$ by def. of ▱. $\angle 2 \cong \angle 3$ and $\angle 1 \cong \angle 4$ because if 2 lines are ∥, then alt. int. ∠s are ≅. $\angle 3 \cong \angle 4$ because if 2 ∠s are each ≅ to 2 ≅ ∠s, then they are ≅ to each other. By def. of bis., $\overline{AC}$ bisects $\angle DCB$. **62a.** Given: 2 sides and the included ∠ of ▱$ABCD$ are ≅ to the corr. parts of ▱$WXYZ$. Let $\angle A \cong \angle W$, $\overline{AB} \cong \overline{WX}$, and $\overline{AD} \cong \overline{WZ}$. Since opp. ∠s of a ▱ are ≅, $\angle A \cong \angle C$ and $\angle W \cong \angle Y$. Thus, $\angle C \cong \angle Y$ by the Trans. Prop. of ≅. Similarly, opp. sides of a ▱ are ≅, thus $\overline{AB} \cong \overline{CD}$ and $\overline{WX} \cong \overline{ZY}$. Using the Trans. Prop. of ≅, $\overline{CD} \cong \overline{ZY}$. Since opp. sides of a ▱ are ∥ and same-side int. ∠s are suppl., $\angle A$ is suppl. to $\angle D$, and $\angle W$ is suppl. to $\angle Z$. Suppls. of ≅ ∠s are ≅, thus $\angle D \cong \angle Z$. The same can be done to prove $\angle B \cong \angle X$. Therefore, since all corr. ∠s and sides are ≅, then ▱$ABCD \cong$ ▱$WXYZ$. **62b.** No; opp. ∠s and sides are not necessarily ≅ in a trapezoid. **63.** Since opp. sides of a ▱ are ≅, $2x = x + 5$, so $x = 5$ and $JM = 2x = 2(5) = 10$. **64.** From Exercise 64, $x = 5$, so $ML = JK = 3x - 4 = 3(5) - 4 = 11$. **65.** Since opp. ∠s of a ▱ are ≅, $9y = 7y + 28; 2y = 28; y = 14. m\angle L = 7y + 28 = 7(14) + 28 = 126$ **66.** From Exercise 65, $y = 14$, so $m\angle J = 9y = 9(14) = 126$. **67.** In a ▱ there are 2 pairs of ≅ ∠s, so the missing ∠ measures 160. **68.** $180 - 32 = 148$
69. Consecutive ∠s are same-side int. ∠s of ∥ lines, so they are suppl.: $(x + 5) + (4x - 10) = 180; 5x - 5 = 180$; $5x = 185; x = 37. x + 5 = 37 + 5 = 42$ and $4x - 10 = 4(37) - 10 = 138$. Since $42 < 138$, the measure of the smaller ∠ is 42. **70.** Since both pairs of opp. sides are ∥, the quad. is a ▱. Since all sides are =, the ▱ is a rhombus. **71.** Since both pairs of opp. sides of the quad. are ∥, it is a ▱. **72.** Any information leading to the conclusion that $\angle ACD$ and $\angle ACB$ are rt. ∠s is needed. Answers may vary. Sample: $\overline{AC} \perp \overline{DB}$ **73.** Since alt. int. ∠s of ∥ lines are ≅, $m\angle 1 = 49$. **74.** Since same-side int. ∠s of ∥ lines are suppl., $m\angle 2 = 180 - 49 = 131$. **75.** Since corr. ∠s of ∥ lines are ≅, $m\angle 3 = 49$. **76.** $m\angle 4 = 180 - 49 = 131$

READING MATH *page 302*

① $LENS$ is a ▱ and $NGTH$ is a ▱. (Given)
② $\angle L \cong \angle N$, $\angle N \cong \angle T$ (Opp. ∠s of a ▱ are ≅.)
③ $\angle L \cong \angle T$ (Trans. Prop. of ≅)

6-3 Proving That a Quadrilateral Is a Parallelogram *pages 303–310*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1. $\left(\frac{5}{2}, \frac{3}{2}\right), \left(\frac{5}{2}, \frac{3}{2}\right)$; they bisect each other. **2.** Slope of $\overline{BC} = \frac{1}{3}$, slope of $\overline{AD} = \frac{1}{3}$. The slopes are =. **3.** Yes; they are vertical lines. **4.** ▱

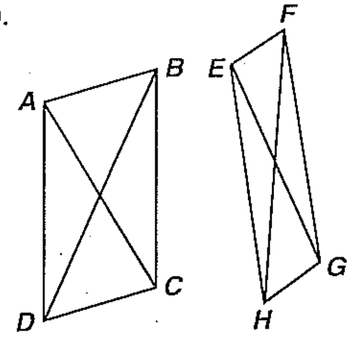
Investigation **1.** The slope of one pair of opp. sides is $\frac{\text{rise}}{\text{run}} = \frac{4}{-1} = -4$. The slope of the other pair of opp. sides is $\frac{\text{rise}}{\text{run}} = \frac{2}{5}$. Since the slopes of opp. sides are =, both pairs of opp. sides are ∥, so the quad. is a ▱. **2.** If the diagonals of a quad. bisect each other, then the quad. is a ▱.
3. The slope of both new sides is $\frac{\text{rise}}{\text{run}} = \frac{2}{-2} = -1$, so it is a ▱. **4.** If a quad. has 1 pair of ≅ and ∥ sides, then the quad. is a ▱.

Check Understanding **1.** If $\overline{QP}$ and $\overline{RS}$ are both ∥ and ≅, then $QPSR$ is a ▱. The same-side int. ∠s must be suppl. for $\overline{QP}$ to be ∥ to $\overline{RS}$, so $(a + 40) + a = 180; 2a + 40 = 180; a + 20 = 90; a = 70$. The opp. sides must be =, so $3c - 3 = c + 1; 2c - 3 = 1; 2c = 4; c = 2$. **2a.** Yes; a pair of opp. sides are ∥ and ≅, so by Thm. 6-6, it is a ▱. **2b.** No; the diagonals do not necessarily bisect each other, so the figure could be a trapezoid. **3.** Once in place, both rulers show the direction and remain ∥. Keep the second ruler in place and move the first ruler to get the compass reading.

Exercises **1.** Since the diagonals of a ▱ bis. each other, $4x - 5 = 3x$, so $x = 5$. **2.** Since the diagonals of a ▱ bis. each other, $2x = 6$, so $x = 3$. Also, $2y - 1 = y + 3$, so $y = 4$. **3.** Since the diagonals of a ▱ bis. each other, $3x = x + 3.2$, so $x = 1.6$. Also, $3y = y + 2$, so $y = 1$. **4.** If a pair of opp. sides is both ≅ and ∥, then a quad. is a ▱, so $6x - 4 = 3x + 1; 3x = 5; x = \frac{5}{3}$. **5.** Since alt. int. ∠s are ≅, $\overline{DC} \parallel \overline{AB}$, so if $\overline{DC} \cong \overline{AB}$, then the quad. is a ▱. $5x - 8 = 2x + 7; 3x - 8 = 7; 3x = 15; x = 5$; the quad. is a ▱ when $x = 5$. **6.** Since a pair of opp. sides are ≅, the quad. is a ▱ if $\overline{AB} \parallel \overline{CD}$, and they are ∥ if the alt. int. ∠s are ≅; $4x - 1 = x + 38; 3x - 1 = 38; 3x = 39; x = 13$. **7.** Yes; since both pairs of opp. sides are ≅, the quad. is a ▱. **8.** No; since only one diagonal is bisected, the quad. could be a kite. **9.** Yes; since both pairs of opp. ∠s are ≅, the quad. is a ▱. **10.** No; since one pair of opp. sides is ≅ and the other pair is ∥, the quad. could be an isosc. trapezoid. **11.** Yes; since alt. int. ∠s are ≅, both pairs of opp. sides are ∥, so the quad. is a ▱ by def. of ▱. **12.** Yes; since one pair of opp. sides is both ∥ and ≅, the quad. is a ▱. **13.** Yes; both pairs of opp. sides are ∥, so it is a ▱ by def. of ▱. **14.** No; since alt. int. ∠s are not necessarily ≅, the figure could be a trapezoid. **15.** No; the quad. could be a kite. **16.** It remains a ▱ because the shelves and connecting pieces remain ∥ and ≅.

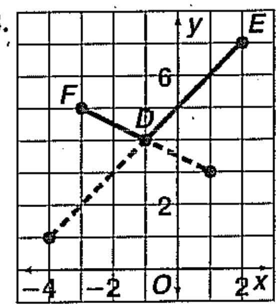
17a. By Thm. 6-5: bisect. **17b.** $\overline{XR}$ **17c.** $\triangle XYR$ **17d.** ASA **17e.** alt. int. **18.** A conditional for Thm. 6-1 is, if a quad. is a ▱, then the opp. sides are ≅. A conditional for Thm. 6-7 is, if both pairs of opp. sides of a quad. are ≅, then the quad. is a ▱. A biconditional is, opp. sides of a quad. are ≅ if and only if the quad. is a ▱. **19a.** Dist. Prop. **19b.** Both sides are divided by 2: Div. Prop. of Eq. **19c.** $\overline{AD} \parallel \overline{BC}, \overline{AB} \parallel \overline{DC}$ **19d.** If same-side int. ∠s are suppl., the lines are ∥. **19e.** Def. of ▱ **20.** Yes; Thm. 6-8: Both pairs of opp. ∠s are ≅. **21.** No; no thm. states that if a diag. bis. opp. ∠s, the quad. is a ▱. The figure could be a kite. **22.** Yes; Thm. 6-6: One pair of opp. sides is both ≅ and ∥. **23.** No; the figure could be an isosc. trapezoid. **24.** Yes; Thm. 6-7: Both pairs of opp. sides are ≅. **25.** Yes; Thm. 6-5: The diagonals bis. each other. **26.** Both pairs of opp. sides must be ∥. Same-side int. ∠s are suppl.: $(3x + 10) + (8x + 5) = 180; 11x + 15 = 180; 11x = 165; x = 15$. Same-side int. ∠s are suppl.: $5y + (3x + 10) = 180; 5y + 3(15) + 10 = 180; 5y + 55 = 180; 5y = 125; y = 25$. **27.** Both pairs of opp. sides are ≅: $3y - 9 = 2y + 2; y - 9 = 2; y = 11$. $3x + 6 = y + 4; 3x + 6 = 11 + 4; 3x + 6 = 15; x + 2 = 5; x = 3$ **28.** One pair of opp. sides is both ∥ and ≅. Alt. int. ∠s are ≅: $4a - 33 = 2a + 15; 2a - 33 = 15; 2a = 48; a = 24$. Opp. sides are ≅: $3c - 5 = 2c + 3; c - 5 = 3; c = 8$. **29.** The diagonals bis. each other. $m = 2.6k$ and $m + 9.1 = 4k - 3.5$. Substitute $2.6k$ for m in the second equation: $2.6k + 9.1 = 4k - 3.5; 12.6 = 1.4k; k = 9, m = 2.6k = 2.6(9) = 23.4$.

30.



Have the diagonals bisect each other, but have each pair of diagonals create different sets of ≅ and suppl. ∠s. Answers may vary. Sample: **31.** The sum of the measures of the ∠s of a quad. is 360 and two of the ∠s have a sum of 240, so the remaining 2 ∠s have a sum of 120. Their measures can be 10 and 110, 20 and 100, 30 and 90, 40 and 80, 50 and 70, and 60 and 60. So, there are 6 possibilities and only one (60 and 60) that makes a ▱. The probability is $\frac{1}{6}$. **32.** Since A is 3 units down and 1 unit left of B, D should be 3 units down and 1 unit left of C: $(4, 0)$. **33.** Since B is 3 units up and 1 unit right of A, E should be 3 units up and 1 unit right of C: $(6, 6)$. **34.** Since B is 2 units up and 3 units left of C, F should be 2 units up and 3 units left of A: $(-2, 4)$. **35.** You can show that a quad. is a ▱ if both pairs of opp. sides are ∥, if both pairs of opp. ∠s are ≅, if both pairs of opp. sides are ≅, if diagonals bis. each other, if all consecutive ∠s are suppl., or if one pair of opp. sides are both ∥ and ≅. **36.** Answers may vary. Sample: ① $\triangle TRS \cong \triangle RTW$ (Given) ② $\overline{RS} \cong \overline{WT}$, $\angle SRT \cong \angle WTR$ (CPCTC) ③ $\overline{SR} \parallel \overline{WT}$ (If alt. int. ∠s are ≅, then lines are ∥.) ④ $RSTW$ is a ▱. (If one pair of opp. sides are ∥ and ≅, then it is a ▱. **37.** ① $\overline{AB} \cong \overline{CD}, \overline{AC} \cong \overline{BD}$ (Given) ② $ACDB$ is a ▱. (If opp. sides are ≅, then it is a ▱.) ③ M is the midpoint of $\overline{BC}$. (The diagonals of a ▱ bisect each other.) ④ $\overline{AM}$ is a median. (def. of a median)

38.



$G(-4, 1)$ and $H(1, 3)$ **39.** $m\angle W = 180 - 128 = 52$. The answer is choice C. **40.** $m\angle S = m\angle N = 128$. The answer is choice F. **41.** The first sentence states that the warp and weft form small rectangles. The answer is choice C. **42.** From the last sentence, it forms nonrectangular ▱s. The answer is choice H. **43.** [2] ① $\triangle NRJ \cong \triangle CPT$ (Given) ② $\overline{NJ} \cong \overline{CT}$ (CPCTC) ③ $\overline{NJ} \parallel \overline{TC}$ (Given) ④ $JNTC$ is a ▱. (If opp. sides of a quad. are both ∥ and ≅, then the quad. is a ▱.) **44** [4] **a.** $7x - 11 = 6x; x = 11$ **b.** Yes; $m\angle ABC = 7x - 11 = 7(11) - 11 = 66$ and $m\angle D = 180 - (5x - 7 + 6x) = 180 - (5(11) - 7 + 6(11)) = 180 - 114 = 66$. Since alt int. ∠s are ≅, $\overline{AF} \parallel \overline{DE}$. **c.** Yes; $\overline{BD} \parallel \overline{FE}$ and $\overline{BF} \parallel \overline{DE}$ so $BDEF$ is a ▱ by def. of ▱. **45.** Since opp. sides of a ▱ are ≅, $a + 15 = 23$, so $a = 8$. Since opp. sides of a ▱ are ∥ and since same-side int. ∠s of ∥ lines are suppl., $4h + 2h = 180$; $6h = 180; h = 30$. Since opp. ∠s of a ▱ are ≅, $k = 4h = 4(30) = 120$. **46.** Since opp. sides of a ▱ are ≅, $3m - 12 = m + 7; 2m - 12 = 7; 2m = 19$;

$m = 9.5$. Since opp. sides of a ▱ are ∥ and since same-side int. $\angle$s of ∥ lines are suppl., $8x + 15 + 3x = 180$; $11x + 15 = 180$; $11x = 165$; $x = 15$. **47.** Since opp. sides of a ▱ are ≅, $f - 3 = 8$, so $f = 11$. Since opp. $\angle$s of a ▱ are ≅, $\frac{c}{2} = 102$, so $c = 204$. Since opp. sides of a ▱ are ∥ and since same-side int. $\angle$s of ∥ lines are suppl., $6e + 102 = 180$; $6e = 78$; $e = 13$. **48.** It is given that $\overline{AD} \cong \overline{BC}$ and $\angle DAB \cong \angle CBA$. By the Reflexive Prop. of ≅, $\overline{AB} \cong \overline{BA}$. Thus, $\triangle DAB \cong \triangle CBA$ by SAS, so $\overline{AC} \cong \overline{BD}$ by CPCTC. **49.** Write a conditional using one part as the hypothesis and the other part as the conclusion. Then write its converse. If a quad. is a ▱, then the diagonals bisect each other. If the diagonals of a quad. bisect each other, then it is a ▱. **50.** Write a conditional using one part as the hypothesis and the other part as the conclusion. Then write its converse. If two lines and a transversal form ≅ corr. $\angle$s, then the two lines are ∥. If two lines are ∥, then a transversal forms ≅ corr. $\angle$s. **51.** Write a conditional using one part as the hypothesis and the other part as the conclusion. Then write its converse. If the prod. of the slopes of two nonvertical lines is -1, then they are ⊥. If two nonvertical lines are ⊥, then the prod. of their slopes is -1.

