Congruent Triangles

DIAGNOSING READINESS page 178

1. $AB = \sqrt{(-1-3)^2 + (1-1)^2} = \sqrt{(-4)^2 + (0)^2} = \sqrt{16} = 4$; $BC = \sqrt{(-1+1)^2 + (-2-1)^2} = \sqrt{(0)^2 + (-3)^2} = \sqrt{9} = 3$; $AC = \sqrt{(-1-3)^2 + (-2-1)^2} = \sqrt{(-4)^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$ **2.** $AB = \sqrt{(-3+3)^2 + (-6-2)^2} = \sqrt{(0)^2 + (-8)^2} = \sqrt{64} = 8$; $BC = \sqrt{(8+3)^2 + (6+6)^2} = \sqrt{(11)^2 + (12)^2} = \sqrt{121 + 144} = \sqrt{265}$; $AC = \sqrt{(8+3)^2 + (6-2)^2} = \sqrt{(11)^2 + (4)^2} = \sqrt{121+16} = \sqrt{137}$ **3.** $AB = \sqrt{(6+1)^2 + (1+2)^2} = \sqrt{(7)^2 + (3)^2} = \sqrt{49+9} = \sqrt{58}$; $BC = \sqrt{(2-6)^2 + (5-1)^2} = \sqrt{(-4)^2 + (4)^2} = \sqrt{16+16} = \sqrt{16 \cdot 2} = 4\sqrt{2}$; $AC = \sqrt{(2+1)^2 + (5+2)^2} = \sqrt{(3)^2 + (7)^2} = \sqrt{9+49} = \sqrt{58}$ **4.** If $m\angle A = x$, then $m\angle B = 180 - x$, and $m\angle C = 180 - (180 - x) = x = m\angle A$. Since $m\angle A = m\angle C = x$, $\angle A \cong \angle C$. **5.** Because the ∠s are supplementary, if $m\angle A = x$, then $m\angle B = 180 - x$. Since the ∠s are equal, then $x = 180 - x$. Solving this equation results in $x = 90$. So, $m\angle A = 90 = \angle B$, so both ∠s are right ∠s. **6.** By def. of complementary ∠s, $m\angle 1 + m\angle 2 = 90$. Since vertical ∠s are congruent, $\angle 1 \cong \angle ACB$. By substitution, $m\angle ACB + m\angle 2 = 90$. Also, $m\angle 1 + m\angle B + m\angle ACB = 180$. By substitution, $m\angle B + 90 = 180$, so $m\angle B = 90$. Therefore, $\angle B$ is a right ∠. **7.** By def. of ⊥, $\angle AFC$ and $\angle BFD$ are right ∠s and measure 90. By the ∠ addition postulate, ① $m\angle AFB + m\angle BFC = 90$ and ② $m\angle BFC + m\angle DFC = 90$. From ① $m\angle BFC = 90 - m\angle AFB$, and from ② $m\angle BFC = 90 - m\angle DFC$. By substitution, $90 - m\angle AFB = 90 - m\angle DFC$, so $m\angle AFB = m\angle DFC$. By def. of ≅, $\angle AFB \cong \angle DFC$. **8.** Since $\overline{AB} \parallel \overline{CD}$, alt. int. ∠s are ≅. So, $\angle A \cong \angle D$ and $\angle B \cong \angle C$. **9.** Since $\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \parallel \overline{DC}$, alt. int. ∠s are ≅. So, $\angle ACD \cong \angle CAB$ and $\angle DAC \cong \angle BCA$. **10.** By the Triangle Angle-Sum Thm., $(x + 9) + (7x + 4) + (6x - 1) = 180$. Combining like terms results in $14x + 12 = 180$. Then solving the equation results in $x = 12$.

4-1 Congruent Figures pages 180–185

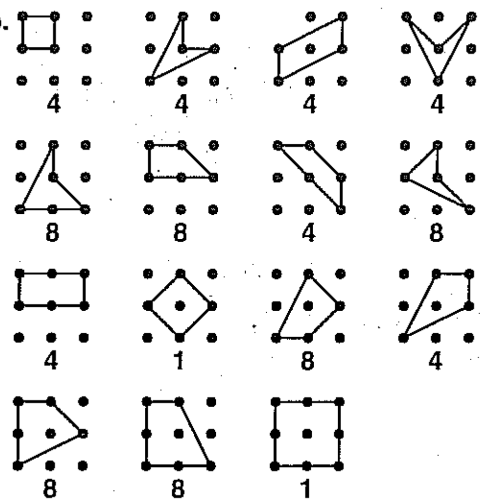
Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1. 19 **2.** 13 **3.** 108 **4.** 10 **5.** 50

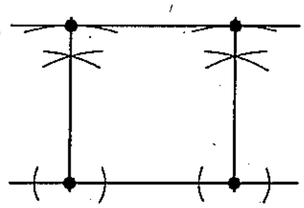
Check Understanding **1.** Use the order of the letters in the △ congruency statement to determine their corresponding parts. $\angle WSY \cong \angle MVK$; $\angle SWY \cong \angle VMK$; $\angle WYS \cong \angle MKV$; $\overline{WY} \cong \overline{MK}$; $\overline{WS} \cong \overline{MV}$; $\overline{YS} \cong \overline{KV}$ **2.** Since Y is the middle letter in the name of $\triangle WYS$, the middle letter of the congruent $\triangle MKV$, which is K, corresponds to it. Thus, $\angle Y \cong \angle K$; so, $35 = m\angle Y = m\angle K$. **3.** Though you can prove that all corresponding ∠s are ≅, you cannot show that corresponding sides are ≅. Thus, you cannot conclude that the △s are ≅. **4.** Show that all pairs of corresponding ∠s are congruent and all pairs of corresponding sides are congruent. **a.** $\angle A \cong \angle D$; $\angle E \cong \angle C$ (Given) **b.** $\angle ABE \cong \angle DBC$ (Vert. ∠s are ≅.) **c.** $\overline{AE} \cong \overline{CD}$; $\overline{AB} \cong \overline{BD}$; $\overline{EB} \cong \overline{BC}$ (Given) **d.** $\triangle ABE \cong \triangle DBC$ (Def. of ≅ △s.)

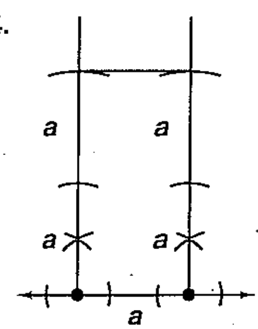
Exercises **1.** Find the corresponding parts according to the order of the letters in the △ congruency statement. $\angle CAB \cong \angle DAB$; $\angle C \cong \angle D$; $\angle ABC \cong \angle ABD$; $\overline{AC} \cong \overline{AD}$; $\overline{AB} \cong \overline{AB}$; $\overline{CB} \cong \overline{DB}$ **2.** Find the corresponding parts according to the order of the letters in the △ congruency statement. $\angle GEF \cong \angle JHI$; $\angle GFE \cong \angle JIH$; $\angle EGF \cong \angle HJI$; $\overline{GE} \cong \overline{JH}$; $\overline{EF} \cong \overline{HI}$; $\overline{FG} \cong \overline{IJ}$ **3.** $\overline{LC} \cong \overline{BK}$ **4.** $\overline{KJ} \cong \overline{CM}$ **5.** $\overline{JB} \cong \overline{ML}$ **6.** $\angle L \cong \angle B$ **7.** $\angle K \cong \angle C$ **8.** $\angle M \cong \angle J$ **9.** $\triangle CML \cong \triangle KJB$ **10.** $\triangle KBJ \cong \triangle CLM$ **11.** $\triangle MLC \cong \triangle JBK$ **12.** $\triangle JKB \cong \triangle MCL$ **13.** A corresponds to E, B corresponds to K, C corresponds to G, and D corresponds to N. **14.** The letter positions in the congruency statement show the corresponding vertices. $\overline{PO} \cong \overline{SI}$; $\overline{OL} \cong \overline{ID}$; $\overline{LY} \cong \overline{SE}$ **15.** The letter positions in the congruency statement show the corresponding vertices. $\angle P \cong \angle S$; $\angle O \cong \angle I$; $\angle L \cong \angle D$; $\angle Y \cong \angle E$ **16.** According to the order of the letters in the congruency statement, $\overline{AD}$ corresponds to $\overline{FI}$, so $AD = 33$, which means 33 in. **17.** According to the order of the letters in the congruency statement, $\overline{HI}$ corresponds to $\overline{CD}$, so $HI = 54$, which means 54 in. **18.** According to the order of the letters in the congruency statement, $\angle FGH$ corresponds to $\angle ABC$, so $m\angle FGH = 105$. **19.** According to the order of the letters in the congruency statement, $\angle ADC$ corresponds to $\angle FIH$, so $m\angle ADC = 77$. **20.** According to the order of the letters in the congruency statement, $\overline{FG}$ corresponds to $\overline{AB}$, so $FG = 36$, which means 36 in. **21.** According to the order of the letters in the congruency statement, $\overline{BC}$ corresponds to $\overline{GH}$, so $BC = 34$, which means 34 in. **22.** According to the order of the letters in the congruency statement, $\angle DCB$ corresponds to $\angle IHG$, so $m\angle DCB = 75$. **23.** According to the order of the letters in the congruency statement, $\angle IFG$ corresponds to $\angle DAB$, so $m\angle IFG = 103$. **24.** They are congruent because three pairs of corresponding sides and ∠s are congruent: $\angle RTK \cong \angle UTK$, $\angle R \cong \angle U$ (Given); $\angle RKT \cong \angle UKT$ (If two ∠s of a △ are ≅ to two ∠s of another △, the third ∠s are ≅.);

$\overline{TR} \cong \overline{TU}, \overline{RK} \cong \overline{UK}$ (Given); $\overline{TK} \cong \overline{TK}$ (Reflexive Prop. of $\cong$); so $\triangle TRK \cong \triangle TUK$ by the def. of $\cong$ $\triangle$s. **25.** No; though $SP = TU = 7$, the other pairs of corresponding sides are not congruent. **26.** No; the corresponding sides are not necessarily congruent. **27.** $m\angle H = 60 = m\angle G, m\angle E = 120 = m\angle F$, $m\angle EJK = 90 = m\angle FJK, m\angle JKH = 90 = m\angle JKG$, $EJ = FJ, EH = 4 = FG, HK = GK$, and $JK = JK$, so they are congruent because all sides and all corresponding $\angle$s are congruent. **28a.** Given **b.** Alt. Int. Angles Thm. **c.** Given **d.** If two $\angle$s of one $\triangle$ are congruent to two $\angle$s of another $\triangle$, then the third $\angle$s are congruent. **e.** Reflexive Prop. of $\cong$ **f.** Given **g.** Def. of $\cong$ $\triangle$s **29.** Triangles are congruent if they are the same size and shape. The congruent pairs are B and G, C and E, and D and F. **30.** Solving for x: Since $m\angle A = 45, m\angle C = 90 - 45 = 45$. Also, $\angle C$ corresponds to $\angle M$, so they are $\cong$. Then $45 = 3x$, so $x = 15$. Solving for t: $\overline{AB}$ corresponds to $\overline{KL}$, so they are $\cong$. Then $4 = 2t$, so $t = 2$. **31.** $\angle DAC$ corresponds to $\angle BAC$, so they are $\cong$. Then $6x = 30$, so $x = 5$. **32.** $\angle A$ and $\angle D$ are corresponding $\angle$s, so their measures are $=$. $x + 10 = 2x$, so $x = 10$. Thus, $m\angle A = x + 10 = 10 + 10 = 20$ and $m\angle D = 2x = 2(10) = 20$. **33.** $\angle B$ and $\angle E$ are corresponding $\angle$s, so their measures are $=$. $3y = 21$, so $y = 7$. $m\angle B = 3y = 3(7) = 21 = m\angle E$. **34.** Since B corresponds to E, and C corresponds to F, $BC = EF$. $3z + 2 = z + 6; 2z + 2 = 6; 2z = 4; z = 2$. $BC = 3z + 2 = 3(2) + 2 = 8$. $EF = z + 6 = (2) + 6 = 8$
35. Since A corresponds to D, and C corresponds to F, $AC = DF$. $7a + 5 = 5a + 9; 2a + 5 = 9; 2a = 4; a = 2$. $AC = 7(2) + 5 = 19$. $DF = 5a + 9 = 5(2) + 9 = 19$.
36. Answers may vary. Sample: The hole and the piece must be the same size and shape, so $PACH \cong OLDE$ to completely fill the hole. **37.** Answers may vary. Sample: When the cards are stacked in a pile, it's easy to see which ones are the like sizes. She could arrange them in a neat pile and pull out the ones of like sizes. **38.** All pairs of corresponding sides and $\angle$s are congruent. Order the corresponding vertices in the same way for each $\triangle$. $\triangle JYB \cong \triangle XCH$ **39.** $CE = ED$ by def. of midpt. All pairs of corresponding sides and $\angle$s are congruent. Order the corresponding vertices in the same way for each $\triangle$. $\triangle BCE \cong \triangle ADE$ **40.** $\angle PTK \cong \angle RTK$ by def. of $\angle$ bisector. $\overline{TK} \cong \overline{TK}$ by the Refl. Prop. of $\cong$. All pairs of corresponding sides and $\angle$s are congruent. Order the corresponding vertices in the same way for each $\triangle$. $\triangle TPK \cong \triangle TRK$ **41.** Vertices L and R correspond. M can correspond to either N or Z. Also, J can correspond to either N or Z. $\triangle JLM \cong \triangle NRZ$; $\triangle JLM \cong \triangle ZRN$
42. Answers may vary. Sample: The die is a mold that is used to make items that are all the same size and shape.
43. Answers may vary. Sample: $\triangle TKR \cong \triangle MJL$: $\overline{TK} \cong \overline{MJ}; \overline{TR} \cong \overline{ML}; \overline{KR} \cong \overline{JL}; \angle TKR \cong \angle MJL; \angle TRK \cong \angle MLJ; \angle KTR \cong \angle JML$ **44a.** Given **b.** If $\parallel$ lines, then alt. int. $\angle$s are $\cong$. **c.** If $\parallel$ lines, then alt. int. $\angle$s are $\cong$. **d.** Vert. $\angle$s are $\cong$. **e.** Given **f.** Given **g.** Def. of segment bisector **h.** Def. of $\cong$ $\triangle$s **45.** Answers may vary. Sample: Since the sum of the $\angle$s of a $\triangle$ is 180, and if 2 $\angle$s of one $\triangle$ are the same as 2 $\angle$s of a second $\triangle$, then their sums must be $=$. Therefore, their sum subtracted from 180 has to be the same. **46.** $GH = \sqrt{(2-2)^2 + (-1-3)^2} = \sqrt{(0)^2 + (4)^2} = 4$; $GJ = \sqrt{(-2-1)^2 + (-1-3)^2} = \sqrt{(-3)^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$; $HJ = \sqrt{(-2-1)^2 + (3-3)^2} = \sqrt{(-3)^2 + (0)^2} = \sqrt{9} = 3$; $KL = GH = 4$, $LM = HJ = 3$, and $KM = GJ = 5$ **47.** $\angle H$ is a rt. $\angle$, so $\angle L$ is also a rt. $\angle$. Since $\overline{LM}$ is horizontal, $\overline{LK}$ must be vertical and, by Exercise 46, K must be 4 units from L, which is $(3, -3)$. So, K can be one of two places, either $(3, -3 + 4)$, which is $(3, -1)$, or $(3, -3 - 4)$, which is $(3, -7)$. **48a.** There are 15 different shapes and sizes (shown in 48b).

48b. 

49. $\overline{KY} \cong \overline{CD}$, so $4a = 6; a = 1.5$. **50.** $\overline{FP} \cong \overline{AB}$, so $4x - 7 = 10; 4x = 17; x = 4.25$. **51.** Since $ABCDE \cong PFKYM$, their corresponding sides are congruent, so they have the same perimeter. The perimeter of $PFKYM$ = the perimeter of $ABCDE = 10 + 11 + 5 + 6 + 8 = 40$.
52. $m\angle H = m\angle G = 66; m\angle L = m\angle S = 42$. $m\angle T = 180 - 42 - 66 = 72$

53. 

54. 

55. By the Triangle Angle-Sum Thm., $m\angle A = 180 - 33 - 47 = 100$. **56.** By the Symmetric Prop. of $=$, if $PQ = RS$, then $RS = PQ$. **57.** By the Reflexive Prop. of $\cong$, $\angle 1 \cong \angle 1$. **58.** By the Addition Prop. of $=$, if $m\angle A - 4 = 8$, then $m\angle A = 8 + 4 = 12$. **59.** By the Transitive Property of $\cong$, if $\overline{AB} \cong \overline{DE}$ and $\overline{DE} \cong \overline{GH}$, then $\overline{AB} \cong \overline{GH}$.

4-2 Triangle Congruence by SSS and SAS pages 186–192

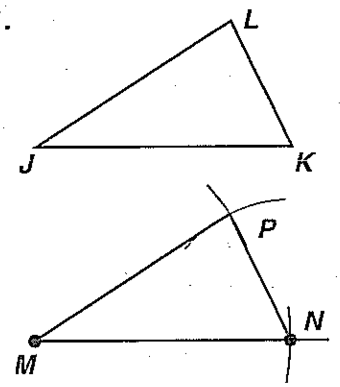
Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM*.
1. $\overline{AB} \cong \overline{DE}$; $\angle C \cong \angle F$ **2.** $\angle Q \cong \angle S$; $\angle QPR \cong \angle SRP$; $\overline{PR} \cong \overline{PR}$ **3.** $\angle M \cong \angle S$; $\angle MON \cong \angle SVT$; $\overline{TO} \cong \overline{NV}$; $\overline{MO} \parallel \overline{VS}$

Investigation **1.** All $\triangle$s created are the same size and shape, so the $\triangle$s are $\cong$. **2.** Check students' work. All $\triangle$s created are the same size and shape, so the $\triangle$s are $\cong$. The conjecture holds.

