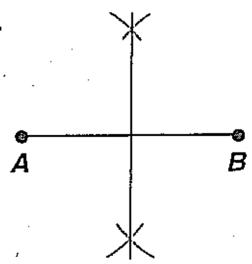
Chapter 5 Relationships Within Triangles

pages 240–285

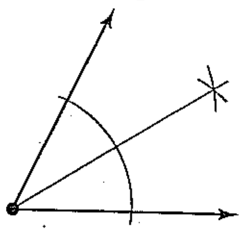
DIAGNOSING READINESS — page 240

1. $3x + 10 \le 22; 3x \le 12; x \le 4$ **2.** $4x - 1 > 2x + 14;$ $2x - 1 > 14; 2x > 15; x > \frac{15}{2}$ **3.** $30 - 5x \ge x + 24;$ $30 - 6x \ge 24; -6x \ge -6; x \le 1$

4.



Open the compass to more than half the length of the segment. With the compass point on one endpt., swing arcs having the same radius above and below the segment. Keep the same setting and, with the compass point on the other endpt., swing arcs to intersect the first two arcs. Draw a line through the arc intersections. **5.** With the compass



point on the angle vertex, swing an arc that intersects both sides of the angle. With the compass point on the intersection of the arc and one of its sides, swing an arc in the interior of the angle. Keep the setting and, with the compass point on the intersection of the arc with the other side of the angle, swing an arc to intersect the previous arc. Draw a ray from the angle vertex through the intersection of the two arcs.

6. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4-1)^2 + (8-4)^2} = \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ **7.** $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-1 - (-6))^2 + (14 - 2)^2} = \sqrt{(-1 + 6)^2 + (14 - 2)^2} = \sqrt{(5)^2 + (12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$
8. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(5 - (-3))^2 + (-6 - (-2))^2} = \sqrt{(5 + 3)^2 + (-6 + 2)^2} = \sqrt{(8)^2 + (-4)^2} = \sqrt{64 + 16} = \sqrt{80} = \sqrt{16 \cdot 5} = 4\sqrt{5}$
9. The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{4 + 6}{2}, \frac{11 + 3}{2}\right) = (5, 7)$. **10.** The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-8 + 2}{2}, \frac{-3 + (-4)}{2}\right) = \left(-3, -\frac{7}{2}\right)$. **11.** The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-7 + (-2)}{2}, \frac{15 + (-10)}{2}\right) = \left(-\frac{9}{2}, \frac{5}{2}\right)$.
12. $\frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - 3}{7 - 8} = \frac{9}{-1} = -9$ **13.** $\frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - (-2)}{0 - 3} = \frac{8}{-3} = -\frac{8}{3}$ **14.** $\frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 4}{-2 - (-5)} = \frac{0}{3} = 0$

TECHNOLOGY — page 242

1. Slopes of midsegments are the same as the slopes of the third side and $\frac{1}{2}$ the length of the third side. **2.** Yes; the slopes are the same and the lengths are $\frac{1}{2}$ the lengths of the $\parallel$ sides. **3a.** $\overline{AD} \cong \overline{DB} \cong \overline{EF}; \overline{AE} \cong \overline{EC} \cong \overline{DF};$ $\overline{DE} \cong \overline{BF} \cong \overline{FC}$ **3b.** $\triangle ADE \cong \triangle FED \cong \triangle DBF \cong \triangle EFC$ by the SSS Post. (Post. 4-1), which says if the three sides of one $\triangle$ are $\cong$ to three sides of another $\triangle$, then the two ⚠ are $\cong$. **4.** The areas of the $\cong$ ⚠ are $=$. **5a.** Since 4 ⚠ of $=$ area comprise $\triangle ABC$, it is 4 times the area of each small $\triangle$. **5b.** The perimeter of $\triangle ABC$ is 2 times the perimeter of each small $\triangle$. **6a.** The area of $\triangle GHI$ is $\frac{1}{16}$ the area of $\triangle ABC$. **6b.** The perimeter of $\triangle GHI$ is $\frac{1}{4}$ the perimeter of $\triangle ABC$. **6c.** For the next midsegment $\triangle$, the area would be $\frac{1}{4} \cdot \frac{1}{16}$, or $\frac{1}{64}$, the area of $\triangle ABC$ and the perimeter would be $\frac{1}{2} \cdot \frac{1}{4}$, or $\frac{1}{8}$, the perimeter of $\triangle ABC$. **7.** The inner quad. will be a parallelogram. The sides of quad. $YXWV$ are $\frac{1}{2}$ the length of the diagonals of quad. $RSTU$.

5-1 Midsegments of Triangles — pages 243–248

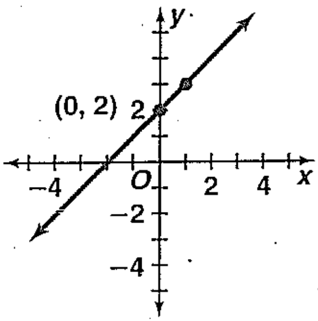
Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1. $(1, 2)$ **2.** $\left(\frac{3}{2}, \frac{11}{2}\right)$ **3.** $\left(-\frac{1}{2}, 8\right)$ **4.** $(1, 1)$ **5.** $-\frac{2}{5}$ **6.** $\frac{1}{3}$ **7.** $\frac{4}{7}$ **8.** $\frac{3}{2}$
Investigation **1.** $LN = \frac{1}{2}AB$; explanations may vary. Sample: Each side of point C on $\overline{AB}$ is folded in half and creates a length $=$ to LN. **2.** Answers may vary. Sample: The midsegment is $\parallel$ to the 3rd side of the $\triangle$ and is half its length.
Check Understanding **1.** By def. of midsegment, $\overline{EB}$ is a midsegment. By the Triangle Midsegment Thm., $EB = \frac{1}{2}DC = \frac{1}{2}(18) = 9$. Since $AB = BC$, $BC = 10$. $AC = AB + BC = 10 + 10 = 20$ **2.** By the Triangle Midsegment Thm., $\overline{UV} \parallel \overline{XY}$. By the Corr. Angles Post., $m\angle VUZ = m\angle YXZ = 65$. **3a.** By def. of midsegment, $\overline{CD}$ is a midsegment. By the Triangle Midsegment Thm., $CD = \frac{1}{2}(2640) = 1320$, which means 1320 ft. **3b.** One mile is 5280 ft, so the bridge is $\frac{1320}{5280}$, or $\frac{1}{4}$ mi.
Exercises **1.** Since x is the length of a midsegment, $x = \frac{1}{2}(18) = 9$. **2.** Since $5x$ is the length of a midsegment, $5x = \frac{1}{2}(70); 5x = 35; x = 7$. **3.** Since $3x$ is the length of a midsegment, $3x = \frac{1}{2}(84); 3x = 42; x = 14$. **4.** Since $x - 1$ is the length of a midsegment, $x - 1 = \frac{1}{2}(45); x - 1 = 22\frac{1}{2};$ $x = 23\frac{1}{2}$. **5.** Since 5 is the length of a midsegment, $5 = \frac{1}{2}(x - 1); 10 = x - 1; x = 11$. **6.** Since 4 is the length of a midsegment, $4 = \frac{1}{2}(5x - 2); 8 = 5x - 2;$ $10 = 5x; x = 2$. **7.** Since HE is the length of a midsegment, $HE = \frac{1}{2}(UV) = \frac{1}{2}(80) = 40$. **8.** Since ED is the length of

a midsegment, $ED = \frac{1}{2}(TV) = \frac{1}{2}(100) = 50$. **9.** Since HD is the length of a midsegment, $HD = \frac{1}{2}(TU)$, so $TU = 2(HD) = 2(80) = 160$. **10.** Since E is a midpt. and, from Exercise 10, $TU = 160$, $TE = \frac{1}{2}(TU) = \frac{1}{2}(160) = 80$. **11.** $\overline{UW}$, $\overline{YW}$, and $\overline{UY}$ are midsegments, so, by the Triangle Midsegment Thm., $\overline{UW} \parallel \overline{TX}$, $\overline{UY} \parallel \overline{VX}$, and $\overline{YW} \parallel \overline{TV}$. **12.** G, J, and L are midpts., so, by the Triangle Midsegment Thm., $\overline{GJ} \parallel \overline{FK}$, $\overline{JL} \parallel \overline{HF}$, and $\overline{GL} \parallel \overline{HK}$. **13a.** S, T, and U are midpts., so, by the Triangle Midsegment Thm., $\overline{ST} \parallel \overline{PR}$, $\overline{SU} \parallel \overline{QR}$, and $\overline{UT} \parallel \overline{PQ}$. **13b.** By the Triangle Midsegment Thm., $\overline{ST} \parallel \overline{PR}$. Since corr. ∠s of ∥ lines are ≅, $m\angle QPR = m\angle QST = 40$. **14.** Since F and E are midpts., by the Triangle Midsegment Thm., $\overline{AB} \parallel \overline{FE}$. **15.** F and G are midpts., so, by the Triangle Midsegment Thm., $\overline{BC} \parallel \overline{FG}$. **16.** E and F are midpts., so, by the Triangle Midsegment Thm., $\overline{EF} \parallel \overline{AB}$. **17.** G and E are midpts., so, by the Triangle Midsegment Thm., $\overline{CA} \parallel \overline{EG}$. **18.** E and G are midpts., so, by the Triangle Midsegment Thm., $\overline{GE} \parallel \overline{AC}$. **19.** F and G are midpts., so, by the Triangle Midsegment Thm., $\overline{FG} \parallel \overline{CB}$. **20a.** The sides of the △ measure 250 strides, $80 + 80 = 160$, which means 160 strides, and $150 + 150 = 300$, which means 300 strides, so the longest side is 300 strides. $(300)(3.5) = 1050$, which means 1050 ft. **20b.** The distance she must paddle is the length of a midsegment. By the Triangle Midsegment Thm., its distance $= \frac{1}{2}(250)(3.5) = 4375$, which means 4375 ft. **21a.** By the Triangle Midsegment Thm., the length of the highlighted segment is $\frac{1}{2}(229.5) = 114.75$, which means 114.75 ft, or 114 ft 9 in. **21b.** Answers may vary. Sample: The highlighted segment is a midsegment of the triangular face of the building. By the Triangle Midsegment Thm., the length of the highlighted segment is half the length of the base. **22.** By the Triangle Midsegment Thm., $\overline{XY} \parallel \overline{VW}$ with transversal $\overline{UV}$, so corr. ∠s are ≅, or have = measures; $m\angle V = m\angle UXY = 60$. **23.** By the Triangle Midsegment Thm., $\overline{XY} \parallel \overline{VW}$ with transversal $\overline{UW}$, so corr. ∠s are ≅, or have = measures; $m\angle UYX = m\angle W = 45$. **24.** By the Triangle Midsegment Thm., $50 = \frac{1}{2}(VW)$, so $VW = 100$. **25.** By the Triangle Midsegment Thm., $XY = \frac{1}{2}(VW) = \frac{1}{2}(110) = 55$. **26a.** $H = \left(\frac{3+1}{2}, \frac{-2+2}{2}\right) = (2, 0)$; $J = \left(\frac{3+5}{2}, \frac{-2+6}{2}\right) = (4, 2)$. **26b.** The slope of $\overline{HJ} = \frac{4-2}{2-0} = \frac{2}{2} = 1$. The slope of $\overline{EF} = \frac{6-2}{5-1} = \frac{4}{4} = 1$. Since the slopes are =, $\overline{HJ} \parallel \overline{EF}$. **26c.** Solve for HJ: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4-2)^2 + (2-0)^2} = \sqrt{(2)^2 + (2)^2} = \sqrt{2(2)^2} = 2\sqrt{2}$. Solve for EF: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(6-2)^2 + (5-1)^2} = \sqrt{(4)^2 + (4)^2} = \sqrt{2(4)^2} = 4\sqrt{2}$. Since $2\sqrt{2} = \frac{1}{2}(4\sqrt{2})$, $HJ = \frac{1}{2}EF$. **27.** By def. of midsegment, I and J are midpts., so $HI = \frac{1}{2}HF = \frac{1}{2}(10) = 5$, and $HJ = \frac{1}{2}HG = \frac{1}{2}(13) = 6\frac{1}{2}$. $P = HI + IJ + JH = 5 + 7 + 6\frac{1}{2} = 18\frac{1}{2}$ **28.** By the Triangle Midsegment Thm., $IJ = \frac{1}{2}FG$; $7 = \frac{1}{2}FG$; $FG = 14$. $P = HF + HG + FG = 10 + 13 + 14 = 37$ **29.** By the Triangle Midsegment Thm., $30 = \frac{1}{2}x$; $x = 60$. **30.** By the Triangle Midsegment Thm., $25 = \frac{1}{2}x$; $x = 50$. **31.** In the small △, the base ∠ of an isosc. △ is 60, so both base ∠s must be 60, and the vertex ∠ must also be 60. So, the △ is equiangular and also equilateral. Each side measures 5. So, $x = 2(5) = 10$. **32.** By the Triangle Midsegment Thm., $x = \frac{1}{2}(3x - 6)$; $2x = 3x - 6$; $-x = -6$; $x = 6$. By the Triangle Midsegment Thm., $y = \frac{1}{2}(2x + 1) = \frac{1}{2}(2(6) + 1) = \frac{1}{2}(12 + 1) = 6\frac{1}{2}$. **33.** The kite consists of 4 overlapping △s, and each segment of ribbon forms a midsegment. The midsegments ∥ to the base of 64 cm are each 32 cm, so 2 pieces of 32-cm ribbon are needed. The midsegments ∥ to the base of 90 cm are each 45 cm, so 2 pieces of 45-cm ribbon are needed. The total amount of ribbon needed is $2(32) + 2(45) = 64 + 90 = 154$ cm. **34.** Since $\overline{DB} \cong \overline{BA}$, $BA = DB = 8$, so $DA = 16$. By the Triangle Midsegment Thm., $6 = \frac{1}{2}AF$, so $AF = 12$. $P = DA + AF + DF = 16 + 12 + 24 = 52$ **35.** By the Triangle Midsegment Thm., $2x + 6 = \frac{1}{2}(5x + 9)$; $2(2x + 6) = 5x + 9$; $4x + 12 = 5x + 9$; $12 = x + 9$; $x = 3$. $DF = 5x + 9 = 5(3) + 9 = 15 + 9 = 24$ **36.** By the Triangle Midsegment Thm., $3x - 1 = \frac{1}{2}(5x + 7)$; $2(3x - 1) = 5x + 7$; $6x - 2 = 5x + 7$; $x - 2 = 7$; $x = 9$. $EC = 3x - 1 = 3(9) - 1 = 27 - 1 = 26$ **37.** Answers may vary. Sample: Draw $\overline{CA}$ and extend $\overrightarrow{CA}$ to P so $CA = AP$. Find B, the midpt. of $\overline{PD}$. Then, by the Triangle Midsegment Thm., $\overline{AB} \parallel \overline{CD}$ and $AB = \frac{1}{2}CD$. **38.** The slope of $\overline{KM} = \frac{3-2}{2-6} = -\frac{1}{4}$ and $\overline{KM}$ is a midsegment ∥ to $\overline{HJ}$, so the slope of $\overline{HJ}$ is also $-\frac{1}{4}$. $L(4, 1)$ lies on $\overline{HJ}$, so 1 unit down and 4 units right of L is $(4 + 4, 1 - 1) = (8, 0)$. Since the distance from L to $(8, 0) = KM$, $J = (8, 0)$. H is 1 unit up and 4 units left of L, so $H = (4 - 4, 1 + 1) = (0, 2)$. Draw rays from H through K and from J through M so that they intersect at $G(4, 4)$. **39.** The fourth ≅ △ is $\triangle UTS$. Proofs may vary. Sample: $\overline{VS} \cong \overline{SY}$, $\overline{YT} \cong \overline{TZ}$, and $\overline{VU} \cong \overline{UZ}$ because S, T, and U are midpts. of the respective sides. By the Triangle Midsegment Thm., $ST = \frac{1}{2}VZ$, so $\overline{ST} \cong \overline{VU} \cong \overline{UZ}$; $SU = \frac{1}{2}YZ$, so $\overline{SU} \cong \overline{YT} \cong \overline{TZ}$; and $TU = \frac{1}{2}VY$, so $\overline{TU} \cong \overline{SY} \cong \overline{SV}$. Therefore, $\triangle YST \cong \triangle TUZ \cong \triangle SVU \cong \triangle UTS$ by the SSS Post. **40.** By the Triangle Midsegment Thm., $x + 85 = \frac{1}{2}(3x + 46)$; $2(x + 85) = 3x + 46$; $2x + 170 = 3x + 46$; $170 = x + 46$; $x = 124$. Solve for PS and RS: $PS = 124 = \frac{1}{2}RS$; $RS = 248$. **41.** From Exercise 40, $x = 124$, so $RQ = x + 50 = 124 + 50 = 174$. $RT = RQ = 174$. **42.** From Exercise 40, $x = 124$, so $TS = 3x + 46 = 3(124) + 46 = 372 + 46 = 418$. **43.** By the Triangle Midsegment Thm., $\overline{BC} \parallel \overline{FD}$ and $\overline{AF}$ is a transversal, so corr. ∠s are ≅. Thus, $m\angle ABC = m\angle BFE = 70$. **44.** $m\angle ACB + m\angle BCD = 180$; $m\angle ACB + 140 = 180$; $m\angle ACB = 40$. By the Triangle Midsegment Thm., $\overline{BC} \parallel \overline{FD}$, so corr. ∠s are ≅. Thus, $m\angle D = m\angle ACB = 40$. **45.** From Exercise 43, $m\angle ABC = 70$. By the Triangle Exterior Angle Thm., $m\angle A + m\angle ABC = 140$; $m\angle A + 70 = 140$; $m\angle A = 70$. **46.** Since $\overline{BE} \parallel \overline{AD}$, same-side int. ∠s are suppl., so