CHECKPOINT QUIZ 1 *page 310*

1. Since same-side int. $\angle$s of ∥ lines are suppl., $m\angle 1 = 180 - 121 = 59$. Since opp. $\angle$s are ≅, $m\angle 2 = 121$, and $m\angle 3 = m\angle 1 = 59$. **2.** Since alt. int. $\angle$s of ∥ lines are ≅, $m\angle 1 = 43$. By the Triangle Angle-Sum Thm., $m\angle 2 = 180 - (43 + 75) = 62$. Since alt. int. $\angle$s of ∥ lines are ≅, $m\angle 3 = m\angle 2 = 62$. **3.** Since opp. sides of a ▱ are ∥ and since alt. int. $\angle$s of ∥ lines are ≅, $m\angle 3 = 26$. By the Triangle Ext. Angle Thm., $m\angle 2 = 26 + 48 = 74$. $m\angle 1 = 180 - m\angle 2 = 180 - 74 = 106$ **4.** Since just one pair of opp. sides is ∥, the quad. is a trapezoid. Since the non-∥ sides are ≅, the trapezoid is an isosc. trapezoid. **5.** Since both pairs of opp. sides are ∥, the quad. is a ▱. Since one $\angle$ is right, then the $\angle$ opp. it is right, and the two remaining $\angle$s must also measure 90, since they are suppl., so the ▱ is a rectangle. **6.** Since same-side alt. int. $\angle$s are suppl., both pairs of opp. sides are ∥, so the quad. is a ▱. Since all 4 $\angle$s are rt. $\angle$s, the ▱ is a rectangle. **7.** Since same-side int. $\angle$s of ∥ lines are suppl., $(2x + 10) + (2x - 10) = 180$; $4x = 180$; $x = 45$. Since opp. $\angle$s are ≅, $y + 20 = 2x - 10$; $y + 20 = 2(45) - 10$; $y + 20 = 90 - 10$; $y + 20 = 80$; $y = 60$. **8.** Since diagonals of a ▱ bis. each other, $4x - 2 = 2x$; $2x - 2 = 0$; $2(x - 1) = 0$; $x = 1$. Also, $2y = 5x - 1$; $2y = 5(1) - 1$; $2y = 4$; $y = 2$. **9.** By Thm. 6-4, $CE = AC = 10.3$. $AE = AC + CE = 10.3 + 10.3 = 20.6$ **10.** Since opp. sides are not ∥ and 2 pairs of adj. sides are ≅, the quad. is a kite.

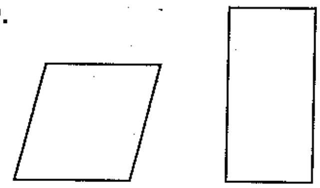
TECHNOLOGY *page 311*

1. The diagonals of a rectangle are ≅. **2.** The diagonals of a rhombus are ⊥ to each other and bis. the $\angle$s. **3.** The diagonals of a square are ≅, ⊥ to each other, and bis. the $\angle$s. **4.** If the diagonals of a ▱ are ⊥, then the ▱ is a rhombus. **5.** If the diagonals of a ▱ are ≅, then the ▱ is a rectangle. **6.** If the diagonals of a ▱ bisect its $\angle$s, then the ▱ is a rhombus. **7.** If the diagonals of a trapezoid are ≅, then the trapezoid is an isosc. trapezoid.

6-4 Special Parallelograms *pages 312–318*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1. 4.6 **2.** 7 **3.** 109 **4.** 4.75 **5.** 3.5 **6.** 9.5 **7.** 71 **8.** 71
9.



The rhombus is not a square because it has no right $\angle$s. The rectangle is not a square because all 4 sides aren't ≅.

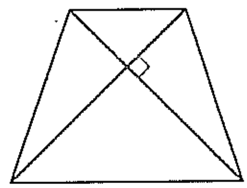
Check Understanding **1.** In a rhombus, the diagonals are ⊥, so the $m\angle 1 = 90$. The rhombus is a ▱, so opp. sides are ∥ and alt. int. $\angle$s are ≅: $m\angle 2 = 50$. The diagonals of a rhombus bisect the $\angle$s, so $m\angle 3 = 50$. By the Triangle Ext. Angle Thm., $m\angle 4 + m\angle 2 = 90$, so $m\angle 4 = 90 - 50 = 40$. **2.** Since the diagonals of a rectangle are ≅, $5y - 9 = y + 5$; $4y - 9 = 5$; $4y = 14$; $y = 3\frac{1}{2}$. $FD = GE = y + 5 = 3\frac{1}{2} + 5 = 8\frac{1}{2}$ **3.** No; if one diagonal bisects two $\angle$s, then the figure is a rhombus and cannot have non-≅ sides. **4.** Yes; if the ropes are ⊥ to each other, then the endpoints of the ropes are the vertices of a square.

Exercises **1.** Since the diagonals of a rhombus bis. the $\angle$s, the diagonal forms an isosc. △, so $m\angle 1 = m\angle 2 = m\angle 3 = m\angle 4 = \frac{180-104}{2} = 38$. **2.** Since the diagonals of a rhombus bis. the $\angle$s and since opp. $\angle$s of a ▱ are ≅, $m\angle 1 = 26$. By the Triangle Angle-Sum Thm., $m\angle 2 = 180 - (26 + 26) = 128$. Since opp. $\angle$s of a ▱ are ≅, $m\angle 3 = m\angle 2 = 128$. **3.** Since opp. $\angle$s of a ▱ are ≅, $m\angle 1 = 118$. Since all sides of a rhombus are ≅, the diagonals form isosc. △s with ≅ base $\angle$s, so $m\angle 1 + 2m\angle 2 = 180$, so $118 + 2m\angle 2 = 180$; $59 + m\angle 2 = 90$; $m\angle 2 = 31$. Since alt. int. $\angle$s are ≅, $m\angle 3 = m\angle 2 = 31$. **4.** Opp. sides of a ▱ are ≅, so $m\angle 3 = 113$. Since same-side int. $\angle$s of ∥ lines are ≅, $m\angle 2 + m\angle 4 = 180 - 113 = 67$. Since the diagonals of a rhombus bis. each other, $m\angle 2 = m\angle 4 = \frac{67}{2} = 33.5$. Since alt. int. $\angle$s of ∥ lines are ≅, $m\angle 1 = m\angle 4 = 33.5$. **5.** Since opp. sides of a ▱ are ≅ and the diagonals of a rhombus bisect the $\angle$s, $m\angle 3 = 58$. Since the diagonals of a rhombus are ⊥, $m\angle 2 = 90$. Since the diagonals of a rhombus bis. the $\angle$s, $m\angle 1 = m\angle 4 = 180 - (90 + 58) = 32$. **6.** Since the diagonals of a rhombus are ⊥, $m\angle 1 = 90$. By the Triangle Angle-Sum Thm., $m\angle 2 = 180 - (90 + 30) = 60$. Since opp. $\angle$s of a ▱ are ≅ and the diagonals bisect the $\angle$s, $m\angle 3 = m\angle 2 = 60$. Since alt. int. $\angle$s of ∥ lines are ≅, $m\angle 4 = 30$. **7.** Since the diagonals of a rhombus are ⊥ and since the sum of the measures of a △ = 180, $m\angle 1 = 180 - (90 + 35) = 180 - 125 = 55$. Since opp. $\angle$s of a ▱ are ≅ and since the diagonals of a rhombus bisect each other, $m\angle 2 = 35$. Since alt. int. $\angle$s of ∥ lines are ≅, $m\angle 3 = m\angle 1 = 55$. Since the diagonals of a rhombus are ⊥, $m\angle 4 = 90$. **8.** Since the diagonals of a rhombus bisect the ∠s, $m\angle 1 = 60$. Since the diagonals are ⊥, $m\angle 2 = 90$. Two opp. ∠s measure 120 each, so their sum is 240. The sum of the measures of the ∠s of a quad. is 360, so the sum of the other two ∠s is $360 - 240$, or 120. Each of the remaining ∠s is 60. Since the diagonals bisect the ∠s, $m\angle 3 = \frac{60}{2}$, or 30. **9.** Since the diagonals of a rhombus are ⊥, $m\angle 1 = m\angle 3 = 90$. Since the diagonals of a rhombus bisect the ∠s, the two acute ∠s measure 2(35), or 70, so the other pair of ∠s measure $180 - 70$, or 110. So, $m\angle 2 = \frac{110}{2}$, or 55. **10.** Since the diagonals of a rectangle are =, $x = 2x - 4$, so $x = 4$. $LN = MP = x = 4$ **11.** Since the diagonals of a rectangle are =, $5x - 8 = 2x + 1; 3x = 9; x = 3$. $LN = MP = 2x + 1 = 2(3) + 1 = 7$ **12.** Since the diagonals of a rectangle are =, $3x + 1 = 8x - 4; -5x = -5; x = 1$. $LN = MP = 3x + 1 = 3(1) + 1 = 4$ **13.** Since the diagonals of a rectangle are =, $9x - 14 = 7x + 4; 2x = 18; x = 9$. $LN = MP = 7x + 4 = 7(9) + 4 = 63 + 4 = 67$ **14.** Since the diagonals of a rectangle are =, $7x - 2 = 4x + 3; 3x = 5; x = \frac{5}{3}$. $LN = MP = 4x + 3 = 4\left(\frac{5}{3}\right) + 3 = \frac{20}{3} + \frac{9}{3} = \frac{29}{3} = 9\frac{2}{3}$ **15.** Since the diagonals of a rectangle are =, $3x + 5 = 9x - 10; -6x = -15; x = \frac{5}{2}$. $LN = MP = 9x - 10 = 9\left(\frac{5}{2}\right) - 10 = \frac{45}{2} - \frac{20}{2} = \frac{25}{2} = 12\frac{1}{2}$ **16.** Impossible; if the ≅ diagonals bisect each other, then it is a ▱ that is a rectangle, so if the quad. has no right ∠s, then the ≅ diagonals must not bisect each other. Thus, it is not a ▱. **17.** Yes; it is possible to have ≅ diagonals in a rectangle. If the diagonals are 3 cm and a pair of sides are 2 cm, then the remaining 2 sides are $\sqrt{5}$ cm. **18.** Impossible; all ∠s must be right ∠s since consecutive ∠s are same-side int. ∠s of ‖ lines, so the ∠s must be suppl. So, all ∠s then must be right ∠s. If all ∠s are right, then the figure is a rectangle. **19.** Impossible; if the diagonal creates ≅ ∠s at one vertex, then since alt. int. ∠s of ‖ lines are ≅, it also creates ≅ ∠s at the other vertex. If a diagonal bisects the ∠s, then the figure is a rhombus and has 4 ≅ sides. The diagram shows that the 4 sides are not ≅. **20.** Yes; the ∠s are bisected. If the bisected ∠s are ≅, then the quad. is a rhombus. **21.** Yes; the diagonals are ⊥, and if they also bis. each other, the quad. is a rhombus. Since the rhombus has a right ∠, the rhombus could be a square. **22.** Both pairs of opp. sides on the frame remain ≅, so by Thm. 6-7, it is a ▱. **23.** She can measure the shelves and sides of the bookshelf. If the shelves are the same length, and the sides are the same height, then since opp. sides of the quad. are ≅, the quad. is a ▱ by Thm. 6-7. Then she can measure the diagonals. If the diagonals are ≅, then the figure is a rectangle by Thm. 6-14. **24.** The diagonals of the ▱ are ≅, so it is a rectangle. The diagonals of the ▱ are ⊥, so it is a rhombus and has ≅ sides. A rectangle that has ≅ sides is a square.

25–34. Symbols may vary. Sample: parallelogram: ▱, rhombus: ◇, rectangle: ▭, square: $\boxed{S}$

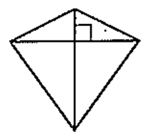
25. ◇, $\boxed{S}$	**26.** ▱, ◇, ▭, $\boxed{S}$
27. ▱, ◇, ▭, $\boxed{S}$	**28.** ▱, ◇, ▭, $\boxed{S}$
29. ▭, $\boxed{S}$	**30.** ▱, ◇, ▭, $\boxed{S}$
31. ▱, ◇, ▭, $\boxed{S}$	**32.** ▭, $\boxed{S}$
33. ◇, $\boxed{S}$	**34.** ◇, $\boxed{S}$

35.



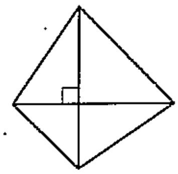
Diagonals are ≅ and ⊥.

36.



Diagonals are ≅ and ⊥.

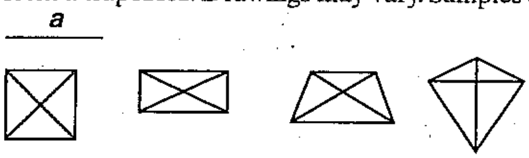
37.



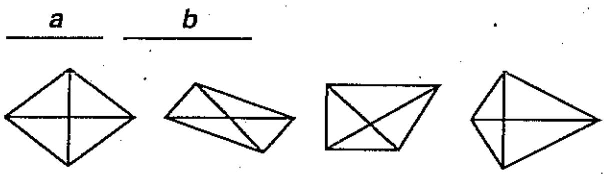
Diagonals are ≅ and ⊥. **38a.** Opp. sides are ≅ and ‖. Diagonals bis. each other. Opp. ∠s are ≅. Consecutive ∠s are suppl. **38b.** All ∠s are rt. ∠s. Diagonals are ≅. **38c.** 4 sides are ≅. Diagonals are ⊥ bis. of each other. Each diagonal bisects 2 ∠s. **39.** Use any of the defs. or theorems relating to the properties of a ▱. Answers may vary. Sample: Using Thm. 6-5, draw diagonal 1 and construct its midpt. Draw a line through the midpt. Construct segments of length of diagonal 2 in opp. directions from the midpt. Then bisect these segments. Connect these midpts. with endpts. of diagonal 1. **40.** Use any of the defs. or theorems relating to the properties of a rectangle. Answers may vary. Sample: Using 6-2 and the fact that rectangles have 4 right ∠s, construct a rt. ∠ and draw diagonal 1 from its vertex. Construct the ⊥ from the endpt. of the opp. end of diagonal 1 to a side of the rt. ∠. At the intersection, repeat to other side. **41.** Use any of the defs. or theorems relating to the properties of a rhombus. Answers may vary. Sample: Using Thm. 6-10, do the same construction as for a rectangle in Exercise 39, but construct a ⊥ line at the midpt. of diagonal 1. **42.** Use any of the defs. or theorems relating to the properties of a square. Answers may vary. Sample: Using Thm. 6-5, 6-10, and 6-14, do the same construction as for 41, except make the diagonals =. **43.** Construct the ⊥ bis. of a segment and arbitrarily choose two points on it that are not equidistant from the midpt. of the segment. Answers may vary. Sample: Draw diagonal 1. Construct a line ⊥ to it at a pt. not the midpt. Construct segments of length of diagonal 2 in opp. directions from the pt. Then bisect these segments. Connect these midpts. to the endpts. of diag. 1. **44.** Use any of the defs. or theorems relating to the properties of a trapezoid. Answers may vary. Sample: Draw an acute ∠ with the smaller diagonal as a side. Construct the line ‖ to the other side through the non-vertex endpt. of the smaller diagonal. Draw an arc with compass set to the length of the larger diagonal from the non-diagonal side of the ∠, passing through the ‖ line. Draw the larger diagonal, and then draw the non-‖ sides of the trapezoid. **45.** Yes; since all rt. ∠s are ≅, the opp. ∠s are ≅, so it is a ▱. Because the ▱ has all rt. ∠s, it is a rectangle. **46.** Yes; 4 sides are ≅, so the opp. sides are ≅. Thus, it is a ▱.