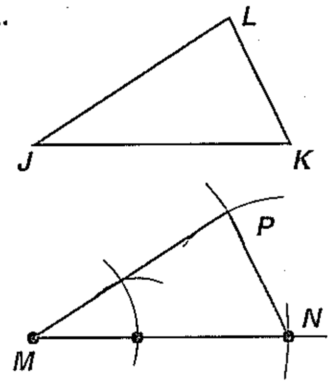
Check Understanding **1.** You are given that $\overline{AB} \cong \overline{CB}$ and $\overline{AD} \cong \overline{CD}$. $\overline{BD} \cong \overline{BD}$ by the Refl. Prop. of $\cong$, so $\triangle ABD \cong \triangle CBD$ by the SSS post. **2.** To use the SAS Post., you are given that $\overline{CD} \cong \overline{AB}$ and $\overline{AC} \cong \overline{CA}$ by the Refl. Prop. of $\cong$. You need $\angle DCA \cong \angle BAC$. To use the SSS Post., you need to know that $CB = 8$. **3.** To use SAS Post., you need to show that the corresponding included $\angle$s, $\angle E$ and $\angle DBC$, are $\cong$. Showing $\angle E \cong \angle DBC$ is not possible from the given information. To use the SSS Post., you need to show that the third pair of corresponding sides are $\cong$. Showing $\overline{AB} \cong \overline{DC}$ is not possible from the given information.

Exercises **1.** Two pairs of corresponding sides are $\cong$ and $\overline{ZD} \cong \overline{DZ}$ by the Refl. Prop. of $\cong$. Use SSS. **2.** Only two pairs of corresponding sides are $\cong$ and no information is given about corresponding $\angle$s. These $\triangle$s cannot be proved $\cong$. **3.** One pair of corresponding sides is $\cong$ and $\overline{AC} \cong \overline{CA}$ by the Refl. Prop. of $\cong$. Also, a pair of corresp. included $\angle$s are $\cong$. Use SAS. **4.** Since F is a midpt., $\overline{IF} \cong \overline{GF}$. The other two pairs of corresp. sides are $\cong$, so use SSS. **5.** Since P, O, and R are equally spaced, $\overline{PO} \cong \overline{RO}$. By the Refl. Prop. of $\cong$, $\overline{OB} \cong \overline{OB}$. Since all right $\angle$s are $\cong$, $\angle POB \cong \angle ROB$. So, two pairs of corresp. sides and their included $\angle$s are $\cong$ by SAS. **6.** $\overline{AC} \cong \overline{DB}$ (Given); $\overline{AE} \cong \overline{BE} \cong \overline{CE} \cong \overline{DE}$ (Def. of midpt.); $\angle AEB \cong \angle CED$ (Vert. $\angle$s are $\cong$.); $\triangle AEB \cong \triangle CED$ by SAS.
7a. $\overline{JM} \cong \overline{LK}$ is given. **7b.** $\overline{KM}$ is $\cong$ to itself by the Refl. Prop. of $\cong$. **7c and d.** $\triangle JKM \cong \triangle LMK$ by SSS.
8. Each of the two sides will have V as an endpt., so they are $\overline{WV}$ and $\overline{VU}$. **9.** The two segments share the endpt. W, so the included $\angle$ is $\angle W$. **10.** The vertex of each $\angle$ is at an endpt. of the segment, so the $\angle$s are $\angle U$ and $\angle V$. **11.** The segment will have endpts. at the $\angle$ vertices, W and U, so the segment is $\overline{WU}$. **12.** The segments share the endpt. X, so the included $\angle$ is $\angle X$. **13.** The sides that include Z will have Z as an endpt., so they are $\overline{XZ}$ and $\overline{YZ}$. **14.** One pair each of corresp. sides and $\angle$s are $\cong$. To prove by SAS, you need $\overline{LG} \cong \overline{MN}$. **15.** Two pairs of corresp. sides are $\cong$ and one pair of nonincluded corresp. $\angle$s is $\cong$. To prove by SAS, you need $\angle T \cong \angle V$. To prove by SSS, you need $\overline{RS} \cong \overline{WU}$. **16.** Since vert. $\angle$s are $\cong$, $\angle ACD \cong \angle ECB$, also one pair of corresp. sides is $\cong$. To prove by SAS, you need $\overline{DC} \cong \overline{BC}$. **17.** By the Refl. Prop. of $\cong$, $\overline{KG} \cong \overline{GK}$, and it's given that $\overline{HK} \cong \overline{GL}$. By addition, $\overline{HG} \cong \overline{LK}$. The other two pairs of corresp. sides are also $\cong$, so the $\triangle$s can be proved $\cong$ by SSS. Additional information is not needed. **18.** Yes; two pairs of corresp. sides and their included angle are $\cong$, so $\triangle ACB \cong \triangle EFD$ by SAS. **19.** Yes; three pairs of corresp. sides are $\cong$, so $\triangle PVQ \cong \triangle STR$ by SSS.
20. Though you can show that $\overline{VW} \cong \overline{VW}$ by the Refl. Prop. of $\cong$, the corresp. included $\angle$s cannot be shown $\cong$ for SAS, or a third pair of corresp. sides cannot be shown $\cong$. To prove the $\triangle$s $\cong$ you need to know that $\angle YVW \cong \angle ZVW$ or that $\overline{YW} \cong \overline{ZW}$. **21.** By the Refl. Prop. of $\cong$, $\overline{MO} \cong \overline{OM}$. So, two pairs of corresp. sides and their included $\angle$s are $\cong$. The $\triangle$s are $\cong$ by SAS. **22.** Two pairs of corresp. sides and their included $\angle$s are $\cong$, so $\triangle ANG \cong \triangle RWT$ by SAS. **23.** Three pairs of corresp. sides are $\cong$, so $\triangle KLJ \cong \triangle MON$ by SSS. **24.** Only two pairs of corresp. sides are $\cong$. To prove by SAS, you need $\angle H \cong \angle P$. To prove by SSS, you need $\overline{DY} \cong \overline{TK}$. It is not possible to prove the $\triangle$s $\cong$. **25.** Either $\overline{JE}$ corresp. to $\overline{SV}$, so $\triangle JEF \cong \triangle SVF$ by SSS, or $\overline{JE}$ corresp. to $\overline{SF}$, so $\triangle JEF \cong \triangle SFV$ by SSS. **26.** The figure shows that two pairs of corresp. sides are congruent according to their $=$ measures. Also, $\overline{BR} \cong \overline{BR}$ by the Refl. Prop. of $\cong$. Since three pairs of corresp. sides are $\cong$, $\triangle BRT \cong \triangle BRS$ by SSS. **27.** Two pairs of corresp. sides and their included $\angle$s are $\cong$, so $\triangle PQR \cong \triangle NMO$ by SAS. **28.** The $\triangle$s are not necessarily $\cong$ because no information is given about two or more sides. Even though the $\angle$s are $\cong$, the sides may not be. **29.** $\angle I$ and $\angle L$ are not included $\angle$s; you would need $\angle H \cong \angle K$ to show $\triangle$s $\cong$ by SAS, or you would need $\overline{GI} \cong \overline{JL}$ to show $\triangle$s $\cong$ by SSS. **30.** Two corresp. sides and their included $\angle$s are $\cong$, so the $\triangle$s are $\cong$ by SAS.

31.



32.



33a. $\angle 1 \cong \angle 2$ because vert. $\angle$s are $\cong$. **33b.** X is the midpt. of $\overline{AG}$ because it's given. **33c.** $\overline{AX} \cong \overline{GX}$ by def. of midpt. **33d.** X is the midpt. of $\overline{NR}$ because it's given. **33e.** $\overline{NX} \cong \overline{RX}$ by def. of midpt. **33f.** $\triangle ANX \cong \triangle GRX$ by SAS.

34. 

Answers may vary. Sample: One pair of $\triangle$s is $\cong$ by the Refl. Prop. of $\cong$, and two other pairs of $\angle$s are $\cong$ by the Corresp. Angles Post. **35a.** Answers may vary. Sample: wallpaper designs; ironwork on a bridge; highway warning signs **35b.** For wallpaper, $\cong$ $\triangle$s produce a well-balanced, symmetric appearance. For bridges in construction, $\cong$ $\triangle$s enhance designs and stabilize the bridge foundation. For highway warning signs, they are more easily identified if they are $\cong$. **36.** By def. of $\angle$ bis., $\angle ISP \cong \angle PSO$. By the Refl. Prop. of $\cong$, $\overline{SP} \cong \overline{SP}$. Since two pairs of corresp. sides and their included $\angle$s are $\cong$, $\triangle ISP \cong \triangle PSO$ by SAS. **37.** By def. of segment bis., $\overline{IP} \cong \overline{PO}$. By the Refl. Prop. of $\cong$, $\overline{SP} \cong \overline{SP}$. Since three pairs of corresp. sides are $\cong$, $\triangle ISP \cong \triangle OSP$ by SSS. **38.** If $\parallel$ lines, then alt. int. $\angle$s are $\cong$, so $\angle ADB \cong \angle DBC$. By the Refl. Prop. of $\cong$, $\overline{BD} \cong \overline{DB}$. Since two pairs of corresp. sides and their included $\angle$s are $\cong$, $\triangle ADB \cong \triangle CBD$ by SAS. **39.** If $\parallel$ lines, then alt. int. $\angle$s are $\cong$, so $\angle DAC \cong \angle ACB$. By the Refl. Prop. of $\cong$, $\overline{AC} \cong \overline{CA}$. Since two pairs of corresp. sides and their included $\angle$s are $\cong$, $\triangle ABC \cong \triangle CDA$ by SAS. **40.** No; $ABCD$ could be a square with side 5 and $EFGH$ could be a polygon with side 5 but no rt. $\angle$s. You would need some information about $\cong$ corresp. $\angle$s to prove the quadrilaterals $\cong$. **41.** Use the $\parallel$ lines to get information about $\angle$s. ① $\overline{FG} \parallel \overline{KL}$ (Given) ② $\angle GFK \cong \angle FKL$ (If $\parallel$ lines, then alt. int. $\angle$s are $\cong$.) ③ $\overline{FG} \cong \overline{KL}$ (Given) ④ $\overline{FK} \cong \overline{FK}$ (Refl. Prop. of $\cong$) ⑤ $\triangle ACB \cong \triangle KLF$ (SAS) **42.** Use def. of bis. to show $\cong$ sides, and use vert. $\angle$s to show $\angle$s $\cong$: $\overline{AE}$ and $\overline{BD}$ bis. each other, so $\overline{AC} \cong \overline{CE}$ and $\overline{BC} \cong \overline{CD}$. $\angle ACB \cong \angle DCE$ because vert. $\angle$s are $\cong$. $\triangle ACB \cong \triangle ECD$ by SAS.

43.

$\boxed{\overline{GK} \text{ bisects } \angle JGM.}$
Given

$\boxed{\angle JGK \cong \angle MGK}$
Def. of bis.

$\boxed{\overline{GJ} \cong \overline{GM}}$
Given

$\boxed{\overline{GK} \cong \overline{GK}}$
Ref. Prop. of $\cong$

$\boxed{\triangle GJK \cong \triangle GMK}$
SAS

Use def. of $\angle$ bis. to show two $\angle$s $\cong$, and use the Refl. Prop. of $\cong$ to show corresp. sides $\cong$. **44.** Use def. of midpt. to show corresp. sides $\cong$: $\overline{AM} \cong \overline{MB}$ because M is the midpt. of $\overline{AB}$. $\angle B \cong \angle AMC$ because all right $\angle$s are $\cong$. $\triangle AMC \cong \triangle MBD$ by SAS. **45.** For $\angle M$ to be an included $\angle$, M must be an endpt. of both segment sides. Similarly, G must be an endpt. of both segment sides. Thus, the needed info. is $\overline{MD} \cong \overline{SG}$. The answer is choice D. **46.** The corresp. included $\angle$s must be $\cong$, or $\angle D \cong \angle S$. The answer is choice G. **47.** One $\triangle$ has known sides of 9, 10, and 11 cm. The other $\triangle$ has known sides of 9 and 11 cm. The information for the unnamed side is GL = 10 cm. The answer is choice C. **48.** [2] **a.** $\overline{AB} \cong \overline{AB}$ by the Refl. Prop. of $\cong$. **b.** The $\triangle$s cannot be proved $\cong$ because $\overline{AB}$ in $\triangle ABW$ does not corresp. to $\overline{AB}$ in $\triangle AZB$. [1] one part correct **49.** Since A is first in $ABCD$, its corresp. part is first in $EFGH$, so $\angle A$ corresp. to $\angle E$. **50.** Since E and F are the first two letters of $EFGH$, they corresp. to the first two letters of $ABCD$, so $\overline{EF}$ corresp. to $\overline{AB}$. **51.** Since B and C are the second and third letters of $ABCD$, they corresp. to the second and third letters of $EFGH$, so $\overline{BC}$ corresp. to $\overline{FG}$. **52.** Since G is the third letter in $EFGH$, it corresp. to the third letter in $ABCD$, so $\angle G$ corresp. to $\angle C$. **53.** The second sentence is a converse of the first. Use the phrase "if and only if" between the two parts of one of the sentences: The product of the slopes of two lines is -1 if and only if the lines are $\perp$. **54.** Remove the "if and only if" phrase, and precede the first part of the sentence with "If" and the second part of the sentence with "then"; then write its converse: If $x = 2$, then $2x = 4$. If $2x = 4$, then $x = 2$. **55.** Statement: "If $x = 3$, then $2x = 6$" is true. Converse: "If $2x = 6$, then $x = 3$" is true. **56.** Statement: "If $x = 3$, then $x^2 = 9$" is true. Converse: "If $x^2 = 9$, then $x = 3$" is false, since 3 is not the only possible value for x. x can be -3.

READING MATH *page 193*

By def. of bis., $\overline{BC} \cong \overline{DC}$. By the Refl. Prop. of $\cong$, $\overline{AC} \cong \overline{AC}$. With the given information, $\triangle ABC \cong \triangle ADC$ by SSS.

4-3 Triangle Congruence by ASA and AAS *pages 194–201*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*
1. $\overline{JH}$ **2.** $\overline{HK}$ **3.** $\angle L$ **4.** $\angle N$ **5.** Refl. Prop. of $\cong$ **6.** If 2 $\angle$s of a $\triangle$ are $\cong$ to 2 $\angle$s of another $\triangle$, then the third pair of $\angle$s are $\cong$.

Investigation **1.** Check students' work. The $\triangle$s should be $\cong$. **2.** The $\triangle$s are the same size and shape, so they are $\cong$.

Check Understanding **1.** No; the $\cong$ side is not the shared side of the two $\angle$s, so it is not the included side. **2.** ① $\angle CAB \cong \angle DAE$; $\overline{AB} \cong \overline{AE}$ (Given) ② $\angle ABC$ and $\angle AED$ are rt. $\angle$s. (Given) ③ $\angle ABC \cong \angle AED$ (All rt. $\angle$s are $\cong$.) ④ $\triangle ABC \cong \triangle AED$ (ASA)

3.

$\overline{RP}$ bisects $\angle SRQ$.
Given

$\angle SRP \cong \angle QRP$
Def. of $\angle$ bis.

$\angle S \cong \angle Q$
Given

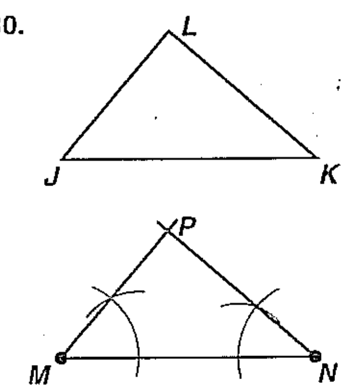
$\overline{RP} \cong \overline{RP}$
Ref. Prop. of $\cong$

$\triangle SRP \cong \triangle QRP$
AAS

4a. Both pairs of $\angle$s are alt. int. $\angle$s of $\parallel$ lines. They are $\cong$ because, if $\parallel$ lines, then alt. int. $\angle$s are $\cong$. **4b.** $\angle Q \cong \angle T$ since, if $\parallel$ lines, then alt. int. $\angle$s are $\cong$. Also, $\overline{QM} \cong \overline{TM}$ by def. of bis. $\angle XMQ \cong \angle RMT$ because vert. $\angle$s are $\cong$. So, $\triangle XMQ \cong \triangle RMT$ by ASA.