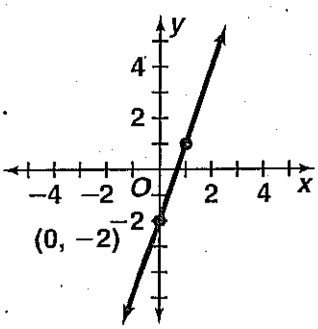
$m\angle CBE + m\angle BCD = 180; m\angle CBE + 140 = 180;$ $m\angle CBE = 40.$ **47.** $\overline{ST} \cong \overline{TS}$ by the Refl. Prop. of $\cong$. Two sides and an included $\angle$ of one $\triangle$ are $\cong$ to the same of another $\triangle$, so $\triangle SXT \cong \triangle TYS$ by SAS Thm.
48. $\angle C \cong \angle C$ by the Refl. Prop. of $\cong$. By the Segment Addition Post., $\overline{AC} \cong \overline{EC}$. Two $\angle$s and an included side of one $\triangle$ are $\cong$ to the same in another $\triangle$, so $\triangle ADC \cong \triangle EBC$ by ASA Thm. **49.** Answers may vary. Sample: By the Segment Addition Post., $\overline{LQ} \cong \overline{NR}$. A hyp. and leg of one rt. $\triangle$ are $\cong$ to the same in another right $\triangle$, so $\triangle KLQ \cong \triangle PNR$ by HL. Another possibility is that, since $\triangle MRQ$ is isosc., its base $\angle$s are $\cong$, so $\triangle KLQ \cong \triangle PNR$ by ASA Thm.

50.



The coefficient of x is 1, so the slope is 1. The y-intercept is 2. Graph (0, 2) and plot another point 1 unit up and 1 unit right at (1, 3). Draw a line through the two points.

51.



The coefficient of x is 3, so the slope is 3. The y-intercept is -2. Graph $(0, -2)$ and plot another point 3 units up and 1 unit right at (1, 1). Draw a line through the two points.

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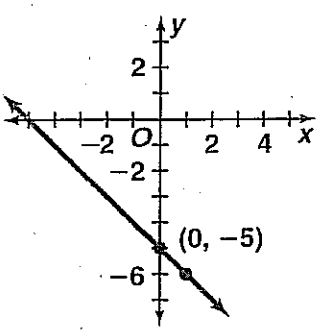
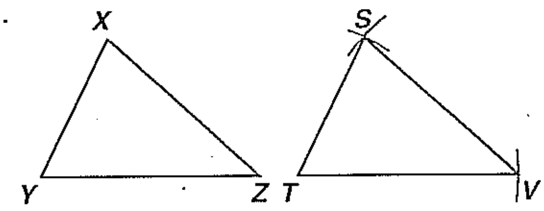


The coefficient of x is -1, so the slope is -1. The y-intercept is -5. Graph $(0, -5)$ and plot another point 1 unit down and 1 unit right at $(1, -6)$. Draw a line through the two points. **53.** Since corr. $\angle$s of $\|$ lines are $\cong$, $3x + 5 = 144; 3x = 139; x = 46\frac{1}{3}$.
54. Since alt. int. $\angle$s of $\|$ lines are $\cong$, $2x = 70$, so $x = 35$. **55.** Since alt. int. $\angle$s of $\|$ lines are $\cong$, $3x - 5 = 115;$ $3x = 120; x = 40.$

5-2 Bisectors in Triangles pages 249–254

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1.



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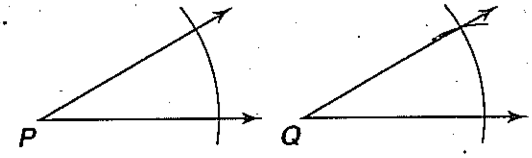

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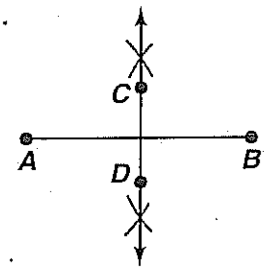

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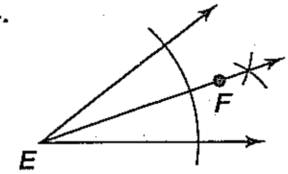


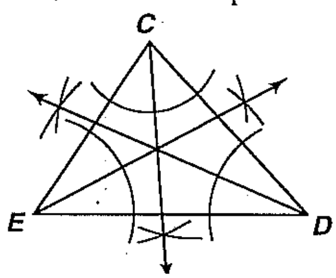
5. 6 **6.** 68

Check Understanding **1.** $\overleftrightarrow{CD}$ is the $\perp$ bis. of $\overline{AB}$, so by the Perpendicular Bisector Thm., $CA = CB = 5$ and $DB = DA = 6$. **2a.** According to the tick marks, $KD = KH = 10$. **2b.** By the Converse of the Angle Bisector Thm., $\overrightarrow{EK}$ is the $\angle$ bisector of $\angle DEH$. **2c.** By the Converse of the Angle Bisector Thm., $\overline{EC}$ is the $\angle$ bisector of $\angle DEH$, so $2x = x + 20; x = 20$. **2d.** $m\angle DEH = 2x + x + 20 = 3x + 20 = 3(20) + 20 = 60 + 20 = 80$

Exercises **1.** $\overline{AC}$ bis. and is $\perp$ to $\overline{BD}$, so $\overline{AC}$ is the $\perp$ bis. of $\overline{BD}$. **2.** By the Perpendicular Bisector Thm., $AB = AD = 15$. **3.** By the Perpendicular Bisector Thm., $BC = DC = 18$. **4.** By the Perpendicular Bisector Thm., $ED = BE = 8$. **5.** By the Converse of the Perpendicular Bisector Thm., the set of points equidistant from H and S is the perpendicular bisector of $\overline{HS}$. **6.** By the Angle Bisector Thm., $2x - 7 = x + 5; x - 7 = 5; x = 12. JK = JM = x + 5 = 12 + 5 = 17$ **7.** By the Angle Bisector Thm., $5y = 3y + 6; 2y = 6; y = 3. ST = TU = 5y = 5(3) = 15$ **8.** By the Converse of the Angle Bisector Thm., $\overrightarrow{HL}$ is the $\angle$ bis. of $\angle JHG$ because a point on $\overrightarrow{HL}$ is equidistant from J and G. **9.** From Exercise 8, $\overrightarrow{HL}$ bis. $\angle FHK$, so $6y = 4y + 18; 2y = 18; y = 9. m\angle FHL = m\angle KHL = 6y = 6(9) = 54$ **10.** By the markings on the diagram, $EF = EK = 27$. **11.** By the Converse of the Angle Bis. Thm., Point E is on the bisector of $\angle KHF$. **12.** $\overline{YW}$ is the $\perp$ bis. of $\overline{TZ}$, so by the Perpendicular Bisector Thm., $2x = 3x - 5; -x = -5; x = 5$. **13.** From Exercise 12, $x = 5$; $TW = 2x = 2(5) = 10$. **14.** From Exercise 12, $x = 5$; $WZ = 3x - 5 = 3(5) - 5 = 15 - 5 = 10$. **15.** Since at least 2 of the sides have = length, the $\triangle$ is isosceles. **16.** By the Perpendicular Bisector Thm., if R is on the perpendicular bisector of $\overline{TZ}$, then r is *equidistant* from T and Z, or $RT = RZ$. **17.** To write a biconditional, separate the hypothesis and conclusion of one of the statements with "if and only if": A point is on the $\perp$ bis. if and only if it is equidistant from the endpts.

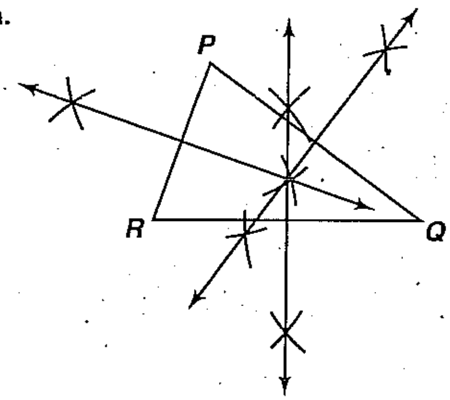
of the segment. **18.** By the Perpendicular Bis. Thm., $CT = CS = 12$. **19.** By the Perpendicular Bis. Thm., $CT = CS = 12$. By subtraction, $TY = CY - CT = 16 - 12 = 4$. **20.** By the Perpendicular Bis. Thm., $CX = CY = 16$. By subtraction, $SX = CX - CS = 16 - 12 = 4$. **21.** By the Perpendicular Bis. Thm., $CX = CY = 16$. **22.** By the Perpendicular Bis. Thm., $MT = MS = 5$. **23.** From Exercise 22, $MT = 5$. By Segment Addition Post., $ST = SM + MT = 5 + 5 = 10$. **24.** By the Perpendicular Bis. Thm., $DY = DX = 7$. **25.** From Exercise 24, $DY = 7$. By Segment Addition Post., $XY = DX + DY = 7 + 7 = 14$. **26.** From Exercise 18, $CS = CT$, so $\triangle SCT$ is isosceles. From Exercise 21, $CX = CY$, so $\triangle XCY$ is isosc. **27.** Answers may vary. Sample: The student needs to know that $\overline{QS}$ bisects $\overline{PR}$. Other possible answers include: S is a midpt., $PS = RS$, $\overline{PS} \cong \overline{RS}$. **28.** A is not on the $\angle$ bisector because, since $8 \neq 9$, A is not equidistant from the sides of $\angle X$. **29.** By def. of $\angle$ bis., A is on the $\angle$ bis. of $\angle TXR$. **30.** By the Converse of the Angle Bis. Thm., A is on the $\angle$ bis. because it is equidistant from the sides of $\angle RXT$ and $\overrightarrow{AR} \perp \overrightarrow{XR}$ and $\overrightarrow{AT} \perp \overrightarrow{XT}$. **31.** The red line is the $\angle$ bis. of the $\angle$ whose vertex is at home plate. From the Real-World Connection caption, second base is 127 ft from home plate, so it is between home plate and second base. The point that fits this description is the pitcher's plate.

32a.



32b. The $\angle$ bisectors intersect at the same point. **32c.** Check students' work.

33a.