Since it has 4 $\cong$ sides, the ▱ is a rhombus. **47.** Yes; from Exercise 46, a quad. with 4 $\cong$ sides is a ▱, and a ▱ having 4 $\cong$ sides and 4 rt. △s is a square. **48.** The ▱ is a rectangle, so the diagonals are = and bis. each other: $SW = 2RZ$, so $5x - 20 = 2(2x + 5)$; $5x - 20 = 4x + 10$; $x - 20 = 10$; $x = 30$. **49.** Since the ▱ has right △s, it is a rectangle. Since the rectangle has $\cong$ adj. sides, it is a square. So, the diagonals bis. the △s, which are 90°; $9x = 45$, so $x = 5$. Since $m\angle 1 = 3y - 6$ and in a square the diagonals are $\perp$, $3y - 6 = 90$; $3y = 96$; $y = 32$. $6z = 45$; $z = 7.5$ **50.** The figure is a rhombus with right △s, so it is a square. The diagonals bis. each other, so $2x - 1 = 3y + 5$; $2x = 3y + 6$. Also, $AC = BD = 4x - y + 1$. So, $(2x - 1) + (3y + 5) = 4x - y + 1$; $2x + 3y + 4 = 4x - y + 1$; $4y + 3 = 2x$. Substitute $3y + 6$ for $2x$: $4y + 3 = 3y + 6$; $y = 3$. Substitute 3 for y in $2x = 3y + 6$: $2x = 3(3) + 6 = 15$, so $x = 7.5$.

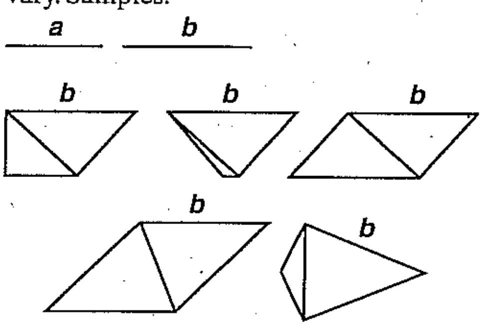
51. Intersecting two straws of the same length arbitrarily and not at either endpt. results in a quadrilateral. If they bisect each other and are $\perp$, they form a square. If they bisect each other and are not $\perp$, they form a rectangle. If only one diagonal bisects the other and they are $\perp$, they form a kite. If they intersect so that one pair of opp. sides is $\parallel$, they form a trapezoid. Drawings may vary. Samples are given:



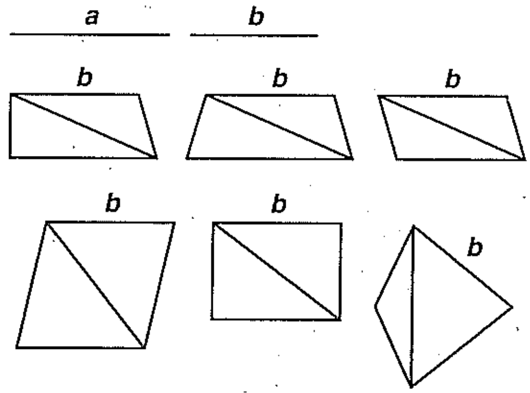
52. Intersecting two straws of different lengths arbitrarily and not at either endpt. results in a quadrilateral. If they bisect each other and are $\perp$, they form a rhombus. If they bisect each other and are not $\perp$, they form a ▱. If only one diagonal bisects the other and they are $\perp$, they form a kite. If they intersect so that one pair of opp. sides is $\parallel$, they form a trapezoid. Drawings may vary. Samples are given:



53. Draw all special quads. and their diags., including a variety of trapezoids. Examine possibilities for $a < b$ and for $a > b$. For $a < b$, possibilities are trapezoid, isosc. trapezoid $\left(a > \frac{b}{2}\right)$, ▱, rhombus and kite. Drawings may vary. Samples:

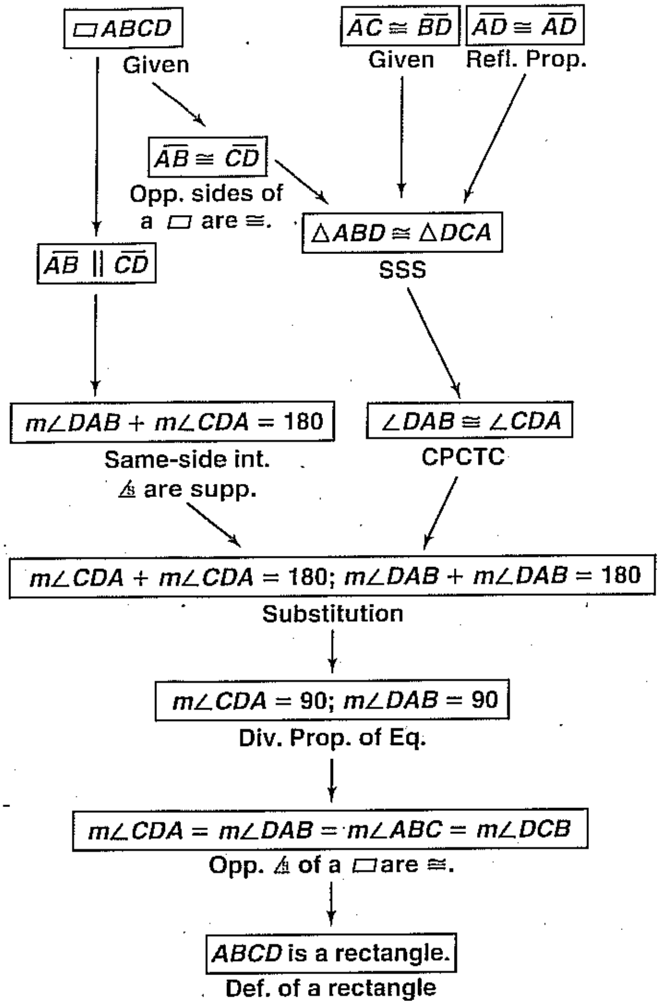


For $a > b$, possibilities are trapezoid, isosc. trapezoid, ▱, rhombus ($a < 2b$), kite, rectangle, and square (if $a = b\sqrt{2}$). Drawings may vary. Samples are given.



54a. The sides of a rhombus are $\cong$: Def. of rhombus. **54b.** A rhombus is a ▱: Diagonals of a ▱ bis. each other. **54c.** Since the reason for Step (e) is SSS, we must be proving △s $\cong$: $\overline{AE} \cong \overline{AE}$. **54d.** Reflexive Prop. of $\cong$ **54e.** $\triangle ABE \cong \triangle ADE$ **54f.** CPCTC **54g.** Together they form a straight $\angle$ whose measure is 180: Angle Addition Post. **54h.** Since the △s are $\cong$ and suppl., each $\angle$ measures 90, so $\angle AEB$ and $\angle AED$ are rt. △s. **54i.** If two △s are $\cong$ and suppl., then each $\angle$ is a rt. $\angle$: Congruent Suppl. Thm. **54j.** If two lines form rt. △s, then they are $\perp$: Def. of $\perp$. **55.** Answers may vary. Sample: If one diagonal of a ▱ bisects a pair of opp. △s, then it forms 2 isosc. △s with $\cong$ base △s. So, the adjacent sides are $\cong$ and the opp. sides are $\cong$, so all sides are $\cong$. Thus, if you know it's a ▱, only one diagonal is needed. **56.** The revised Thm. 6-12 reads, if one diagonal of a ▱ bisects one $\angle$ of the ▱, then the ▱ is a rhombus. Proofs may vary. Sample: Given $ABCD$ with diagonal $\overline{AC}$. Let $\overline{AC}$ bisect $\angle BAD$. Because alt. int. △s of $\parallel$ lines are $\cong$, $\triangle ABC \cong \triangle DAC$ by ASA, so $\overline{AB} \cong \overline{DA}$ by CPCTC. But since opp. sides of a ▱ are $\cong$, $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$. So, $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$, and ▱$ABCD$ is a rhombus. The new statement is true. **57.** The diagonals of a rectangle are =, so $2(x - 3) = x + 5$; $2x - 6 = x + 5$; $x - 6 = 5$; $x = 11$. $AC = BD = x + 5 = 11 + 5 = 16$ **58.** The diagonals of a rectangle are =, so $2(5a + 1) = 2(a + 1)$; $5a + 1 = a + 1$; $4a = 0$; $a = 0$. $AC = BD = 2(a + 1) = 2(0 + 1) = 2$ **59.** The diagonals of a rectangle are =, so $\frac{3y}{5} = 3y - 4$; $3y = 15y - 20$; $-12y = -20$; $y = \frac{5}{3}$. $AC = BD = 3y - 4 = 3\left(\frac{5}{3}\right) - 4 = 5 - 4 = 1$ **60.** The diagonals of a rectangle are =, so $\frac{3c}{9} = 4 - c$; $3c = 36 - 9c$; $12c = 36$; $c = 3$. $AC = BD = 4 - c = 4 - 3 = 1$ **61.** (4) $\triangle ABC \cong \triangle ADC$ (ASA) (5) $\overline{AB} \cong \overline{AD}$ (CPCTC) (6) $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$ (Opp. sides of a ▱ are $\cong$.) (7) $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$ (Trans. Prop. of $\cong$) **62.** Answers may vary. Sample: The diagonals of a ▱ bisect each other, so $\overline{AE} \cong \overline{CE}$. Both $\angle AED$ and $\angle CED$ are $\cong$ rt. △s because $\overline{AC} \perp \overline{BD}$, and since $\overline{DE} \cong \overline{DE}$ by the Refl. Prop. of $\cong$, $\triangle AED \cong \triangle CED$ by SAS. By CPCTC, $\overline{AD} \cong \overline{CD}$, and opp. sides of a ▱ are $\cong$, so $ABCD$ is a rhombus because it has 4 $\cong$ sides.

63.



64. If the diagonals bis. each other, then the quad is a ▱. If they also are ⊥, then the ▱ is a rhombus. The answer is choice D. **65.** The quadrilateral must be symmetric on both diagonals, so it must be a rhombus. The answer is choice I. **66.** [2] Since diagonals of a rhombus bisect each other, $QS = 9$ cm. Also, since all sides are ≅, $RS = 9$ cm. So, $\triangle ARS$ is an equilateral △ and each interior ∠ measures $\frac{180}{3}$, or 60. $\triangle QTS$ is also an equilateral △, so its ∠s are 60°. By the Angle Add. Post., $(m\angle PST + m\angle PSR = m\angle RST)$, $m\angle RST = 60 + 60$, or 120. [1] no work shown OR a response that is only partially correct **67.** Yes; Thm. 6-7 is satisfied because both pairs of opp. sides are ≅. **68.** No; there are not necessarily 2 pairs of ≅ opp. sides, 2 pairs of ∥ opp. sides, or one pair of ≅ and ∥ opp. sides. **69.** Yes; Thm. 6-5 is satisfied because the diagonals bisect each other. **70.** By the Triangle Midsegment Thm., $TQ = 6$. **71.** By the Triangle Midsegment Thm., $PQ = 2(8) = 16$. **72.** By the Triangle Midsegment Thm., $SR = 5$ and $TU = \frac{1}{2}PR = \frac{1}{2}(5 + 5) = 5$. **73.** By the Triangle Midsegment Thm., $\overline{SU} \parallel \overline{RQ}$. **74.** By the Triangle Midsegment Thm., $\overline{TU} \parallel \overline{RP}$. **75.** By the Triangle Midsegment Thm., $\overline{PQ} \parallel \overline{ST}$. **76.** By the Polygon Exterior Angle Thm., $c + (c + 65) + (c + 28) = 360$; $3c + 93 = 360$; $c + 31 = 120$; $c = 89$.

TECHNOLOGY *page 319*

1a. Answers may vary. Sample: The figure formed by connecting the midpoints of a quad. is a ▱. **1b.** yes **1c.** no **2a.** rectangle **2b.** rhombus **2c.** square **3.** For $MNOP$ and $EFGH$, the ratio of the sides and perimeters is 1 : 2 and the ratios of the areas is 1 : 4. The sides of $MNOP$ and $EFGH$ are ∥. **4.** Answers may vary. Sample: By the Midsegment Thm., both $\overline{EF}$ and $\overline{HG}$ are ∥ to $\overline{AC}$ and each is half the length of $\overline{AC}$. Thus, $\overline{EF} \parallel \overline{HG}$ and $\overline{EF} \cong \overline{HG}$, so $EFGH$ is a ▱ by Thm. 6-6.

6-5 Trapezoids and Kites *pages 320–325*

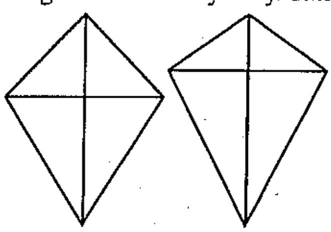
Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*
1. $a = 5.6, b = 6.8$; 4.5, 4.2, 4.5, 4.2 **2.** 3; 4.8, 16.4, 18, 18 **3.** $m = 5, n = 15$; 15, 15, 21, 21

Check Understanding **1.** The base ∠s of an isosc. trapezoid are ≅, so $m\angle R = m\angle S = 70$. Since $\overline{PQ} \parallel \overline{SR}$, same-side int. ∠s of ∥ lines are suppl, so $m\angle P = 180 - 70 = 110 = m\angle Q$. **2.** The measure of each ∠ at the center of the ceiling is $\frac{360}{18}$, or 20. $m\angle 1$ is $\frac{20}{2}$, or 10. The measure of each acute base ∠ is $\frac{180 - 10}{2} = 85$. The measure of each obtuse base ∠ is 180 − 85, or 95. **3.** The figure is a kite, so its diags. are ⊥ and $m\angle 1 = 90$. So, each of the 4 △s inside the kite are rt. △s. By the Converse of the Perp. Bis. Thm., the horizontal diag. bis. the vertical diag., so the △s on the left side are ≅ by SSS. By CPCTC, $m\angle 2 = 46$. By the Triangle Angle-Sum Thm., $m\angle 3 = 180 - (90 + 46) = 44$.