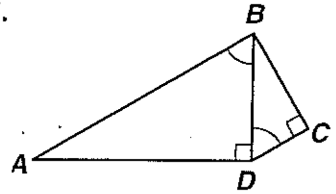
Exercises **1.** For ASA, there must be 2 corresp. $\angle$s and the included side, so $\triangle PQR \cong \triangle VXW$. **2.** For ASA, there must be 2 corresp. $\angle$s and the included side, so $\triangle ACB \cong \triangle EFD$. **3.** The endpts. must be the vertices of the $\angle$s, so the included side is $\overline{RS}$. **4.** The vertices of the $\angle$s are the endpts. of the included side, so the $\angle$s are $\angle N$ and $\angle O$. **5.** The corresp. included sides can be proved $\cong$ by the Refl. Prop. of $\cong$, so, yes, they can be proved $\cong$ by ASA. **6.** The figure gives no information of any corresp. $\cong$ sides, so it is not possible to prove the $\triangle$s $\cong$. **7.** One pair of corresp. $\angle$s can be proved $\cong$ because vert. $\angle$s are $\cong$. Another pair are $\cong$ because their measures are $=$. The included corresp. sides are $\cong$ because their measures are $=$, so, yes, the $\triangle$s are $\cong$ by ASA.
8a. $\overline{KM}$ is $\cong$ to itself by the Refl. Prop. of $\cong$. **8b.** Since two corresp. $\angle$s and their included corresp. sides are $\cong$, the $\triangle$s are $\cong$ by the ASA Post. **9.** One pair of corresp. $\angle$s is $\cong$ because rt. $\angle$s are $\cong$. Another pair of corresp. $\angle$s is $\cong$ because vert. $\angle$s are $\cong$. A pair of nonincluded sides is also $\cong$. So, the $\triangle$s are $\cong$ by AAS. **10.** Two pairs of corresp. $\angle$s are $\cong$ because if $\parallel$ lines, then alt. int. $\angle$s are $\cong$. The included corresp. sides are $\cong$ by the Refl. Prop. of $\cong$. So, the $\triangle$s are $\cong$ by ASA. **11.** One side of one $\triangle$ is $\cong$ to one side of another $\triangle$, and 2 pairs of $\angle$s are $\cong$. However, the side is the included side in one $\triangle$ but not in the other. So, it is not possible to prove the $\triangle$s $\cong$. **12.** To prove by AAS, only one pair of $\cong$ corresp. $\angle$s can have a vertex at an endpt. of the given side, so the vertices of the missing $\angle$s must be D and H; $\angle FDE \cong \angle GHI$. To prove by ASA, both pairs of $\cong$ corresp. $\angle$s have a vertex at an endpt. of the given side, so the vertices of the missing $\angle$s must be F and G; $\angle DFE \cong \angle HGI$.
13a. $\triangle UWT \cong \triangle UWV$ by AAS if $\angle T \cong \angle V$, $\angle UWT \cong \angle UWV$. **13b.** Then $\overline{UW} \cong \overline{UW}$. **13c.** $\angle UWT \cong \angle UWV$ because all right $\angle$s are congruent. **13d.** $\overline{UW} \cong \overline{UW}$ by the Refl. Prop. of $\cong$. **14.** By the Refl. Prop. of $\cong$, $\overline{AC} \cong \overline{CA}$. $\angle BAC \cong \angle DCA$ because if $\parallel$ lines, then alt. int. $\angle$s are $\cong$. For AAS, the corresp. vertices of the other pair of $\angle$s must not be an endpt. of the given corresp. sides. So, the missing information is $\angle B \cong \angle D$. **15.** By the Refl. Prop. of $\cong$, $\overline{OU} \cong \overline{OU}$. $\angle OUM \cong \angle OUN$ because all rt. $\angle$s are $\cong$. For SAS, the corresp. sides must share an endpt. with the vertex of the included $\angle$s. So, the missing information is $\overline{MU} \cong \overline{NU}$. **16.** One pair of $\cong$ $\angle$s is given. Another pair is $\cong$ because vert. $\angle$s are $\cong$. For ASA, the endpts. of the corresp. sides must be the vertices of both $\angle$s, P and Q in one $\triangle$ and S and Q in the other. So, the missing information is $\overline{PQ} \cong \overline{SQ}$. **17.** By the Refl. Prop. of $\cong$, $\overline{WZ} \cong \overline{WZ}$. For AAS, the corresp. vertices of the other pair of $\angle$s must be an endpt. of the given corresp. sides. So, the missing information is $\angle WZV \cong \angle WZY$.
18a. $\angle NQT \cong \angle SQR$ because vert. $\angle$s are $\cong$. **18b.** The line bis. $\overline{TR}$ at Q is given. **18c.** By def. of bis., $\overline{TQ} \cong \overline{QR}$.
18d. Since two pairs of corresp $\angle$s and the nonincluded sides are $\cong$, $\triangle NQT \cong \triangle SQR$ by AAS. **19.** $\overline{MO} \cong \overline{MO}$ by the Refl. Prop. of $\cong$. Two pair of corresp. $\angle$s and the included sides are $\cong$, so $\triangle PMO \cong \triangle NMO$ by ASA.
20. $\overline{TS} \cong \overline{TS}$ by the Refl. Prop. of $\cong$. $\angle TSU \cong \angle STR$ because if $\parallel$ lines, then alt. int. $\angle$s are $\cong$. So, 2 pair of corresp. $\angle$s and a nonincluded side are $\cong$. $\triangle UTS \cong \triangle RST$ by AAS. **21.** $\overline{VY} \cong \overline{VY}$ by the Refl. Prop. of $\cong$. So, 2 pair of corresp. $\angle$s and a nonincluded side are $\cong$. $\triangle ZVY \cong \triangle WVY$ by AAS. **22.** Two pair of corresp. $\angle$s and a nonincluded side are $\cong$. $\triangle UTX \cong \triangle EDO$ by AAS. **23.** The $\triangle$s are not $\cong$ because no pair of corresp. sides are $\cong$. **24.** Two pair of corresp. $\angle$s and an included side are $\cong$. $\triangle UTX \cong \triangle EOD$ by ASA. **25.** The $\triangle$s are not $\cong$ because the corresp. $\cong$ $\angle$s are not included $\angle$s.
26. Yes; if 2 $\angle$s of a $\triangle$ are $\cong$ to 2 $\angle$s of another $\triangle$, then the 3rd pair of $\angle$s are $\cong$. So, by including this step, an AAS proof can be rewritten as an ASA proof.
27a. $\triangle QPR \cong \triangle SRP$ by AAS if $\angle Q \cong \angle S$, $\angle QPR \cong \angle SRP$, and **27b.** $\overline{PR} \cong \overline{PR}$. $\angle QPR \cong \angle SRP$ because they are **27c.** alt. int. $\angle$s for the given parallel lines and the transversal **27d.** $\overline{PR}$. $\overline{PR} \cong \overline{PR}$ by the **27e.** Refl. Prop. of $\cong$. **28a.** It's given that $\overline{SQ}$ bisects $\angle PSR$.
28b. By def. of $\angle$ bis., $\angle PSQ \cong \angle RSQ$. **28c.** It's given that $\angle P \cong \angle R$. **28d.** By the Refl. Prop. of $\cong$, $\overline{SQ} \cong \overline{SQ}$.
28e. Since two pairs of corresp. $\angle$s and a nonincluded side are $\cong$, the $\triangle$s are $\cong$ by AAS.
29. ① $\overline{PQ} \perp \overline{QS}$; $\overline{RS} \perp \overline{QS}$ (Given) ② $\angle Q$ and $\angle S$ are rt. $\angle$s. (Def of $\perp$) ③ $\angle Q \cong \angle S$ (All rt. $\angle$s are $\cong$.) ④ $\angle QTP \cong \angle STR$ (Vert. $\angle$s are $\cong$.) ⑤ T is the midpt. of $\overline{PR}$. (Given) ⑥ $\overline{PT} \cong \overline{RT}$ (Def. of midpt.) ⑦ $\triangle PQT \cong \triangle RST$ (AAS)

30.



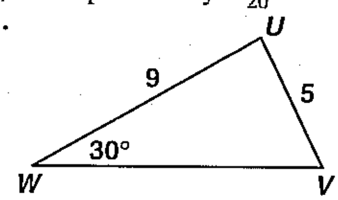
Construct $\overline{MN} \cong \overline{JK}$. Then construct $\angle M \cong \angle J$ and $\angle N \cong \angle K$. Extend the sides of the $\angle$s to P. **31.** $\angle MON \cong \angle QOP$ because vert. $\angle$s are $\cong$. So, 2 pairs of corresponding $\angle$s and a nonincluded side are $\cong$, so the $\triangle$s are $\cong$ by AAS. **32.** $\angle FGJ \cong \angle HJG$ because if $\parallel$ lines, then alt. int. $\angle$s are $\cong$. Also, by the Refl. Prop. of $\cong$, $\overline{GJ} \cong \overline{JG}$. So, 2 pairs of corresponding $\angle$s and a nonincluded side are $\cong$, so the $\triangle$s are $\cong$ by AAS. **33.** $\angle EAB \cong \angle DBC$ because if $\parallel$ lines, then corresp. $\angle$s are $\cong$. So, 2 pairs of corresponding $\angle$s and an included side are $\cong$, so the $\triangle$s are $\cong$ by ASA. **34.** $\angle BDH \cong \angle FDH$ by def. of $\angle$ bis. $\overline{DH} \cong \overline{DH}$ by the Refl. Prop. of $\cong$. So, 2 pairs of corresponding $\angle$s and an included side are $\cong$, so the $\triangle$s are $\cong$ by ASA.

35.



Answers may vary. Sample: Make one of the congruent sides included in one $\triangle$ and not an included side in the other. **36a.** The result should look like an isosc. $\triangle$ with a median from the vertex or a kite and a segment on its line of symmetry. Check students' work. **36b.** Answers may vary. The most likely answer is ASA. **37.** Since opp. sides are $\parallel$, the figure is a $\square$, so opp. sides and $\angle$s are $\cong$. $\triangle AEB \cong \triangle CED$, $\triangle BEC \cong \triangle DEA$, $\triangle ABC \cong \triangle CDA$, $\triangle BCD \cong \triangle DAB$ **38.** The list should include all pairs from Ex. 37, as well as all pairs of right $\triangle$s formed in the figure. $\triangle AEB \cong \triangle CED$, $\triangle BEC \cong \triangle DEA$, $\triangle ABC \cong \triangle CDA$, $\triangle BCD \cong \triangle DAB$, $\triangle ABD \cong \triangle DCA$, $\triangle ABD \cong \triangle BAC$, $\triangle ABD \cong \triangle CDB$, $\triangle DCA \cong \triangle BAC$, $\triangle DCA \cong \triangle CDB$, $\triangle BAC \cong \triangle CDB$ **39.** They are $\angle$ bisectors. By def. of $\cong$ $\triangle$s, all their corresp. parts are $\cong$, so $\angle JKL \cong \angle MNP$. Then $\angle JKQ \cong \angle MNR$ by division. So, $\triangle JKQ \cong \triangle MNR$ by ASA. **40.** Select three congruencies that satisfy SSS, SAS, ASA, or AAS. There is only 1 way to select SSS, 3 ways to select SAS, 3 ways to select ASA, and 6 ways to select AAS. $1 + 3 + 3 + 6 = 13$, so the probability is $\frac{13}{20}$.

41.



Suppose R is the center of a circle with radius $\overline{RS}$. The circle intersects $\overrightarrow{TS}$ in two places, creating a new $\triangle$ having two sides and a nonincluded $\angle \cong$ to two sides and a nonincluded $\angle$ of the original $\triangle$, but the $\triangle$s are not $\cong$. **42.** If only one pair of $\cong$ $\angle$s are given, then the $\angle$s must be included $\angle$s, so SSA is not a method used to prove $\triangle$s $\cong$. The answer is choice D. **43.** A pair of $\cong$ $\angle$s is missing. The vertices of the $\angle$s must be the endpoints of the given segments, so the missing information is $\angle T \cong \angle D$. The answer is choice F. **44.** [2] **a.** $\angle RPQ \cong \angle SPQ$ and $\angle RQP \cong \angle SQP$ by def. of $\angle$ bis. **b.** $\overline{PQ} \cong \overline{PQ}$ by the Refl. Prop. of $\cong$. So, two corresp. pairs of $\angle$s and a corresp. included side are $\cong$. The $\triangle$s are $\cong$ by ASA. [1] one part correct **45.** [4] **a.** The segments are $\cong$ because the midpt. of a segment is a point that divides it into two $\cong$ segments. **b.** Yes, they can be proved $\cong$ by ASA. $\angle JLM \cong \angle KGM$ because they are alt. int. $\angle$s of $\parallel$ lines, and $\angle LMJ \cong \angle GMK$ because vert. $\angle$s are $\cong$. Since two pairs of corresp. $\angle$s and their included sides are $\cong$, the $\triangle$s are $\cong$ by ASA. **c.** Yes, they can be proved $\cong$ by AAS. $\angle LJM \cong \angle GKM$ because they are alt. int. $\angle$s of $\parallel$ lines, and $\angle LMJ \cong \angle GMK$ because vert. $\angle$s are $\cong$. Since two pairs of corresp. $\angle$s and their included sides are $\cong$, the $\triangle$s are $\cong$ by AAS. Or, they can be proved by AAS using the same congruencies as in part ⓑ and then using the theorem that says "If 2 $\angle$s of one $\triangle$ are $\cong$ to 2 $\angle$s of another, the third $\angle$s are $\cong$." [3] incorrect $\angle$s for part ⓑ or ⓒ, but otherwise correct [2] correct conclusions but incomplete explanations for parts ⓑ and ⓒ [1] at least one part correct **46.** By the Refl. Prop. of $\cong$, $\overline{LN} \cong \overline{NL}$, so two pairs of corresp. sides and their included $\angle$s are $\cong$. $\triangle ONL \cong \triangle MLN$ by SAS. **47.** Since vert. $\angle$s are $\cong$, $\angle PQT \cong \angle SQR$, but they are not included $\angle$s, so it is not possible to prove the $\triangle$s $\cong$. **48.** The only included side is the one whose endpts. are the $\angle$ vertices, $\overline{AB}$. The sides not included, then, are $\overline{AC}$ and $\overline{BC}$. **49.** The $\angle$s are corresp. $\angle$s, so the theorem is: If corresp. $\angle$s are $\cong$, then the lines are $\parallel$. **50.** The 2 ft-by-3 ft poster is 24 in. by 36 in. The greatest number of photos can be placed on the poster by setting the short side of the photo parallel to the short side of the poster. $24 \div 3 = 8$, so 8 photos go across the short side, and $36 \div 5 = 7.2$, so there will be 7 full rows of photos. This array is 7 by 8, which is 56 photos. **51.** The 2 ft-by-3 ft poster is 24 in. by 36 in. The result is the same no matter which way the photos align. $24 \div 4 = 6$, so 6 photos go across the short side, and $36 \div 6 = 6$, so 6 rows of photos go across the long side. This array is 6 by 6, which is 36 photos. **52.** The paper used by the small photo is (3)(5), or 15 in.^{2}. The paper used by the large photo is (4)(6), or 24 in.^{2}. $24 \div 15 = 1.6$, so the large photo uses 1.6 times, or 160% of the paper for a small photo. $160\% - 100\% = 60\%$, so the large photo uses 60% more paper than the small photo.

CHECKPOINT QUIZ 1 — page 201

1. There will be three pairs each of sides and $\angle$s. $\overline{RS} \cong \overline{JK}$, $\overline{ST} \cong \overline{KL}$, $\overline{RT} \cong \overline{JL}$, $\angle R \cong \angle J$, $\angle S \cong \angle K$, $\angle T \cong \angle L$ **2.** Two pairs of corresp. $\angle$s and their included side are $\cong$, so the $\triangle$s are $\cong$ by ASA. **3.** Three pairs of corresp. sides are $\cong$, so the $\triangle$s are $\cong$ by SSS. **4.** Two pairs of corresp. sides and their included $\angle$ are $\cong$, so the $\triangle$s are $\cong$ by SAS. **5.** There are no pairs of corresp. sides $\cong$, so it is not possible to prove the $\triangle$s $\cong$. **6.** Two pairs of corresp. $\angle$s and their nonincluded side are $\cong$, so the $\triangle$s are $\cong$ by AAS. **7.** Two pairs of corresp. sides are $\cong$, but the corresp. $\angle$s are not included, so it is not possible to prove the $\triangle$s $\cong$. **8.** The $\angle$s are alt. int. $\angle$s. If $\parallel$ lines, then alt. int. $\angle$s are $\cong$. **9.** The $\angle$s are vert. $\angle$s. Vert. $\angle$s are $\cong$. **10.** Since all pairs of corresp. $\angle$s and one included side are $\cong$, the $\triangle$s are $\cong$ by ASA or AAS.

TECHNOLOGY — page 202

1. No, there is not an AAA $\cong$ theorem. There are many noncongruent $\triangle$s with all 3 pairs of $\angle$s $\cong$. At least one pair of corresp. sides must be $\cong$ to establish the size of the $\triangle$s. **2.** No, there is no SSA $\cong$ theorem. Explanations may vary. Sample: There may be two different $\triangle$s whose corresp. parts satisfy SSA. **3.** No, the circle intersects $\overrightarrow{AB}$ just once, so only one $\triangle$ is formed. If the $\cong$ $\angle$s are obtuse, then there could be an SSA congruency since a $\triangle$ can have only one obtuse $\angle$. **4.** Consider the cases when $\angle A$ is acute, right, or obtuse. If $\angle A$ is obtuse or right, there will be exactly one $\triangle$, provided that CE and AC are large enough to form $\triangle ACE$. If it is acute, there may be more than one $\triangle$ with the given conditions.