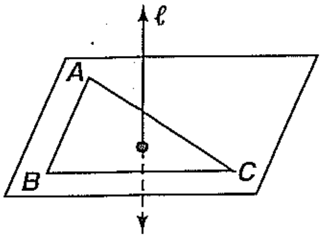


33b. The $\perp$ bis. intersect at the same point. **33c.** Check students' work. **34.** Any points that satisfy the equation $y = 2$ are on the $\perp$ bisector. Answers may vary. Sample: $C(0, 2)$ and $D(1, 2)$; the distance formula shows that $AC = BC = 2$, and $AD = BD = \sqrt{5}$. **35.** Any points satisfying the equation $x = 3$ are on the $\perp$ bis. Answers may vary. Sample: $C(3, 2)$ and $D(3, 0)$; the distance formula shows that $AC = BC = 2$, and $AD = BD = \sqrt{13}$. **36.** Any points satisfying the equation $y = 0$ are on the $\perp$ bis. Answers may vary. Sample: $C(3, 0)$ and $D(0, 0)$; the distance formula shows that $AC = BC = 3$, and $AD = BD = 3\sqrt{2}$. **37.** Any points satisfying the equation $y = x$ are on the $\perp$ bis. Answers may vary. Sample: $C(0, 0)$ and $D(1, 1)$; the distance formula shows that $AC = BC = 3$, and $AD = BD = \sqrt{5}$. **38.** Any points satisfying the equation $y = \frac{1}{2}x + 1$ are on the $\perp$ bis. Answers may vary. Sample: $C(2, 2)$ and $D(4, 3)$; the distance formula shows that $AC = BC = \sqrt{5}$, and $AD = BD = \sqrt{10}$. **39.** Any points satisfying the equation $y = \frac{1}{5}x + 2$ are on the $\perp$ bis. Answers may vary. Sample: $C\left(\frac{5}{2}, \frac{5}{2}\right)$ and $D(5, 3)$; the distance formula shows that $AC = BC = \frac{\sqrt{26}}{2}$, and $AD = BD = \sqrt{13}$. **40a.** The slope of $\overleftrightarrow{OA} = \frac{8-0}{6-0} = \frac{4}{3}$, so the slope of a $\perp$ line is $-\frac{3}{4}$; an equation for ℓ through $(6, 8)$ is $y - 8 = -\frac{3}{4}(x - 6)$ or $y = -\frac{3}{4}x + \frac{25}{2}$. The slope of $\overleftrightarrow{OB} = \frac{0-0}{10-0} = 0$. The slope of a $\perp$ line is of the form $x = c$. The equation for m through $(10, 0)$ is $x = 10$. **40b.** Substitute 10 for x in $y = -\frac{3}{4}x + \frac{25}{2}$: $y = -\frac{3}{4}(10) + \frac{25}{2} = -\frac{15}{2} + \frac{25}{2} = \frac{10}{2} = 5$; so when $x = 10$, $y = 5$. The lines intersect at $(10, 5)$. **40c.** $CA = \sqrt{(10-6)^2 + (5-8)^2} = \sqrt{(4)^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$; $CB = \sqrt{(10-10)^2 + (5-0)^2} = \sqrt{(5)^2} = 5$; so $CA = CB = 5$. **40d.** C is equidistant from $\overrightarrow{OA}$ and $\overrightarrow{OB}$, the sides of $\angle AOB$, so by the Converse of the Perpendicular Bis. Thm., C is on the bisector of $\angle AOB$. **41.** $\overline{AC} \cong \overline{BC}$ by definition of *bisector*. $\overleftrightarrow{CD} \perp \overline{AB}$, so $\angle DCA$ and $\angle DCB$ are *right* $\triangle$. Therefore, $\angle DCA \cong \angle DCB$. $\overline{DC} \cong \overline{DC}$ by the *Reflexive* Property of Congruence. Therefore, $\triangle CDA \cong \triangle CDB$ by SAS. $\overline{DA} \cong \overline{DB}$ because CPCTC, so $DA = DB$. **42.** Prove: $\overline{AB}$ is the perpendicular bisector of $\overline{PQ}$. $\triangle ABP$ and $\triangle ABQ$ are right $\triangle$ with a common leg and congruent hypotenuses. Thus, $\triangle BAP \cong \triangle BAQ$ by the HL Theorem. $\overline{PB} \cong \overline{BQ}$ using CPCTC, so $\overline{AB}$ bisects $\overline{PQ}$ by the definition of *bisector*.

43. Answers may vary. Sample:

Statement	Reason
$\angle ABP$ and $\angle ABQ$ are right $\triangle$.	Given
$\overline{AB} \cong \overline{AB}$	Reflexive Prop. of $\cong$
$\overline{AP} \cong \overline{AQ}$	Given
$\triangle ABP \cong \triangle ABQ$	HL
$\overline{PB} \cong \overline{BQ}$	CPCTC
$\overline{AB}$ bisects $\overline{PQ}$.	Def. of bisector

44. The midpt. of $\overline{AB}$ is $\left(\frac{0+6}{2}, \frac{0+0}{2}\right) = (3, 0)$. The slope of $\overline{AB}$ is $\frac{0-0}{6-0}$, or 0, so $\overline{AB}$ is horizontal. The line $\perp$ to it is vertical, so the equation is in the form $x = c$. The $\perp$ bis. passes through $(3, 0)$, so the equation is $x = 3$. **45.** The $\perp$ bis. passes through the midpt. of $\overline{AB}$, which is $\left(\frac{1+3}{2}, \frac{-1+1}{2}\right) = (2, 0)$. The slope of $\overline{AB} = \frac{1-(-1)}{3-1} = \frac{2}{2} = 1$, so the slope of the $\perp$ bis. $= -1$. The line through $(2, 0)$ is $y - 0 = -1(x - 2)$, or $y = -(x - 2)$, or $y = -x + 2$. **46.** The $\perp$ bis. passes through the midpt. of $\overline{AB}$, which is $\left(\frac{-2+2}{2}, \frac{0+8}{2}\right) = (0, 4)$. The slope of $\overline{AB} = \frac{8-0}{2-(-2)} = \frac{8}{4} = 2$, so the slope of the $\perp$ bis. $= -\frac{1}{2}$. The equation of the $\perp$ bis. through $(0, 4)$ is $y - 4 = -\frac{1}{2}(x - 0)$, or $y = -\frac{1}{2}x + 4$.

47. 

Find all points in the plane that are equidistant from the 3 points. Do this by drawing segments connecting the points. Then find the $\perp$ bis. of the segments. They intersect in one point, which is equidistant from the given points, according to the Perpendicular Bis. Thm. The line equidistant from the given points must pass through this point, but it will not be coplanar with them. Line ℓ is equidistant from points A, B, and C if it is $\perp$ to the plane determined by A, B, and C and if it goes through the point that is the intersection of the $\perp$ bisectors of the sides of $\triangle ABC$. **48.** $\overrightarrow{BP} \perp \overrightarrow{AB}$ and $\overrightarrow{PC} \perp \overrightarrow{AC}$, thus $\angle ABP$ and $\angle ACP$ are rt. $\triangle$s. Since $\overrightarrow{AP}$ bisects $\angle BAC$, $\angle BAP \cong \angle CAP$. $\overline{AP} \cong \overline{AP}$ by the Reflexive Prop. of $\cong$. Thus, $\triangle ABP \cong \triangle ACP$ by AAS and $\overline{PB} \cong \overline{PC}$ by CPCTC. Therefore, $PB = PC$ by the def. of $\cong$. **49.** ① $\overline{SP} \perp \overrightarrow{QP}$ and $\overline{SR} \perp \overrightarrow{QR}$ (Given) ② $\angle QPS$ and $\angle QRS$ are rt. $\triangle$s. (Def. of $\perp$) ③ $\angle QPS \cong \angle QRS$ (All rt. $\triangle$s are $\cong$.) ④ $SP = SR$ (Given) ⑤ $\overline{SP} \cong \overline{SR}$ (Def. of $\cong$) ⑥ $\overline{QS} \cong \overline{QS}$ (Reflexive Prop. of $\cong$) ⑦ $\triangle QPS \cong \triangle QRS$ (HL) ⑧ $\angle PQS \cong \angle RQS$ (CPCTC) ⑨ $\overrightarrow{QS}$ bisects $\angle PQR$. (Def. of $\angle$ bis.) **50.** $\triangle CPR \cong \triangle GKR$ by HL, so $GK = CP = 5$. $TK = TG + GK = 20 + 5 = 25$. The answer is choice D. **51.** $\triangle CPR \cong \triangle GKR$ by HL. $\angle P \cong \angle K$ by CPCTC. By the Converse to the Angle Bis. Thm., $\overrightarrow{TR}$ bis. $\angle PTK$, so $m\angle RTK = 27$. $\triangle PTR \cong \triangle KTR$ by AAS. $\angle TRP \cong \angle TRK$ and they are suppl., so $m\angle TRK = 90$. $m\angle K = 180 - (27 + 90) = 180 - 117 = 63$. The answer is choice H. **52.** From Exercise 51, $\overline{TR}$ bis. $\angle QPK$ and is also the $\perp$ of $\overline{PK}$, so it is on the line of symmetry. By symmetry, $CT = GT = 20$, $GK = CP = 5$, and $PR = RK = 8$. The perimeter is $RP + CP + CT + TG + GK + RK = 8 + 5 + 20 + 20 + 5 + 8 = 66$. The answer is choice D. **53.** [2] Since $\overline{MK} \cong \overline{MR}$, $\overline{MK} \perp \overline{KV}$, and $\overline{MR} \perp \overline{RV}$, by the Angle Bis. Thm., $\overline{MV}$ is the $\angle$ bis. of $\angle KVR$.
[1] partially correct logical argument **54.** [4] It's given that $\overline{MK} \cong \overline{MR}$. By the Reflexive Prop. of $\cong$, $\overline{MV} \cong \overline{MV}$. It is given that $\angle MKV$ and $\angle MRV$ are rt. $\triangle$s. By HL, $\triangle MKV \cong \triangle MRV$. By CPCTC, $\overline{KV} \cong \overline{RV}$. By the Converse of the Perp. Bis. Thm., points M and V lie on the $\perp$ bisector of $\overline{KR}$. [3] appropriate steps with one logical error OR one incorrect reason statement
[2] two logical errors OR two incorrect reasons statements [1] proved $\cong$ $\triangle$s but failed to reach desired conclusion **55.** By the Triangle Midsegment Thm., $12 = \frac{1}{2}(3x)$; $24 = 3x$; $x = 8$. **56.** By the Triangle Midsegment Thm., $3x = \frac{1}{2}(5x + 4)$; $6x = 5x + 4$; $x = 4$.
57. By the Triangle Midsegment Thm., $5x = \frac{1}{2}(60)$; $5x = 30$; $x = 6$. **58.** Each side of the = symbol is the same: Reflexive Prop. of =. **59.** Both sides are divided by 2: Div. Prop. of =. **60.** x is added to both sides: Add. Prop. of =. **61.** 3 is distributed across $(4x - 1)$; Distr. Prop. **62.** $m\angle 3$ in the first equation is substituted for $m\angle 4$ in the second equation: Subst. or, since value is transferred from $m\angle 3$ to $m\angle 5$: Transitive Prop. of =. **63.** The congruence is transferred from $\angle 3$ to $\angle 5$ through $\angle 4$: Trans. Prop. of $\cong$. **64.** Solve for C:
$\left(\frac{0+6}{2}, \frac{5+8}{2}\right) = \left(3, \frac{13}{2}\right)$. $AC = \sqrt{(0-3)^2 + \left(5-\frac{13}{2}\right)^2}$
$\sqrt{(-3)^2 + \left(-\frac{3}{2}\right)^2} = \sqrt{9+\frac{9}{4}} = \sqrt{\frac{45}{4}} = \sqrt{\frac{9\cdot 5}{4}} = \frac{3\sqrt{5}}{2}$.
$BC = \sqrt{(6-3)^2 + \left(8-\frac{13}{2}\right)^2} = \sqrt{(3)^2 + \left(\frac{3}{2}\right)^2} = \sqrt{9+\frac{9}{4}} = \frac{3\sqrt{5}}{2}$. $\frac{1}{2}AB = \frac{1}{2}\sqrt{(0-6)^2 + (5-8)^2} = \frac{1}{2}\sqrt{(6)^2 + (-3)^2} = \frac{1}{2}\sqrt{36+9} = \frac{1}{2}\sqrt{45} = \frac{1}{2}(3\sqrt{5}) = \frac{3\sqrt{5}}{2}$. So, $AC = CB = \frac{1}{2}AB$. **65.** $C = \left(\frac{-2+2}{2}, \frac{8+(-1)}{2}\right) = \left(0, \frac{7}{2}\right)$. $AC = \sqrt{(-2-0)^2 + \left(8-\frac{7}{2}\right)^2} = \sqrt{(-2)^2 + \left(\frac{9}{2}\right)^2} = \sqrt{4+\frac{81}{4}} = \sqrt{\frac{16}{4}+\frac{81}{4}} = \sqrt{\frac{97}{4}} = \frac{\sqrt{97}}{2}$. $BC = \sqrt{(2-0)^2 + \left(-1-\frac{7}{2}\right)^2} = \sqrt{(2)^2 + \left(-\frac{9}{2}\right)^2} = \sqrt{4+\frac{81}{4}} = \frac{\sqrt{97}}{2}$. $\frac{1}{2}AB = \frac{1}{2}\sqrt{(-2-2)^2 + (8-(-1))^2} = \frac{1}{2}\sqrt{(-4)^2 + (9)^2} = \frac{1}{2}\sqrt{16+81} = \frac{1}{2}\sqrt{97} = \frac{\sqrt{97}}{2}$. So, $AC = CB = \frac{1}{2}AB$. **66.** $C = \left(\frac{5+6}{2}, \frac{3+7}{2}\right) = \left(\frac{11}{2}, 5\right)$. $AC = \sqrt{\left(5-\frac{11}{2}\right)^2 + (3-5)^2} = \sqrt{\left(-\frac{1}{2}\right)^2 + (-2)^2} = \sqrt{\frac{1}{4}+4} = \sqrt{\frac{17}{4}} = \frac{\sqrt{17}}{2}$. $BC = \sqrt{\left(6-\frac{11}{2}\right)^2 + (7-5)^2} = \sqrt{\left(\frac{1}{2}\right)^2 + (2)^2} = \sqrt{\frac{1}{4}+4} = \frac{\sqrt{17}}{2}$. $\frac{1}{2}AB = \frac{1}{2}\sqrt{(5-6)^2 + (3-7)^2} = \frac{1}{2}\sqrt{(-1)^2 + (-4)^2} = \frac{1}{2}\sqrt{1+16} = \frac{\sqrt{17}}{2}$. So, $AC = CB = \frac{1}{2}AB$.

TECHNOLOGY — page 255

1. Of the four sets of 3 lines, the 3 lines intersect in one point. **2.** Yes, the property still holds. **3.** The 3 $\perp$ bisectors of the sides of a $\triangle$ intersect in one point. The 3 $\angle$ bisectors of a $\triangle$ intersect in one point. The 3 altitudes of a $\triangle$ intersect in one point. The 3 medians of a $\triangle$ intersect in one point.
4.

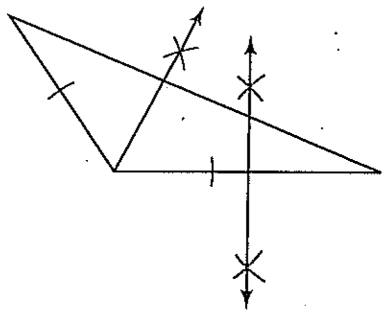
	Perpendicular Bisectors	Angle Bisectors	Lines Containing the Altitudes	Medians
Acute triangle	inside	inside	inside	inside
Right triangle	on	inside	on	inside
Obtuse triangle	outside	inside	outside	inside

5. The location of the intersection point for obtuse isosc., acute isosc., and rt. isosc. $\triangle$s follow the same patterns as for all other $\triangle$s. Since equilateral $\triangle$s are also acute $\triangle$s, all intersection points will be inside the $\triangle$. **6.** By the Converse of the Perp. Bis. Thm., the point is the intersection of the $\perp$ bis. of the sides of the $\triangle$.

5-3 Concurrent Lines, Medians, and Altitudes pages 256–263

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1–2.



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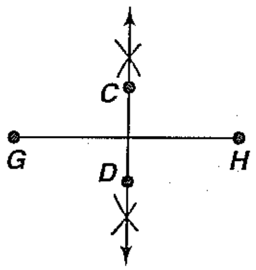

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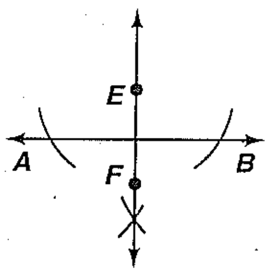
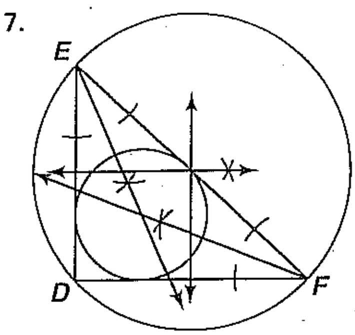


Investigation **1.** The bisectors of the $\angle$s of a $\triangle$ meet at a point inside the $\triangle$. **2.** The $\perp$ bis. of the sides of a $\triangle$ intersect at a point that might fall inside, outside, or on the $\triangle$.