Exercises **1.** Since the base ∠s of an isosc. trapezoid are ≅, $m\angle 1 = 77$. Since the 2 ∠s that share a leg are suppl., $m\angle 2 = 180 - 77 = 103 = m\angle 3$. **2.** Since base ∠s of an isosc. trapezoid are ≅, $m\angle 3 = 111$. Since the 2 ∠s that share a leg are suppl., $m\angle 1 = m\angle 2 = 180 - 111 = 69$. **3.** Since the base ∠s of an isosc. trapezoid are ≅, $m\angle 1 = 49$. Since the 2 ∠s that share a leg are suppl., $m\angle 2 = m\angle 3 = 180 - 49 = 131$. **4.** Since the base ∠s of an isosc. trapezoid are ≅, $m\angle Y = 105$. Since the 2 ∠s that share a leg are suppl., $m\angle W = m\angle Z = 180 - 105 = 75$. **5.** Since the base ∠s of an isosc. trapezoid are ≅, $m\angle S = 65$. Since the 2 ∠s that share a leg are suppl., $m\angle R = m\angle S = 180 - 65 = 115$. **6.** Since the base ∠s of an isosc. trapezoid are ≅, $m\angle D = 60$. Since the 2 ∠s that share a leg are suppl., $m\angle B = m\angle C = 180 - 60 = 120$.
7a. Since exactly one pair of sides is ∥ and the non-∥ sides are ≅, the quad. is an isosc. trapezoid. **7b.** The long base of each trapezoid forms an isosc. △ with the vertex ∠ of 42°, so its base ∠s measure $\frac{180 - 42}{2}$, or 69. The other ∠s are suppl., so they each measure $180 - 69 = 111$. The measures of the ∠s are 69, 69, 111, and 111. **8.** The diags. of a kite are ⊥, so $m\angle 1 = 90$. The diags. of a kite form 2 pairs of ≅ rt. △s, so $m\angle 2 = 180 - (90 + 22) = 68$. **9.** The diags. of a kite are ⊥, so $m\angle 1 = 90$. $m\angle 2 = 180 - (90 + 45) = 45$. The diags. of a kite create 2 pairs of ≅ rt. △s, so

$m\angle 3 = 45$. **10.** A horizontal diag. would create 2 $\cong$ $\triangle$s by SSS, so $m\angle 1 = m\angle 2 = \frac{360 - (90 + 54)}{2} = \frac{360 - 144}{2} = 108$. **11.** Since the diags. of a kite are $\perp$, $m\angle 1 = m\angle 3 = 90$. The short diag. creates 2 isosc. $\triangle$s with = base $\angle$s and one vertex $\angle$ measuring 2(64), or 128. So, $m\angle 2 = \frac{180 - 128}{2} = 26$. **12.** The diags. of a kite are $\perp$, so $m\angle 1 = m\angle 3 = 90$. The vertical diag. creates 2 isosc. $\triangle$s with = base $\angle$s and one vertex $\angle$s measuring 2(50), or 100. $m\angle 2 = \frac{180 - 100}{2} = 40$ **13.** The diags. of a kite are $\perp$, so $m\angle 1 = m\angle 3 = 90$. The vertical diag. creates 2 isosc. $\triangle$s with = base $\angle$s and a vertex $\angle$ of 2(35), or 70, so $m\angle 2 = m\angle 4 = \frac{180 - 70}{2} = 55$. $m\angle 5 = 35$ **14.** The diags. of a kite are $\perp$, so $m\angle 1 = 90$. The vertical diag. forms 2 isosc. $\triangle$s with = base $\angle$s and a vertex $\angle$ of 2(38), or 76, and a second vertex $\angle$ of 106. $m\angle 2 = \frac{180 - 76}{2} = 52$. $m\angle 3 = 38$. $m\angle 4 = \frac{180 - 106}{2} = 37$. $m\angle 5 = 53$ **15.** The diags. of a kite are $\perp$, so $m\angle 1 = m\angle 2 = m\angle 3 = m\angle 4 = 90$. $m\angle 5 = 46$. $m\angle 6 = 34$. $m\angle 8 = m\angle 10 = \frac{180 - 2(46)}{2} = 90 - 46 = 44$. $m\angle 7 = m\angle 9 = \frac{180 - 2(34)}{2} = 90 - 34 = 56$ **16.** $m\angle 1 = m\angle 2 = \frac{360 - (90 + 46)}{2} = 112$

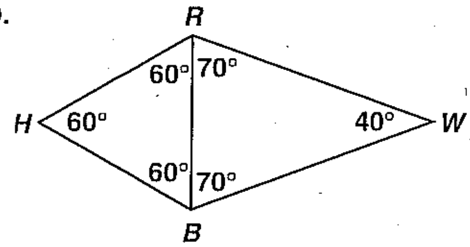
17. To create a kite, make $\perp$ diagonals such that only one bisects the other. Then to make the second kite, reposition the $\perp$ bis. so it is not bisected by the other diag. Answers may vary. Sample:



18. A kite has 2 different lengths for its sides, so 2 of the sides measure n and the other two measure $2n - 3$. $66 = 2(n) + 2(2n - 3)$; $66 = 2n + 4n - 6$; $66 = 6n - 6$; $72 = 6n$; $n = 12$. So, 2 sides measure n, or 12, and the other two sides measure $2n - 3 = 2(12) - 3$, or 21. **19.** No; explanations may vary. Sample: If both $\angle$s are bisected, then this combined with $\overline{KM} \cong \overline{KM}$ by the Reflexive Prop. means $\triangle KLM \cong \triangle KNM$ by SAS. So, by CPCTC, opp. $\angle$s L and N are $\cong$, so it is not an isosc. trapezoid. **20.** Since base $\angle$s of an isosc. trapezoid are $\cong$, $5x = 60$; $x = 12$. **21.** Since base $\angle$s of an isosc. trapezoid are $\cong$, $3x = 45$; $x = 15$. **22.** Since base $\angle$s of an isosc. trapezoid are $\cong$, $3x + 15 = 60$; $x + 5 = 20$; $x = 15$. **23.** Since diags. of an isosc. trapezoid are $\cong$, $2x - 1 = x + 2$; $x - 1 = 2$; $x = 3$. **24.** Since diags. of an isosc. trapezoid are $\cong$, $x + 1 = 2x - 3$; $x = 4$. **25.** Since diags. of an isosc. trapezoid are $\cong$, $3x + 3 = x + 5$; $2x = 2$; $x = 1$.
26. ① $ABCD$ is an isosc. trapezoid, $\overline{AB} \cong \overline{DC}$. (Given) ② Draw $\overline{AE} \parallel \overline{DC}$. (Two pts. determine a line.) ③ $\overline{AD} \parallel \overline{EC}$ (Def. of trapezoid) ④ $AECD$ is a ▱. (Def. of a ▱) ⑤ $\angle C \cong \angle 1$ (Corr. $\angle$s are $\cong$.) ⑥ $\overline{DC} \cong \overline{AE}$ (Opp. sides of a ▱ are $\cong$.) ⑦ $\overline{AB} \cong \overline{AE}$ (Trans. Prop. of $\cong$) ⑧ $\triangle AEB$ is an isosc. $\triangle$. (Def. of an isosc. $\triangle$) ⑨ $\angle B \cong \angle 1$ (Base $\angle$s of an isosc. $\triangle$ are $\cong$.) ⑩ $\angle B \cong \angle C$ (Trans. Prop. of $\cong$) ⑪ $\angle B$ and $\angle BAD$ are suppl., $\angle C$ and $\angle CDA$ are suppl. (Same-side int. $\angle$s are suppl.) ⑫ $\angle BAD \cong \angle CDA$ (Suppl. of $\cong$ $\angle$s are $\cong$.) **27.** Since the diags. are $\perp$, $(x + 6) + 2x = 90$; $3x + 6 = 90$; $x + 2 = 30$; $x = 28$. **28.** $4x - 30 = 3x + 5$; $x - 30 = 5$; $x = 35$. So, the measures of those two $\angle$s is $4(35) - 30$, or 110. The sum of the other two $\angle$s is $2(y + 2y - 20) = 360 - 2(110)$; $y + 2y - 20 = 180 - 110$; $3y - 20 = 70$; $3y = 90$; $y = 30$. **29.** By symmetry, $y = 6x$. The sum of the $\angle$s is $y + 90 + 6x + \frac{3x}{2} + \frac{3x}{2} = 360$; $6x + 90 + 6x + 3x = 360$; $15x + 90 = 360$; $15x = 270$; $x = 18$. $y = 6x = 6(18) = 108$ **30.** Isosc. trapezoid; all the large rt. $\triangle$s appear $\cong$. **31.** Since base $\angle$s of an isosc. trapezoid are =, one other $\angle$ is 112. The other two $\angle$s are suppl.: $180 - 112 = 68$. So, the other $\angle$s measure 112, 68, and 68. **32.** Yes; one of the diags. of a kite divides it into 2 $\cong$ $\triangle$s, so one pair of opp. kite $\angle$s are $\cong$ by CPCTC; the $\cong$ $\angle$s can be obtuse. **33.** Yes; one diag. of a kite creates 2 $\cong$ $\triangle$s, so one pair of opp. kite $\angle$s are $\cong$ by CPCTC. The 2 $\cong$ $\angle$s and one other $\angle$ can be obtuse if their sum is less than 360°. **34.** Yes; one diag. creates 2 $\cong$ $\triangle$s, so one pair of opp. kite $\angle$s are $\cong$ by CPCTC. The 2 $\cong$ $\angle$s can each be 90°, so they are suppl. The other two $\angle$s are also suppl. **35.** No; one diag. creates 2 $\cong$ $\triangle$s, so one pair of opp. kite $\angle$s are $\cong$ by CPCTC. If 2 consecutive $\angle$s are suppl., then another pair must be also, because one pair of opp. $\angle$s are $\cong$. Therefore, both pairs of opp. $\angle$s would be $\cong$, so the quad. would be a parallelogram and not a kite. **36.** Yes; one diag. divides the kite into 2 $\cong$ $\triangle$s, so one pair of opp. kite $\angle$s are $\cong$. The sum of the $\angle$ measures of a quad. is 360. If each $\cong$ $\angle$ measures a, and if one of the other $\angle$s measures b, then the fourth $\angle$ measures $360 - (2a + b)$. If the $\cong$ $\angle$s are complementary, then $2a = 90$, so $a = 45$. Then the sum of the $\angle$s is $2a + b + [360 - (2a + b)] = 360$; $2(45) + b + [360 - (2(45) + b)] = 360$; $90 + b + 270 - b = 360$; $360 = 360$, which is always true so the 2 $\cong$ $\angle$s can each = 45. If the noncongruent $\angle$s are complementary, then the $\angle$ measures are a, b, a, and $90 - b$, so $2a + 90 = 360$; $2a = 270$; $a = 135$. So, the congruent $\angle$s are either 45° each or 135° each. **37.** Answers may vary. Sample: No; if 2 consec. $\angle$s were compl., the kite would be concave. **38.** Rhombuses and squares have 2 pairs of adj. sides $\cong$, so they could also be kites. **39.** D is any point on $\overleftrightarrow{BN}$ such that $ND \neq BN$ and N is between B and D. **40.** ① $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$ (Given) ② $\overline{BD} \cong \overline{BD}$ (Refl. Prop. of $\cong$) ③ $\triangle ABD \cong \triangle CBD$ (SSS) ④ $\angle A \cong \angle C$ (CPCTC) **41.** Answers may vary. Sample: Draw $\overline{TA}$ and $\overline{RP}$. ① isosc. trapezoid $TRAP$ (Given) ② $\overline{TA} \cong \overline{PR}$ (Diags. of an isosc. trap. are $\cong$.) ③ $\overline{TR} \cong \overline{PA}$ (Given) ④ $\overline{RA} \cong \overline{AR}$ (Refl. Prop. of $\cong$) ⑤ $\triangle TRA \cong \triangle PAR$ (SSS) ⑥ $\angle RTA \cong \angle APR$ (CPCTC) **42.** Draw $\overline{BI}$, the $\perp$ bis. of $\overline{RA}$ intersecting $\overline{RA}$ at B and $\overline{TP}$ at I. Then draw $\overline{BT}$ and $\overline{BP}$. ① $\overline{TR} \cong \overline{PA}$ (Given) ② $\angle R \cong \angle A$ (Base $\angle$s of an isosc. trap. are $\cong$.) ③ $\overline{RB} \cong \overline{AB}$ (Def. of bis.) ④ $\triangle TRB \cong \triangle PAB$ (SAS) ⑤ $\overline{BT} \cong \overline{BP}$ (CPCTC) ⑥ $\angle RBT \cong \angle ABP$ (CPCTC) ⑦ $\angle TBI \cong \angle PBI$ (Compl. of $\cong$ $\angle$s are $\cong$.)

⑧ $\overline{BI} \cong \overline{BI}$ (Refl. Prop. of $\cong$) ⑨ $\triangle TBI \cong \triangle PBI$ (SAS) ⑩ $\angle BIT \cong \angle BIP$ (CPCTC) ⑪ $\angle BIT$ and $\angle BIP$ are rt. ∠s. ($\cong$ suppl. ∠s are rt. ∠s.) ⑫ $\overline{TI} \cong \overline{PI}$ (CPCTC) ⑬ $\overline{BI}$ is the ⊥ bis. of $\overline{TP}$. (Def. of ⊥ bis.)
43. It is $\frac{1}{2}$ the sum of the lengths of the 2 ∥ bases. Justifications may vary. Sample: Draw a diagonal to form 2 △s sharing the bases of the trapezoid. Then the segment joining the midpts. of the nonparallel sides of the trapezoid is the sum of the midsegments of the 2 △s. By the Triangle Midsegment Thm., the midsegment is half the length of its base. If the bases are b and B, then the segment connecting the midpts. of the non-∥ sides of the trap. is $\frac{1}{2}B + \frac{1}{2}b$, or $\frac{1}{2}(B + b)$. **44.** It is $\frac{1}{2}$ the difference of the lengths of the bases. Justifications may vary. Sample: From Exercise 43, the length of the segment joining the midpts. of the non-∥ sides is $\frac{1}{2}(B + b)$. The segment joining the midpts. of the diagonals is the middle part of that segment divided by the 2 diags. Each outer segment measures $\frac{1}{2}b$. So, the length of the middle segment is $\frac{1}{2}(B + b) - \frac{1}{2}b - \frac{1}{2}b =$ $\frac{1}{2}B + \frac{1}{2}b - \frac{1}{2}b - \frac{1}{2}b = \frac{1}{2}B - \frac{1}{2}b$, or $\frac{1}{2}(B - b)$. **45.** By def. of trapezoid, the answer is choice B. **46.** By Thm. 6-17, the answer is choice I. **47.** Since base ∠s of a trapezoid are $\cong$, the answer is choice C. **48.** In an isosc. trap. the diags. are $\cong$. The answer is choice C. **49.** Length cannot be assumed from a diagram. The answer is choice D.

50.

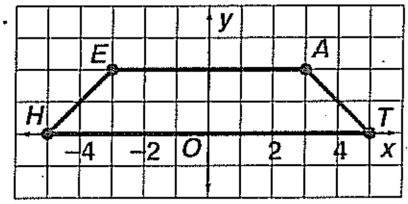


[2] 60 + 70 = 130, so the measure of the greatest ∠ is 130. [1] incorrect diagram OR no work shown **51.** Opp. ∠s of a parallelogram are $\cong$: 126. **52.** The diag. forms an isosc. △ with $\cong$ base ∠s. From Exercise 51, the vertex ∠ is 126°, so $126 + 2m\angle 2 = 180$; $63 + m\angle 2 = 90$; $m\angle 2 = 27$. **53.** From Exercise 52, $m\angle 2 = 27$. Since alt. int. ∠s of ∥ lines are $\cong$, $m\angle 3 = m\angle 2 = 27$. **54a.** $\triangle FGE \cong \triangle HGE$ by HL, so $FG = GH$; $2a - 3 = a + 1$; $a - 3 = 1$; $a = 4$. **54b.** From part ⓐ, $a = 4$. $FG = 2a - 3 = 2(4) - 3 = 8 - 3 = 5$
54c. From part ⓐ, $a = 4$. $GH = a + 1 = 4 + 1 = 5$
55a. The 2 △s are $\cong$ by AAS, so their corr. parts are $\cong$. $DC = BC$; $2x + 24 = 7x + 9$; $-5x = -15$; $x = 3$
55b. From part ⓐ, $x = 3$. $CD = 2x + 24 = 2(3) + 24 = 6 + 24 = 30$ **55c.** From part ⓐ, $x = 3$. $BC = 7x + 9 = 7(3) + 9 = 21 + 9 = 30$ **56.** Two sides and their included ∠ are $\cong$: SAS.