4-4 Using Congruent Triangles: CPCTC pages 203–208

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*
1. $\angle J \cong \angle H$; $\angle R \cong \angle V$; $\angle C \cong \angle G$ **2.** $\overline{JR} \cong \overline{HV}$; $\overline{RC} \cong \overline{VG}$; $\overline{JC} \cong \overline{HG}$ **3.** $\angle T \cong \angle L$; $\angle I \cong \angle O$; $\angle C \cong \angle K$ **4.** $\overline{TI} \cong \overline{LO}$; $\overline{IC} \cong \overline{OK}$; $\overline{TC} \cong \overline{LK}$

Check Understanding 1a. By CPCTC, $\angle CLS \cong \angle CRS$. Then $\angle 5 \cong \angle 6$, because suppl. of $\cong$ $\angle$s are $\cong$. **1b.** As point S is moved toward point C, the given relationships between the two stretchers and between the two $\angle$s remains the same. The $\triangle$s change shape, but they are always $\cong$ to each other by SAS. **2.** $20(2.5) = 50$; 50 ft

Exercises 1. By CPCTC, all other unmarked corresp. parts are $\cong$. $\angle PSQ \cong \angle SPR$; $\overline{SQ} \cong \overline{RP}$; $\overline{PQ} \cong \overline{SR}$ **2.** A second pair of vert. $\angle$s is $\cong$, so, since two pairs of corresp. $\angle$s and included side are $\cong$, $\triangle ABC \cong \triangle EBD$ by AAS. Other $\cong$ parts by CPCTC are $\angle A \cong \angle E$, $\overline{CB} \cong \overline{DB}$, and $\overline{DE} \cong \overline{CA}$. **3.** Since two pairs of corresp. sides and included $\angle$ are $\cong$, $\triangle KLJ \cong \triangle OMN$ by SAS. Other $\cong$ parts by CPCTC are $\angle K \cong \angle O$, $\angle J \cong \angle N$, and $\overline{KJ} \cong \overline{ON}$. **4.** Since three pairs of corresp. sides are $\cong$, $\triangle HUG \cong \triangle BUG$ by SSS. Other $\cong$ parts by CPCTC are $\angle H \cong \angle B$, $\angle HUG \cong \angle BUG$, and $\angle UGH \cong \angle UGB$. **5.** The $\triangle$s are $\cong$ by AAS, so $\angle T \cong \angle S$ by CPCTC. Their exterior $\angle$s are $\cong$ because supplements of $\cong$ $\angle$s are $\cong$. **6a.** It is given that the three sides of the $\triangle$s are congruent, so $\triangle ARC \cong \triangle EHV$ by SSS. **6b.** Thus, $\angle ARC \cong \angle EHV$ by CPCTC. **7.** $\triangle ABD \cong \triangle CBD$ by ASA because $\overline{BD} \cong \overline{BD}$ by the Refl. Prop. of $\cong$. $\overline{AB} \cong \overline{CB}$ by CPCTC. **8.** $\triangle MOE \cong \triangle REO$ by SSS because $\overline{OE} \cong \overline{OE}$ by the Refl. Prop. of $\cong$. $\angle M \cong \angle R$ by CPCTC. **9.** $\triangle SPT \cong \triangle OPT$ by SAS because $\overline{TP} \cong \overline{TP}$ by the Refl. Prop. of $\cong$. $\angle S \cong \angle O$ by CPCTC. **10.** $\triangle PNK \cong \triangle MNL$ by SAS because $\angle KNP \cong \angle LNM$ by vert. $\angle$s are $\cong$. $\overline{KP} \cong \overline{LM}$ by CPCTC. **11.** $\triangle CYT \cong \triangle RYP$ by AAS. $\overline{CT} \cong \overline{RP}$ by CPCTC. **12.** $\triangle ATM \cong \triangle RMT$ by SAS because $\angle ATM \cong \angle RMT$, because if $\parallel$ lines, then alt. int. $\angle$s are $\cong$. $\angle AMT \cong \angle RTM$ by CPCTC. **13.** Yes, because $BD = BD$ by the Refl. Prop. of $=$. Now all corresp. sides are $\cong$ by def. of $\cong$, and $\triangle ABD \cong \triangle CBD$ by SSS. So, $\angle A \cong \angle C$ by CPCTC. **14a.** $\angle QPS \cong \angle RSP$ because it's given. **14b.** $\angle Q \cong \angle R$ because it's given. **14c.** $\overline{PS} \cong \overline{PS}$ by the Refl. Prop. of $\cong$. **14d.** $\triangle PQS \cong \triangle SRP$ by AAS. **15.** $\overline{KL} \cong \overline{KL}$ by the Refl. Prop. of $\cong$. Since two pairs of corresp. sides and their included $\angle$ are $\cong$, the $\triangle$s are $\cong$ by SAS. $\angle P \cong \angle Q$ by CPCTC. **16.** $\overline{KL} \cong \overline{KL}$ by the Refl. Prop. of $\cong$. By def. of $\perp$ bis., $\angle KLP$ and $\angle KLQ$ are right $\angle$s and $\overline{PL} \cong \overline{QL}$. $\angle KLP \cong \angle KLQ$ because all right $\angle$s are $\cong$. Since two pairs of corresp. sides and their included $\angle$ are $\cong$, the $\triangle$s are $\cong$ by SAS. $\angle P \cong \angle Q$ by CPCTC. **17.** $\overline{KL} \cong \overline{KL}$ by the Refl. Prop. of $\cong$. By def. of $\perp$, $\angle KLP$ and $\angle KLQ$ are right $\angle$s. $\angle KLP \cong \angle KLQ$ because all right $\angle$s are $\cong$. By def of $\angle$ bis., $\angle PKL \cong \angle QKL$. Since two pairs of corresp. $\angle$s and their included side are $\cong$, the $\triangle$s are $\cong$ by ASA. $\angle P \cong \angle Q$ by CPCTC. **18.** One pair of vert. $\angle$s is $\cong$, so the $\triangle$s are $\cong$ by SAS. By CPCTC, the distance across the sinkhole is 26.5 yd. **19a.** Given **19b.** Def. of $\perp$: $\perp$ lines form rt. $\angle$s. **19c.** Right $\angle$s are $\cong$. **19d.** Given **19e.** Def. of segment bis.: a point, segment, ray, or line that divides a segment into two congruent segments **19f.** Refl. Prop. of $\cong$ **19g.** Since two pairs of corresp. sides and their included $\angle$s are $\cong$, the $\triangle$s are $\cong$ by SAS. **19h.** CPCTC: Corresp. Parts of $\cong$ $\triangle$s are $\cong$. **20.** $\overline{AX} \cong \overline{AX}$ by the Refl. Prop. of $\cong$. Since three pairs of corresp. sides are $\cong$, $\triangle ABX \cong \triangle ACX$ by SSS. So, $\angle BAX \cong \angle CAX$ by CPCTC. Thus, $\overrightarrow{AX}$ bisects $\angle BAC$ by the Def. of $\angle$ bis. **21.** Since $\overline{AF} \cong \overline{EC}$ (Given) and $\overline{EF} \cong \overline{FE}$ (Refl. Prop. of $\cong$), $\overline{AE} \cong \overline{FC}$ by subtraction. So, $\triangle ABE \cong \triangle CDF$ by SAS. **22.** $\angle P \cong \angle J$ and $\angle K \cong \angle Q$ because if $\parallel$ lines, then alt. int. $\angle$s are $\cong$. So, $\triangle KJM \cong \triangle QPM$ by ASA. **23.** ① $\angle A \cong \angle C$ (Given; b or e) ② $\overline{BD}$ bis. $\angle ABC$ (Given; b or e) ③ $\angle 1 \cong \angle 2$ (Def. of $\angle$ bis.; d) ④ $\overline{BD} \cong \overline{BD}$ (Refl. Prop. of $\cong$; c) ⑤ $\triangle ABD \cong \triangle CBD$ (AAS Thm.; f) ⑥ $\overline{AB} \cong \overline{CB}$ (CPCTC; a) **24.** $\overline{BA} \cong \overline{BC}$ is given. $\overline{BD} \cong \overline{BD}$ by the Refl. Prop. of $\cong$ and since $\overline{BD}$ bisects $\angle ABC$, $\angle ABD \cong \angle CBD$ by def. of an $\angle$ bisector. Thus, $\triangle ABD \cong \triangle CBD$ by SAS. $\overline{AD} \cong \overline{DC}$ by CPCTC, so $\overline{BD}$ bisects $\overline{AC}$ by Def. of a bis. $\angle ADB \cong \angle CDB$ by CPCTC and $\angle ADB$ and $\angle CDB$ are suppl. Thus, $\angle ADB$ and $\angle CDB$ are right $\angle$s and $\overline{BD} \perp \overline{AC}$ by def. of $\perp$. **25a.** $\overline{AP} \cong \overline{BP}$ since they are radii of the same circle whose center is P. $\overline{AC} \cong \overline{BC}$ since they are radii of $\cong$ circles. **25b.** The diagram is constructed in such a way that the $\triangle$s are $\cong$ by SSS. $\angle CPA \cong \angle CPB$ by CPCTC. Since these $\angle$s are $\cong$ and supplementary, they are right $\angle$s. Thus $\overleftrightarrow{CP}$ is $\perp$ ℓ. **26.** Plan: Draw a diagonal and use the $\parallel$ lines to conclude that alt. int. $\angle$s are $\cong$. Show the $\triangle$s are $\cong$ by ASA. Proof 1: ① $\overline{PR} \parallel \overline{MG}$; $\overline{MP} \parallel \overline{GR}$ (Given) ② Draw $\overline{PG}$. (2 pts. determine a line.) ③ $\angle RPG \cong \angle PGM$ and $\angle RGP \cong \angle GPM$ (If $\parallel$ lines, then alt. int. $\angle$s are $\cong$.) ④ $\triangle PGM \cong \triangle GPR$ (ASA)
Proof 2: ① $\overline{PR} \parallel \overline{MG}$; $\overline{MP} \parallel \overline{GR}$ (Given) ② Draw $\overline{MR}$. (2 pts. determine a line.) ③ $\angle PRM \cong \angle GMR$ and $\angle GRM \cong \angle PMR$ (If $\parallel$ lines, then alt. int. $\angle$s are $\cong$.) ④ $\triangle PMR \cong \triangle GRM$ (ASA) **27.** From Exercise 26, $\triangle PGM \cong \triangle GPR$ (or $\triangle PMR \cong \triangle GRM$), so $\overline{PR} \cong \overline{MG}$ and $\overline{MP} \cong \overline{GR}$ by CPCTC. **28.** The order of the letters in the $\triangle$ congruency statement relates the corresp. parts. $\overline{WX}$ corresp. to $\overline{TX}$, but not necessarily to $\overline{JX}$. The answer is choice C. **29.** According to the given, $\triangle ABC \cong \triangle ADC$, $BC = DC$. The answer is choice C.

30. According to the given, $\triangle ABC \cong \triangle ADC$, there is no known relationship between $m\angle ABC$ and $m\angle DAB$. The answer is choice D. **31.** The figure shows that E is between A and C, so $AE < AC$. The answer is choice B. **32.** Since $\triangle ABC \cong \triangle ADC$, $\overline{BC} \cong \overline{DC}$ and $\angle BCE \cong \angle DCE$ by CPCTC. $\overline{CE} \cong \overline{CE}$ by the Refl. Prop. of $\cong$. $\triangle BCE \cong \triangle DCE$ by SAS. $BE = DE$ by CPCTC. The answer is choice C. **33.** [2] **a.** You need to show that $\triangle KBV \cong \triangle KBT$. Since $\overline{KB}$ is an $\angle$ bis., $\angle VKB \cong \angle TKB$. By the Refl. Prop. of $\cong$, $\overline{KB} \cong \overline{KB}$. The $\triangle$s are $\cong$ by SAS. **b.** Since $\triangle KBV \cong \triangle KBT$, $\overline{VB} \cong \overline{TB}$ by CPCTC. **34.** Since two corresp. $\angle$s and their included sides are $\cong$, the $\triangle$s are $\cong$ by ASA. **35.** The $\triangle$s share a side that can be proved $\cong$ to itself by the Refl. Prop. of $\cong$. Since two corresp. $\angle$s and their nonincluded sides are $\cong$, the $\triangle$s are $\cong$ by AAS. **36.** Let x be the measure of the $\angle$ and $180 - x$ be the measure of its supplement. $x = 10 + (180 - x) \rightarrow x = 190 - x \rightarrow 2x = 190 \rightarrow x = 95$. So, the $\angle$ is 95, and its suppl. is $180 - 95$, or 85. **37.** The Law of Detachment says that if $p \rightarrow q$ is a true statement and p is true, then q is also true. Assume the first statement is true; p = Line m is nonvertical and parallel to line n; q = their slopes are equal. Conclusion: The slope of line m is the same as the slope of line n. **38.** The Law of Detachment says that if $p \rightarrow q$ is a true statement and p is true, then q is also true. Assume the first statement is true. The second statement does not match the first part of the first statement because a 5-sided figure is not a quadrilateral. A conclusion is not possible. **39.** The Law of Detachment says that if $p \rightarrow q$ is a true statement and p is true, then q is also true. Assume the first statement is true. The second statement matches the q part of the first statement, so a conclusion is not possible.

ALGEBRA 1 REVIEW *page 209*

1. Substitute $3x + 2$ for y in $y = x - 4$: $3x + 2 = x - 4 \rightarrow 2x + 2 = -4 \rightarrow 2x = -6 \rightarrow x = -3$. Solve for y: $y = x - 4 = -3 - 4 = -7$; $(x, y) = (-3, -7)$. **2.** Solve for x in the second equation: $x + 2y = 9 \rightarrow x = -2y + 9$. Substitute $-2y + 9$ for x in $2x - y = 8$: $2(-2y + 9) - y = 8 \rightarrow -4y + 18 - y = 8 \rightarrow -5y + 18 = 8 \rightarrow -5y = -10 \rightarrow y = 2$. Solve for x in the second equation: $x + 2(2) = 9 \rightarrow x + 4 = 9 \rightarrow x = 5$; $(x, y) = (5, 2)$. **3.** Solve for y in the first equation: $3x + y = 4 \rightarrow y = -3x + 4$. Substitute $-3x + 4$ for y in the second equation: $-6x - 2y = 12 \rightarrow -6x - 2(-3x + 4) = 12 \rightarrow -6x + 6x - 8 = 12 \rightarrow -8 = 12$. Since this statement is false, there is no solution. **4.** Solve for y in the first equation: $2x = y + 3 \rightarrow 2x - 3 = y$. Substitute $2x - 3$ for y in the second equation: $2x + y = -3 \rightarrow 2x + (2x - 3) = -3 \rightarrow 4x - 3 = -3 \rightarrow 4x = 0 \rightarrow x = 0$. To solve for y, substitute 0 for x in the second equation: $2(0) + y = -3 \rightarrow y = -3$; $(x, y) = (0, -3)$. **5.** Substitute $x + 1$ for y in the second equation: $x = y - 1 \rightarrow x = (x + 1) - 1 \rightarrow x = x$. This result is true for all (x, y) where $y = x + 1$. So, there are infinitely many solutions. **6.** Solve for x in the first equation: $x - y = 4 \rightarrow x = y + 4$. Substitute $y + 4$ for x in the second equation: $3x - 3y = 6 \rightarrow 3(y + 4) - 3y = 6 \rightarrow 3y + 4 - 3y = 6 \rightarrow 4 = 6$. Since this statement is false, there is no solution. **7.** Substitute $-x + 2$ for y in $2y = 4 - 2x$: $2(-x + 2) = 4 - 2x \rightarrow -2x + 4 = 4 - 2x$. This result is true for all (x, y) where $y = -x + 2$. So, there are infinitely many solutions. **8.** Substitute $2x + 1$ for y in $y = 3x - 7$: $2x + 1 = 3x - 7 \rightarrow -x + 1 = -7 \rightarrow -x = -8 \rightarrow x = 8$. Substitute 8 for x in $y = 2x + 1$: $y = 2(8) + 1 = 16 + 1 = 17$; $(x, y) = (8, 17)$. **9.** Solve $x + y = 1$ for y: $y = -x + 1$. Substitute $-x + 1$ for y in $x - y = 2$: $x - (-x + 1) = 2 \rightarrow x + x - 1 = 2 \rightarrow 2x - 1 = 2 \rightarrow 2x = 3 \rightarrow x = \frac{3}{2}$. Substitute $\frac{3}{2}$ for x in $x + y = 1$: $x + \left(\frac{3}{2}\right) = 1 \rightarrow x = 1 - \frac{3}{2} = \frac{2}{2} - \frac{3}{2} = \frac{2-3}{2} = -\frac{1}{2}$; $(x, y) = \left(\frac{3}{2}, -\frac{1}{2}\right)$.

4-5 Isosceles and Equilateral Triangles *pages 210–216*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*
1. $\angle C$ **2.** $\angle A$ **3.** $\overline{BC}$ **4.** $\overline{BA}$ **5.** 105

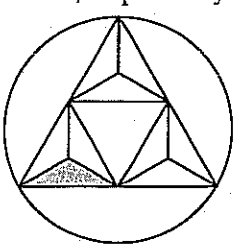
Investigation 1. $\angle A \cong \angle B$; conjecture: Isosc. $\triangle$s have $\cong$ base $\angle$s. **2a.** The $\angle$ bisector of the vertex $\angle$ of an isosc. $\triangle$ creates right $\angle$s with the base. **2b.** The $\angle$ bis. of the vertex $\angle$ divides the base of the isosc. $\triangle$ into 2 $\cong$ segments: $\overline{AD} \cong \overline{BD}$. **2c.** The $\angle$ bis. of the vertex $\angle$, $\overline{CD}$, is the $\perp$ bisector of the base, $\overline{AB}$.