Check Understanding **1a.** The equation for the $\perp$ bis. of the segment with endpts. $(0,0)$ and $(-8,0)$ is $x = -4$. The equation for the $\perp$ bis. of the segment with endpts. $(0,0)$ and $(0,6)$ is $y = 3$. These lines intersect at $(-4,3)$, which is the center of the circle. **1b.** By Thm. 5-6, all of the $\perp$ bisectors of the sides of a $\triangle$ are concurrent. **2a.** By the Perp. Bis. Thm., the points on the $\perp$ bis. of a segment are equidistant from the segment's endpoints. Draw segments connecting the towns. Build the library at the intersection pt. of the $\perp$ bisectors of the segments. **2b.** Thm. 5-3: The $\perp$ bisectors of the sides of a $\triangle$ are concurrent at a point equidistant from the vertices. **3.** From Example 3, $DE = 6$. By Thm. 5-8, $BD = \frac{2}{3}BE$, so $DE = \frac{1}{3}BE$. Since $\frac{2}{3}$ is twice $\frac{1}{3}$, $BD = 2DE = 2(6) = 12$. **4.** $\overline{UW}$ is a median because it connects a vertex to the midpt. of the opposite side.

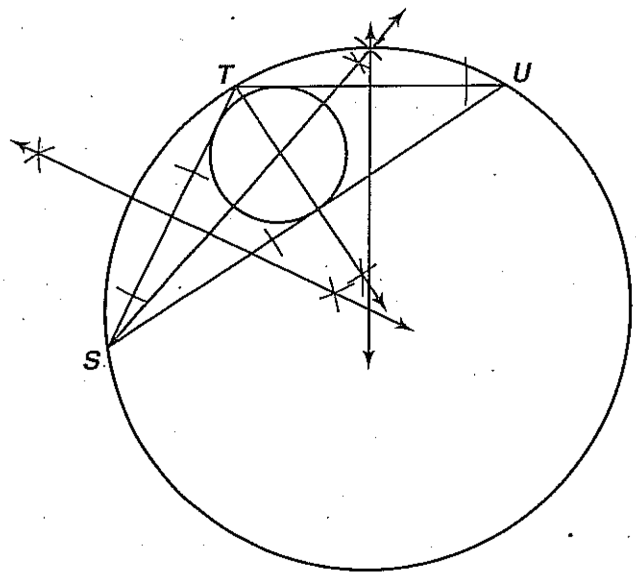
Exercises **1.** The circumcenter is the intersection of 2 $\perp$ bisectors of the $\triangle$. The equation of the $\perp$ bis. of the segment whose endpts. are $(-4,0)$ and $(0,0)$ is $x = -2$. The equation of the $\perp$ bis. of the segment whose endpts. are $(0,-6)$ and $(0,0)$ is $y = -3$. Their intersection point is $(-2,-3)$. **2.** The circumcenter is the intersection of two $\perp$ bis. of the $\triangle$. The slope and midpt. of the segment whose endpts. are $(4,0)$ and $(0,4)$ are -1 and $(2,2)$, respectively, so the equation of the $\perp$ bis. of that segment is $y = x$. The equation of the $\perp$ bis. of the segment whose endpts. are $(-4,0)$ and $(4,0)$ is $y = 0$. The intersection of lines of $y = x$ and $y = 0$ is $(0,0)$. **3.** The circumcenter is the intersection of any two perp. bis. of the $\triangle$. The slope and midpt. of $\overline{AB}$ are 0 and $\left(1\frac{1}{2}, 0\right)$, respectively, so the equation of the perp. bis. of $\overline{AB}$ is $x = 1\frac{1}{2}$. The slope and midpt. of $\overline{BC}$ are undefined and $(3,1)$, respectively, so the equation of the perp. bis. of $\overline{BC}$ is $y = 1$. The intersection of the lines of $x = 1\frac{1}{2}$ and $y = 1$ is $\left(1\frac{1}{2}, 1\right)$. **4.** The circumcenter is the intersection of any two perp. bis. of the $\triangle$. The slope and midpt. of $\overline{AB}$ are 0 and $(2,0)$, respectively, so the equation of the perp. bis. of $\overline{AB}$ is $x = 2$. The slope and midpt. of $\overline{BC}$ are undefined and $\left(4, -1\frac{1}{2}\right)$, respectively, so the equation of the perp. bis. of $\overline{BC}$ is $y = -1\frac{1}{2}$. The intersection of the lines of $x = 2$ and $y = -1\frac{1}{2}$ is $\left(2, -1\frac{1}{2}\right)$. **5.** The circumcenter is the intersection of any two perp. bis. of the $\triangle$. The slope and midpt. of $\overline{AB}$ are 0 and $(-3,5)$, respectively, so the equation of the perp. bis. of $\overline{AB}$ is $x = -3$. The slope and midpt. of $\overline{BC}$ are undefined and $\left(-2, 1\frac{1}{2}\right)$, respectively, so the equation of the perp. bis. of $\overline{AB}$ is $y = 1\frac{1}{2}$. The intersection of the lines of $x = -3$ and $y = 1\frac{1}{2}$ is $\left(-3, 1\frac{1}{2}\right)$. **6.** The circumcenter is the intersection of any two perp. bis. of the $\triangle$. The slope and midpt. of $\overline{AB}$ are 0 and $(-3,-2)$, respectively, so the equation of the perp. bis. of $\overline{AB}$ is $x = -3$. The slope and midpt. of $\overline{AC}$ are undefined and $\left(-1, -4\frac{1}{2}\right)$, respectively, so the equation of the perp. bis. of $\overline{AC}$ is $y = -4\frac{1}{2}$. The intersection of the lines of $x = -3$ and $y = -4\frac{1}{2}$ is $\left(-3, -4\frac{1}{2}\right)$. **7.** The circumcenter is the intersection of any two perp. bis. of the $\triangle$. The slope and midpt. of $\overline{AB}$ are undefined and $(1,3)$, respectively, so the equation of the perp. bis. of $\overline{AB}$ is $y = 3$. The slope and midpt. of $\overline{BC}$ are 0 and $\left(3\frac{1}{2}, 2\right)$, respectively, so the equation of the perp. bis. of $\overline{BC}$ is $x = 3\frac{1}{2}$. The intersection of the lines of $x = 3\frac{1}{2}$ and $y = 3$ is $\left(3\frac{1}{2}, 3\right)$. **8.** The point of concurrency of the $\angle$ bis. is in the interior of the $\triangle$ at C. **9.** Points X and Y are on an altitude. The point of concurrency of the $\angle$ bis. is in the interior of the $\triangle$ at Z. **10.** Find the $\perp$ bis. of 2 of the sides of the $\triangle$ formed by the tennis court, the playground, and the volleyball court. That pt. will be equidistant from the vertices of the $\triangle$. **11.** The centroid is the point of concurrency of the medians and is $\frac{2}{3}$ the median length from the vertex and $\frac{1}{3}$ the median length from the side. Since $\frac{2}{3} = 2 \cdot \frac{1}{3}$, $TY = 2YW = 2(9) = 18$. $TW = TY + YW = 18 + 9 = 27$ **12.** The centroid is the point of concurrency of the medians and is $\frac{2}{3}$ the median length from the vertex and $\frac{1}{3}$ the median length from the side. Since $\frac{1}{3} = \frac{1}{2} \cdot \frac{2}{3}$, $ZY = \frac{1}{2}YU = \frac{1}{2}(9) = 4\frac{1}{2}$. $YU = \frac{2}{3}ZY$; $9 = \frac{2}{3}ZY$; $ZY = \frac{3}{2} \cdot 9 = 13\frac{1}{2}$. **13.** The centroid is the point of concurrency of the medians and is $\frac{2}{3}$ the median length from the vertex and $\frac{1}{3}$ the median length from the side. $VX = \frac{2}{3}(9) = 6$; $YX = \frac{1}{3}(9) = 3$ **14.** Since it connects a vertex with the midpt. A of the opp. side, it is a median. **15.** Medians and altitudes each have an endpt. at a vertex. Since neither A nor B is a vertex, $\overline{AB}$ is *neither* a median nor an altitude. **16.** Since it is a segment $\perp$ to a side and has an endpt. on a vertex, it is an altitude.

17.



The center of the inscribed circle is the intersection of any 2 $\angle$ bisectors, and the center of the circumscribed circle is the intersection of any 2 perp. bisectors.

18.

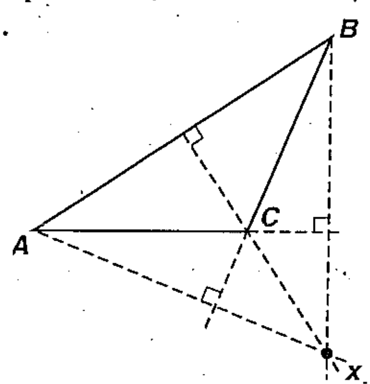


The center of the inscribed circle is the intersection of any 2 $\angle$ bisectors, and the center of the circumscribed circle is the intersection of any 2 perp. bisectors. **19.** The segment that divides an $\angle$ into 2 $\cong$ $\triangle$s is $\overline{BE}$. **20.** A segment that connects a vertex with the midpt. of the opp. side is $\overline{FC}$. **21.** A ray that intersects the midpt. of a side and is $\perp$ to that side is $\overrightarrow{CA}$. **22.** A segment having an endpt. on a vertex and that is $\perp$ to the opp. side is $\overline{DG}$. **23.** The centroid is $\frac{2}{3}$ the length of the median from the vertex and $\frac{1}{3}$ the length from the opp. side. The ratio is $\frac{1}{3}:\frac{2}{3}$ which is the same as $1:2$, or the ratio is $\frac{2}{3}:\frac{1}{3}$, which is the same as $2:1$. **24.** The 3 pts. form a $\triangle$. Points equidistant from the vertices are the $\perp$ bis. of the sides. Their intersection, the circumcenter, is equidistant from all three vertices. So, find the circumcenter of the $\triangle$ formed by the three pines. **25.** Check students' work. **26.** Check students' work. **27.** Since the segment bisects an $\angle$, it is an $\angle$ bisector. **28.** The segment is a midsegment having no endpts. as vertices, so it is not an $\angle$ bisector, a median, or an altitude. Since it is not $\perp$ to either of the sides, it is also not a $\perp$ bisector. So, it is *none of these*. **29.** The segment does not bisect a side, so it is not a $\perp$ bis. or median. It does not bisect an $\angle$, so it is not an $\angle$ bisector. Since at least one of its endpts., namely A, is a vertex and since the segment is $\perp$ to the opp. side, it is an altitude. **30a.** $\overline{AB}$ (Given from the diagram.) **30b.** $\overline{BC}$ (Given from the diagram.) **30c.** XC (Perp. Bis. Thm.) **30d.** Perpendicular Bis. **31a.** Angle Bis. Thm. (The $\angle$ bisectors are equidistant from the sides of the $\triangle$s.) **31b.** Angle Bis. Thm. (The $\angle$ bisectors are equidistant from the sides of the $\triangle$s.) **31c.** Transitive **31d.** Angle Bis. (If a pt. is equidistant from the sides of an $\angle$, then it is on the $\angle$ bis. of the $\angle$.)

32a. $L = \left(\frac{2+0}{2}, \frac{6+0}{2}\right) = (1, 3)$; $M = \left(\frac{2+8}{2}, \frac{6+0}{2}\right) = (5, 3)$; $N = \left(\frac{8+0}{2}, \frac{0+0}{2}\right) = (4, 0)$ **32b.** The slope of $\overleftrightarrow{AM} = \frac{3-0}{5-0} = \frac{3}{5}$ and its y-intercept is 0, so its equation is $y = \frac{3}{5}x$. The slope of $\overleftrightarrow{BN} = \frac{6-0}{2-4} = -3$, so the point-slope equation through $(4, 0)$ is $y - 0 = -3(x - 4)$. The equation in slope-intercept form is $y = -3x + 12$. The slope of $\overleftrightarrow{CL} = \frac{3-0}{1-8} = -\frac{3}{7}$, so the point-slope equation through $(8, 0)$ is $y - 0 = -\frac{3}{7}(x - 8)$. The equation in slope-intercept form is $y = -\frac{3}{7}x + \frac{24}{7}$. **32c.** From Exercise 32b, the equation for $\overleftrightarrow{AM}$ is $y = \frac{3}{5}x$ and the equation for $\overleftrightarrow{BN}$ is $y = -3x + 12$. Substitute $\frac{3}{5}x$ for y in $y = -3x + 12$; $\frac{3}{5}x = -3x + 12$; $3x = -15x + 60$; $18x = 60$; $x = \frac{10}{3}$. To solve for y, substitute $\frac{10}{3}$ for x in $y = \frac{3}{5}x = \frac{3}{5}\left(\frac{10}{3}\right) = 2$. So, they intersect at $\left(\frac{10}{3}, 2\right)$. **32d.** Substitute $\left(\frac{10}{3}, 2\right)$ in the equation for $\overleftrightarrow{CL}$: $y = -\frac{3}{7}x + \frac{24}{7}$; $2 = -\frac{3}{7}\left(\frac{10}{3}\right) + \frac{24}{7}$; $2 = -\frac{10}{7} + \frac{24}{7} = \frac{14}{7} = 2$. **32e.** $AM = \sqrt{(5-0)^2 + (3-0)^2} = \sqrt{25+9} = \sqrt{34}$ and $AP = \sqrt{\left(\frac{10}{3} - 0\right)^2 + (2-0)^2} = \sqrt{\frac{100}{9} + 4} = \sqrt{\frac{100}{9} + \frac{36}{9}} = \sqrt{\frac{136}{9}} = \sqrt{\frac{4 \cdot 34}{9}} = \frac{2}{3}\sqrt{34}$.
$BN = \sqrt{(2-4)^2 + (6-0)^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$ and $BP = \sqrt{\left(2 - \frac{10}{3}\right)^2 + (6-2)^2} = \sqrt{\left(\frac{6}{3} - \frac{10}{3}\right)^2 + (6-2)^2} = \sqrt{\left(-\frac{4}{3}\right)^2 + (4)^2} = \sqrt{\frac{16}{9} + 16} = \sqrt{\frac{16}{9} + \frac{144}{9}} = \sqrt{\frac{160}{9}} = \sqrt{\frac{4 \cdot 10}{9}} = \frac{2}{3}\sqrt{10}$. $CL = \sqrt{(8-1)^2 + (0-3)^2} = \sqrt{49+9} = \sqrt{58}$ and $CP = \sqrt{\left(8 - \frac{10}{3}\right)^2 + (0-2)^2} = \sqrt{\left(\frac{24}{3} - \frac{10}{3}\right)^2 + (0-2)^2} = \sqrt{\left(\frac{14}{3}\right)^2 + (-2)^2} = \sqrt{\frac{196}{9} + 4} = \sqrt{\frac{196}{9} + \frac{36}{9}} = \sqrt{\frac{232}{9}} = \sqrt{\frac{4 \cdot 58}{9}} = \frac{2}{3}\sqrt{58}$.
33. To help with visual estimation, measure the sides to locate their midpts. A is a vertex and not in the interior, so it cannot be on all perp. bisectors, all $\angle$ bisectors, or all medians. A must be the lines containing the altitudes. B is in the interior, but is not near any midpts., so it is not on all perp. bis. or all medians, and since A is on all alts., B must be on all the $\angle$ bisectors. C is now either on all perp. bis. or on all medians. Since medians must be concurrent in the interior of a $\triangle$, C must be on all medians. That leaves D on all perp. bis. I-D, II-B, III-C, and IV-A **34.** To help with visual estimation, measure the sides to locate their midpts. A is in the ext. of the $\triangle$, so it cannot be on all $\angle$ bis. or all medians. Segments to