6-6 Placing Figures in the Coordinate Plane pages 326–330

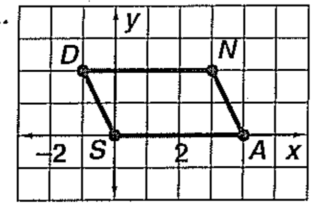
Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1.



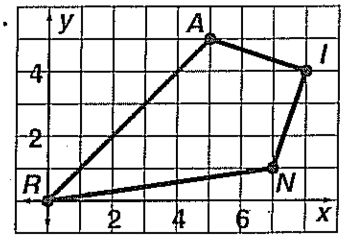
isosc. trapezoid

2.



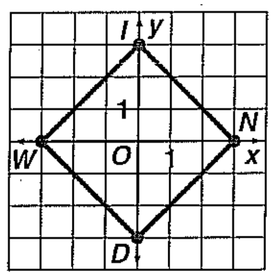
parallelogram

3.



kite

4.



square

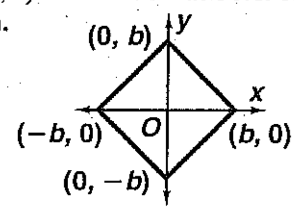
Investigation 1. Answers may vary. Sample: Squares with sides on the axes; it's easiest to find horiz. and vertical slopes. **2a.** Answers may vary. Sample: Place one side on the x-axis and another on the y-axis. **2b.** Answers may vary. Sample: Place one side on the x-axis. Determine the height. Place the opp. side ∥ to the x-axis at the required height, so that one vertex is on the y-axis.

Check Understanding 1. Since the slope of $\overline{OP} = 0$, the slope of $\overline{RQ} = 0$. Let the coordinates of Q be (x, y). Then $\frac{y - c}{x - b} = 0$, so $y - c = 0$; $y = c$. The slope of $\overline{RO} = \frac{c - 0}{b - 0} = \frac{c}{b}$, so the slope of $\overline{QP}$ is also $\frac{c}{b} = \frac{y - 0}{x - s}$; $c(x - s) = b(y - 0)$; $c(x - s) = b(c - 0)$; $cx - cs = bc$; $x - s = b$; $x = s + b$: $Q(s + b, c)$. **2.** Use any related theorems or definitions to show that a quad. is a ▱. Answers may vary. Sample: The diagonals of a ▱ bisect each other. The midpt. of $\overline{TV} = \left(\frac{a + c + e}{2}, \frac{b + d}{2}\right)$ and the midpt. of $\overline{UW} = \left(\frac{a + c + e}{2}, \frac{b + d + 0}{2}\right) = \left(\frac{a + c + e}{2}, \frac{b + d}{2}\right)$. Since the midpts. are the same, the diagonals bis. each other, so $TWVU$ is a ▱.

Exercises 1. W is h units up on the y-axis: $W(0, h)$. Z is b units right on the x-axis: $Z(b, 0)$. **2.** Since the figure is a square and one side is a, W is a units right on the x-axis and a units up on the y-axis: $W(a, a)$. Z is a units right on the x-axis: $Z(a, 0)$. **3.** The figure is a square centered at the origin, so all coordinates will be either b or $-b$. W is b units left on the x-axis and b units up on the y-axis: $W(-b, b)$. Z is b units left on the x-axis and b units

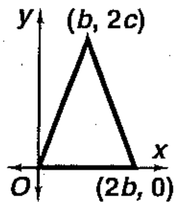
down on the y-axis: $Z(-b,-b)$. **4.** The figure is a ▱ with 2 sides ∥ to the x-axis. W is b units up on the y-axis: $W(0,b)$. Since $(c,0)$ is a units right of $(c-a,b)$, Z is a units right of W, so $Z(a,0)$. **5.** The figure is a rhombus centered at the origin. The diagonals are on the axes and bisect each other at $(0,0)$. W is r units left on the x-axis: $W(-r,0)$. Z is t units down on the y-axis: $Z(0,-t)$. **6.** The figure is an isosc. trapezoid centered about the y-axis with one base on the x-axis. W is b units left on the x-axis and c units up on the y-axis: $W(-b,c)$. Z is c units up on the y-axis: $Z(0,c)$. **7.** $W(0,h)$ and $Z(b,0)$, so the midpt. is $\left(\frac{b+0}{2},\frac{0+h}{2}\right)=\left(\frac{b}{2},\frac{h}{2}\right)$. The slope is $\frac{0-h}{b-0}=-\frac{h}{b}$. **8.** $W(a,a)$ and $Z(a,0)$, so the midpt. is $\left(\frac{a+a}{2},\frac{0+a}{2}\right)=\left(a,\frac{a}{2}\right)$. The slope is $\frac{a-0}{a-a}=\frac{a}{0}$, so the slope is undefined. **9.** $W(-b,b)$ and $Z(-b,-b)$, so the midpt. is $\left(\frac{-b+(-b)}{2},\frac{-b+b}{2}\right)=(-b,0)$. The slope is $\frac{-b-b}{-b-(-b)}=\frac{2b}{0}$, so the slope is undefined. **10.** $W(0,b)$ and $Z(a,0)$, so the midpt. is $\left(\frac{a+0}{2},\frac{0+b}{2}\right)=\left(\frac{a}{2},\frac{b}{2}\right)$. The slope is $\frac{0-b}{a-0}=-\frac{b}{a}$.
11. $W(-r,0)$ and $Z(0,-t)$, so the midpt. is $\left(\frac{-r+0}{2},\frac{0-t}{2}\right)=\left(-\frac{r}{2},-\frac{t}{2}\right)$. The slope is $\frac{-t-0}{0-(-r)}=-\frac{t}{r}$. **12.** $W(-b,c)$ and $Z(0,c)$, so the midpt. is $\left(\frac{0+(-b)}{2},\frac{c+c}{2}\right)=\left(-\frac{b}{2},c\right)$. The slope is $\frac{c-c}{0-(-b)}=0$. **13a.** Since A is on the x-axis, the y-coordinate is 0: $(2a,0)$. **13b.** Since B is on the y-axis, the x-coordinate is 0: $(0,2b)$. **13c.** $\left(\frac{2a+0}{2},\frac{0+2b}{2}\right)$, or (a,b)
13d. $\sqrt{(2a-a)^2+(0-b)^2}=\sqrt{(a)^2+(-b)^2}=\sqrt{a^2+b^2}$ **13e.** $\sqrt{(0-a)^2+(2b-b)^2}=\sqrt{(-a)^2+(b)^2}=\sqrt{a^2+b^2}$
13f. $\sqrt{(0-a)^2+(0-b)^2}=\sqrt{a^2+b^2}$ **13g.** $MA=MB=MC$ **14.** Assign values to q and p such that $q>p>0$ and plot the coordinate pairs on a coordinate plane. The vertices of a ▱ are A, C, H, and F. **15.** Assign values to q and p such that $q>p>0$ and plot the coordinate pairs on a coordinate plane. Answers may vary. Samples: A, C, G, E and B, D, H, F. **16.** Assign values to q and p such that $q>p>0$ and plot the coordinate pairs on a coordinate plane. Answers may vary. Samples: The vertices of a rectangle are A, B, F, E; A, C, G, E; A, D, H, E; B, C, G, F; B, D, H, F; C, D, H, G. **17.** Assign values to q and p such that $q>p>0$ and plot the coordinate pairs on a coordinate plane. Answers may vary. Samples: The vertices of a square are A, C, G, E; B, D, H, F.
18. Assign values to q and p such that $q>p>0$ and plot the coordinate pairs on a coordinate plane. Answers may vary. Samples: The vertices of a trapezoid are E, A, B, G; E, A, B, H; E, A, C, H; E, A, D, F; E, A, D, G; F, A, B, G; F, A, B, H; F, A, C, G; F, A, C, H; F, A, D, G; F, A, D, H; A, C, F, E. **19.** Assign values to q and p such that $q>p>0$ and plot the coordinate pairs on a coordinate plane. Answers may vary. Samples: The vertices of an isosc. trapezoid are E, B, C, H; A, D, G, F. **20.** The coordinates are doubled, so $W(0,2h)$ and $Z(2b,0)$. **21.** The coordinates are doubled, so $W(2a,2a)$ and $Z(2a,0)$. **22.** The coordinates are doubled, so $W(-2b,2b)$ and $Z(-2b,-2b)$. **23.** The x-coordinates are doubled, so $W(0,b)$ and $Z(2a,0)$.

24. The coordinates are doubled, so $W(-2r,0)$ and $Z(0,-2t)$. **25.** The coordinates are doubled, so $W(-2b,2c)$ and $Z(0,2c)$. **26a.** Just as the coordinate axes are ⊥, so are the diagonals of a rhombus.
26b. The diagonals of a ▱ that is not a rhombus are not ⊥. **27.** Choose different values for r and t, preferably such that one is not a multiple of the other. Answers may vary. Sample: $r=3$, $t=2$; slopes are $\frac{2-0}{0-3}=-\frac{2}{3}$ and $\frac{0-(-2)}{3}=\frac{2}{3}$; all lengths are $\sqrt{(0-3)^2+(2-0)^2}=\sqrt{9+4}=\sqrt{13}$. **28.** P is a units to the left of $(c,0)$ and b units above it, so the coordinates of P are $(c-a,0+b)$: $P(c-a,b)$. **29.** P is a units to the right of the origin: $P(a,0)$. **30.** P is b units left of the origin: $P(-b,0)$.
31a.

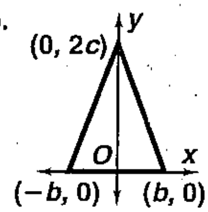


Center the square on the origin. **31b.** $(-b,0)$, $(0,b)$, $(b,0)$, $(0,-b)$

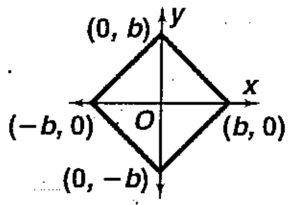
31c. $\sqrt{(0-b)^2+(b-0)^2}=\sqrt{2b^2}=b\sqrt{2}$
31d. Slope from $(-b,0)$ to $(0,b)$ is $\frac{b-0}{0-(-b)}=1$; slope from $(b,0)$ to $(0,b)$ is $\frac{b-0}{0-b}=-1$. **31e.** Yes; the product of the slopes is -1.
32a.



32b.



32c. $\sqrt{(b-2b)^2+(2c-0)^2}=\sqrt{(-b)^2+(2c)^2}=\sqrt{b^2+4c^2}$; $\sqrt{(b-0)^2+(2c-0)^2}=\sqrt{b^2+4c^2}$
32d. $\sqrt{(0-(-b))^2+(2c-0)^2}=\sqrt{(b)^2+(2c)^2}=\sqrt{b^2+4c^2}$; $\sqrt{(0-b)^2+(2c-0)^2}=\sqrt{b^2+4c^2}$
32e. The lengths are =.
33.

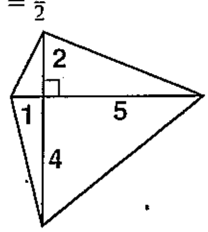


34. Step 1: Since adding zero does not change a value and multiplying by 0 always results in 0, choose $(0,0)$. Step 2: C is a units to the right of the origin directly below A, so C is at $(a,0)$. Step 3: Since $m\angle 1+m\angle 2+90=180$, $\angle 1$ and $\angle 2$ must be compl., and $\angle 2$ and $\angle 3$ are the acute ∠s of a rt. △, so they are also compl. Step 4: So $OD=AC$, D should be $(-b,0)$. Step 5: So that $OA=OB$, B should be $(-b,a)$. Step 6: The slope for $\ell_1=\frac{a-0}{-b-0}=-\frac{a}{b}$; the slope for $\ell_2=\frac{b-0}{a-0}=\frac{b}{a}$. Mult. the slopes $\left(-\frac{a}{b}\right)\left(\frac{b}{a}\right)=-\frac{ab}{ba}=-\frac{ab}{ab}=-1$. **35.** In Quadrant II, the x-values are negative and the y-values are positive. So, the midpt. of $(-a,0)$ and $(0,b)$ is $\left(-\frac{a}{2},\frac{b}{2}\right)$. The answer is choice B.
36. In Quadrant IV, the x-values are positive and the y-values are negative. So, the slope of $(b,0)$ and $(0,-c)$ is $\frac{0-(-c)}{b-0}=\frac{c}{b}$. The answer is choice F. **37.** A diagonal

goes from (a, a) to $(0, 0)$ and that distance is $\sqrt{(a-0)^2 + (a-0)^2} = a\sqrt{2}$. The answer is choice C. **38.** $(r, s) = \left(\frac{3p + p}{2}, \frac{3p + (p + 2)}{2}\right) = (2p, 2p + 1)$, so $r = 2p$. The answer is choice C. **39.** $(r, s) = \left(\frac{3p + p}{2}, \frac{3p + (p + 2)}{2}\right) = (2p, 2p + 1)$, so $s = 2p + 1$. Thus, $s > 2p$. The answer is choice A. **40.** The slope of $\overline{AC} = \frac{3p - (p + 2)}{3p - p} = \frac{2p - 2}{2p} = \frac{p - 1}{p} = \frac{p}{p} - \frac{1}{p} = 1 - \frac{1}{p}$. The answer is choice C. **41.** [2] From $(0, 2a)$ to $(2b, 0)$, the midpt. is (b, a). From $(2b, 2a)$ to $(0, 0)$, the midpt. is (b, a). The diagonals of a rectangle bisect each other. [1] no conclusion given **42.** Since base $\angle$s of an isosc. trapezoid are $\cong$, $m\angle 1 = 62$. Since same-side int. $\angle$s of $\parallel$ lines are suppl., and base $\angle$s of an isosc. trap. are $\cong$, $m\angle 2 = m\angle 3 = 180 - 62 = 118$. Since the legs of an isosc. trap. are $=$, $x + 2 = 5x - 8$; $-4x = -10$; $x = 2.5$. **43.** The center of the circle is the point where the $\perp$ bisectors of the sides of the $\triangle$ intersect. The eq. for the $\perp$ bis. of $\overline{AC}$ is $x = 3$. The eq. for the $\perp$ bis. of $\overline{BC}$ is $y = 2$. They intersect at $(3, 2)$. **44.** The center of the circle is the point where the $\perp$ bisectors of the sides of the $\triangle$ intersect. The eq. for the $\perp$ bis. of $\overline{AC}$ is $x = -3$. The eq. for the $\perp$ bis. of $\overline{BC}$ is $y = -4$. They intersect at $(-3, -4)$. **45a.** Reflexive **45b.** Two $\angle$s and a nonincluded side are $\cong$: AAS.