Check Understanding 1. The Converse of the Isosc. $\triangle$ Thm. states that if 2 $\angle$s of a $\triangle$ are $\cong$, then the sides opp. the $\angle$s are $\cong$. Given: In $\triangle XYZ$, $\angle Z \cong \angle Y$. Prove: $\triangle XYZ$ is isosc. Draw $\overline{XB}$, the bis. of $\angle YXZ$. Then $\angle ZXB \cong \angle YXB$ by def. of $\angle$ bis. Since $\angle Z \cong \angle Y$, and $\overline{XB} \cong \overline{XB}$ by the Refl. Prop. of $\cong$, $\triangle ZXB \cong \triangle YXB$ by AAS. Thus, $\overline{XZ} \cong \overline{XY}$ by CPCTC. So, $\triangle XYZ$ is isosc. by def. of isosc. **2.** No measures indicate any one particular position for point U on $\overline{RT}$, so $\angle RVU$ and $\angle RUV$ can't be shown $\cong$ to $\angle R$. **3.** By Theorem 4-5, you know that $\overline{MO} \perp \overline{LN}$, so $x = 90$. By the Isosc. $\triangle$ Thm., $m\angle N = m\angle L = 43$. Substitute 90 for x and 43 for $m\angle N$: $m\angle N + x + y = 180 \rightarrow 43 + 90 + y = 180 \rightarrow 133 + y = 180 \rightarrow y = 47$. **4.** The outside corner of the path consists of one $\angle$ of an equilateral $\triangle$, 60, and one $\angle$ of a rectangle, 90. $60 + 90 = 150$

Exercises 1a. $\overline{RS}$ **1b.** $\overline{RS}$ **1c.** Given **1d.** Def. of $\angle$ bis. **1e.** Refl. Prop. of $\cong$ **1f.** Since 2 corresp. $\angle$s and a nonincluded side are $\cong$, AAS. **2a.** $\overline{KM}$ **2b.** $\overline{KM}$ **2c.** By construction **2d.** Def. of segment bis. **2e.** Refl. Prop. of $\cong$ **2f.** Since 3 corresp. sides are $\cong$, SSS. **2g.** CPCTC **3.** $\overline{VX}$ by the Converse of the Isosc. $\triangle$ Thm. **4.** $\overline{UW}$ by the Converse of the Isosc. $\triangle$ Thm. **5.** Since 3 corresp. sides are $\cong$, SSS. **6.** CPCTC **7.** By the Isosc. $\triangle$ Thm., both base $\angle$s of the $\triangle$ on the left are 50: $50 + 50 + x = 180 \rightarrow x = 80$. By the Isosc. $\triangle$ Thm., both base $\angle$s of the $\triangle$ on the right are y; $2y + 100 = 180 \rightarrow 2y = 80 \rightarrow y = 40$. **8.** By the Triangle Exterior $\angle$ Thm., $x + y = 110$. By the Isosc. $\triangle$ Thm., the other base $\angle$ is y. Substitute 110 for $x + y$ in $x + y + y =$

$180 \to 110 + y = 180 \to y = 70$. Substitute 70 for y in $x + y = 110 \to x + 70 = 110 \to x = 40$. **9.** The $\angle$ bis. is $\perp$ to the base of the isosc. $\triangle$, so $90 + 52 + x = 180 \to 142 + x = 180 \to x = 38$. The $\angle$ bis. divides the base of an isosc. $\triangle$ into 2 $\cong$ segments, so $y = 4$. **10.** $60 + 60 + y = 180 \to 120 + y = 180 \to y = 60$. $4x + 4x + 4x = 54 \to 12x = 54 \to x = 4\frac{1}{2}$ **11.** By the Polygon Exterior Angle-Sum Thm. and by division, each ext. $\angle$ measures $\frac{360}{5}$, or 72. So, $m\angle E = 180 - 72 = 108$. Since $ABCDE$ is regular, $ED = AD$, so $\triangle EAD$ is an isosc. $\triangle$ and $m\angle EDA = y$; $2y + 108 = 180 \to 2y = 72 \to y = 36$. By symmetry, $\angle CDB = 36$. Since $ABCDE$ is regular, $\angle E \cong \angle EDC$, so $36 + x + 36 = 108 \to x + 72 = 108 \to x = 36$. **12.** $x + 44 + 44 = 180 \to x + 88 = 180 \to x = 92$. By the Converse of the Isosc. $\triangle$ Thm., $y = 7$. **13.** By the Isosc. $\triangle$ Thm., $m\angle KJL = 58$. $58 + 58 + m\angle KJL = 180 \to 116 + m\angle LKJ = 180 \to m\angle LKJ = 64$ **14.** The $\angle$ bis. of the vertex $\angle$ of an isosc. $\triangle$ bis. the base side, so $ML = \frac{1}{2}(5) = 2\frac{1}{2}$. **15.** By the Isosc. $\triangle$ Thm., $m\angle J = m\angle L$, so $2m\angle J + 48 = 180 \to 2m\angle J = 132 \to m\angle J = 66$. **16.** The $\angle$ bis. of the vertex of an isosc. $\triangle$ is $\perp$ to its base, so $m\angle KMJ = 90$. $90 + 55 + m\angle JKM = 180 \to 145 + m\angle JKM = 180 \to m\angle JKM = 35$ **17.** Each $\angle$ in the square is 90 and each $\angle$ in the hexagon is $\frac{180(n-2)}{n} = \frac{180(6-2)}{6} = \frac{180(4)}{6} = 120$; $m\angle SHA + 90 + 120 = 360 \to m\angle SHA + 210 = 360 \to m\angle SHA = 150$. The measure of each side of the square = the measure of each side of the hexagon, so $\triangle SHA$ is an isosc. $\triangle$. By the Isosc. $\triangle$ Thm., $\angle HAS \cong \angle HSA$. So, $2m\angle HAS + 150 = 180 \to 2m\angle HAS = 30 \to m\angle HAS = 15$. **18.** They encircle a point and have a sum of 360°. $x + 2x + 3x + 4x + 5x = 360 \to 15x = 360 \to x = 24$. So, the $\angle$ measures of x, $2x$, $3x$, $4x$, and $5x$ are 24, 48, 72, 96, and 120, respectively.

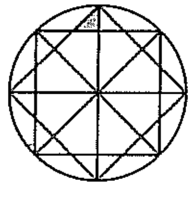
19a.



Answers may vary. Sample: There are 3 $\cong$ obtuse $\angle$s encircling a point, so their sum is 360 and each measures $\frac{360}{3}$, or 120. By the Isosc. $\triangle$ Thm., the two base $\angle$s are $\cong$, so $2x + 120 = 180$ and $x = 30$. So, the $\angle$ measures are 30, 30, and 120. **19b.** The logo consists of 1 large and 3 $\cong$ smaller equilateral $\triangle$s. They are also equiangular. Each $\angle$ of the equilateral $\triangle$ measures $\frac{180}{3}$, or 60. From part (a), the $\angle$s in the obtuse $\triangle$s are 30, 30 and 120. Thus, including $\angle$s formed by adjacent $\angle$s, there are 5 different sizes of $\angle$s. They are 30, 60, 90, 120, and 150. **19c.** Check students' work. **20.** Let x represent the measure of a base $\angle$. $2x + 40 = 180 \to 2x = 140 \to x = 70$ **21.** The figure shows two isosc. $\triangle$s. By the Isosc. $\triangle$ Thm., the base $\angle$s of the upper $\triangle$ are both 65. The vert. are also both 65, so the base $\angle$s of the lower $\triangle$ are both 65. $65 + 65 + x = 180 \to 130 + x = 180 \to x = 50$ **22.** The rt. $\triangle$ consists of 2 isosc. $\triangle$s, one acute and one obtuse. By the Isosc. $\triangle$ Thm., the base $\angle$s of the acute isosc. $\triangle$ are each 70. So, the base $\angle$s of the obtuse isosc. $\triangle$ are each $90 - 70$, or 20. $20 + 20 + x = 180 \to 40 + x = 180 \to x = 140$ **23.** The $\cong$ legs of the isosc. $\triangle$ are both $2x - 5$. Solve for the perimeter: $20 = (2x - 5) + (2x - 5) + x = 5x - 10 \to 30 = 5x \to x = 6$. **24.** By the Isosc. $\triangle$ Thm., both base $\angle$s of the upper $\triangle = 60$; $2(60) + x = 180 \to 120 + x = 180 \to x = 60$. The vertex $\angle$ of the lower isosc. $\triangle = 180 - 60$, or 120. Both base $\angle$s are y, so $2y + 120 = 180 \to 2y = 60 \to y = 30$. **25.** Solve for x: By the Isosc. $\triangle$ Thm., both base $\angle$s are 58. So, $2(58) + x = 180 \to 116 + x = 180 \to x = 64$. Solve for y: The right $\angle$ is the vertex $\angle$ of the middle $\triangle$, so the base $\angle$s are each 45; $x + 45 + y = 180 \to 64 + 45 + y = 180 \to 109 + y = 180 \to x = 71$. **26.** The large $\triangle$ is equilateral, so, by the Corollary to Thm. 4-3, each $\angle$ is =. Therefore each measures $\frac{180}{3}$, or 60, so $x + 30 = 60$ resulting in $x = 30$. By the Isosc. $\triangle$ Thm., each $\angle$ in the smaller $\triangle$ is 30, 30, and y, so $2(30) + y = 180 \to 60 + y = 180 \to y = 120$. **27.** The Isosc. $\triangle$ Thm. is "If 2 sides of a $\triangle$ are $\cong$, then the $\angle$s opposite those sides are $\cong$." Its converse is "If 2 $\angle$s of a $\triangle$ are $\cong$, then the sides opposite the $\angle$s are $\cong$." Since $p \to q$ and $q \to p$ are both true, remove "If" and "then" from either statement, replacing "then" with "if and only if": Two sides of a $\triangle$ are $\cong$ if and only if the $\angle$s opp. those sides are $\cong$. **28.** If the exterior $\angle$ is at one of the base $\angle$s, then one base $\angle$ measures $180 - 100 = 80$. If the vertex $\angle$ measures a, then $2(80) + a = 180 \to 160 + a = 180 \to a = 20$. So, one possible set of $\angle$s is 80, 80, 20. If the exterior $\angle$ is at the vertex $\angle$, then the vertex $\angle$ measures $180 - 100 = 80$. If a base $\angle$ measures b, then $2b + 80 = 180 \to 2b = 100 \to b = 50$. So, the other possible set of $\angle$s is 80, 50, 50. **29a.** The radio tower is $\perp$ to the ground, so it forms 90-degree $\angle$s with it. The radio tower is $\cong$ to itself by the Refl. Prop. of $\cong$. The ground distances from the tower to the cables on either side of it are the same. Thus, the $\triangle$s are $\cong$ by SAS. By CPCTC, the cables are the same length. Therefore, the $\triangle$s formed by two cables of the same height and the ground are isosc. $\triangle$s. **29b.** $450 + 450 = 900$; $550 + 550 = 1100$. The different base lengths are 900 ft and 1100 ft. **29c.** The tower is $\perp$ to the ground and halves the distance between the cables, so it is the $\perp$ bis. of the base of each $\triangle$. **30.** No; the $\triangle$ can be positioned in ways such that the base is not on the bottom. **31.** The base $\angle$s of an isosc. right $\triangle$ are = and the vertex $\angle$ is 90, so, if x is the measure of one base $\angle$, then $2x + 90 = 180 \to 2x = 90 \to x = 45$. They are = and have a sum of 90. **32.** Answers may vary. Sample: Corollary to Thm. 4-3: Since $\overline{XY} \cong \overline{YZ}$, $\angle X \cong \angle Z$ by Thm. 4-3. $\overline{YZ} \cong \overline{ZX}$, so $\angle Y \cong \angle X$ by Thm. 4-3 also. By the Trans. Prop. of $\cong$, $\angle Y \cong \angle Z$, so $\angle X \cong \angle Y \cong \angle Z$. Corollary to Thm. 4-4: Since $\angle X \cong \angle Z$, $\overline{XY} \cong \overline{YZ}$ by Thm. 4-4. $\angle Y \cong \angle X$, so $\overline{YZ} \cong \overline{ZX}$ by Thm. 4-4 also. By the Trans. Prop. of $\cong$, $\overline{XY} \cong \overline{ZX}$, so $\overline{XY} \cong \overline{YZ} \cong \overline{ZX}$. **33a.** Given **33b.** $\angle A$ and $\angle B$ are base $\angle$s, so $\angle A \cong \angle B$. **33c.** Given **33d.** SAS Post. implies $\cong$ $\triangle$s, so $\triangle ABE \cong \triangle DCE$. **34.** Solve for m: By the Triangle Ext. $\angle$ Thm., $90 + m = 126 \to m = 36$. By the Isosc. $\triangle$ Thm., both base $\angle$s of the other $\triangle$ measure n, so $2n + 126 = 180 \to 2n = 54 \to n = 27$.

35. Solve for n: By Corollary to the Isosc. $\triangle$ Thm., the equilateral $\triangle$ is equiangular, so each $\angle$ measures $\frac{180}{3}$, or 60; $n + 60 = 90$, so $n = 30$. Solve for m: $m + n + 90 = 180 \rightarrow m + 30 + 90 = 180 \rightarrow m + 120 = 180 \rightarrow m = 60$. **36.** The sum of the base $\angle$s of the isosc. right $\triangle$ is 90 and they are $=$, so $n = \frac{90}{2}$, or 45. Each of the 2 base $\angle$s of the larger isosc. $\triangle$ measures $m + n$, so $2(m + n) + 50 = 180 \rightarrow 2(m + 45) + 50 = 180 \rightarrow 2(m + 45) = 130 \rightarrow m + 45 = 65 \rightarrow m = 20$. **37.** For this solution, call the vertex of the vertex $\angle$ of the isosc. $\triangle$ the "apex." If the given pts. are base vertices, then the apex could be at (0, 0) or (4, 4). If the apex is at (4, 0), then the missing base vertex could be at (8, 4) or (0, −4). If the apex is at (0, 4), then the missing base vertex could be at (4, 8) or (−4, 0). **38.** For this solution, call the vertex of the vertex $\angle$ of the isosc. $\triangle$ the "apex." If the given pts. are base vertices, then the apex could be at (5, 3) or (2, 6). If the apex is at (5, 6), then the missing base vertex could be at (2, 9) or (8, 3). If the apex is at (2, 3), then the missing base vertex could be at (−1, 6) or (5, 0).
39. For this solution, call the vertex of the vertex $\angle$ of the isosc. $\triangle$ the "apex." If the given pts. are base vertices, then the apex could be at (5, 0) or (0, 5). If the apex is at (0, 0), then the missing base vertex could be at (−5, 5) or (5, −5). If the apex is at (5, 5), then the missing base vertex could be at (0, 10) or (10, 0).
40a. $(x + 15) + (3x - 35) + 4x = 180 \rightarrow 8x - 20 = 180 \rightarrow 8x = 200 \rightarrow x = 25$ **40b.** Substitute 25 for x in each of the algebraic expressions. $x + 15 = 25 + 15 = 40$; $3x - 35 = 3(25) - 35 = 75 - 35 = 40$; $4x = 4(25) = 100$ **40c.** Since it has an obtuse $\angle$ and the acute $\angle$s are $\cong$, it is an obtuse isosc. $\triangle$. **41.** $\overline{AC} \cong \overline{BC}$ and $\overline{CD}$ bis. $\angle ACB$ are given. $\angle ACD \cong \angle BCD$ by def. of $\angle$ bis. $\overline{CD} \cong \overline{CD}$ by the Refl. Prop. of $\cong$, so $\triangle ACD \cong \triangle BCD$ by SAS. So, $\overline{AD} \cong \overline{BD}$ by CPCTC, and $\overline{CD}$ bis. $\overline{AB}$ by def. of segment bis. Also, $\angle ADC \cong \angle BDC$ by CPCTC and $m\angle ADC + m\angle BDC = 180$ by the $\angle$ Add. Post. So, $m\angle ADC = m\angle BDC = 90$ by the Subst. Prop. So, $\overline{CD} \perp$ to $\overline{AB}$. Since $\overline{CD} \perp \overline{AB}$ and $\overline{CD}$ bis. $\overline{AB}$, $\overline{CD}$ is the $\perp$ bisector of $\overline{AB}$. **42.** Theorem 4-5 states "The bisector of the vertex $\angle$ of an isosceles $\triangle$ is the perpendicular bisector of the base." Reworded in if-then form it becomes "If a segment, ray, or line bisects the vertex $\angle$ of an isosceles $\triangle$, then the bisector is the perpendicular bisector of the base." To write the converse, switch the hypothesis and conclusion: If a bisector is the perpendicular bisector of the base of an isosc. $\triangle$, then it bisects the vertex $\angle$." The converse is true. Proof: Given $\triangle ABC$ with vertex $\angle C$ and $\perp$ bis. $\overline{CD}$, $\angle ADC \cong \angle CDB$ and $\overline{AD} \cong \overline{DB}$ by def of $\perp$ bis. Since $\overline{CD} \cong \overline{CD}$ by Refl. Prop. of $\cong$, $\triangle ACD \cong \triangle BCD$ by SAS. So, $\angle ACD \cong \angle BCD$ by CPCTC, and $\overline{CD}$ bis. $\angle ACB$.
43a. The lengths of the hypotenuses of the right $\triangle$s are: diameter, radius, length of side of largest square, half the length of the side of the largest square (the $\triangle$ is the tip of the "star"), and a fourth of the length of the side of the largest square (the $\triangle$ is half the tip of the "star"), so there are 5 different sizes of isosc. rt. $\triangle$s. See part ⓑ for examples.

43b.



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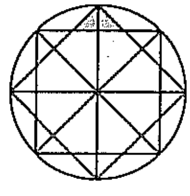

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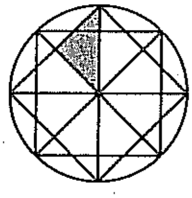

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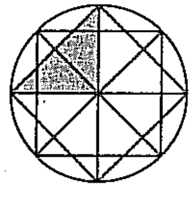

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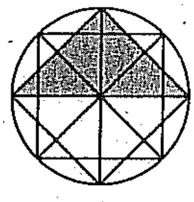

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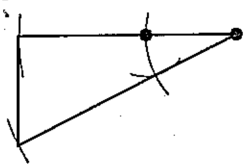
44. The measure of an obtuse $\angle$ is between 90 and 180. So, the sum of the base $\angle$s must be between 0 and 90. Since the base $\angle$s are $=$, $0 <$ measure of base $\angle < 45$. **45.** The measure of an acute $\angle$ is between 0 and 90. So, the sum of the base $\angle$s must be between 90 and 180. Since the base $\angle$s are $=$, $45 <$ measure of base $\angle < 90$. **46.** Since $\angle A$ is the vertex $\angle$, $\angle B$ and $\angle C$ are the base $\angle$s. The answer is choice C. **47.** The two $\triangle$s whose longest side is horizontal are $\cong$ (by SSS) isosc. $\triangle$s with $\cong$ base $\angle$s measuring 40 each. The horizontal segment is the $\angle$ bis. of the vertex $\angle$s of 2 other $\cong$ (by SSS) isosc. $\triangle$s, so it is also $\perp$ to their vertical base. So, each of the 4 small $\triangle$s is $\cong$ by AAS and has 2 $\angle$s measuring 90 and 40. Thus, $m\angle 2 = 180 - (90 + 40) = 50$. The answer is choice G. **48.** $4x + (2x + 10) + (2x + 10) = 180 \rightarrow 8x + 20 = 180 \rightarrow 8x = 160 \rightarrow x = 20$. The measure of the vertex $\angle = 4x = 4(20) = 80$. The answer is choice D. **49.** [2] **a.** By the Isosc. $\triangle$ Thm., $m\angle PAB = m\angle PBA$, so $2m\angle PAB + 60 = 180 \rightarrow 2m\angle PAB = 120 \rightarrow m\angle PAB = 60$. So, $\triangle APB$ is equiangular and equilateral. **b.** $60 + m\angle QAB = 90$, so $m\angle QAB = 30$; $\triangle QAB$ is isosc. with base $\angle$s measuring 30 each, so $30 + 30 + m\angle AQB = 180 \rightarrow 60 + m\angle AQB = 180 \rightarrow m\angle AQB = 120$ [1] one part correct **50.** $m\angle C = 180 - (93 + 59) = 180 - 152 = 28$, so $\triangle RTC \cong \triangle GHV$ by ASA, so $\overline{RC} \cong \overline{GV}$ by CPCTC. **51.** A second pair of $\angle$s are $\cong$ vertical $\angle$s, so two pairs of corresp. $\angle$s and their nonincluded side are $\cong$. The $\triangle$s are $\cong$ by AAS. **52.** A third pair of sides are $\cong$ by the Refl. Prop. of $\cong$, so the $\triangle$s are $\cong$ by SSS. **53.** Since the polygon is regular, each exterior $\angle$ measures 15. Also, the sum of the measures of the exterior $\angle$s of a polygon is 360. The number of $\angle$s = the number of sides of the polygon, so the number of sides is $\frac{360}{15}$, or 24.