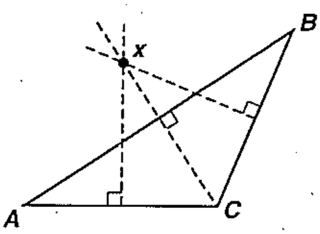
the midpts. of each of the sides appear to be $\perp$, so A must be on all $\perp$ bis. of the sides. Segments from each vertex through B appear to have endpts. at the segment midpts., so B is on all medians. Now, C is either on all $\angle$ bis. or on all lines containing altitudes. Since it is the only point left in the interior, it must be on all $\angle$ bis., leaving D to be on all lines containing altitudes. I-A, II-C, III-B, and IV-D **35.** Answers may vary. Sample: Let $\triangle ABC$ be isosc. with base $\angle$s B and C. If $\overline{AD}$ bis. $\angle A$, then it is also the $\perp$ bis. of $\overline{BC}$ and an altitude. Since it bis. $\overline{BC}$, it is also a median. Since $\angle$ bisectors, perp. bisectors, altitudes, and medians are each concurrent, their pts. of concurrency must all lie on $\overleftrightarrow{AD}$. So, $\overleftrightarrow{AD}$ contains the circumcenter, incenter, centroid, and orthocenter. **36.** Look at the points of the $\triangle$s in Exercises 33 and 34. The point that is not collinear with the others in Exercise 33 is B, the intersection of the $\angle$ bisectors, and the point that is not collinear with the others in Exercise 34 is C, the intersection of the $\angle$ bisectors. So, the point of concurrency not necessarily on Euler's line is that of the $\angle$ bisectors. **37.** $RL = \frac{1}{3}RD$; $54 = \frac{1}{3}RD$; $RD = 3(54) = 162$, or 162 cm. The answer is choice C. **38.** $WL = \frac{2}{3}WJ = \frac{2}{3}(210) = 140$, or 140 mm. The answer is choice H. **39.** $WL = \frac{2}{3}(WJ)$ and $WJ = WL + LJ = 15x + (5x + 3) = 20x + 3$. So, $WL = \frac{2}{3}(20x + 3)$; $15x = \frac{2}{3}(20x + 3)$; $45x = 2(20x + 3)$; $45x = 40x + 6$; $5x = 6$; $x = 1.2$. The answer is choice D. **40.** [2] any acute $\angle$, OR a list that contains all of the following: equiangular $\triangle$s, equilateral $\triangle$s, acute isosceles $\triangle$s, acute scalene $\triangle$s [1] a list that does not contain equiangular $\triangle$s, equilateral $\triangle$s, acute isosceles $\triangle$s, or scalene $\triangle$s

41.

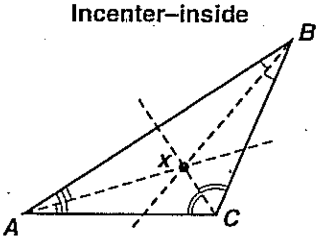


[4] Since the orthocenter is outside, the $\triangle$ must be obtuse.

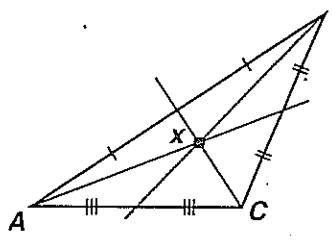
Circumcenter–outside



Incenter–inside



Centroid–inside



[3] one diagram partially or completely incorrect [2] two diagrams partially or completely incorrect [1] three diagrams partially or completely incorrect **42.** $\overrightarrow{TB}$ divides $\angle T$ into 2 $\cong$ $\triangle$s, so it is the $\angle$ bisector. T is a pt. on $\overrightarrow{TB}$, so T is on the bisector of $\angle T$. **43.** B is equidistant from both sides of the $\angle$, so it is on the $\angle$ bisector. **44.** Since there is no indication of perpendicularity, the given lengths may or may not be distances from B to the sides of the $\triangle$. B is not necessarily equidistant from the sides. **45.** Since $m\angle L = 90$, it is a right $\angle$, so the $\triangle$ is a right $\triangle$. **46.** Since $m\angle K > 90$, the $\angle$ is obtuse, so the $\triangle$ is an obtuse $\triangle$. **47.** A skew line is not parallel to and does not intersect the given line. Answers may vary. Possible answers: $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ **48.** A skew line is not parallel to and does not intersect the given line. Answers may vary. Possible answers: $\overleftrightarrow{AD}$, $\overleftrightarrow{CD}$ **49.** Any two planes intersect in a line. Answers may vary. Sample: ABC and ADE **50.** Parallel segments are contained in coplanar lines that do not intersect. Answers may vary. Possible answers: $\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{AD}$ **51.** Planes intersect in a line. The line that is in both ABC and BCE is $\overleftrightarrow{BC}$.

CHECKPOINT QUIZ 1 *page 263*

1. $12 = \frac{1}{2}(4x)$; $24 = 4x$; $x = 6$ **2.** $3 = \frac{1}{2}(2x)$; $6 = 2x$; $x = 3$ **3a.** $52 = \frac{1}{2}YZ$; $YZ = 104$ **3b.** $AY = AX = 26$; $BX = BZ = 36$; from part (a), $YZ = 104$; $P = AX + AY + YZ + BZ + BX = 26 + 26 + 104 + 36 + 36 = 228$. **4.** $\angle ADC$ is a straight $\angle$, so it measures 180. $m\angle CDB = 180 - 90 = 90$, so $\angle CDB$ is a right $\angle$. **5.** From Exercise 4, $\angle CDB$ is a rt. $\angle$. Since $\overline{BD}$ is $\cong$ to itself by the Refl. Prop. of $\cong$, $\triangle ABD \cong \triangle CBD$ by HL. **6.** From Exercise 5, $\triangle ABD \cong \triangle CBD$, so $\overline{AD} \cong \overline{DC}$ by CPCTC. **7.** Since y is equidistant from the sides of $\angle ZXW$, $\overrightarrow{XY}$ bisects $\angle ZXW$. **8.** Since $\overline{XY} \cong \overline{XY}$ by the Refl. Prop. of $\cong$, $\triangle XYZ \cong \triangle XYW$ by HL, so $\overline{XZ} \cong \overline{XW}$ by CPCTC. By def. of $\cong$, $XZ = XW = 21$. **9.** Answers may vary. Sample: Bisect a side of the $\triangle$ by constructing the $\perp$ bisector of the side. Then connect the intersection of the $\perp$ bis., which is the midpt., to the opp. vertex. **10.** With the compass point on a vertex of the $\triangle$, swing an arc that intersects the line containing the opposite side in two places. With the compass point on one of the arc intersections with the side, swing an arc above and below the line. Keep the same setting and, with the compass point on the other arc intersection with the line, swing arcs to intersect the previous two arcs. Draw a line through the two arc intersections.

5-4 Inverses, Contrapositives, and Indirect Reasoning pages 264–270

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1. If we go skiing, then it snows tomorrow. **2.** If 2 lines do not intersect, then they are parallel. **3.** If $x^2 = 1$, then $x = -1$. **4.** If a point is on the bisector of an ∠, then it is equidistant from the sides of the ∠. If a point is equidistant from the sides of an ∠, then it is on the bisector of the ∠. **5.** If a point is on the ⊥ bis. of a segment, then it is equidistant from the endpoints of the segment. If a point is equidistant from the endpoints of a segment, then it is on the ⊥ bis. of the segment. **6.** If you will pass a geometry course, then you are successful with your homework. If you are successful with your homework, then you will pass a geometry course.

Check Understanding **1a.** To negate a statement, insert the word *not* if it's not in the original statement, or remove it if it is: The measure of $\angle XYZ$ is not more than 70. **1b.** To negate a statement, insert the word *not* if it's not in the original statement, or remove it if it is: Today is Tuesday. **2a.** To write the inverse, negate the hypothesis and conclusion: If you stand for something you won't fall for anything. **2b.** To write the contrapositive, of a statement, write the inverse and then switch the hypothesis and conclusion: If you won't fall for anything, then you stand for something. **3a.** Assume the negation of the conclusion is true: The shoes cost more than \$20. **3b.** Assume the negation of the conclusion is true: $m\angle A$ is not greater than $m\angle B$, which is the same as $m\angle A \leq m\angle B$. **4.** Two segments cannot be both ∥ and ⊥, since ∥ lines never intersect and ⊥ lines must intersect. The statements that contradict are I and II. **5.** There are 4 types of ∠s in geometry: acute, right, obtuse, and straight. Straight ∠s can be assumed from a diagram, so they do not need to be proved. What was overlooked, therefore, was that $\angle X$ could be a right ∠.