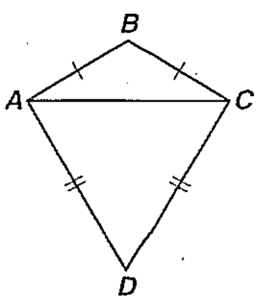
CHECKPOINT QUIZ 2 — *page 331*

1. The quad. is a rhombus and in a rhombus the diags. bis. each other, so $x = y$. The diag. forms an isosc. $\triangle$, so $78 + 2x = 180$; $39 + x = 90$; $x = 51 = y$. **2.** The quad. is a rhombus and in a rhombus the diags. bis. each other, so $x = 58$. Then, y is half the suppl. of $58 + 58$, so $y = \frac{180 - (58 + 58)}{2} = 32$. **3.** Since diagonals of a $\square$ bis. each other, $2y - 2 = 6$, so $y = 4$. Also, $2x = y$, so $2x = 4$, or $x = 2$. **4.** $5x - 1 = 3x + 5$; $2x - 1 = 5$; $2x = 6$; $x = 3$ **5.** The figure is a kite, and the diags. of a kite are $\perp$, so $b = 90$. $6x + 1 = 3x + 6$; $3x + 1 = 6$; $3x = 5$; $x = \frac{5}{3}$. $4y - 3 = 2y + 6$; $2y - 3 = 6$; $2y = 9$; $y = \frac{9}{2}$
6. False, because it is only sometimes true. Counterexamples may vary. Sample:



7. False, because it is only sometimes true. Counterexamples may vary. Sample: A kite can have $\cong$ $\perp$ diagonals.

8.



False, because only one diagonal bisects two $\angle$s of the kite. $\angle BAC$ is not $\cong$ to $\angle CAD$. **9.** A is n units to the right of the origin and m units above the origin: (n, m). **10.** A is k units to the right of the origin on the x-axis: $(k, 0)$.

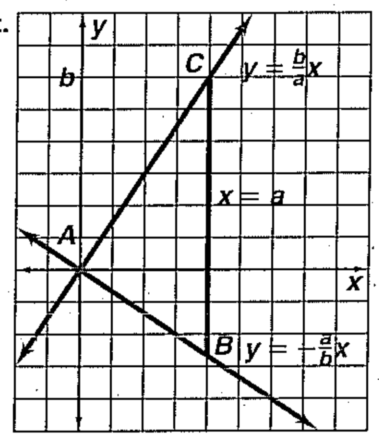
6-7 Proofs Using Coordinate Geometry — *pages 332–337*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*
1. The quad. is a rectangle. **2.** (a, c) **3.** $(-a, 0)$
Check Understanding **1a.** $M = \left(\frac{2b + 0}{2}, \frac{2c + 0}{2}\right) = (b, c)$. $N = \left(\frac{2d + 2a}{2}, \frac{2c + 0}{2}\right) = (d + a, c)$. By starting with multiples of 2, you eliminate fractions when using the midpt. formula. **1b.** The slope of $\overline{MN}$ whose endpts. are (b, c) and $(d + a, c)$ is $\frac{c - c}{(d + a) - b} = 0$. The slope of $\overline{TP}$ whose endpts. are $(0, 0)$ and $(2a, 0)$ is $\frac{0 - 0}{0 - 2a} = 0$. The slope of $\overline{RA}$ whose endpts. are $(2b, 2c)$ and $(2d, 2c)$ is $\frac{2c - 2c}{2d - 2b} = 0$. The slopes are $=$. **1c.** $MN = \sqrt{((d + a) - b)^2 + (c - c)^2} = \sqrt{(d + a - b)^2 + (0)^2} = \sqrt{(d + a - b)^2} = d + a - b$. $TP = \sqrt{(2a - 0)^2 + (0 - 0)^2} = \sqrt{(2a)^2} = 2a$. $RA = \sqrt{(2d - 2b)^2 + (2c - 2c)^2} = \sqrt{(2d - 2b)^2 + (0)^2} = \sqrt{(2d - 2b)^2} = 2d - 2b$. $RA + TP = 2d - 2b + 2a$, or $2d + 2a - 2b$, which is $2(d + 2a - 2b)$ or $2 \cdot MN$.
1d. The base along the x-axis allows us to calculate horizontal length by subtracting x-values. **2.** Using multiples of 2 in the coordinates for M, N, P, and O eliminates the use of fractions when finding midpts., since finding midpts. requires division by 2.
Exercises **1a.** $W = \left(\frac{a + 0}{2}, \frac{b + 0}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}\right)$. $Z = \left(\frac{c + e}{2}, \frac{d + 0}{2}\right) = \left(\frac{c + e}{2}, \frac{d}{2}\right)$ **1b.** $W = \left(\frac{2a + 0}{2}, \frac{2b + 0}{2}\right) = (a, b)$. $Z = \left(\frac{2c + 2e}{2}, \frac{2d + 0}{2}\right) = (c + e, d)$ **1c.** $W = \left(\frac{4a + 0}{2}, \frac{4b + 0}{2}\right) = (2a, 2b)$. $Z = \left(\frac{4c + 4e}{2}, \frac{4d + 0}{2}\right) = (2c + 2e, 2d)$ **1d.** c; it uses multiples of 2 to name the coordinates of W and Z. **2a.** origin **2b.** x-axis **2c.** 2 **2d.** coordinates **3a.** y-axis **3b.** distance
4a. right $\angle$ **4b.** legs **4c.** multiples of 2
4d. M **4e.** N **4f.** Midpt. **4g.** Distance **5a.** isosceles
5b. x-axis **5c.** y-axis **5d.** midpoints **5e.** $\cong$ sides
5f. slopes **5g.** the Distance Formula
6a. $\sqrt{(b - (-a))^2 + (c - 0)^2} = \sqrt{(b + a)^2 + c^2}$
6b. $\sqrt{(-b - a)^2 + (c - 0)^2} = \sqrt{((-1)(b + a))^2 + (c - 0)^2} = \sqrt{(-1)^2(b + a)^2 + (c - 0)^2} = \sqrt{1(b + a)^2 + (c - 0)^2} = \sqrt{(b + a)^2 + c^2}$
7a. $\sqrt{(b - 0)^2 + (a - 0)^2} = \sqrt{b^2 + a^2}$
7b. $\sqrt{(0 - 2b)^2 + (2a - 0)^2} = \sqrt{(-2b)^2 + (2a)^2} = \sqrt{4b^2 + 4a^2} = \sqrt{4(b^2 + a^2)} = 2\sqrt{b^2 + a^2}$
8a. $D = \left(\frac{-2b + (-2a)}{2}, \frac{2c + 0}{2}\right) = (-a - b, c)$. $E = (0, 2c)$. $F = \left(\frac{2a + 2b}{2}, \frac{2c + 0}{2}\right) = (a + b, c)$. $G = (0, 0)$
8b. $\sqrt{(0 - (-a - b))^2 + (2c - c)^2} = \sqrt{(a + b)^2 + c^2}$
8c. $\sqrt{(0 - (a + b))^2 + (2c - c)^2} = \sqrt{(a + b)^2 + c^2}$
8d. $\sqrt{(0 - (a + b))^2 + (0 - c)^2} = \sqrt{(a + b)^2 + c^2}$

8e. $\sqrt{(0-(-a-b))^2+(0-c)^2}=\sqrt{(a+b)^2+c^2}$
8f. $\frac{2c-c}{0-(-a-b)}=\frac{c}{a+b}$ 8g. $\frac{c-0}{a+b-0}=\frac{c}{a+b}$
8h. $\frac{2c-c}{0-(a+b)}=-\frac{c}{a+b}$ 8i. $\frac{-a-b-0}{c-0}=\frac{-a-b}{c}=-\frac{a+b}{c}$ 8j. sides 8k. $DEFG$ 9a. $\left(\frac{2a+0}{2},\frac{2b+0}{2}\right)=(a,b)$ 9b. $\left(\frac{0+2a}{2},\frac{2b+0}{2}\right)=(a,b)$ 9c. the same point
10. Any proofs that might involve midpoints, slope ($\parallel$ or $\perp$ lines), or distance involve algebra, so they may be easier to solve on the coordinate plane. Answers may vary. Sample: The Triangle Midsegment Thm.; the segment connecting the midpts. of 2 sides of the $\triangle$ is $\parallel$ to the 3rd side and half its length. You can use the Midpoint Formula and the Distance Formula to prove the statement directly. 11a. $\cong$ 11b. midpoints 11c. It's in Quadrant II where the x-coordinates are negative and the y-coordinates are positive: $(-2b, 2c)$. 11d. $L=\left(\frac{0+2b}{2},\frac{2a+2c}{2}\right)=(b,a+c)$. $M=\left(\frac{2b+0}{2},\frac{2c+0}{2}\right)=(b,c)$. $N=\left(\frac{-2b+0}{2},\frac{2c+0}{2}\right)=(-b,c)$. $K=\left(\frac{0+(-2b)}{2},\frac{2a+2c}{2}\right)=(-b,a+c)$ 11e. They are horizontal, $\parallel$ to the x-axis, so their slopes are 0.
11f. vertical 11g. parallel 11h. perpendicular
12. Answers may vary. Sample: yes; by the Distance Formula 13. Answers may vary. Sample: yes; by showing the lines containing the segments have the same slope
14. Answers may vary. Sample: yes; by showing that the product of the slopes is -1 15. Answers may vary. Sample: No; there is no intersection point, so the segments may not intersect. 16. Answers may vary. Sample: no; may need $\angle$ measures 17. Answers may vary. Sample: no; may need $\angle$ measures 18. Answers may vary. Sample: yes; by showing the product of the slopes of the $\angle$ sides $=-1$ 19. Answers may vary. Sample: by using the Distance Formula 20. Answers may vary. Sample: yes; by using the Distance Formula to show that 2 sides are $=$ 21. Answers may vary. Sample: no; may need $\angle$ measures 22. Answers may vary. Sample: yes; by finding the intersection point of the lines containing the segments 23. Answers may vary. Sample: yes; by showing that the slope of $\overline{AB}$ = the slope of $\overline{BC}$
24. Answers may vary. Sample: yes; by using the Distance Formula to show 4 = sides 25. $|10-(-2)|=12$ and $12\div4=3$. Each segment is 3 units long, so the coordinates are $-2+3$, or 1; $1+3$, or 4; $4+3$, or 7: 1, 4, 7.
26. $|10-(-2)|=12$ and $12\div6=2$. Each segment is 2 units long, so the coordinates are $-2+2$, or 0; $0+2$, or 2; $2+2$, or 4; $4+2$, or 6; $6+2$, or 8: 0, 2, 4, 6, 8.
27. $|10-(-2)|=12$ and $12\div10=1.2$. Each segment is 1.2 units long, so the coordinates are $-2+1.2$, or -0.8; $-0.8+1.2$, or 0.4; $0.4+1.2$, or 1.6; $1.6+1.2$, or 2.8; $2.8+1.2$, or 4; $4+1.2$, or 5.2; $5.2+1.2$, or 6.4; $6.4+1.2$, or 7.6; $7.6+1.2$, or 8.8: -0.8, 0.4, 1.6, 2.8, 4, 5.2, 6.4, 7.6, 8.8. 28. $|10-(-2)|=12$ and $12\div50=0.24$. Each segment is 0.24 units long, so the coordinates are $-2+0.24$, or -1.76; $-1.76+0.24$, or -1.52; $-1.52+0.24$, or -1.28; $-1.28+0.24$, or -1.04; $-1.04+0.24$, or -0.8; $-0.8+0.24$, or -0.56; ... 9.52, 9.76. 29. $|10-(-2)|=12$ and $12\div n=\frac{12}{n}$. Each segment is $\frac{12}{n}$ units long, so the coordinates are $-2+\frac{12}{n}, -2+2\left(\frac{12}{n}\right), -2+3\left(\frac{12}{n}\right),\ldots, -2+(n-1)\left(\frac{12}{n}\right)$. 30. The x-distance is $|9-(-3)|$, or 12, and the y-distance is $|15-5|$, or 10. So, add $12\div4$, or 3 to the x-values starting with -3, and add $10\div4$, or 2.5 to the y-values starting with 5: $(-3+3, 5+2.5)=(0,7.5)$, $(0+3, 7.5+2.5)=(3,10)$, $(3+3,10+2.5)=(6,12.5)$. 31. The x-distance is $|9-(-3)|$, or 12, and the y-distance is $|15-5|$, or 10. So, add $12\div6$, or 2 to the x-values starting with -3, and add $10\div6$, or $1\frac{2}{3}$ to the y-values starting with 5: $\left(-3+2, 5+1\frac{2}{3}\right)=\left(-1,6\frac{2}{3}\right)$, $\left(-1+2, 6\frac{2}{3}+1\frac{2}{3}\right)=\left(1,8\frac{1}{3}\right)$, $\left(1+2, 8\frac{1}{3}+1\frac{2}{3}\right)=(3,10)$, $\left(3+2,10+1\frac{2}{3}\right)=\left(5,11\frac{2}{3}\right)$, $\left(5+2,11\frac{2}{3}+1\frac{2}{3}\right)=\left(7,13\frac{1}{3}\right)$.
32. The x-distance is $|9-(-3)|$, or 12, and the y-distance is $|15-5|$, or 10. So, add $12\div10$, or 1.2 to the x-values starting with -3, and add $10\div10$, or 1 to the y-values starting with 5: $(-3+1.2, 5+1)=(-1.8,6)$, $(-1.8+1.2,6+1)=(-0.6,7)$, $(-0.6+1.2,7+1)=(0.6,8)$, $(0.6+1.2,8+1)=(1.8,9)$, $(1.8+1.2,9+1)=(3,10)$, $(3+1.2,10+1)=(4.2,11)$, $(4.2+1.2,11+1)=(5.4,12)$, $(5.4+1.2,12+1)=(6.6,13)$, $(6.6+1.2,13+1)=(7.8,14)$. 33. The x-distance is $|9-(-3)|$, or 12, and the y-distance is $|15-5|$, or 10. So, add $12\div50$, or 0.24 to the x-values starting with -3, and add $10\div50$, or 0.2 to the y-values starting with 5: $(-3+0.24, 5+0.2)=(-2.76,5.2)$, $(-2.76+0.24, 5.2+0.2)=(-2.52,5.4)$, $(-2.52+0.24,5.4+0.2)=(-2.28,5.6),\ldots,(8.52,14.6),(8.76,14.8)$. 34. The x-distance is $|9-(-3)|$, or 12, and the y-distance is $|15-5|$, or 10. So, add $12\div n$, or $\frac{12}{n}$ to the x-values starting with -3, and add $10\div n$, or $\frac{10}{n}$ to the y-values starting with 5: $\left(-3+\frac{12}{n}, 5+\frac{10}{n}\right), \left(-3+2\left(\frac{12}{n}\right), 5+2\left(\frac{10}{n}\right)\right),\ldots,\left(-3+(n-1)\left(\frac{12}{n}\right), 5+(n-1)\left(\frac{10}{n}\right)\right)$.
35. Assume $b>a$. Divide the distance $b-a$ by n and add it to a, then add it to the sum, and repeat a total of $n-1$ times: $a+\frac{b-a}{n}, a+2\left(\frac{b-a}{n}\right),\ldots,a+(n-1)\left(\frac{b-a}{n}\right)$.
36. Assume $b>a$ and $d>c$. Divide the x-distance $|b-a|$ by n and the y-distance $|d-c|$ by n and add each to the x- and y-coordinates of (a,c), respectively: $\left(a+\frac{b-a}{n}, c+\frac{d-c}{n}\right), \left(a+2\left(\frac{b-a}{n}\right), c+2\left(\frac{d-c}{n}\right)\right),\ldots, \left(a+(n-1)\left(\frac{b-a}{n}\right), c+(n-1)\left(\frac{d-c}{n}\right)\right)$. Repeat for $b>a$ and $d<c$, except subtract $\frac{c-d}{n}$ from c: $\left(a+\frac{b-a}{n}, c-\frac{d-c}{n}\right), \left(a+2\left(\frac{b-a}{n}\right), c-2\left(\frac{d-c}{n}\right)\right),\ldots, \left(a+(n-1)\left(\frac{b-a}{n}\right), c-(n-1)\left(\frac{d-c}{n}\right)\right)$.
37a. The ▲s with bases d and b, and heights c and a, respectively, have the same area. They share the small right $\triangle$ with base d and height c, and the remaining areas are ▲s with base c and height $(b-d)$. So, $\frac{1}{2}ad=\frac{1}{2}bc$. Mult. both sides by 2 gives $ad=bc$. b. The diagram shows that