4-6 Congruence in Right Triangles pages 217–223

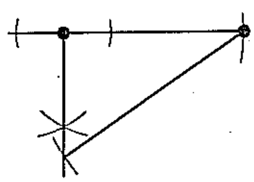
Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*
1. yes **2.** yes **3.** no **4.** yes **5.** yes **6.** no **7.** yes; SAS **8.** yes; SAS

Check Understanding **1.** The 2 $\triangle$s having a hypotenuse of 5 and one leg of 3 are congruent: $\triangle LMN \cong \triangle OQP$. **2.** The 2 $\triangle$s are right $\triangle$s. $\overline{CB} \cong \overline{EB}$ and $m\angle CBD = m\angle EBA$ because $\overline{AD}$ is the $\perp$ bis. of $\overline{CE}$. It is given that $\overline{CD} \cong \overline{EA}$. So, $\triangle CBD \cong \triangle EBA$ by HL. **3.** The corresp. sides and included rt. $\angle$s are $\cong$, so the $\triangle$s are $\cong$ by SAS.

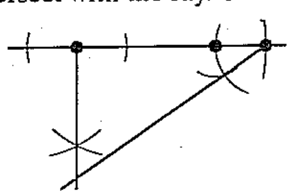
Exercises **1.** Both $\triangle$s are rt. $\triangle$s, $\overline{AC} \cong \overline{DF}$, and $\overline{CB} \cong \overline{FE}$. A leg and hypotenuse are $\cong$, so $\triangle ABC \cong \triangle DEF$ by HL. **2.** Both $\triangle$s are rt. $\triangle$s, $\overline{SP} \cong \overline{QR}$, and $\overline{PR} \cong \overline{PR}$ by the Refl. Prop. of $\cong$. In the rt. $\triangle$s, a leg and hypotenuse are $\cong$, so $\triangle SPR \cong \triangle QRP$ by HL. **3.** Both $\triangle$s are rt. $\triangle$s because vert. $\angle$s are $\cong$; $\overline{LP} \cong \overline{ON}$ and $\overline{LM} \cong \overline{OM}$. In the rt. $\triangle$s, a leg and hypotenuse are $\cong$, so $\triangle LMP \cong \triangle OMN$ by HL. **4.** B is the midpt. of $\overline{AD}$ and $\overline{EC}$, so $\overline{AB} \cong \overline{DB}$ and $\overline{EB} \cong \overline{CB}$ by def. of midpt. In the rt. $\triangle$s, a leg and hypotenuse are $\cong$, so $\triangle AEB \cong \triangle DCB$ by HL. **5.** You need to know that both $\triangle$s are right $\triangle$s, that the legs of both $\triangle$s are $\perp$, or that $\angle T$ and $\angle Q$ are rt. $\angle$s. **6.** The hypotenuses are $\cong$ by the Refl. Prop. of $\cong$, so you need to know that $\overline{RX} \cong \overline{RT}$ or that $\overline{XV} \cong \overline{TV}$. **7.** The hypotenuses are $\cong$ by the Refl. Prop. of $\cong$, so you need to know that $\overline{TY} \cong \overline{ER}$ or that $\overline{RT} \cong \overline{YE}$. **8.** The HL Thm. can only be used with rt. $\triangle$s, so you need to know that two of the $\triangle$s are rt. A pair of right, nonincluded $\angle$s are needed, either $\angle A$ and $\angle G$ or $\angle AQC$ and $\angle GJC$. **9.** To use HL, the $\triangle$s must both be rt. $\triangle$s, hypotenuses and one pair of legs must be $\cong$. The missing information is the hypotenuses: $\overline{BC} \cong \overline{FA}$. **10.** To use HL, the $\triangle$s must both be rt. $\triangle$s, hypotenuses and one pair of legs must be $\cong$. The missing information is the hypotenuses: $\overline{RT} \cong \overline{NQ}$. **11a.** Given **11b.** Definition of rt. $\triangle$ **11c.** Refl. Prop. of $\cong$ **11d.** Given **11e.** The $\triangle$s are rt., and a leg and hyp. of one $\triangle$ is $\cong$ to a leg and hyp. of the other $\triangle$: HL. **12a.** $\cong$ supp. $\angle$s are rt. $\angle$s. **12b.** Def. of rt. $\triangle$ **12c.** Given **12d.** Refl. Prop. of $\cong$ **12e.** The $\triangle$s are rt., and a leg and hyp. of one $\triangle$ is $\cong$ to a leg and hyp. of the other $\triangle$: HL. **13.** Answers may vary. Sample: $\overline{PS} \cong \overline{PT}$, so $\angle S \cong \angle T$ by the Isosc. $\triangle$ Thm. $\angle PRS \cong \angle PRT$, so $\triangle PRS \cong \triangle PRT$ by AAS. Or, show that $\overline{PR} \perp \overline{ST}$, then $\overline{PR}$ bis. $\angle SPT$. Thus, $\angle RPS \cong \angle TPS$ by def. of bis., so the $\triangle$s are $\cong$ by AAS. **14.** By def. of midpt., the hypotenuses are $\cong$, also corresp. legs are $\cong$, and both $\triangle$s are right, so the $\triangle$s are $\cong$ by HL. **15.** Since $\overline{JP} \parallel \overline{MW}$, $\angle PMW$ is a rt. $\angle$ (if $\parallel$ lines, then alt. int. $\angle$s $\cong$). By the Refl. Prop. of $\cong$, $\overline{PM} \cong \overline{MP}$. Both $\triangle$s are right, and corresp. hyp. and legs are $\cong$, so they are $\cong$ by HL. **16a.** Given **16b.** Def. of $\perp$ **16c.** $\triangle MLJ$ and $\triangle KJL$ are rt. $\triangle$s. **16d.** Given **16e.** $\overline{LJ} \cong \overline{JL}$ **16f.** Both $\triangle$s are right, and corresp. hyp. and legs are $\cong$, so they are $\cong$ by HL. **17a.** Given **17b.** $\triangle IGH$ **17c.** Def. of rt. $\triangle$ **17d.** I is the midpt. of $\overline{HV}$. **17e.** Def. of midpt. **17f.** $\triangle IGH \cong \triangle ITV$ **18.** The original polygon is regular, so the hypotenuses are $\cong$. Each right $\triangle$ has a $\cong$ hyp. and side, so the $\triangle$s are $\cong$ by HL. **19.** In the right $\triangle$s, the legs must be $\cong$, so $x = y + 1$, and the hypotenuses must be $\cong$, so $3y = x + 3$. Substitute $y + 1$ for x in $3y = x + 3$: $3y = (y + 1) + 3 \rightarrow 3y = y + 4 \rightarrow 2y = 4 \rightarrow y = 2$. Substitute 2 for y in $x = y + 1$: $x = 2 + 1 = 3$. **20.** In the right $\triangle$s, the legs must be $\cong$, so $y - x = x + 5$. Thus, $y = 2x + 5$ and the hypotenuses must be $\cong$, so $3y + x = y + 5$. Substitute $2x + 5$ for y in $3y + x = y + 5$: $3(2x + 5) + x = (2x + 5) + 5 \rightarrow 6x + 15 + x = 2x + 10 \rightarrow 7x + 15 = 2x + 10 \rightarrow 5x + 15 = 10 \rightarrow 5x = -5 \rightarrow x = -1$. Substitute -1 for x in $3y + x = y + 5$: $3y + (-1) = y + 5 \rightarrow 2y - 1 = 5 \rightarrow 2y = 6 \rightarrow y = 3$. **21.** The longest side of a rt. $\triangle$ is the hypotenuse, so you need to know if the 7-yd side is the hyp. or a leg. **22.** Rt. $\triangle$s are $\triangle ABX$, $\triangle ABC$, $\triangle BXC$, $\triangle ADX$, $\triangle ADC$, and $\triangle DXC$. $\overline{AX} \cong \overline{AX}$ by the Refl. Prop. of $\cong$, so $\triangle ABX \cong \triangle ADX$ by HL. $\overline{AC} \cong \overline{AC}$ by the Refl. Prop. of $\cong$, so $\triangle ABC \cong \triangle ADC$ by SSS or SAS (since rt. $\angle$s are $\cong$). $\overline{XC} \cong \overline{XC}$ by the Refl. Prop. of $\cong$, so $\triangle BXC \cong \triangle DXC$ by HL. **23a.** Show that $\angle AXB$ is suppl. and $\cong$ to $\angle AXD$ which are corresp. parts of $\triangle ABX$ and $\triangle ADX$. Use SAS to show that $\triangle ABX \cong \triangle ADX$ by first showing that $\angle BAX \cong \angle DAX$. These $\angle$s are corresp. parts of $\triangle BAC$ and $\triangle DAC$, which are $\cong$ by SSS. **23b.** By the Refl. Prop. of $\cong$, $\overline{AC} \cong \overline{AC}$, so $\triangle ABC \cong \triangle ADC$ by SSS. Then, $\angle BAC \cong \angle DAC$ by CPCTC. By the Refl. Prop. of $\cong$, $\overline{AX} \cong \overline{AX}$, so $\triangle ABX \cong \triangle ADX$ by SAS. Then, $\angle AXB \cong \angle AXD$ by CPCTC. Thus, $\angle AXB$ is suppl. and $\cong$ to $\angle AXD$, so they are both rt. $\angle$s.

24. 

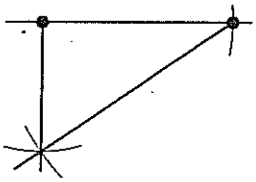
Draw a ray. Construct an $\angle$ congruent to one of the $\angle$s of the $\triangle$. Measure each of its adjacent sides with the compass and mark each length with an arc on the two rays of the constructed $\angle$. Connect the intersections of the arcs and their respective rays.

25. 

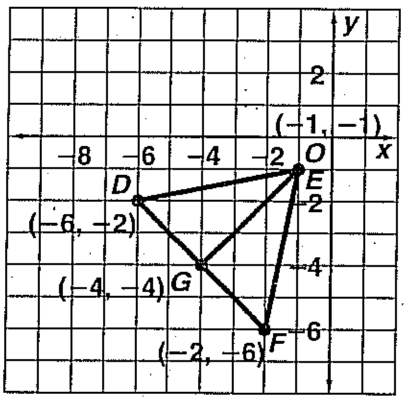
Draw a line. Construct a ray $\perp$ to it. Measure one of the legs of the $\triangle$ with the compass and mark the length with an arc on the line. Measure the hypotenuse and, with the compass at the same setting and the point on the intersection of the line and the arc, swing an arc to intersect with the ray. Connect the two arc intersections.

26. 

Draw a line. Construct a ray $\perp$ to it. Measure one of the legs of the $\triangle$ with the compass and mark the length with an arc on the line. Construct an $\angle$ congruent to one of the acute $\angle$s of the $\triangle$. Extend the two rays until they intersect.

27. 

Draw a line. Measure one of the legs of the △ with the compass and mark the length with an arc on the line. Measure the other leg of the △ and swing an arc from one of the ends of the first segment. Measure the hypotenuse and swing an arc from the other end of the first segment. Connect the last two arc intersections with the endpts. of the first segment. **28.** ① $\overline{EB} \cong \overline{DB}$; ∠A and ∠C are rt. ∠s. (Given) ② △BEA and △BDC are rt. △s. (Def. of rt. △) ③ B is the midpt. of $\overline{AC}$. (Given) ④ $\overline{AB} \cong \overline{CB}$ (Def. of midpt.) ⑤ △BEA ≅ △BDC (HL) **29.** ① $\overline{LO}$ bis. ∠MLN; $\overline{OM} \perp \overline{LM}$; $\overline{ON} \perp \overline{LN}$ (Given) ② ∠M and ∠N are rt. ∠s. (Def. of ⊥) ③ ∠MLO ≅ ∠NLO (Def. of ∠ bis.) ④ ∠M ≅ ∠N (All rt. ∠s are ≅.) ⑤ $\overline{LO} \cong \overline{LO}$ (Refl. Prop. of ≅) ⑥ △LMO ≅ △LNO (AAS)
30. Answers may vary. Sample: Measure 2 sides of the △ formed by the amp. and the platform's right corner. Since the △s will be ≅ by HL (The hyp. is the width of the amp.) or by SAS, the ∠s will be the same by CPCTC.

31a. 

31b. slope of $\overline{DG} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-2 - (-4)}{-6 - (-4)} = \frac{-2 + 4}{-6 + 4} = \frac{2}{-2} = -1$; slope of $\overline{GF} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-6 - (-4)}{-2 - (-4)} = \frac{-6 + 4}{-2 + 4} = \frac{-2}{2} = -1$; slope of $\overline{GE} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-1 - (-4)}{-1 - (-4)} = \frac{-1 + 4}{-1 + 4} = \frac{3}{3} = 1$ **31c.** The slope of $\overline{EG}$ is 1 and the slope of $\overline{DF}$ is −1. The product of their slopes is (1)(−1), or −1. So, $\overline{EG} \perp \overline{DF}$. By def. of ⊥, ∠EGD and ∠EGF are rt. ∠s.
31d. $DE = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(-6 - (-1))^2 + (-2 - (-1))^2} = \sqrt{(-6 + 1)^2 + (-2 + 1)^2} = \sqrt{(-5)^2 + (-1)^2} = \sqrt{25 + 1} = \sqrt{26}$; $FE = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(-2 - (-1))^2 + (-6 - (-1))^2} = \sqrt{(-2 + 1)^2 + (-6 + 1)^2} = \sqrt{(-1)^2 + (-5)^2} = \sqrt{1 + 25} = \sqrt{26}$ **31e.** From part ⓒ, both △s are rt. △s. From part ⓓ, $\overline{DE} \cong \overline{FE}$ by def. of ≅. Also, $\overline{EG} \cong \overline{EG}$ by the Refl. Prop. of ≅. Since corresp. legs and hyp. are ≅, △EGD ≅ △EGF by HL. **32.** If a corresp. hyp. and acute ∠s are ≅ in a right △, then two corresp. ∠s and a nonincluded side are ≅. So, an HA Thm. is the same as AAS with AAS corresp. to the rt. ∠, an acute ∠, and the hyp. **33.** Since $\overline{BE} \perp \overline{EA}$ and $\overline{BE} \perp \overline{EC}$, the segments form rt. ∠s, so △AEB and △CEB are both rt. △s. $\overline{AB} \cong \overline{BC}$ because △ABC is equilateral, and $\overline{BE} \cong \overline{BE}$ by the Refl. Prop. of ≅. Since the right △s have a leg and hyp. ≅, △AEB ≅ △CEB by HL. **34.** Since △AEB ≅ △CEB, $\overline{AB} \cong \overline{BC}$ by CPCTC. m∠AEB could be any measure between 0° and 180°, so $0 < AC < 2AE$. Thus, $\overline{AC}$ does not have to be ≅ to $\overline{AB}$ or to $\overline{CB}$. **35.** The 3 components to proving △s ≅ by HL are rt. △s, one pair of legs ≅, and hyp. ≅. Given are 2 rt. △s whose hyp. are ≅. Missing is the corresp. legs. The answer is choice A. **36.** The 3 components to proving △s ≅ by HL are rt. △s, one pair of legs ≅, and hyp. ≅. Given are 2 rt. △s whose corresp. legs are ≅. Missing is the hyp. The answer is choice H.
37. The hyp. are ≅ by the Refl. Prop. of ≅, and one set of legs is ≅. The △s are ≅ by HL. The answer is choice D.
38. [2] **a.** Though 3 pairs of △s can be proved ≅, the only pair that can be proved ≅ by HL is △TFW ≅ △TGW.
38b. A set of ≅ legs is given. The longest sides (hyp.) are ≅ by the Refl. Prop. of ≅. The ∠s opposite the hyp. must be rt. ∠s, so ∠RFW and ∠RTW are rt. ∠s. [1] one part correct **39.** The △ is ≅ to itself, but the order of the letters is different, so $\overline{XY} \cong \overline{ZY}$, but $\overline{XZ}$ is not ≅ to any other segment. Since 2 sides of the △ are ≅, the △ is isosceles. **40.** The △ is ≅ to itself, but the order of the letters is different, so $\overline{XY} \cong \overline{ZX} \cong \overline{YZ}$. Since 3 sides of the △ are ≅, the △ is equilateral. **41.** Lines or segments are ∥ if and only if their slopes are =. The slope of $\overline{AB}$ is $\frac{y_1 - y_2}{x_1 - x_2} = \frac{5 - 3}{5 - 3} = \frac{2}{2} = 1$. The slope of $\overline{BC} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{5 - 1}{5 - 9} = \frac{4}{-4} = -1$. The slope of $\overline{CD} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{1 - (-3)}{9 - 9} = \frac{4}{0}$, which does not exist, so $\overline{CD}$ has no slope. The slope of $\overline{AD} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{3 - (-3)}{3 - 9} = \frac{3 + 3}{3 - 9} = \frac{6}{-6} = -1$. Thus, $\overline{BC} \parallel \overline{AD}$. Two lines or segments are ⊥ if and only if the product of their slopes is −1. Since (1)(−1) = −1, $\overline{AB} \perp \overline{BC}$ and $\overline{AD} \perp \overline{BC}$. **42.** If ∥ lines, then alt. int. ∠s are ≅. **43.** If 2 lines are ∥, then same-side int. ∠s are suppl. **44.** Vert. ∠s are ≅. **45.** If 2 lines are ∥, then corresp. ∠s are ≅. **46.** If 2 lines are ∥, then corresp. ∠s are ≅. **47.** If 2 lines are ∥, then same-side interior ∠s are suppl.