Exercises **1.** To negate a statement, insert the word *not* if it's not in the original statement, or remove it if it is: Two ∠s are not congruent. **2.** To negate a statement, insert the word *not* if it's not in the original statement, or remove it if it is: You are sixteen years old. **3.** To negate a statement, insert the word *not* if it's not in the original statement, or remove it if it is: The ∠ is obtuse. **4.** To negate a statement, insert the word *not* if it's not in the original statement, or remove it if it is: The soccer game is not on Friday. **5.** To negate a statement, insert the word *not* if it's not in the original statement, or remove it if it is: The figure is not a △. **6.** To negate a statement, insert the word *not* if it's not in the original statement, or remove it if it is: $m\angle A \geq 90$. **7a.** To write the inverse, negate the hypothesis and conclusion: If you don't eat all of your vegetables, then you won't grow. **7b.** To write the contrapositive, switch the hypothesis and conclusion of the inverse: If you won't grow, then you don't eat all of your vegetables. **8a.** To write the inverse, negate the hypothesis and conclusion: If a figure is not a square, then at least one of its ∠s is not a right ∠. **8b.** To write the contrapositive, switch the hypothesis and conclusion of the inverse: If at least one ∠ is not a right ∠, then the figure isn't a square. **9a.** To write the inverse, negate the hypothesis and conclusion: If a figure is not a rectangle, then it doesn't have four sides. **9b.** To write the contrapositive, switch the hypothesis and conclusion of the inverse: If a figure doesn't have four sides, then it isn't a rectangle. **10.** Assume the negation of the conclusion is true: Assume that it is not raining outside. **11.** Assume the negation of the conclusion is true: Assume that $\angle J$ is a right ∠. **12.** Assume the negation of the conclusion is true: Assume that $\triangle PEN$ is not isosc. **13.** Assume the negation of the conclusion is true: Assume that none of the ∠s is obtuse. **14.** Assume the negation of the conclusion is true: Assume that $\overline{XY}$ is not ≅ to $\overline{AB}$. **15.** Assume the negation of the conclusion is true: Assume that $m\angle 2 \neq 90$, or assume that $m\angle 2 \leq 90$. **16.** In an equilateral △, all ∠s measure 60. In a rt. △, one ∠ measures 90. The statements that contradict each other are I and II. **17.** In a rt. △, one ∠ measures 90 and the two acute ∠s are complementary, so their sum is 90. If $\angle A \cong \angle C$, then $m\angle C = m\angle A = 60$, but $60 + 60 \neq 90$. The statements that contradict each other are I and II. **18.** Skew lines lie in different planes and ∥ lines are coplanar. The statements that contradict one another are I and III. **19.** If Val spent \$34 for 2 items, then their mean cost was \$17 each. If one cost less, then the other cost even more. So, at least one of the items cost more than \$15. The statements that contradict each other are II and III. **20a.** 20 or more members (10 members of the Chess Club + at least 10 members of the Debate Club ≥ 20) **20b.** The Debate Club and the Chess Club have fewer than 20 members. **20c.** The Debate Club has fewer than 10 members (what you are trying to prove). **21a.** right ∠ (Assume the opposite of what you want to prove.) **21b.** right ∠s **21c.** 90 (def. of rt. ∠) **21d.** 180 (The sum of the ∠s of a △ is 180.) **21e.** 90 (Substitute 90 for $m\angle M$.) **21f.** 90 (Substitute 90 for $m\angle N$.) **21g.** 0 ($m\angle L + 90 + 90 = 180$; $m\angle L + 180 = 180$; $m\angle L = 0$) **21h.** more than one right ∠ (what you assumed to be true as a first step) **21i.** at most one right ∠ (what you are trying to prove) **22a.** To write the inverse, negate the hypothesis and conclusion: If you don't live in Sarasota, then you don't live in Florida. A counterexample is that you could live in Miami, Florida, so the inverse is false. **22b.** To write the contrapositive, switch the hypothesis and conclusion of the inverse: If you don't live in Florida, then you don't live in Sarasota. Since the only city named Sarasota is in Florida, the statement is true. **23a.** To write the inverse, negate the hypothesis and conclusion: If four points aren't collinear, then they aren't coplanar. A counterexample is that any 4 points on a circle are coplanar, so the inverse is false. **23b.** To write the contrapositive, switch the hypothesis and conclusion of the inverse: If four points aren't coplanar, then they aren't collinear. The contrapositive is true. **24.** Answers may vary. Sample: Conditional: If a figure is a square, then it has four right ∠s. Inverse: If a figure is not a square, then it doesn't have four right ∠s. A counterexample is a rectangle, so the inverse is false. **25.** Answers may vary. Sample: Conditional: If today is Sunday, then tomorrow is Monday. Inverse: If today is not Sunday, then tomorrow is not Monday. **26.** Since the truth value of a conditional and its contrapositive are the same, it is *not possible* to write a false contrapositive statement from a true conditional. **27.** Write any true statement as a conditional. Answers may vary. Sample: Conditional: If two sides of a △ are congruent, then the △ is isosc. Contrapositive: If a △ is not isosc., then no two sides of the △ are congruent. **28.** Angie assumed that the inverse, "If you drink Muscle Rex, then you will build muscles," was true, but a conditional and its inverse may not have the same truth value. The conditional in this case subtly implies that you won't build muscles regardless of whether or not you drink Muscle Rex. **29.** Use indirect reasoning. Assume that the driver did not apply the brakes. Then there would be no skid marks. This contradicts the fact that fresh skid marks appear. Thus, the green car applied the brakes is a true statement. **30.** Use indirect reasoning. Assume that the temperature outside is more than 32°F. Then ice would not be forming on the sidewalk. This contradicts the fact that ice is forming. Thus, the statement that the temperature must be 32°F or less is true. **31.** Use indirect reasoning. Assume that an obtuse △ can contain a right ∠. Then the sum of the measures of the obtuse ∠ and the right ∠ is more than 180. This contradicts the fact that the sum of the 3 ∠s of a △ is 180. Thus, the statement that an obtuse △ cannot contain a right ∠ is true. **32.** Use indirect reasoning. Assume $\overleftrightarrow{XY}$ and $\overleftrightarrow{XZ}$ are two different lines ⊥ to $\overleftrightarrow{AX}$, with Y and Z on the same side of $\overleftrightarrow{AX}$. If B is on $\overleftrightarrow{AX}$ opposite pt. A from X, then $m\angle AXY + m\angle YXB + m\angle ZXB = 180$. But $m\angle AXY = m\angle ZXB = 90$, so $m\angle YXZ = 0$. Thus, X, Y, and Z are collinear. **33.** The hypothesis is inside the conclusion of the conditional: If the animals are kittens, then they are cats. To write the contrapositive, negate both hypothesis and conclusion and switch their positions: If the animals aren't cats, then they aren't kittens. **34.** The hypothesis is inside the conclusion of the conditional: If the ∠s measure 120, then they are obtuse. To write the contrapositive, negate both hypothesis and conclusion and switch their positions: If the ∠s aren't obtuse, then they don't measure 120. **35.** The hypothesis is inside the conclusion of the conditional: If the numbers are whole numbers, then they are integers. To write the contrapositive, negate both hypothesis and conclusion and switch their positions: If the numbers are not integers, then they are not whole numbers. **36.** Check students' work. **37a.** Earl's conclusion of "it must be later than 5:00" is what he is trying to prove. **37b.** He starts with the assumption that it is not later than 5:00; he assumes it is before 5:00. **37c.** He doesn't hear construction noise. **38.** There are only 5 possibilities involving how the culprit came to be in the room at a certain time. Either he was already in the room or he entered by door, window, chimney, or hole in the roof. All possibilities were eliminated except entry through the hole in the roof. **39.** Assume the opposite of what you are trying to prove is true. Assume that $\angle A \cong \angle B$. By the Converse of the Isosc. Triangle Thm., $\overline{BC} \cong \overline{AC}$. By def. of ≅, $BC = AC$. This contradicts the given, that $BC > AC$. Thus, $\angle A$ is not ≅ to $\angle B$. **40.** Assume the opposite of what you want to prove is true. Assume that one base ∠ is a rt. ∠. Then, by the Isosc. Triangle Thm., the other base ∠ is also a rt. ∠. But a △ can have at most one right ∠. So, neither base ∠ is a rt. ∠. **41.** Assume the opposite of what you want to prove is true. Assume $\overline{XB} \perp \overline{AC}$. Then $\angle AXB$ and $\angle CXB$ are rt. ∠s. Since $m\angle ABX = m\angle CBX = 36$, then $\angle A \cong \angle B$, because if two ∠s of a △ are congruent, then the third ∠s are congruent. Then $AB = BC$ since sides opp. ≅ ∠s are ≅, so $\triangle ABC$ is an isosc. △. But this contradicts the given statement that $\triangle ABC$ is scalene. Thus, $\overline{XB}$ is not perpendicular to $\overline{AC}$. **42.** The three possibilities for comparison are <, >, and =. The negation of $x \leq 10$ eliminates < and =, leaving only >. The answer is choice D. **43.** The three possibilities for comparison are <, >, and =. The negation of $y > 8$ eliminates >, leaving < and =, or ≤. The answer is choice G. **44.** The inverse negates the hypothesis and conclusion. The answer is choice D. **45.** The contrapositive negates both the hypothesis and conclusion and switches them. The answer is choice H. **46.** [2] Assume that a △ can have more than one obtuse ∠. Since the measure of an obtuse ∠ is greater than 90, the sum of the measures of the two obtuse ∠s is greater than 180. This contradicts the Triangle Angle-Sum Thm. So, a △ can have at most one obtuse ∠. [1] partially incorrect logical argument **47.** It is ⊥ to and bis. the base of the △, so it is a ⊥ bis., a median, and an altitude. By SAS, the 2 small △s are ≅, so by CPCTC, the two smaller ∠s having vertex X are ≅. So, $\overline{XY}$ is also an ∠ bisector. **48.** $\overline{XY}$ is ⊥ to a side, but does not bisect any side or ∠, so it is not a perp. bis., ∠ bis., or median. Since it connects a vertex with the line containing the opp. side, it is an altitude. **49.** $\overline{XY}$ is not necessarily ⊥ to a side and it doesn't necessarily bis. an ∠, but it does bisect a side. So, it cannot be a ⊥ bis., ∠ bis., or altitude, but it is a median. **50.** They are both on the same side of the transversal, and one ∠ is between the ∥ lines and the other is not. The ∠s are corresponding ∠s. **51.** They are on the same side of the transversal and between the parallel lines, so they are same-side interior ∠s. **52.** They are on opp. sides of the transversal and both between the ∥ lines, so they are alt. int. ∠s. **53.** They are on the same side of the transversal, one is between the ∥ lines and the other is not, so they are corr. ∠s. **54.** Add 10 to both sides: 35. **55.** Reverse the statement: $45 = m\angle ABC$

READING MATH *page 271*

a. 20 or more members (Debate Club members + Chess Club members = 10 or more members + 10 members = 20 or more members) **b.** The Debate Club and the Chess Club have fewer than 20 members. (Given information) **c.** It is true that the Debate Club has fewer than 10 members (what you are trying to prove).

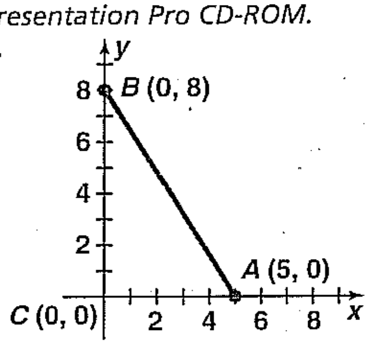
ALGEBRA 1 REVIEW *page 272*

1. $7x - 13 \le -20; 7x \le -7; x \le -1$ **2.** $3x + 8 > 16; 3x > 8; x > \frac{8}{3}$ **3.** $-2x - 5 < 16; -2x < 21; x > -\frac{21}{2}$ **4.** $8y + 2 \ge 14; 8y \ge 12; y \ge \frac{12}{8}; y \ge \frac{3}{2}$ **5.** $5a + 1 \le 91; 5a \le 90; a \le 18$ **6.** $-x - 2 > 17; -x > 19; x < -19$ **7.** $-4z - 10 < -12; -4z < -2; z > \frac{2}{4}; z > \frac{1}{2}$ **8.** $9x - 8 \ge 82; 9x \ge 90; x \ge 10$ **9.** $6n + 3 \le -18; 6n \le -21; n \le -\frac{21}{6}; n \le -\frac{7}{2}$ **10.** $c + 13 > 34; c > 21$ **11.** $3x - 5x + 2 < 12; -2x + 2 < 12; -2x < 10; x > -5$ **12.** $x - 19 < -78; x < -59$ **13.** $-n - 27 \le 92; -n \le 119; n \ge -119$ **14.** $-9t + 47 < 101; -9t < 54; t > -6$ **15.** $8x - 4 + x > -76; 9x - 4 > -76; 9x > -72; x > -8$ **16.** $2(y - 5) > -24; y - 5 > -12; y > -7$ **17.** $8b + 3 \ge 67; 8b \ge 64; b \ge 8$ **18.** $-3(4x - 1) \ge 15; 4x - 1 \le -5; 4x \le -4; x \le -1$ **19.** $r - 9 \le -67; r \le -58$ **20.** $\frac{1}{2}(4x - 7) \ge 19; 4x - 7 \ge 38; 4x \ge 45; x \ge \frac{45}{4}$ **21.** $5x - 3x + 2x < -20; 4x < -20; x < -5$ **22.** $9x - 10x + 4 < 12; -x + 4 < 12; -x < 8; x > -8$ **23.** $-3x - 7x \le 97; -10x \le 97; x \ge 9.7$ **24.** $8y - 33 > -1; 8y > 32; y > 4$ **25.** $4a + 17 \ge 13; 4a \ge -4; a \ge -1$ **26.** $-4(5z + 2) > 20; 5z + 2 < -5; 5z < -7; z < -\frac{7}{5}$ **27.** $x + 78 \ge -284; x \ge -362$ **28.** $6c \ge -12 - 24; 6c \ge -36; c \ge -6$ **29.** $27 - 12 < 3x; 15 < 3x; 5 < x; x > 5$ **30.** $8y - 4y + 11 \le -33; 4y + 11 \le -33; 4y \le -44; y \le -11$ **31.** $5x - 2x + 13 > -8; 3x + 13 > -8; 3x > -21; x > -7$ **32.** $4(5a + 3) \le -8; 5a + 3 \le -2; 5a \le -5; a \le -1$ **33.** $8c + 2c + 7 < -10 - 3; 10c + 7 < -13; 10c < -20; c < -2$

5-5 Inequalities in Triangles *pages 273–279*

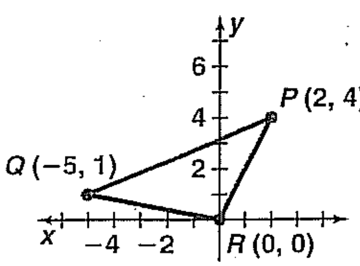
Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1.



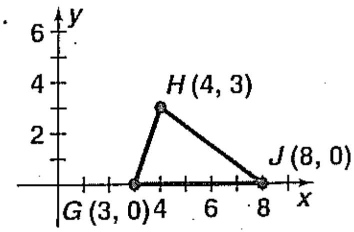
$AC < BC < AB$

2.



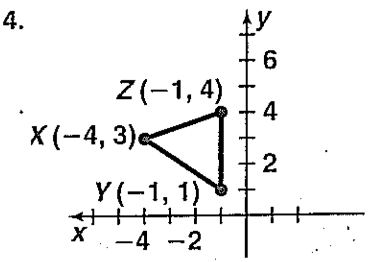
$RP < RQ < QP$

3.



$GJ = JH$; GH is the shortest.

4.



$ZY < XZ < XY$

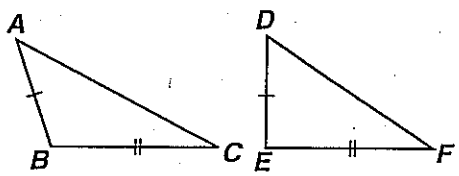
5. Assume that $m\angle A \le m\angle B$. **6.** $AB < AC$

Check Understanding **1.** Since $m\angle OTY = m\angle 4 + m\angle 2$, by the Comparison Prop. of Ineq., $m\angle OTY > m\angle 2$. From Example 1, $m\angle 2 > m\angle 3$, so by the Trans. Prop., $m\angle OTY > m\angle 3$. **2.** Since $18 < 21 < 27$, $m\angle A < m\angle C < m\angle B$. **3.** By the Triangle Angle-Sum Thm., $m\angle Y = 80$, so, since $40 < 60 < 80$, $YZ < XY < XZ$. From shortest to longest the segments are $\overline{YZ}$, $\overline{XY}$, and $\overline{XZ}$. **4a.** $7 + 9 > 2$, $2 + 9 > 7$, but $2 + 7$ is not greater than 9, so a $\triangle$ cannot have these lengths. **4b.** $4 + 6 > 9$, $4 + 9 > 6$, $6 + 9 > 4$, so a $\triangle$ can have these lengths. **5.** $12 - 3 < x < 12 + 3$; $9 < x < 15$

Exercises **1.** $\angle 3 \cong \angle 2$ because vert. $\angle$s are $\cong$, so $m\angle 3 = m\angle 2$, and $m\angle 1 > m\angle 3$ by the Corrollary to the Ext. Angle Thm. So, $m\angle 1 > m\angle 2$ by substitution. **2.** The measure of an ext. $\angle$ is greater than the measure of each of its remote int. $\angle$s. **3.** $m\angle 1 > m\angle 4$ by the Corollary to the Ext. Angle Thm. Since the lines are $\parallel$, alt. int. $\angle$s 2 and 4 are $\cong$. By substitution, $m\angle 1 > m\angle 2$. **4.** By Thm. 5-10, the smallest $\angle$ is opp. the shortest side, and the largest is opp. the longest side. Since $2.7 < 4.3 < 5.8$, the $\angle$s from smallest to largest are $\angle M$, $\angle L$, and $\angle K$. **5.** By Thm. 5-10, the smallest $\angle$ is opp. the shortest side, and the largest is opp. the longest side. Since 105 is obtuse and there can be only one obtuse $\angle$ in a $\triangle$, it must be the largest $\angle$. Also, x must be positive, so $3x > x$. The order of the $\angle$s from smallest to largest is $\angle D$, $\angle C$, and $\angle E$. **6.** By Thm. 5-10, the smallest $\angle$ is opp. the shortest side, and the largest is opp. the longest side. The $\triangle$ is a right $\triangle$, so the hyp. is the longest side. Since $HI < GI < HG$, the order of the $\angle$s from smallest to largest is $\angle G$, $\angle H$, and $\angle I$. **7.** By Thm. 5-10, the smallest $\angle$ is opp. the shortest side, and the largest is opp. the longest side. Since $5 < 7 < 8$, then $BC < AC < AB$, so the $\angle$s in order from smallest to largest are $\angle A$, $\angle B$, $\angle C$. **8.** By Thm. 5-10, the smallest $\angle$ is opp. the shortest side, and the largest is opp. the longest side. Since $5 < 15 < 18$, then $DF < DE < EF$, so the $\angle$s in order from smallest to largest are $\angle E$, $\angle F$, $\angle D$. **9.** By Thm. 5-10, the smallest $\angle$ is opp. the shortest side, and the largest is opp. the longest side. Since $12 < 24 < 30$, then $XY < YZ < ZX$, so the $\angle$s in order from smallest to largest are $\angle Z$, $\angle X$, $\angle Y$. **10.** By Thm. 5-11, the