$\frac{a}{b} = \frac{c}{d}$, since both represent the slope of the top segment of the △. So, by part (a), $ad = bc$. **38.** The centroid is the point of concurrency of the medians of a △. So, divide the quad. into 2 △s by drawing a diagonal. Find the centroid for each △ and connect them with a segment. Now divide the quad. into 2 other △s and follow the same steps. Where the two lines meet connecting the centroids of the 4 △s is the centroid of the quad. **39a.** The midpt. of $A(0,0)$ and $B(2b,2d)$ is $L\left(\frac{0+2b}{2}, \frac{0+2d}{2}\right)$, or $L(b, d)$. The midpt. of $B(2b, 2d)$ and $C(2c, 0)$ is $M\left(\frac{2b+2c}{2}, \frac{2d+0}{2}\right)$, or $M(b+c, d)$. The midpt. of $A(0,0)$ and $C(2c,0)$ is $N\left(\frac{0+2c}{2}, \frac{0+0}{2}\right)$, or $N(c, 0)$. **39b.** Write an equation for $\overleftrightarrow{AM}$: The slope of $\overleftrightarrow{AM} = \frac{d-0}{b+c-0}$, or $\frac{d}{b+c}$, so the equation is $y - 0 = \frac{d}{b+c}(x-0)$, or $y = \frac{d}{b+c}x$. Write an equation for $\overleftrightarrow{BN}$: The slope of $\overleftrightarrow{BN} = \frac{2d-0}{2b-c}$, or $\frac{2d}{2b-c}$, so the equation is $y - 0 = \frac{2d}{2b-c}(x-c)$, or $y = \frac{2d}{2b-c}(x-c)$. Write an equation for $\overleftrightarrow{CL}$: The slope of $\overleftrightarrow{CL} = \frac{b-2c}{d-0}$, or $\frac{b-2c}{d}$, so the equation is $y - 0 = \frac{d-0}{b-2c}(x-2c)$, or $y = \frac{d}{b-2c}(x-2c)$. **39c.** Since the equation for $\overleftrightarrow{AM}$ is $y = \frac{d}{b+c}x$ and the equation for $\overleftrightarrow{BN}$ is $y = \frac{2d}{2b-c}(x-c)$, by substitution, $\frac{d}{b+c}x = \frac{2d}{2b-c}(x-c)$. Multiplying both sides by $(b+c)(2b-c)$ results in $(2b-c)dx = (b+c)2d(x-c)$. Then $2bdx - cdx = 2bdx - 2bcd + 2cdx - 2c^2d$; $0 = -2bcd + 3cdx - 2c^2d$; $0 = -2b + 3x - 2c$; $3x = 2b + 2c$; $x = \frac{2b+2c}{3} = \frac{2}{3}(b+c)$. Then solve for y: $y = \frac{d}{b+c}x$; $y = \frac{d}{b+c}\left(\frac{2}{3}\right)(b+c) = \frac{2}{3}d$. So, the coordinates for P are $\left(\frac{2}{3}(b+c), \frac{2}{3}d\right)$. **39d.** Since the equation for $\overleftrightarrow{AM}$ is $y = \frac{d}{b+c}x$ and the equation for $\overleftrightarrow{CL}$ is $y = \frac{d}{b-2c}(x-2c)$, by substitution, $\frac{d}{b+c}x = \frac{d}{b-2c}(x-2c)$. Dividing both sides by d results in $\frac{x}{b+c} = \frac{x-2c}{b-2c}$; $(b-2c)x = (b+c)(x-2c)$; $bx - 2cx = bx - 2bc + cx - 2c^2$; $-3cx = -2bc - 2c^2$; $3x = 2b + 2c$; $x = \frac{2}{3}(b+c)$. Then solve for y: $y = \frac{d}{b+c}x$; $y = \frac{d}{b+c}\left(\frac{2(b+c)}{3}\right) = \frac{2}{3}d$. So, the coordinates for P are $\left(\frac{2}{3}(b+c), \frac{2}{3}d\right)$. **39e.** Show that AP $= \frac{2}{3}AM$: $AP = \sqrt{\left(\frac{2}{3}(b+c) - 0\right)^2 + \left(\frac{2d}{3} - 0\right)^2} = \sqrt{\frac{4}{9}(b+c)^2 + \frac{4d^2}{9}} = \frac{2}{3}\sqrt{(b+c)^2 + d^2}$; $AM = \sqrt{((b+c) - 0)^2 + (d-0)^2} = \sqrt{(b+c)^2 + d^2}$, so $AP = \frac{2}{3}AM$. Show that $BP = \frac{2}{3}BN$: $BP = \sqrt{\left(\frac{2}{3}(b+c) - 2b\right)^2 + \left(\frac{2d}{3} - 2d\right)^2} = \sqrt{\left(\frac{2}{3}(-2b+c)\right)^2 + \left(\frac{2}{3}(-2d)\right)^2} = \sqrt{\left(\frac{2}{3}\right)^2(-2b+c)^2 + \left(\frac{2}{3}\right)^2(-2d)^2} = \frac{2}{3}\sqrt{(-2b+c)^2 + (-2d)^2} = \frac{2}{3}\sqrt{(2b-c)^2 + (2d)^2}$; $BN = \sqrt{(2b-c)^2 + (2d-0)^2} = \sqrt{(2b-c)^2 + (2d)^2}$, so $BP = \frac{2}{3}BN$. Show that $CP = \frac{2}{3}CL$: $CP = \sqrt{\left(\frac{2}{3}(b+c) - 2c\right)^2 + \left(\frac{2}{3}d - 0\right)^2} = \sqrt{\left(\frac{2}{3}(b-2c)\right)^2 + \left(\frac{2}{3}d\right)^2} = \frac{2}{3}\sqrt{(b-2c)^2 + d^2}$; $CL = \sqrt{(b-2c)^2 + (d-0)^2} = \sqrt{(b-2c)^2 + d^2}$, so $CP = \frac{2}{3}CL$. **40a.** The slope of p is the negative reciprocal of the slope of $\overline{BC}$: $-\frac{-b}{c} = \frac{b}{c}$. **40b.** Answers may vary. Samples: Use the point-slope form of eq. with $(a, 0)$: $y - 0 = \frac{b}{c}(x-a)$, or $y = \frac{b}{c}(x-a)$. Or let (x, y) be a pt. on line p. Then the eq. of p is $\frac{y-0}{x-a} = \frac{b}{c}$ or $y = \frac{b}{c}(x-a)$. **40c.** Line q is on the y-axis so its eq. is $x = 0$. **40d.** Substitute 0 for x in $y = \frac{b}{c}(x-a)$: $y = \frac{b}{c}(-a) = -\frac{ab}{c}$. So, the point of intersection is $\left(0, -\frac{ab}{c}\right)$. **40e.** The slope of line r is the negative reciprocal of $\frac{c}{-a}$: $-\frac{-a}{c} = \frac{a}{c}$. **40f.** Answers may vary. Sample: Let (x, y) be a pt. on line r. Then the eq. of r is $\frac{y-0}{x-b} = \frac{a}{c}$, or $y = \frac{a}{c}(x-b)$. **40g.** Substitute $\left(0, -\frac{ab}{c}\right)$ into $y = \frac{a}{c}(x-b)$: $\frac{-ab}{c} \stackrel{?}{=} \frac{a}{c}(0-b)$; $\frac{-ab}{c} = \frac{-ab}{c}$. The answer checks. It also checks for $x = 0$ since the x-coordinate of $\left(0, \frac{-ab}{c}\right)$ is 0. **40h.** $\left(0, -\frac{ab}{c}\right)$ **41a.** Horiz. lines have slope 0, and vert. lines have undef. slope. Neither could be mult. to get -1. **41b.** Assume that the lines do not intersect. Then they have the same slope, say m. Then $m \cdot m = m^2 = -1$. This is impossible, so the lines must intersect.

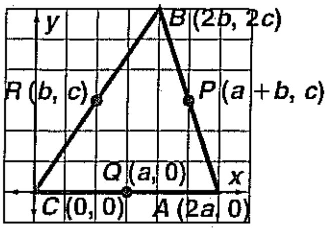
41c.



Let the eq. for ℓ_1 be $y = \frac{b}{a}x$, the eq. for ℓ_2 be $y = -\frac{a}{b}x$, and the origin be the intersection pt. Define $C(a, b)$, $A(0, 0)$, and $B\left(a, -\frac{a^2}{b}\right)$. Using the Distance Formula, $AC = \sqrt{a^2 + b^2}$, $BA = \sqrt{a^2 + \frac{a^4}{b^2}}$, and $CB = b - \left(-\frac{a^2}{b}\right) = b + \frac{a^2}{b}$. Then $\sqrt{a^2 + b^2}^2 + \sqrt{a^2 + \frac{a^4}{b^2}}^2 = \left(a^2 + b^2\right) + \left(a^2 + \frac{a^4}{b^2}\right) = b^2 + 2a^2 + \frac{a^4}{b^2} = b^2 + 2a^2\frac{b^2}{b^2} + \frac{a^4}{b^2} = \left(b + \frac{a^2}{b}\right)^2$, so $AC^2 + BA^2 = CB^2$, and $m\angle A = 90$ by the Converse of the Pythagorean Thm. So, $\ell_1 \perp \ell_2$. **42.** The slope of the first line is $\frac{10-2}{-7-9}$, or $-\frac{1}{2}$. To be $\parallel$, the other line must have an $\cong$ slope, so $\frac{4-(-3)}{x-1} = -\frac{1}{2}$; $\frac{7}{x-1} = -\frac{1}{2}$; $2(7) = -(x-1)$; $14 = -x + 1$; $x = -13$. The answer is choice A. **43.** The slope of the first line is $\frac{2}{3}$. The slope of the line $\perp$ to it is $-\frac{3}{2}$. Use the slope formula: $\frac{y-(-1)}{6-8} = -\frac{3}{2}$; $\frac{y+1}{-2} = \frac{3}{-2}$; $y + 1 = 3$; $y = 2$. The answer is choice G. **44.** [2] a. Use the Midpt. Formula: $\left(\frac{a+7}{2}, \frac{b+(-3)}{2}\right) =$

$(3, 4)$, so $\frac{a + 7}{2} = 3$, thus $a = -1$, and $\frac{b + (-3)}{2} = 4$, thus $b = 11$, so $(a, b) = (-1, 11)$. **b.** The endpts. of the segment are $(7, -3)$ and $(-1, 11)$. Use the Distance Formula: $\sqrt{(7 - (-1))^2 + (-3 - 11)^2} = \sqrt{8^2 + (-14)^2} = \sqrt{64 + 196} = \sqrt{260} = 2\sqrt{65} \approx 16.12$. [1] minor computational error OR no work shown

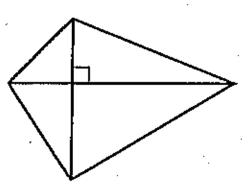
45. [4] **a-b.** Answers may vary. Sample:



c. $AP = \sqrt{(b - a)^2 + c^2} = RQ$, $PQ = QP$, and $AQ = a = RP$, so $\triangle APQ \cong \triangle RQP$ by SSS. **46.** It will be a units left and b units up, so the coordinates are $(-a, b)$. **47a.** To write the inverse, negate both the hypothesis and conclusion: If the sum of the ∡ of a polygon is 360°, then the polygon is a quadrilateral. **47b.** To write the contrapositive, switch the hypothesis and conclusion of the inverse: If a polygon is a quadrilateral, then the sum of the ∡ is 360°. **48a.** To write the inverse, negate both the hypothesis and conclusion: If $x \neq 51$, then $2x \neq 102$. **48b.** To write the contrapositive, switch the hypothesis and conclusion of the inverse: If $2x \neq 102$, then $x \neq 51$. **49a.** To write the inverse, negate both the hypothesis and conclusion: If $a \neq 5$, then $a^2 \neq 25$. **49b.** To write the contrapositive, switch the hypothesis and conclusion of the inverse: If $a^2 \neq 25$, then $a \neq 5$. **50a.** To write the inverse, negate both the hypothesis and conclusion: If $b \geq -4$, then b is not negative. **50b.** To write the contrapositive, switch the hypothesis and conclusion of the inverse: If b is not negative, then $b \geq -4$. **51a.** To write the inverse, negate both the hypothesis and conclusion: If $c \leq 0$, then c is not positive. **51b.** To write the contrapositive, switch the hypothesis and conclusion of the inverse: If c is not positive, then $c \leq 0$. **52.** $\angle A \cong \angle C$, $\overline{AD} \cong \overline{CD}$, and $\angle ADB \cong \angle CDB$, so two ∡ and an included side are $\cong$. Thus, $\triangle ADB \cong \triangle CDB$ by ASA, and by CPCTC, $\overline{AB} \cong \overline{CB}$. **53.** $\overline{HE} \cong \overline{FG}$, $\overline{EF} \cong \overline{GH}$, and $\overline{HF} \cong \overline{FH}$ by the Refl. Prop. of $\cong$, so $\triangle HEF \cong \triangle FGH$ by SSS. By CPCTC, $\angle 1 \cong \angle 2$. **54.** $\overline{LM} \cong \overline{NK}$, $\overline{LN} \cong \overline{NL}$ by the Refl. Prop. of $\cong$, and $\angle LNK \cong \angle NLM$ because all rt. ∡ are $\cong$. So, $\triangle LNK \cong \triangle NLM$ by SAS, and $\angle K \cong \angle M$ by CPCTC.

TEST-TAKING STRATEGIES *page 338*

1.