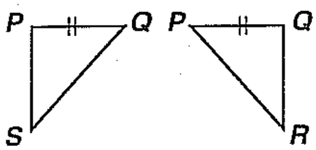
CHECKPOINT QUIZ 2 *page 223*

1. If 2 △s are ≅, then all corresp. parts are ≅, so $\overline{PR} \cong \overline{SQ}$, ∠P ≅ ∠S, and ∠PRQ ≅ ∠SQR. **2a.** Isosc. △ **2b.** ≅ **2c.** Converse of the Isosc. △ Thm. **3.** The six △s are △EAB, △EAC, △EAD, △EBC, △EBD, and △ECD. The △s with ≅ base ∠s are isosc. by the Converse of the Isosc. △ Thm. ∠EBC ≅ ∠ECB since suppl. of ≅ ∠s are ≅, so △EBC is isosc. and so is △EAD. **4.** Since a set of legs and hyp. are ≅ in rt. △s, the △s are ≅ by HL. **5.** Since $\overline{WT} \cong \overline{TW}$ by the Refl. Prop. of ≅, ∠WTG ≅ ∠TWS, and $\overline{GT} \cong \overline{SW}$, then △GTW ≅ △SWT by SAS. So, $\overline{GW} \cong \overline{ST}$ by CPCTC.

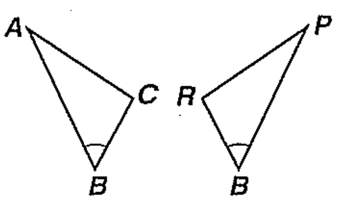
4-7 Using Corresponding Parts of Congruent Triangles pages 224–230

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*
1. 15; 31 **2a.** yes; SAS **2b.** yes; AAS **2c.** yes; Trans. Prop. of $\cong$
Check Understanding **1a.** Two common vertices in the congruence statement show that $\overline{CD}$ is the common side. **1b.** Answers may vary. Sample: $\triangle ABD$ and $\triangle CBD$. Two common vertices in the congruence statement show that $\overline{BD}$ is the common side. **2.** Prove that the base $\angle$s of $\triangle CED$ are $\cong$ and use the Converse of the Isosc. $\triangle$ Thm. to show that the sides opp. them are $\cong$. The $\angle$s are corresp. parts of the congruent $\triangle$s. ① $\triangle ACD \cong \triangle BDC$ (Given) ② $\angle ADC \cong \angle BCD$ (CPCTC) ③ $\overline{CE} \cong \overline{DE}$ (If base $\angle$s are $\cong$, then the opp. sides are $\cong$.) **3.** First show $\triangle PSQ \cong \triangle RSQ$ by SAS. Then use CPCTC to show $\triangle QPT \cong \triangle QRT$. ① $\overline{PS} \cong \overline{RS}$; $\angle PSQ \cong \angle RSQ$ (Given) ② $\overline{QS} \cong \overline{QS}$ (Refl. Prop. of $\cong$) ③ $\triangle PSQ \cong \triangle RSQ$ (SAS) ④ $\overline{PQ} \cong \overline{RQ}$ (CPCTC) ⑤ $\angle PQT \cong \angle RQT$ (CPCTC) ⑥ $\overline{QT} \cong \overline{QT}$ (Refl. Prop. of $\cong$) ⑦ $\triangle PQT \cong \triangle RQT$ (SAS) **4.** Answers may vary. Sample: $\overline{BD}$ and $\overline{FD}$ are corresp. parts of $\triangle BDC$ and $\triangle FDE$, respectively. Show $\triangle BDC \cong \triangle FDE$ using CPCTC from $\triangle$s ACD and AED. ① $\angle CAD \cong \angle EAD$; $\angle C \cong \angle E$ (Given) ② $\overline{AD} \cong \overline{AD}$ (Refl. Prop. of $\cong$) ③ $\triangle ACD \cong \triangle AED$ (AAS) ④ $\overline{CD} \cong \overline{ED}$ (CPCTC) ⑤ $\angle BDC \cong \angle FDE$ (Vert. $\angle$s are $\cong$.) ⑥ $\triangle BDC \cong \triangle FDE$ (ASA) ⑦ $\overline{BD} \cong \overline{FD}$ (CPCTC)

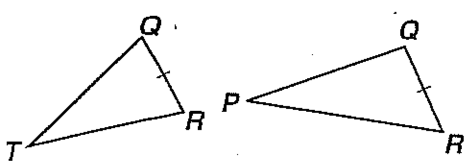
Exercises **1.** Triangles KNM and PLM have only one vertex in common, so they have an $\angle$ but no sides in common: $\angle M$. **2.** Triangles DEF and FDG have 2 vertices, F and D, in common so they have a side in common: $\overline{FD}$. **3.** Triangles TXY and ZYX have 2 vertices, X and Y, in common, so they have a side in common: $\overline{XY}$.

4. 

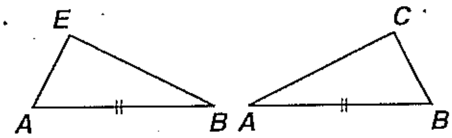
Triangles PQS and QPR have 2 vertices, Q and P, in common, so they have a side in common: $\overline{QP}$.

5. 

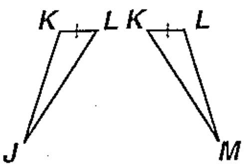
Triangles ACB and PRB have one vertex, B, in common, so they have an $\angle$ in common: $\angle B$.

6. 

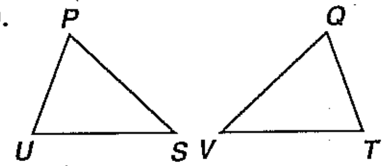
Triangles TRQ and PQR have 2 vertices, Q and R, in common, so they have a side in common: $\overline{QR}$.

7. 

Triangles ABE and BAC have 2 vertices, A and B, in common, so they have a side in common: $\overline{AB}$.

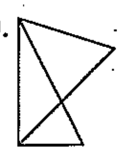
8. 

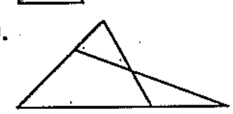
Triangles JKL and MLK have 2 vertices, K and L, in common, so they have a side in common: $\overline{KL}$.

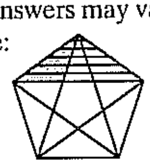
9. 

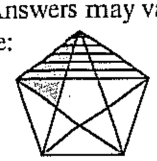
Triangles PSU and QVT have no vertices in common, so they have no side or vertex in common.

10a. Given **10b.** Refl. Prop. of $\cong$ **10c.** Given **10d.** Two $\angle$s and a nonincluded side are $\cong$: AAS. **10e.** CPCTC **11.** Since $\overline{LM}$ and $\overline{PQ}$ are $\perp$ to $\overline{LP}$, $\triangle LQP$ and $\triangle PML$ are rt. $\triangle$s. $\overline{LP} \cong \overline{PL}$ by the Refl. Prop. of $\cong$. It's given that $\overline{MP} \cong \overline{QL}$. Since the rt. $\triangle$s have corresp. legs and hyp. $\cong$, $\triangle LQP \cong \triangle PML$ by HL. **12.** $\overline{ST} \cong \overline{TS}$ by the Refl. Prop. of $\cong$. Since 3 pairs of sides are $\cong$, $\triangle RST \cong \triangle UTR$ by CPCTC. **13.** $\overline{DA} \cong \overline{AD}$ by the Refl. Prop. of $\cong$. Since 2 corresp. sides and their included $\angle$s are $\cong$, $\triangle QDA \cong \triangle UAD$ by SAS. **14.** If $\|$ lines, then corresp. $\angle$s are $\cong$, so $\angle P \cong \angle RUS$ and $\angle QTP \cong \angle S$. Two pairs of corresp. $\angle$s and nonincluded sides are $\cong$, so $\triangle QPT \cong \triangle RUS$ by AAS. **15.** Answers may vary. Sample: $\overline{TD}$ and $\overline{RO}$ are corresp. parts of $\triangle$s TDI and ROE. The $\triangle$s are $\cong$ by AAS if $\angle TID \cong \angle REO$. The $\angle$s are corresp. parts of $\triangle$s TEI and RIE. $\triangle TEI \cong \triangle RIE$ by SSS. **16.** $\overline{AE}$ and $\overline{DE}$ are corresp. parts of $\triangle$s AEB and DEC. The $\triangle$s are $\cong$ by AAS if $\angle AEB \cong \angle DEC$ because vert. $\angle$s are $\cong$ and if $\angle A \cong \angle D$. The $\angle$s are corresp. parts of $\triangle$s ABC and DCB. $\triangle ABC \cong \triangle DCB$ by HL. **17.** Answers may vary. Sample: $\triangle QET \cong \triangle QEU$ by ASA, using the Isosc. $\triangle$ Thm., if $\overline{QT} \cong \overline{QU}$. The segments are corresp. parts of $\triangle QTB$ and $\triangle QUB$, which are $\cong$ by ASA. **18.** Answers may vary. Sample: Since vert. $\angle$s are $\cong$ and by $\angle$ addition, $\angle ADC \cong \angle EDG$, so $\triangle ADC \cong \triangle EDG$ by ASA if $\angle A \cong \angle E$. These 2 $\angle$s are corresp. parts of $\triangle$s ADB and EDF, which are $\cong$ by SAS.

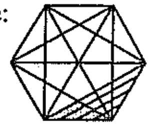
19. 
Draw the segment and make two points to the right of the segment. Draw segments from each endpt. to each point.

20. 
Draw an $\angle$ with two segments. Mark a point on each side of the $\angle$ and connect the endpts. of the $\angle$ to each point.

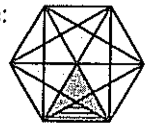
21a. Answers may vary. Sample:


21b. Answers may vary. Sample:


22a. Answers may vary. Sample:

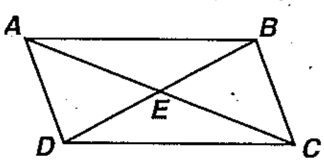


22b. Answers may vary. Sample:



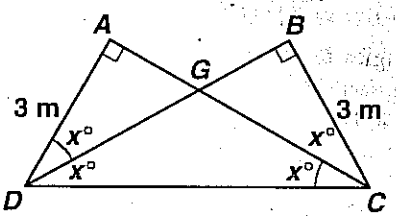
23. Plan: The shared $\angle$ is $\cong$ to itself by the Refl. Prop. of $\cong$. Two $\angle$s and an included side are $\cong$, so $\triangle ACE \cong \triangle BCD$ by ASA. Proof: $\overline{AC} \cong \overline{BC}$ and $\angle A \cong \angle B$ (Given); $\angle C \cong \angle C$ (Refl. Prop. of $\cong$); $\triangle ACE \cong \triangle BCD$ (ASA) **24.** Plan: $\triangle WYX \cong \triangle ZXY$ by HL. Proof: $\overline{WY} \perp \overline{YX}$, $\overline{ZX} \perp \overline{RX}$, and $\overline{WX} \cong \overline{ZY}$ (Given); $\angle WYX$ and $\angle ZXY$ are rt. $\angle$s. (Def. of $\perp$); $\overline{XY} \cong \overline{YX}$ (Refl. Prop. of $\cong$); $\triangle WYX \cong \triangle ZXY$ (HL) **25.** If $\|$ lines, then corresp. $\angle$s are $\cong$, so $56 = m\angle 1$ and $m\angle 2$. By def. of $\perp$, $\angle B$ and $\angle GCF$ are rt. $\angle$s, so $m\angle 4 = 90$. In $\triangle ABC$, $m\angle 3 = 180 - (56 + 90) = 180 - 146 = 34$. By the Angle Addition Post. and by the Isosc. $\triangle$ Thm., $m\angle 3 + m\angle 5 = 56$, so $m\angle 5 = 56 - 34 = 22$. By the Angle Subtraction Post., $m\angle 6 = 90 - 56 = 34$. In $\triangle FGC$, $m\angle 7 = 180 - (56 + 90) = 180 - 146 = 34$. By the Triangle Angle-Sum Thm., in $\triangle DEC$, $m\angle 8 = 180 - 2(56) = 180 - 112 = 68$. **26.** $\triangle ABC$ and $\triangle FCG$ are rt. $\triangle$s each having an acute $\angle$ measure of 56. Since two corresp. $\angle$s and their included sides are $\cong$, $\triangle ABC \cong \triangle FCG$ by ASA. **27a.** Given **27b.** Refl. Prop. of $\cong$ **27c.** Given **27d.** $\triangle ETI$ **27e.** $\triangle IRE$ **27f.** CPCTC **27g.** Given **27h.** All rt. $\angle$s are $\cong$. **27i.** $\triangle TDI$ **27j.** $\triangle ROE$ **27k.** CPCTC **28a.** Given **28b.** Def. of $\perp$ **28c.** Def. of rt. $\triangle$ **28d.** Given **28e.** $\overline{BC} \cong \overline{BC}$ **28f.** Reflexive **28g.** Since the hyp. and a set of legs are $\cong$, the $\triangle$s are $\cong$ by HL. **28h.** CPCTC **28i.** DEC **28j.** Vert. $\angle$s are $\cong$. **28k.** Since two $\angle$s and a nonincluded side are $\cong$, the $\triangle$s are $\cong$ by AAS. **28l.** $\overline{AE} \cong \overline{DE}$ **28m.** CPCTC **29.** Proofs may vary. Sample: It is given that $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. Since $\overline{QB} \cong \overline{QB}$ by the Refl. Prop. of $\cong$, $\triangle QTB \cong \triangle QUB$ by ASA. So, $\overline{QT} \cong \overline{QU}$ by CPCTC. Since $\overline{QE} \cong \overline{QE}$ by the Refl. Prop. of $\cong$, then $\triangle QET \cong \triangle QEU$ by SAS. **30.** Proofs may vary. Sample: ① $\overline{AD} \cong \overline{ED}$ (Given) ② D is the midpt. of $\overline{BF}$. (Given) ③ $\overline{FD} \cong \overline{DB}$ (Def. of midpt.) ④ $\angle FDE \cong \angle BDA$ (SAS) ⑤ $\triangle FDE \cong \triangle BDA$ (SAS) ⑥ $\angle E \cong \angle A$ (CPCTC) ⑦ $\angle GDE \cong \angle CDA$ (Vert. $\angle$s are $\cong$.) ⑧ $\triangle ADC \cong \triangle EDG$ (ASA)

31a.

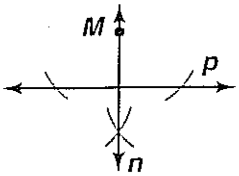


$\overline{AD} \cong \overline{BC}$, $\overline{AB} \cong \overline{DC}$, $\overline{AE} \cong \overline{EC}$, $\overline{DE} \cong \overline{EB}$ **31b.** If $\|$ lines, then alt. int. $\angle$s are $\cong$, so $\angle ABD \cong \angle CDB$ and $\angle ADB \cong \angle CBD$. $\overline{BD} \cong \overline{DB}$ by the Refl. Prop. of $\cong$. So, $\triangle ABD \cong \triangle CDB$ by ASA. By CPCTC, $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$. Because vert. $\angle$s are $\cong$, $\angle AED \cong \angle CEB$. Thus, $\triangle AED \cong \triangle CEB$ by AAS. By CPCTC, $\overline{AE} \cong \overline{CE}$ and $\overline{ED} \cong \overline{EB}$. **32.** ① $\overline{AC} \cong \overline{EC}$; $\overline{CB} \cong \overline{CD}$ (Given) ② $\angle C \cong \angle C$ (Refl. Prop. of $\cong$) ③ $\triangle ACD \cong \triangle ECB$ (SAS) ④ $\angle A \cong \angle E$ (CPCTC) **33.** Proofs may vary. Sample: $\overline{PQ} \cong \overline{RQ}$ and $\angle PQT$ and $\angle RQT$ are rt. $\angle$s by def. of $\perp$ bis. Since all rt. $\angle$s are $\cong$, $\angle PQT \cong \angle RQT$. By the Refl. Prop. of $\cong$, $\overline{QT} \cong \overline{QT}$, so $\triangle PQT \cong \triangle RQT$ by SAS. $\angle P \cong \angle R$ by CPCTC. $\overline{QT}$ bis. $\angle VQS$, so $\angle VQT \cong \angle SQT$. Since rt. $\angle$s measure 90, $\angle PQV$ and $\angle VQT$ are complementary $\angle$s and so are $\angle RQS$ and $\angle SQT$. Thus, $\angle VQP \cong \angle SQR$ because complements of $\cong$ $\angle$s are $\cong$. Then, $\triangle PQV \cong \triangle RQS$ by ASA, so $\overline{QV} \cong \overline{QS}$ by CPCTC. Or, after showing $\triangle PQT \cong \triangle RQT$ by SAS, then $\angle QTP \cong \angle QTR$ by CPCTC. Thus, $\triangle QVT \cong \triangle QST$ by ASA and $\overline{VQ} \cong \overline{SQ}$ by CPCTC. **34.** By the Triangle Angle-Sum Thm., $m\angle LKJ = 180 - (90 + 25) = 180 - 115 = 65$. The answer is choice C. **35.** $\triangle LMN \cong \triangle KMJ$ by SAS, so $m\angle LJM = 30$ and $m\angle L = 60$ by CPCTC. By the Angle Addition Post., $m\angle LJK = 60$, so $\triangle LKJ$ is equiangular. By the Corollary to the Converse of the Isosc. $\triangle$ Thm., $\triangle LKJ$ is equilateral. Since $MK = 7.4$, $LK = 14.8 = LJ = JK$, so the perimeter is $3(14.8) = 44.4$. The answer is choice F. **36.** $\triangle LMN \cong \triangle KMJ$ by SAS, so $\angle LJM \cong \angle KJM$ by CPCTC. By the Angle Division Post., $m\angle LJM = 47 \div 2 = 23.5$. The answer is choice A. **37a.** $\triangle HBC \cong \triangle HED$ **37b.** $\overline{HB} \cong \overline{HE}$ by CPCTC if $\triangle HBC \cong \triangle HED$. Since $\triangle BDC \cong \triangle CED$ by AAS, then $\angle DBC \cong \angle CED$ by CPCTC and $\angle CHB \cong \angle DHE$ because vert. $\angle$s are $\cong$. **38.** [4] **a.** Their hyp. are $\cong$ by the Refl. Prop. of $\cong$ and a set of legs is $\cong$ because they have = measures. Both are rt. $\triangle$s, so they are $\cong$ by HL.