shortest side is opp. the smallest $\angle$, and the longest is opp. the largest $\angle$. By the Triangle Angle-Sum Thm., $m\angle M = 180 - (45 + 75) = 60$. Since $45 < 60 < 75$, the order of the sides from shortest to longest is $\overline{MN}, \overline{ON}, \overline{MO}$. **11.** By Thm. 5-11, the shortest side is opp. the smallest $\angle$, and the longest is opp. the largest $\angle$. By the Triangle Angle-Sum Thm., $m\angle H = 180 - (28 + 110) = 42$. Since $28 < 42 < 110$, the order of the sides from shortest to longest is $\overline{FH}, \overline{GF}, \overline{GH}$. **12.** By Thm. 5-11, the shortest side is opp. the smallest $\angle$, and the longest is opp. the largest $\angle$. By the Triangle Angle-Sum Thm., $m\angle M = 180 - (90 + 30) = 60$. Since $30 < 60 < 90$, the order of the sides from shortest to longest is $\overline{TU}, \overline{UV}, \overline{TV}$. **13.** By Thm. 5-11, the shortest side is opp. the smallest $\angle$, and the longest is opp. the largest $\angle$. Since $40 < 50 < 90$, then $m\angle B < m\angle C < m\angle A$, so the order of the sides from shortest to longest is $\overline{AC}, \overline{AB}, \overline{CB}$. **14.** By Thm. 5-11, the shortest side is opp. the smallest $\angle$, and the longest is opp. the largest $\angle$. Since $20 < 40 < 120$, then $m\angle D < m\angle F < m\angle E$, so the order of the sides from shortest to longest is $\overline{EF}, \overline{DE}, \overline{DF}$. **15.** By Thm. 5-11, the shortest side is opp. the smallest $\angle$, and the longest is opp. the largest $\angle$. Since $51 < 59 < 70$, then $m\angle X < m\angle Y < m\angle Z$, so the order of the sides from shortest to longest is $\overline{ZY}, \overline{XZ}, \overline{XY}$. **16.** $6 + 3 > 2$, $6 + 2 > 3$, but $3 + 2$ is not greater than 6, so a $\triangle$ is not possible with these measures. **17.** $15 + 12 > 11$, $15 + 11 > 12$, and $11 + 12 > 15$, so a $\triangle$ can be made with these measures. **18.** $19 + 10 > 8$, $19 + 8 > 10$, but $8 + 10$ is not greater than 19, so a $\triangle$ is not possible with these measures. **19.** $15 + 15 > 1$, $15 + 1 > 15$, and $1 + 15 > 15$, so a $\triangle$ is possible with these measures. **20.** $10 + 9 > 2$, $10 + 2 > 9$, and $2 + 9 > 10$, so a $\triangle$ can be made with these measures. **21.** $9 + 5 > 4$, $9 + 4 > 5$, but $4 + 5$ is not greater than 9, so a $\triangle$ is not possible with these measures. **22.** $12 - 8 < s < 12 + 8$; $4 < s < 20$ **23.** $16 - 5 < s < 16 + 5$; $11 < s < 21$ **24.** $6 - 6 < s < 6 + 6$; $0 < s < 12$ **25.** $23 - 18 < s < 23 + 18$; $5 < s < 41$ **26.** $7 - 4 < s < 7 + 4$; $3 < s < 11$ **27.** $35 - 20 < s < 35 + 20$; $15 < s < 55$ **28.** Answers may vary. Sample: Avi's location, Wichita, and Topeka may be vertices of a $\triangle$. If y is the distance between Wichita and Topeka, then $110 - 90 < y < 110 + 90$, so $20 < y < 200$. If it's possible that the 3 points are collinear, then $20 \leq y \leq 200$.
29. Let the distance between the peaks be d and the distances from the hiker to each of the peaks be a and b. Then $d + a > b$ and $d + b > a$. Thus, $d > b - a$ and $d > a - b$.

30a.



Draw 2 $\triangle$s having 2 pairs of $\cong$ sides. Make the included $\angle$ of the first $\triangle$ greater than the included $\angle$ of the second $\triangle$. **30b.** The third side of the first $\triangle$ is longer than the third side of the second $\triangle$. **30c.** See diagram in part ⓐ. **30d.** The included $\angle$ of the first $\triangle$ is greater than the included $\angle$ of the second $\triangle$. **31.** Answers may vary. Sample: The shortcut across the grass is shorter than the sum of the two sidewalk paths. **32.** By Thm. 5-11, the longest side lies opp. the largest $\angle$. By the Triangle Angle-Sum Thm., $70 + (2x - 10) + (3x + 20) = 180$; $5x + 80 = 180$; $5x = 100$; $x = 20$. $m\angle B = 2x - 10 = 2(20) - 10 = 40 - 10 = 30$. $m\angle C = 3x + 20 = 3(20) + 20 = 60 + 20 = 80$. Since $30 < 70 < 80$, $\angle C$ is the largest $\angle$. The side opp. $\angle C$ is the longest side: $\overline{AB}$. **33a.** $m\angle OTY$ (the $\angle$ opp. $\overline{YO}$) **33b.** $m\angle 3$ (the $\angle$ opp. $\overline{YT}$) **33c.** Isosceles Triangle Thm.: Base $\angle$s of an isosc. $\triangle$ are $\cong$. **33d.** Angle Add. Post. **33e.** Comparison Prop. of Ineq. (Since $\angle$ measures of a $\triangle$ are always positive, subtracting $m\angle 4$ from only the right side decreases the value on that side of the = symbol.) **33f.** Substitution (from Step 2) **33g.** Corollary to the Exterior Angle Thm.: An ext. $\angle$ of a $\triangle$ is greater than either remote int. $\angle$. **33h.** Trans. Prop. of Ineq. (Steps 5 and 6) **34.** By the Triangle Angle-Sum Thm. and the Isosc. Triangle Thm., $m\angle PQR = m\angle PRQ = 75$. By Thm. 5-11, the shortest side in $\triangle PQR$ is $\overline{QR}$. By the Triangle Angle-Sum Thm., $m\angle S = 50$, so the shortest side of $\triangle QRS$ is the shortest side of the figure, $\overline{RS}$. **35.** By the Triangle Angle-Sum Thm., $m\angle C = 38$ and $m\angle BDA = 36$. The shortest side of $\triangle CBD$ is $\overline{CD}$, and the shortest side of $\triangle BDA$ is $\overline{BD}$. Since, in $\triangle CBD$ $CD < BD$, the shortest side is $\overline{CD}$.
36. By the Triangle Angle-Sum Thm., $m\angle XYW = 85$ and $m\angle YWZ = 40$. By Thm. 5-11, the shortest side of $\triangle WXY$ is $\overline{XY}$, and the shortest side of $\triangle WYZ$ is $\overline{WY}$ and $\overline{YZ}$, since they are $\cong$. But, in $\triangle WXY$, $XY < WY$, so the shortest side in the figure is $\overline{XY}$. **37.** If she picks the 3-cm straw, she cannot form a $\triangle$ since $3 + 6$ is not greater than 9. If she picks the 5-cm straw, she can form a $\triangle$ because the sum of the lengths of every two straws is greater than the length of the third. If she picks the 11-cm straw, she can form a $\triangle$ because the sum of the lengths of every two straws is greater than the length of the third. If she picks the 15-cm straw, she cannot form a $\triangle$ since $6 + 9$ is not greater than 15. There are 2 favorable outcomes out of 4 possibilities, so the probability is $\frac{2}{4}$, or $\frac{1}{2}$. **38.** $x = \{11, 12, 13\}$ and $y = \{9, 10, 11, 12, 13\}$. If $x = 11$, then y must be between 6 and 16 to satisfy Thm. 5-12, so y can be 9, 10, 11, 12, or 13. If $x = 12$, then y must be between 7 and 17 to satisfy Thm. 5-12, so y can be 9, 10, 11, 12, or 13. If $x = 13$, then y must be between 8 and 18, so y can be 9, 10, 11, 12, or 13. No values for x or y are excluded, so x can be 11, 12, or 13, and y can be 9, 10, 11, 12, or 13. **39.** Since $10 < x < 14$, $x = \{11, 12, 13\}$, and since $8 < y < 14$, $y = \{9, 10, 11, 12, 13\}$. One side is 5 cm. Use Thm. 5-12. If $x = 11$, then $6 < y < 16$, so y can be 9, 10, 11, 12, or 13, of which only 11 cm creates an isosc. $\triangle$. If $x = 12$, then $7 < y < 17$, so y can be 9, 10, 11, 12, or 13, of which only 12 cm creates an isosc. $\triangle$. If $x = 13$, then $8 < y < 18$, so y can be 9, 10, 11, 12, or 13, of which only 13 cm creates an isosc. $\triangle$. For each of the 3 possible values of x there are 5 possible outcomes with only 1 favorable outcome each. So, there are 3 favorable outcomes from 15 possibilities, so the probability is $\frac{3}{15}$, or $\frac{1}{5}$. **40.** $CD = AC$ is given, so $\triangle ACD$ is isosc. by def. of isosc. $\triangle$. This means $m\angle D = m\angle CAD$.

The $m\angle DAB > m\angle CAD$ by the Comparison Prop. of Ineq. So, by the Trans. Prop., $m\angle DAB > m\angle D$ and by Thm. 5-11, $DB > AB$. Since $DC + CB = DB$, by subst. $DC + CB > AB$. Using subst. again, $AC + CB > AB$. **41.** Since $\angle T$ is a rt. $\angle$, it is the largest $\angle$ in $\triangle PTA$. Thus, $PA > PT$ because the longest side of a $\triangle$ is opp. the largest $\angle$. **42.** By the Corollary to the Ext. Angle Thm., $2a + 12 + 4a = 114$; $6a = 102$; $a = 17$. Solve for $m\angle P$: $m\angle P = 2a + 12 = 2(17) + 12 = 34 + 12 = 46$; $m\angle Q = 4a = 4(17) = 68$; $m\angle PRQ = 180 - 114 = 66$. $\overline{PR}$ is opp. the largest $\angle$, so it is the longest side. It must be > 184. The only choice > 184 is 187. The answer is choice D. **43.** $x = 180 - 50 = 130$; by the Corollary to the Ext. Angle Thm., $x = 4c + (c - 25)$; $130 = 4c + (c - 25)$; $130 = 5c - 25$; $155 = 5c$; $c = 31$. Since $130 > 31$, $x > c$. The answer is choice A. **44.** From Exercise 43, $c = 31$, so $m\angle J = 4c = 4(31) = 124$, $m\angle B = c - 25 = 31 - 25 = 6$, and $m\angle JNB = 50$. Since $6 < 124$, then $m\angle B < m\angle J$, so $JN < BN$. The answer is choice B. **45.** By the Corollary to the Ext. Angle Thm., $x = 4c + (c - 25)$. Add $4c$ to both sides: $x - 4c = c - 25$. The answer is choice C. **46.** From Exercise 44, $m\angle J = 124$, $m\angle B = 6$, and $m\angle JNB = 50$. By Thm. 5-11, since $m\angle JNB < m\angle J$, $JB < BN$. The answer is choice B. **47** [2] **a.** Since $m\angle A > m\angle C > m\angle B$, the lengths of the sides opp. them are related in the same way: $BC > AB > AC$. Of $\overline{AB}$ and $\overline{AC}$, $\overline{AB}$ is longer than $\overline{AC}$. Since 9 in. $>$ 5 in., $AB = 9$ in. and $AC = 5$ in. **b.** $\overline{BC}$ is the longest side, so $9 < BC < 9 + 5$, or $9 < BC < 14$. The possible whole-number measurements for $\overline{BC}$ are 10 in., 11 in., 12 in., and 13 in. [1] part ⓐ OR part ⓑ incorrect **48.** When comparing two measures, there are just 3 choices: $<$, $>$, and $=$. So, $m\angle A$ is not less than or equal to $m\angle B$ means that $m\angle A > m\angle B$. **49.** When comparing two measures, there are just 3 choices: $<$, $>$, and $=$. So, $m\angle X$ is not greater than $m\angle B$ means that $m\angle X \leq m\angle B$. **50.** To negate a sentence, insert the word *not* if the original statement doesn't include it and remove it if it does. The $\angle$ is not a right $\angle$. **51.** To negate a sentence, insert the word *not* if the original statement doesn't include it and remove it if it does. The $\triangle$ is obtuse. **52.** Since $\angle EDH$ is a straight $\angle$, its measure is 90, so $m\angle ADH = 180 - 90 = 90$. **53.** Since vert. $\angle$s are $\cong$, $m\angle GDH = m\angle EDC = 35$. **54.** Since $\angle EDH$ is a straight $\angle$, $m\angle CDH = 180 - 35 = 145$. **55.** Since $\angle CDG$ is a straight $\angle$, $m\angle ADG = 180 - (35 + 90) = 55$. **56.** $A = \pi r^2 = \pi(1.6)^2 = 2.56\pi \approx 8.0$, which means 8.0 ft^2. **57.** $r = \frac{d}{2} = \frac{35}{2} = 17.5$, which means 17.5 mm; $A = \pi r^2 = \pi(17.5)^2 = 306.25\pi \approx 962.1$, which means 962.1 mm^2. **58.** $A = \pi r^2 = \pi(0.5)^2 = \pi(0.25) \approx 0.8$, which means 0.8 m^2. **59.** $r = \frac{d}{2} = \frac{20}{2} = 10$, which means 10 mi; $A = \pi r^2 = \pi(10)^2 = 100\pi = 314.2$, which means 314.2 mi^2.

TEST-TAKING STRATEGIES *page 280*

1. Let $BC = x$. Then $AB = x + 15$, $CD = x + 8$, and $AD = (x + 15) + x + (x + 8) = 3x + 23$. $AD = 2AB - 5$; $3x + 23 = 2(x + 15) - 5$; $3x + 23 = 2x + 30 - 5$; $3x + 23 = 2x + 25$; $x + 23 = 25$; $x = 2$. $AB = x + 15 = 2 + 15 = 17$; $BC = x = 2$; $CD = x + 8 = 2 + 8 = 10$ **2.** The slope of $\overline{AB}$ is $\frac{4}{3}$, so the slope of $\overline{BN}$, which is $\perp$ to $\overline{AB}$, is $-\frac{3}{4}$. In point-slope form, an equation for $\overleftrightarrow{BN}$ through $(3, 4)$ is $y - 4 = -\frac{3}{4}(x - 3)$. Because N is on the x-axis, solve for x when $y = 0$: $0 - 4 = -\frac{3}{4}(x - 3)$; $-16 = -3(x - 3)$; $-16 = -3x + 9$; $-25 = -3x$; $x = 8\frac{1}{3}$. So, the coordinates of point N are $(8\frac{1}{3}, 0)$. **3.** Let a be any integer. Then $2a$ is an even integer and two even integers next to it are 2 integers less and 2 integers more. The three integers are $2a - 2$, $2a$, and $2a + 2$. By the Triangle Angle-Sum Thm., $(2a - 2) + 2a + (2a + 2) = 180$; $6a = 180$; $a = 30$. The $\angle$ measures are $2a - 2 = 2(30) - 2 = 60 - 2 = 58$, $2a = 2(30) = 60$, and $2a + 2 = 2(30) + 2 = 60 + 2 = 62$. **4.** Let x represent the x-coordinate of P. Since P is on the x-axis, $y = 0$. Use the distance formula: $13 = \sqrt{(x - 7)^2 + (5 - 0)^2} = \sqrt{(x^2 - 14x + 49) + 25} = \sqrt{x^2 - 14x + 74}$; $169 = x^2 - 14x + 74$; $0 = x^2 - 14x - 95$; $(x - 19)(x + 5) = 0$; $x = 19$ or $x = -5$. The possible coordinates for point P are $(19, 0)$ and $(-5, 0)$. **5.** Let x represent the x-coordinate of P. Since P is on the x-axis, $y = 0$. The slope of $\overleftrightarrow{AP} = \frac{4 - 0}{1 - x} = \frac{4}{1 - x}$. The slope of $\overleftrightarrow{BP} = \frac{10 - 0}{9 - x} = \frac{10}{9 - x}$. Then $\frac{4}{1 - x} = 2\left(\frac{10}{9 - x}\right)$; $\frac{4}{1 - x} = \frac{20}{9 - x}$; $4(9 - x) = 20(1 - x)$; $36 - 4x = 20 - 20x$; $16x + 36 = 20$; $16x = -16$; $x = -1$. So, the coordinates of point P are $(-1, 0)$.