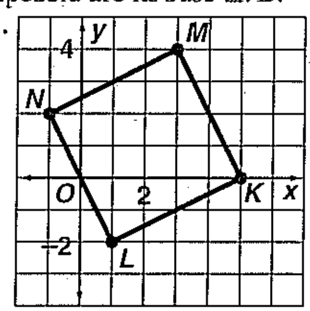


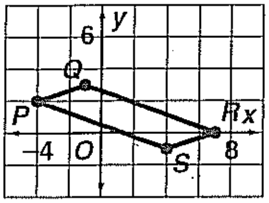
Show two $\perp$ diags. such that neither bisects the other. **2.** D must be the vertex between A and C. Draw $\overrightarrow{AD}$ with slope $\frac{5 - 2}{8 - 5}$, or 1 and $\overrightarrow{CD}$ with slope $\frac{2}{5}$. They intersect at $(3, 3)$. **3.** The missing vertex can be between any two of the given points. Opp. sides of a *parallelogram* are $\parallel$, so the slopes of the opp. sides are =. Find the vertex (a, b) between $(0, 0)$ and $(8, 5)$: The slope of the segment from $(0, 0)$ to (a, b) is $\frac{b}{a}$ and the slope of the side opp. it is $\frac{5 - 2}{8 - 5}$, so $\frac{b}{a} = \frac{5 - 2}{8 - 5}$, or $a = b$. The slope of the segment from $(8, 5)$ to (a, b) is $\frac{b - 5}{a - 8}$ and the slope of the side opp. it is $\frac{2}{5}$, so $\frac{b - 5}{a - 8} = \frac{2}{5}$; $5b - 25 = 2a - 16$; $5b - 2a = 9$; by substitution, $5a - 2a = 9$, so $a = 3 = b$. Thus, $(a, b) = (3, 3)$. Find the vertex (c, d) between $(5, 2)$ and $(8, 5)$: The slope of the segment from $(5, 2)$ to (c, d) is $\frac{d - 2}{c - 5}$ and the slope of the side opp. it is $\frac{5}{8}$, so $\frac{d - 2}{c - 5} = \frac{5}{8}$; $8d - 16 = 5c - 25$; $8d + 9 = 5c$; $c = 1.6d + 1.8$. The slope of the segment from $(8, 5)$ to (c, d) is $\frac{d - 5}{c - 8}$ and the slope of the side opp. it is $\frac{2}{5}$, so $\frac{d - 5}{c - 8} = \frac{2}{5}$; $5d - 25 = 2c - 16$; $5d - 9 = 2c$; $c = 2.5d - 4.5$; by substitution, $1.6d + 1.8 = 2.5d - 4.5$; $16d + 18 = 25d - 45$; $9d = 63$; $d = 7$, so $c = 2.5(7) - 4.5 = 13$. Thus, $(c, d) = (13, 7)$. Find the vertex (e, f) between $(0, 0)$ and $(5, 2)$: The slope of the segment from $(0, 0)$ to (e, f) is $\frac{f}{e}$ and the slope of the side opp. it is $\frac{5 - 2}{8 - 5}$, so $\frac{f}{e} = \frac{5 - 2}{8 - 5}$, or $e = f$. The slope of the segment from $(5, 2)$ to (e, f) is $\frac{f - 2}{e - 5}$ and the slope of the side opp. it is $\frac{5}{8}$, so $\frac{f - 2}{e - 5} = \frac{5}{8}$; $8f - 16 = 5e - 25$; $8f - 5e = -9$; by substitution, $8f - 5f = -9$, $3f = -9$, so $e = -3 = f$. Thus, $(e, f) = (-3, -3)$. **4.** Find the midpt. M of $\overline{AC}$ and draw the $\perp$ bis. having slope $-\frac{3}{2}$. Measure $\overline{MC}$ on the $\perp$ bis. each way from M. Those 2 pts. are B and D: $(1, 5)$ and $(5, -1)$. **5.** Rhombus; a pair of sides is $\parallel$ to each of the 2 diags. and is half their length and since the diags. of an isosc. trap. are $\cong$, the sides of the ▱ are all $\cong$.

CHAPTER REVIEW *pages 339–341*

1. A *rectangle* is a ▱ with four right ∡: F. **2.** A *kite* is a quadrilateral with two pairs of adjacent sides congruent and no opposite sides congruent: H. **3.** Angles of a polygon that share a common side are *consecutive* ∡: G. **4.** A *trapezoid* is a quadrilateral with exactly one pair of parallel sides: B. **5.** A *rhombus* is a ▱ with four congruent sides: I. **6.** The *midsegment* of a trapezoid is the segment that joins the midpoints of the nonparallel sides: J. **7.** A *parallelogram* is a quadrilateral with both pairs of opposite sides parallel: A. **8.** A *square* is a ▱ with four congruent sides and four right ∡: C. **9.** An *isosceles trapezoid* is a trapezoid whose nonparallel sides are congruent: E. **10.** The two ∡ that share a base of a trapezoid are its *base* ∡: D.

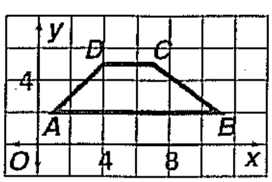
11.



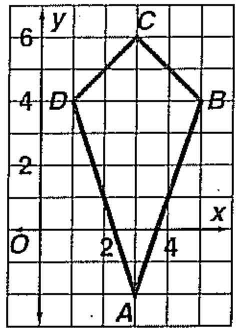
12. 

13. $AD = BC$, so $2x - 7 = x + 1$; $x - 7 = 1$; $x = 8$. $AB = 2x - 2 = 2(8) - 2 = 16 - 2 = 14$. $AD = BC = x + 1 = 8 + 1 = 9$. $CD = x - 1 = 8 - 1 = 7$ **14.** $KL = KN$, so $3m - 5 = m + 3$; $2m - 5 = 3$; $2m = 8$; $m = 4$. $LM = NM$, so $m + 2t = t + 9$; $4 + 2t = t + 9$; $4 + t = 9$; $t = 5$. $KL = KN = m + 3 = 4 + 3 = 7$. $LM = NM = t + 9 = 5 + 9 = 14$ **15.** Since same-side int. ∠s of ∥ lines are suppl., $m\angle 1 = m\angle 3 = 180 - 79 = 101$. Since opp. ∠s of a ▱ are ≅, $m\angle 2 = 79$. **16.** Since alt. int. ∠s of ∥ lines are ≅, $m\angle 1 = 38$. Since same-side int. ∠s are suppl., $m\angle 2 + 38 + 99 = 180$; $m\angle 2 + 137 = 180$; $m\angle 2 = 43$. Since opp. ∠s of a ▱ are ≅, $m\angle 3 = 99$. **17.** Since alt. int. ∠s of ∥ lines are ≅, $m\angle 1 = 37$. By the Triangle Ext. Angle Thm., $m\angle 2 + 37 = 63$; $m\angle 2 = 26$. Since alt. int. ∠s of ∥ lines are ≅, $m\angle 3 = m\angle 2 = 26$. **18.** yes, by Thm. 6-8 **19.** yes, by Thm. 6-7 **20.** No; there is no information about the diagonals, ∠s or sides. **21.** yes, by Thm. 6-5 **22.** Since same-side int. ∠s are suppl., $4x + 2x + 6 = 180$; $6x + 6 = 180$; $x + 1 = 30$; $x = 29$. Since opp. ∠s are ≅, $4x = 4y + 4$; $4(29) = 4y + 4$; $29 = y + 1$; $y = 28$. **23.** The diags. of a ▱ bis. each other, so $3x = 3y - 3$; $x = y - 1$. Substitute $y - 1$ for x in $4x - 2 = 3y - 1$: $4(y - 1) - 2 = 3y - 1$; $4y - 4 - 2 = 3y - 1$; $4y - 6 = 3y - 1$; $y - 6 = -1$; $y = 5$. $x = y - 1 = 5 - 1 = 4$ **24.** $m\angle 1 = 180 - 56 = 124$. $2m\angle 2 + m\angle 1 = 180$; $2m\angle 2 + 124 = 180$; $2m\angle 2 = 56$; $m\angle 2 = 28$. $m\angle 2 + m\angle 3 = 90$; $28 + m\angle 3 = 90$; $m\angle 3 = 62$ **25.** The diags. of a rhombus bis. the ∠s, so $m\angle 1 = 60$. The diags. of a rhombus are ⊥, so $m\angle 2 = 90$. By the Triangle Ext. Angle Thm., $m\angle 3 + 60 = 90$, so $m\angle 3 = 30$. **26.** The diags. of a kite are ⊥, so $m\angle 1 = 90$. By the Triangle Ext. Angle Thm., $m\angle 2 + 65 = m\angle 1$; $m\angle 2 + 65 = 90$, so $m\angle 2 = 25$. **27.** The diags. of a ▱ bis. each other, so $AC = 2(13) = 26$ in. **28.** The diags. of a ▱ bis. each other, and the diags. of a rectangle are ≅, so $AC = 2(10) = 20$ cm. **29.** The diags. of an isosc. trapezoid are ≅, so $AC = 7 + 12 = 19$ ft. **30.** P is a units right and b units up from the origin: (a, b). **31.** Since the diags. of a square bis. each other, P is c units up from the origin: $(0, c)$. **32.** The distance P is to the right of the origin is $-b - (-a)$, or $a - b$. The distance P is up from the origin is c units. So, $P = (a - b, c)$. **33a.** $\frac{a - 0}{0 - a} = -1$ **33b.** $\frac{a}{a} = 1$ **33c.** $-1(1) = -1$; the prod. of the slopes is -1. **34a.** C is a units to the right of the origin: a. **34b.** D is b units up from the origin: b. **34c.** $\sqrt{a^2 + b^2}$ **34d.** $\sqrt{a^2 + b^2}$ **34e.** BD

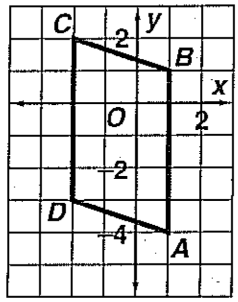
CHAPTER TEST *page 342*

1. 

Since exactly 2 sides are ∥, it is a trapezoid.

2. 

Since 2 distinct pair of adj. sides are ≅, it is a kite.

3. 

Since one pair of opp. sides is both ∥ and ≅, it is a ▱. **4a.** The product of the slopes of the adj. sides must $= -1$. Answers may vary. Sample: $(0, 0)$, $(3, 0)$, $(3, 3)$, $(0, 3)$ **4b.** One pair of opp. sides must be ∥ and ≅. Answers may vary. Sample: $(0, 0)$, $(3, 0)$, $(4, 3)$, $(1, 3)$ **4c.** Adj. sides must be ⊥. Answers may vary. Sample: $(0, 0)$, $(5, 0)$, $(5, 3)$, $(0, 3)$ **4d.** Exactly one pair of opp. sides can be ∥. Answers may vary. Sample: $(0, 0)$, $(5, 0)$, $(4, 3)$, $(1, 3)$ **5.** Since opp. sides of a ▱ are ≅, $AN = ML = 6$ in. **6.** Since the diags. of a ▱ bis. each other, $AN = AR = 9$ cm. **7.** Make one ▱ and then make the second one by "squishing" the first one. Check students' work. **8.** $x = 180 - 80 = 100$. The ▱ is a rectangle, and the diags. divide it into 4 ▵s, or 2 pair of ≅ isosc. ▵s. So, $z = 90 - y$, or $y + z = 90$. One △ has ∠ measures of 100, z, and $90 - y$, so by the Triangle Angle-Sum Thm., $y - z = 10$. Adding the equations $y - z = 10$ and $y + z = 90$ results in $2y = 100$, so $y = 50$. Then $z = 40$. **9.** The figure is a rhombus and the diags. of a rhombus bis. the ∠s. Thus, $y = 57$. Because base ∠s of an isosc. △ are ≅, $x = 57$. Then $z = 180 - (x + y) = 180 - 2(57) = 66$ **10.** Since the diags. of a ▱ bis. each other, $4x - 4 = 3x + 2$; $x - 4 = 2$; $x = 6$. $2y + 2 = 2x$; $2y + 2 = 2(6)$; $y + 1 = 6$; $y = 5$ **11.** Since alt. int. ∠s of ∥ lines are ≅, $x = 36$. The ▱ is a rectangle. The diags. of a rectangle are ≅ and bis. each other so they form 4 isos. ▵s. Thus, the sum of the measures of the ∠s in the "top" △ is $2x + y = 180$; $2(36) + y = 180$; $y = 108$. $z = 180 - 108 = 72$. **12.** No; both diags. must bis. each other. **13.** Yes; by Thm. 6-6, if one pair of opp. sides of a quad. is ∥ and ≅, then the quad. is a ▱. **14.** Yes; by Thm. 6-7, if both pairs of opp. sides of a quad. are ≅, then the quad. is a ▱. **15.** Yes; by Thm. 6-8, if both pairs of opp. ∠s of a quad. are ≅, then the quad. is a ▱. **16.** Answers may vary. Sample: A square has 4 ≅ sides and a kite has no opp. sides ≅. **17.** Both pairs of opp. sides are ≅, so $7x - 2 = 5x + 2$; $2x - 2 = 2$; $x = 2$. $7x - 1 = 6x + y$; $7(2) - 1 = 6(2) + y$; $13 = 12 + y$; $y = 1$ **18.** The diags. of a kite are ⊥, so $m\angle 1 = 90$. The ⊥ bis. of an isosc. △ bis. the vertex ∠, so the vertex ∠ measures $60 + 60 = 120$ and $2m\angle 2 + 120 = 180$; $2m\angle 2 = 60$; $m\angle 2 = 30$. **19.** Base ∠s of an isosc. trapezoid are ≅, so $m\angle 1 = 50$. The ∠s of a trapezoid whose sides share a leg are suppl., so $m\angle 2 = 180 - 50 = 130$. **20.** The axes are lines of symmetry, so S is a units left and b units down from the origin: $S(-a, -b)$ and T is a units left and b units up from the origin: $T(-a, b)$. The midpt. of $\overline{ST}$ is $\left(\frac{-a + (-a)}{2}, \frac{-b + b}{2}\right) = (-a, 0)$. $\overline{ST}$ is vertical, so it has no slope. **21.** S is at the origin: $S(0, 0)$ and T is c units right of b and d units up from the origin: $T(b + c, d)$. The slope of $\overline{ST}$ is $\frac{d}{b + c}$. **22.** The midsegment of a trapezoid is half the sum of the bases: $\frac{1}{2}(25 + 15) = \frac{1}{2}(40) = 20$, or 20 ft. **23a.** $\sqrt{a^2 + a^2} = \sqrt{2a^2} = a\sqrt{2}$ **23b.** $\sqrt{(0 - a)^2 + (a - 0)^2} = \sqrt{a^2 + a^2} = a\sqrt{2}$ **23c.** AC

STANDARDIZED TEST PREP *page 343*

1. Both pairs of opp. sides are $\parallel$, so it is a ▱. It has 4 right ∡, so it is a rectangle. The answer is choice C. **2.** By the Triangle Angle-Sum Thm., the 3rd $\angle$ measures $180 - (48 + 84) = 48$. The answer is choice I. **3.** $DE = \frac{1}{2}(23) = 11.5$. The answer is choice A. **4.** By the Alt. Int. Angles Thm., the answer is choice G. **5.** Two sides and an included $\angle$ are $\cong$: SAS. The answer is choice C. **6.** $m\angle MOQ + 90 + 30 = 180$, so $m\angle MOQ = 60$. The answer is choice I. **7.** The smallest $\angle$ is opp. the shortest side and the largest $\angle$ is opp. the longest side. Since $13 > 11$, $m\angle G > m\angle H$. The answer is choice A. **8.** The smallest $\angle$ is opp. the shortest side and the largest $\angle$ is opp. the longest side. Since $13 < 14$, $m\angle G < m\angle J$. The answer is choice B. **9.** The smallest $\angle$ is opp. the shortest side and the largest $\angle$ is opp. the longest side. Since $11 < 14$, $m\angle H < m\angle J$. The answer is choice B. **10.** The diags. of a ▱ bis. each other, so $DE = \frac{1}{2}DB = \frac{1}{2}(15) = 7.5$. **11.** The sum of the exterior ∡ of a polygon is 360, so the mean $\angle$ measure of the pentagon is $\frac{360}{5}$, or 72. So, the mean $\angle$ measure of the interior ∡ is $180 - 72$, or 108. Then the sum of the measures of the 5 ∡ is 5(108), or 540. So, $x + 138 + 86 + 93 + 135 = 540$; $x + 452 = 540$; $x = 88$. **12.** [2] Since opp. sides of a ▱ are $\cong$, $5(x - 2) = 3(x + 2)$; $5x - 10 = 3x + 6$; $2x - 10 = 6$; $2x = 16$; $x = 8$. [1] one computational error **13.** Answer includes any of the points (0, 8), (6, −2), or (−4, 2), and an explanation involving same slopes for opp. sides, or same side lengths using Dist. Formula. [3] a correct point from a sketch [2] an incorrect point, but correct approach with a computational error [1] no work shown

REAL-WORLD SNAPSHOTS *pages 344–345*

a. The spinning triangle should not wobble. **b.** The spinning triangle wobbles or falls off the pencil. **c.** The medial of (16, 0) and (20, 18) is (18, 9). The centroid is $\frac{2}{3}$ the distance from (0, 0) to (18, 9) which is $\left(18 \cdot \frac{2}{3}, 9 \cdot \frac{2}{3}\right)$, or (12, 6). The triangle should not wobble with the pencil point at (12, 6). **d.** The results are the same.