CHAPTER REVIEW *pages 281–283*

1. A *median of a* $\triangle$ is a segment whose endpoints are a vertex and the midpoint of the side opposite the vertex. **2.** The length of the perpendicular segment from a point to a line is the *distance from the point to the line*. **3.** If T is a point on the perpendicular bisector of $\overline{FG}$, then $TF = TG$ because of the *Perpendicular Bis. Thm.* **4.** The *altitude* of a $\triangle$ is a perpendicular segment from a vertex to the line containing the side opposite the vertex. **5.** The notation $\sim q \rightarrow \sim p$ is the *contrapositive* of $p \rightarrow q$. **6.** To write an *indirect proof*, you start by assuming that the opposite of what you want to prove is true. **7.** In $\triangle ABC$, $AB + BC > AC$ because of the *Triangle Inequality Thm.* **8.** The *incenter* of a $\triangle$ is the point of concurrency of the $\angle$ bisectors of the $\triangle$. **9.** The *Angle Bis. Thm.* says that if a point is on the bisector of an $\angle$, then it is equidistant from the sides of the $\angle$. **10.** A point where three lines intersect is a *point of concurrency*. **11.** $x = \frac{1}{2}(30) = 15$ **12.** $x + 5 = \frac{1}{2}(3x - 1)$; $2(x + 5) = 3x - 1$; $2x + 10 = 3x - 1$; $-x + 10 = -1$; $-x = -11$; $x = 11$ **13.** $m\angle BEF = m\angle DEB = 180 - (90 + 50) = 40$ **14.** $\triangle FBE \cong \triangle DBE$ by AAS, so $FE = DE = DC = 7$. **15.** $EC = ED + DC = 7 + 7 = 14$ **16.** $m\angle CEA = 2m\angle DEB = 2(180 - (90 + 50)) = 2(180 - 140) = 2(40) = 80$ **17.** The circumcenter is the pt. of concurrency of the $\perp$ bis. of a $\triangle$. The equation for the $\perp$ bis. of a segment whose endpts. are $(2, 3)$ and $(2, -3)$ is $y = 0$. The equation for the $\perp$ bis. of a segment having endpts. at $(-4, -3)$ and $(2, -3)$ is $x = -1$. The

intersection of $x = -1$ and $y = 0$ is $(-1, 0)$. **18.** The centroid is the pt. of concurrency of the medians. The midpt. $\overline{BC}$ is $X(-1, -3)$ and the slope of $\overleftrightarrow{AX}$ is $\frac{-3-3}{-1-2} = 2$, so an equation through $(-1, -3)$ is $y + 3 = 2(x + 1)$, or $y = 2x - 1$. The midpt. of $\overline{AC}$ is $Y(2, 0)$ and the slope of $\overleftrightarrow{BY}$ is $\frac{0+3}{2+4} = \frac{1}{2}$, so an equation through $(2, 0)$ is $y - 0 = \frac{1}{2}(x - 2)$, or $y = \frac{1}{2}x - 1$. The lines for the equations $y = 2x - 1$ and $y = \frac{1}{2}x - 1$ intersect at $(0, -1)$. **19.** The orthocenter is the pt. of concurrency of the altitudes. $\overline{AC}$ is vertical and $\overline{BC}$ is horizontal, so they are altitudes of the $\triangle$. They meet at $C(2, -3)$. **20.** Since it bis. an $\angle$, $\overline{AB}$ is an $\angle$ bis. **21.** Since it is a segment with an endpt. at a vertex and $\perp$ to the opp. side, it is an altitude. **22.** Since it is a segment with an endpt. at a vertex and bisects the opp. side, it is a median. **23.** To write the inverse, negate both the hypothesis and conclusion: If it is not snowing, then it is not cold outside. To write the contrapositive, switch the hypothesis and conclusion of the inverse: If it is not cold outside, then it is not snowing. **24.** To write the inverse, negate both the hypothesis and conclusion: If an $\angle$ is not obtuse, then its measure is not greater than 90 and less than 180. To write the contrapositive, switch the hypothesis and conclusion of the inverse: If an $\angle$'s measure is not greater than 90 and less than 180, then it is not obtuse. **25.** To write the inverse, negate both the hypothesis and conclusion: If a figure is not a square, then its sides are not congruent. To write the contrapositive, switch the hypothesis and conclusion of the inverse: If a figure's sides are not congruent, then it is not a square. **26.** To write the inverse, negate both the hypothesis and conclusion: If you are not in Australia, then you are not south of the equator. To write the contrapositive, switch the hypothesis and conclusion of the inverse: If you are not south of the equator, then you are not in Australia. **27.** Assume that both numbers are odd. Since all even numbers have a factor of 2, let a be any integer; then $2a$ is even and $2a + 1$ and $2a - 1$ are both odd integers whose product is $4a^2 - 1 = 2 \cdot 2a^2 - 1$, which is odd. So, the product of 2 odd numbers is always odd, which contradicts that the product is even. Therefore, at least one number must be even. **28.** Assume that a right $\angle$ can be formed by non-perp. lines. Then the $\angle$s formed by the lines are not right $\angle$s necessary to form $\perp$ lines, which contradicts the assumption. Therefore, the assumption is false. **29.** Assume that a $\triangle$ has 2 obtuse $\angle$s. Then these $\angle$s, by def., measure more than 90, which makes their sum greater than 180. But the sum of the measures of the $\angle$s of a $\triangle = 180$, so the assumption must be false. **30.** Assume one $\angle$ is obtuse; therefore, its measure is greater than 90. Since a prop. of an equilateral $\triangle$ is that all 3 $\angle$s are $\cong$, then each $\angle$ would be greater than 90. Thus, the sum of the 3 $\angle$s would be greater than 180, which contradicts the Triangle Angle-Sum Thm. **31.** $m\angle T = 180 - (80 + 70) = 30$; since $30 < 70 < 80$; the $\angle$s in order from smallest to largest are $\angle T, \angle R, \angle S$. The sides opp. the $\angle$s follow the same pattern, so the sides in order from shortest to longest are $\overline{RS}, \overline{TS}, \overline{TR}$. **32.** The smallest $\angle$ is opp. the shortest side and the largest $\angle$ is opp. the longest side. Since $4 < 5 < 8$, the $\angle$s from smallest to largest are $\angle G, \angle O, \angle F$, and the sides from shortest to longest are $\overline{OF}, \overline{FG}, \overline{OG}$. **33.** $15 + 8 > 5$, $15 + 5 > 8$, but $8 + 5$ is not greater than 15, so these lengths do not form a $\triangle$. **34.** $20 + 12 > 10, 20 + 10 > 12, 10 + 12 > 20$, so these lengths can form a $\triangle$. **35.** $24 + 22 > 20, 24 + 20 > 22$, and $22 + 20 > 24$, so these lengths can form a $\triangle$. **36.** $8 + 6 > 3, 8 + 3 > 6$, and $3 + 6 > 8$, so these lengths can form a $\triangle$. **37.** $3 + 4 > 1$, but $3 + 1$ is not greater than 4 and $1 + 1$ is not greater than 3, so these lengths cannot form a $\triangle$. **38.** $7 + 6 > 5, 7 + 5 > 6$, and $5 + 6 > 7$, so these lengths can form a $\triangle$.
39. $7 - 4 < x < 7 + 4; 3 < x < 11$ **40.** $15 - 8 < x < 15 + 8; 7 < x < 23$ **41.** $8 - 2 < x < 8 + 2; 6 < x < 10$
42. $13 - 12 < x < 13 + 12; 1 < x < 25$

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1a. To write the inverse, negate the hypothesis and conclusion: If a polygon does not have eight sides, then it is not an octagon. **1b.** To write the contrapositive switch the hypothesis and conclusion of the inverse: If a polygon is not an octagon, then it does not have eight sides.
2a. To write the inverse, negate the hypothesis and conclusion: If it is not a leap year, then it is not an even-numbered year. **2b.** To write the contrapositive switch the hypothesis and conclusion of the inverse: If it is not an even-numbered year, then it is not a leap year. **3a.** To write the inverse, negate the hypothesis and conclusion: If it is not snowing then it is summer. **3b.** To write the contrapositive switch the hypothesis and conclusion of the inverse: If it is summer, then it is not snowing.
4. Answers may vary. Samples: $\overline{DE}$ is a midsegment by def. of midsegment; $DE = \frac{1}{2}BC$ and $\overline{DE} \parallel \overline{BC}$ by the Triangle Midsegment Thm. **5.** Only 1 $\angle$ in a $\triangle$ can be non-acute, so I and II are contradictory. **6.** Vert. $\angle$s can never share a side, so II and III are contradictory. **7.** The smallest $\angle$s are opp. the shortest sides, and the largest are opp. the longest sides: $\angle A, \angle C, \angle B$. **8.** The smallest $\angle$s are opp. the shortest sides, and the largest are opp. the longest sides: $\angle B, \angle C, \angle A$. **9.** The smallest $\angle$s are opp. the shortest sides, and the largest are opp. the longest sides: $\angle C, \angle B, \angle A$. **10.** Choose any 2 lengths. Then make the third side less than or equal to their difference or greater than or equal to their sum. Answers may vary. Sample: 2, 4, 8 because $2 + 4$ is not greater than 8. **11.** The shortest side is opp. the smallest $\angle$ and the longest side is opp. the longest $\angle$. $m\angle T = 180 - (80 + 30) = 70$, and $30 < 70 < 80$, so the sides from shortest to longest are $\overline{ST}, \overline{SR}, \overline{RT}$. **12.** The shortest side is opp. the smallest $\angle$ and the longest side is opp. the longest $\angle$. $m\angle M = 180 - (40 + 110) = 30$, and $30 < 40 < 110$, so the sides from shortest to longest are $\overline{KV}, \overline{VM}, \overline{KM}$. **13.** $x = \frac{1}{2}(13) = 6.5$ **14.** $3x = \frac{1}{2}(5x + 12); 6x = 5x + 12; x = 12$
15. Assume that, if an isosc. $\triangle$ is obtuse, then the obtuse $\angle$ is a base $\angle$. Then, by the def. of an isosc. $\triangle$, the other base $\angle$ must be obtuse since they are $\cong$. By the def. of obtuse, each $\angle$ is greater than 90, so their sum would be greater than 180. This contradicts the Triangle

Angle-Sum Thm., so the assumption is false. So, the obtuse $\angle$ must be the vertex $\angle$. **16.** Each pt. on an $\angle$ bis. is equidistant from the sides of the $\angle$, so $KM = PM$; $5x - 8 = 2x + 13$; $3x - 8 = 13$; $3x = 21$; $x = 7$. $KM = 5x - 8 = 5(7) - 8 = 35 - 8 = 27$ **17.** Each pt. on an $\angle$ bis. is equidistant from the sides of the $\angle$, so $KM = PM$; $4x = 7x - 2$; $-3x = -2$; $x = \frac{2}{3}$. $KM = 4\left(\frac{2}{3}\right) = \frac{8}{3}$, or $2\frac{2}{3}$
18. The center of the circle that circumscribes a $\triangle$ is the circumcenter, which is the pt. of concurrency of the perp. bisectors of the $\triangle$. The equation of the $\perp$ bis. of $\overline{BC}$ is $x = -2\frac{1}{2}$. The equation of the $\perp$ bis. of $\overline{AB}$ is $y = -1\frac{1}{2}$. The intersection of these two lines is $\left(-2\frac{1}{2}, -1\frac{1}{2}\right)$.
19. The center of the circle that circumscribes a $\triangle$ is the circumcenter, which is the pt. of concurrency of the perp. bisectors of the $\triangle$. The equation of the $\perp$ bis. of $\overline{AB}$ is $x = -2$. The equation of the $\perp$ bis. of $\overline{BC}$ is $y = 1$. The intersection of these two lines is $(-2, 1)$. **20.** The center of the circle that circumscribes a $\triangle$ is the circumcenter, which is the pt. of concurrency of the perp. bisectors of the $\triangle$. The equation of the $\perp$ bis. of $\overline{AB}$ is $x = \frac{1}{2}$. The equation of the $\perp$ bis. of $\overline{AC}$ is $y = -4\frac{1}{2}$. The intersection of these two lines is $\left(\frac{1}{2}, -4\frac{1}{2}\right)$. **21.** Name (6, 1) pt. X. Using the distance formula, $AX = BX = CX = 4\sqrt{2}$, so X is equidistant from the vertices. X must be the pt. of concurrency of the $\perp$ bis., so it must be the circumcenter.
22a. QA (Perp. Bis. Thm.) **22b.** QA (Perp. Bis. Thm.)
22c. Transitive Prop. of = **23.** Y is on $\overleftrightarrow{TW}$, the $\perp$ bis. of $\overline{XZ}$ because Y is equidistant from the endpts. of $\overline{XZ}$. (Converse of the Perp. Bis. Thm.) **24.** A is on $\overrightarrow{CK}$, the $\angle$ bis. of $\angle SCD$. (Converse of the Angle Bis. Thm.)

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1. The first sentence states that most kitchen activities are between these three appliances. The answer is choice B. **2.** The longest side is 280 between the sink and fridge, so the vertex of the largest $\angle$ is at the stove. The answer is choice G. **3.** The shortest side is between the sink and fridge, so the vertex of the smallest $\angle$ is at the stove. The answer is choice B. **4.** Apartment B does not form a $\triangle$ because $145 + 165$ is not greater than 320.
5. $P_A = 250 + 280 + 240 = 770$; $P_B = 630$; $P_C = 665$; $P_D = 732$. The perimeter of the work $\triangle$ in Apt. B is the least, so two people are more likely to get in each other's way. **6.** Since the microwave and can opener are endpts. of a midsegment, the distance is $\frac{1}{2}(240)$, or 120 cm. The answer is choice F. **7.** A midsegment is $\parallel$ to the 3rd side.
8. The perimeters of A, C, and D are 770, 665, and 732, respectively. Since C has the least perimeter, its midsegment $\triangle$ is the least. The answer is choice C.
9. Only 2 sides of the $\triangle$ are needed to go to each vertex once, so add the two smaller segments for each $\triangle$.
A: $240 + 250 = 490$, which means 490 cm; C: $115 + 220 = 335$, which means 335 cm; D: $152 + 270 = 422$, which means 422 cm; so C has the least sum. The answer is choice G. **10.** $270 + 152 = 422$, which means 422 cm; the answer is choice C.