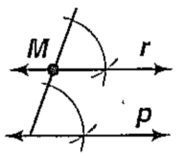
38b.



It's given that $x = m\angle ADG = m\angle ACD$. So, $x = m\angle ACD = m\angle BDC$ by CPCTC and substitution. Also, $2x = m\angle ADC = m\angle BCD$ by CPCTC and $\angle$ addition. By subtraction, $m\angle BCG = x$. **38c.** In $\triangle ADC$, $m\angle A + m\angle ADC + m\angle ACD = 180$. By substitution, $90 + 2x + x = 180 \rightarrow 3x + 90 = 180 \rightarrow 3x = 90 \rightarrow x = 30$. **38d.** $\angle AGB$ is an exterior $\angle$ of $\triangle AGD$, so it is the sum of the two remote interior $\angle$s: $m\angle AGB = 90 + 30 = 120$. **38e.** [3] 4 parts answered correctly [2] 3 parts answered correctly [1] 2 parts answered correctly **39a.** right **39b.** $\cong$ **39c.** Reflexive **39d.** HL

40. 

With the point of the compass on M, swing an arc that intersects p in two places. With the point of the compass on one of the arc intersections, swing another arc. With the point of the compass on the other of the arc intersections with p, swing an arc that intersects the previous arc. Draw a line through the intersection of the two arcs and M and label it n.

41. 

Draw a line through M that intersects p. With the point of the compass on the intersection of the two lines, swing an arc that intersects both lines. With the point of the compass on M and keeping the same setting, swing another arc that intersects the new line. Place the point of the compass on the intersection of the first arc with the new line and swing an arc that intersects the intersection of the first arc with p. Keeping the same setting, place the point of the compass on the intersection of the new line and the second arc and swing an arc that intersects that arc. Draw a line that passes through that intersection and M. **42.** $x_1 = 2$, $y_1 = -6$, and $m = \frac{1}{2}$, so $y - y_1 = m(x - x_1) \rightarrow y - (-6) = \frac{1}{2}(x - 2) \rightarrow y + 6 = \frac{1}{2}(x - 2)$. **43.** $x_1 = 0, y_1 = 5$, and $m = 1$, so $y - y_1 = m(x - x_1) \rightarrow y - 5 = 1(x - 0)$. **44.** $x_1 = -3, y_1 = 6$, and $m = -2$, so $y - y_1 = m(x - x_1) \rightarrow y - 6 = -2(x - (-3)) \rightarrow y - 6 = -2(x + 3)$. **45.** $x_1 = 0, y_1 = 0$, and $m = -\frac{1}{3}$, so $y - y_1 = m(x - x_1) \rightarrow y - 0 = -\frac{1}{3}(x - 0)$. **46.** Equation written using point A: $m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{4 - 2}{1 - 0} = 2, x_1 = 1$, and $y_1 = 4$, so $y - y_1 = m(x - x_1) \rightarrow y - 4 = 2(x - 1)$. Equation written using point B: $m = 2, x_1 = 0$, and $y_1 = 2$, so $y - y_1 = m(x - x_1) \rightarrow y - 2 = 2(x - 0)$. **47.** Equation written using point E: $m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-5 - 0}{3 - 6} = \frac{-5}{-3} = \frac{5}{3}$, $x_1 = 3$, and $y_1 = -5$, so $y - y_1 = m(x - x_1) \rightarrow y - (-5) = \frac{5}{3}(x - 3) \rightarrow y + 5 = \frac{5}{3}(x - 3)$. Equation written using point F: $m = \frac{5}{3}, x_1 = 6$, and $y_1 = 0$, so $y - y_1 = m(x - x_1) \rightarrow y - 0 = \frac{5}{3}(x - 6)$. **48.** Equation written using point X: $m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-3 - (-8)}{-4 - 2} = \frac{5}{-6} = -\frac{5}{6}, x_1 = -4$, and $y_1 = -3$, so $y - y_1 = m(x - x_1) \rightarrow y - (-3) = -\frac{5}{6}(x - (-4)) \rightarrow y + 3 = -\frac{5}{6}(x + 4)$. Equation written using point Y: $m = -\frac{5}{6}, x_1 = 2$, and $y_1 = -8$, so $y - y_1 = m(x - x_1) \rightarrow y - (-8) = -\frac{5}{6}(x - 2) \rightarrow y + 8 = -\frac{5}{6}(x - 2)$.

EXTENSION

1.

$\boxed{E \text{ is the midpt. of } \overline{AD}.}$ Given

$\boxed{\overline{AE} \cong \overline{ED}}$ Def. of midpt.

$\boxed{\overline{CE} \cong \overline{BE}}$ Given

$\boxed{\overline{AB} \cong \overline{DC}}$ Given

$\boxed{\triangle ABE \cong \triangle DCE}$ SSS

$\boxed{m\angle ABE = m\angle DCE}$ CPCTC

$\boxed{m\angle ABC = m\angle ABE + m\angle EBC}$ Ang. Add. Post.

$\boxed{m\angle EBC = m\angle ECB}$ Base ∠s of isos. △s are ≅.

$\boxed{m\angle ABC = m\angle DCE + m\angle ECB}$ Substitution

$\boxed{m\angle DCE + m\angle ECB = m\angle DCB}$ Ang. Add. Post.

$\boxed{m\angle ABC = m\angle DCB}$ Trans. Prop of =

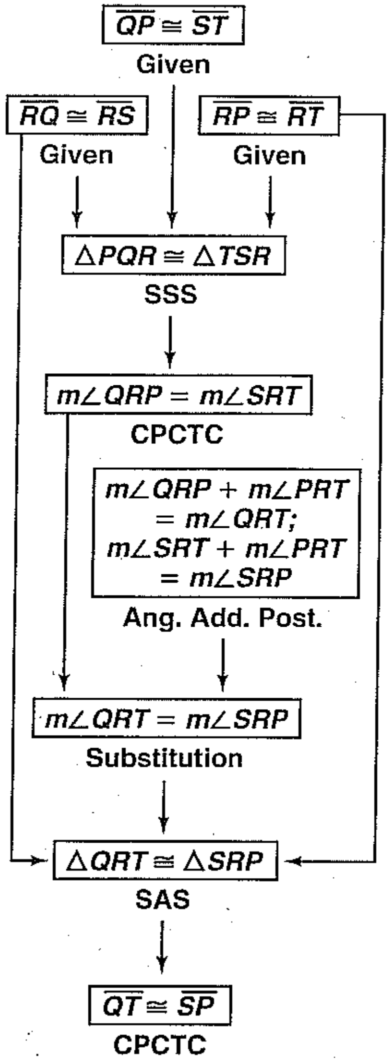
$\boxed{\overline{BC} \cong \overline{BC}}$ Ref. Prop. of ≅

$\boxed{\triangle ABC \cong \triangle DCB}$ SAS

$\boxed{\overline{AC} \cong \overline{DB}}$ CPCTC

Answers may vary. Sample: Another possible proof involves showing $\triangle ABD \cong \triangle DCE$.

2.



TEST-TAKING STRATEGIES *page 232*

1. The diagram shows no actual relationship among the sides. Though the figure looks like a square, we only know it is a rectangle, so the answer is choice D. **2.** If ∥ lines, then alt. int. ∠s are ≅, so $2x = x + 10$. Thus, $x = 10$. Since $10 > 5$, The answer is choice A. **3.** $\overline{XZ} \cong \overline{QR}$ only if $\triangle PQR$ is equilateral. There is no way to show that, so the relationship cannot be determined. The answer is choice D. **4.** By the Isosc. △ Thm., the measures of the ∠s of the △ on the left are 40, x, and x, so $40 + 2x = 180$. Thus, $x = 70$. By the Isosc. △ Thm., the measures of the ∠s of the △ on the right are $\frac{1}{2}x$, y, and y, so $\frac{1}{2}x + 2y = 180$. Substituting 70 for x results in $\frac{1}{2}(70) + 2y = 180$, so $y = 72.5$. Since $70 < 72.5$, the answer is choice B.

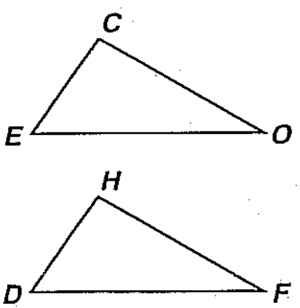
CHAPTER REVIEW *page 233*

1. The two congruent sides of an isosceles △ are the *legs*. **2.** The two congruent sides of an isosceles △ form the *vertex* ∠. **3.** If you know that two △s are congruent, then the corresponding sides and ∠s of the △s are congruent because *CPCTC*. **4.** The side opposite the right ∠ of a right △ is the *hypotenuse*. **5.** The ∠s of an isosceles △ that are not the vertex ∠ are called the *base* ∠s. **6.** A *corollary* to a theorem is a statement that follows immediately from the theorem. **7.** The *legs* are the two sides of a right △ that are not the hypotenuse. **8.** *Congruent polygons* have congruent corresponding parts. **9.** The side of an isosceles △ that is not a leg is called the *base*. **10.** T and S are the second 2 letters in $RSTUV$. They correspond to the second 2 letters in $KLMNO$, so $\overline{TS} \cong \overline{ML}$. **11.** N is the second to the last letter in $KLMNO$ and it corresponds to the second to the last letter in $RSTUV$, so $\angle N \cong \angle U$. **12.** L and M are the second 2 letters in $KLMNO$ and they correspond to the second 2 letters in $RSTUV$, so $\overline{LM} \cong \overline{ST}$. **13.** $VUTSR$ is $RSTUV$ backwards, so it corresponds to $KLMNO$ backwards: $ONMLK$. **14.** $\angle P$ corresponds to $\angle W$, so its measure is 80. **15.** $\overline{QR}$ corresponds to $\overline{XY}$, so its measure is 3. **16.** $\overline{WX}$ corresponds to $\overline{PQ}$, so its measure is 5. **17.** $\angle Z$ corresponds to $\angle S$, so its measure is 35. **18.** Since $m\angle Z$ is 35, it is suppl. to $\angle Y$ and they are same-side interior ∠s, so $\overline{WZ} \parallel \overline{XY}$. Thus, $m\angle X + 80 = 180$, so $m\angle x = 100$. **19.** The vertical segment is ≅ to itself, making 3 pairs of ≅ corresp. sides. The △s are ≅ by SSS. **20.** Another ∠ is needed for AAS or another side is needed for HL. Neither is available, so it is not possible to prove the △s ≅. **21.** Vert. ∠s ≅ provide an included ∠, so the △s are ≅ by SAS. **22.** The shared side can be proved ≅ to itself by the Refl. Prop. of ≅, but the third pair of sides or included ∠s must be ≅. That information is not available, so it is not possible to prove the △s ≅. **23.** Two corresp. ∠s and a nonincluded side are ≅, so the △s are ≅ by AAS. **24.** Since the vert. ∠s are ≅, 2 ∠s and an included side are ≅. Thus, the △s are ≅ by ASA. **25.** $\overline{WC} \cong \overline{CW}$ by the Refl. Prop. of ≅. Two ∠s and a nonincluded side are ≅, so the △s are ≅ by AAS. **26.** Two sides and their included ∠s are ≅, so the △s are ≅ by SAS. **27.** Two ∠s and their included sides are ≅, so the △s are ≅ by ASA. **28.** Two ∠s and their nonincluded sides are ≅, so $\triangle VTY \cong \triangle WYX$ by AAS. Thus, $\overline{TV} \cong \overline{YW}$ by CPCTC. **29.** Two ∠s and their included sides are ≅, so $\triangle BCE \cong \triangle DCE$ by ASA. Thus, $\overline{BE} \cong \overline{DE}$ by CPCTC. **30.** The corresp. hyp. and leg of 2 right △s are ≅, so $\triangle KNM \cong \triangle MLK$ by HL. **31.** $x = 4$; by the Isosc. △ Thm., the 2 base ∠s measure y, so $50 + 2y = 180$. Solving results in $y = 65$. **32.** $x + 125 = 180$, so $x = 55$. By vert. ∠s ≅, the vertex ∠ of the isosc. △ on the right is also 55. The two base ∠s measure y. So, $55 + 2y = 180$, and solving the equation results in $y = 62.5$. **33.** The vertex ∠ of the outer isosc. △ measures 25 + 25, or 50, so the two base ∠s are =; $50 + 2x = 180$, so $x = 65$. Since the bisector of the vertex ∠ is ⊥ to the base of an isosc. △, $y = 90$. **34.** ⊥ segments form right ∠s, so $\triangle PSQ$ and $\triangle RQS$ are rt. △s. $\overline{QS} \cong \overline{QS}$ by the Refl. Prop. of ≅. Also, $\overline{PQ} \cong \overline{RS}$. Since the hyp. and legs of the right △s are ≅, $\triangle PSQ \cong \triangle RQS$ by HL. **35.** Since $\overline{LN} \perp \overline{KM}$, $\angle LNK$ and $\angle LNM$ are rt. ∠s. It's given that $\overline{KL} \cong \overline{ML}$, and $\overline{LN} \cong \overline{LN}$ by the Refl. Prop. of ≅. So, $\triangle KLN \cong \triangle MLN$ by HL. **36.** Since 2 pair of corresp. sides and their included ∠s are ≅, $\triangle AEC \cong \triangle ABD$ by SAS. **37.** $\overline{IH} \cong \overline{HI}$ by the Refl. Prop. of $\cong$, so since 2 pair of corresp. sides and their included $\angle$s are $\cong$, $\triangle FIH \cong \triangle GHI$ by SAS. **38.** $\angle PTS \cong \angle RTA$ by the Refl. Prop. of $\cong$. Since 2 pair of corresp. $\angle$s and their included sides are $\cong$, $\triangle PTS \cong \triangle RTA$ by ASA. **39.** $\overline{FE} \cong \overline{EF}$ by the Refl. Prop. of $\cong$. Since 2 pair of corresp. $\angle$s and their included sides are $\cong$, $\triangle CFE \cong \triangle DEF$ by ASA.

CHAPTER TEST *page 236*

1. Y corresp. to L, and A in $\triangle PAY$ corresp. to P in $\triangle APL$, so $\triangle PAY \cong \triangle APL$. **2.** N corresp. to S, O corresp. to O, and E corresp. to E, so $\triangle ONE \cong \triangle OSE$. **3.** The shared side is $\cong$ to itself. Since 2 pairs of corresp. sides and their included $\angle$s are $\cong$, the $\triangle$s are $\cong$ by SAS. **4.** Since the hyp. of both $\triangle$s is shared, it is $\cong$ to itself by the Refl. Prop. of $\cong$. Since the hyp. and a set of legs are $\cong$ in the rt. $\triangle$s, they are $\cong$ by HL. **5.** Only one pair of corresp. sides and one pair of corresp. $\angle$s are $\cong$. Another pair of corresp. sides or $\angle$s $\cong$ is needed. It is not possible to prove the $\triangle$s $\cong$ on this information. **6.** The shared side is $\cong$ by the Refl. Prop. of $\cong$. Since 3 pairs of sides are $\cong$, the $\triangle$s are $\cong$ by SSS. **7.** Since vert. $\angle$s are $\cong$, 2 pairs of corresp. $\angle$s and their included sides are $\cong$, so the $\triangle$s are $\cong$ by ASA. **8.** Since two pair of $\angle$s and their corresp. nonincluded sides are $\cong$, the $\triangle$s are $\cong$ by AAS. **9.** Answers may vary. Sample: Information about the sides establishes size. The corr. sides of the two $\triangle$s may not be $\cong$.

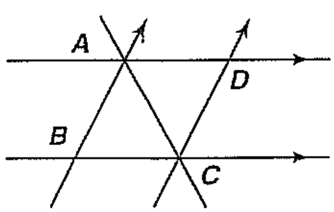
10.



Answers may vary. Sample: The order of the letters in the congruence statement $\triangle CEO \cong \triangle HDF$ shows that C corresp. to H, E corresp. to D, and O corresp. to F. So, $\overline{CE} \cong \overline{HD}$, $\overline{CO} \cong \overline{HF}$, $\overline{EO} \cong \overline{DF}$, $\angle C \cong \angle H$, $\angle E \cong \angle D$, and $\angle O \cong \angle F$.

11. They are not necessarily $\cong$. One could be square and the other could be a rectangle with unequal sides. **12.** $3x = 108$, so $x = 36$. **13.** If $\parallel$ lines, then alt. int. $\angle$s are $\cong$, so $\angle ATG \cong \angle SGT$. It's given that $\overline{AT} \cong \overline{GS}$, and $\overline{GT} \cong \overline{GT}$ by the Refl. Prop. of $\cong$, so $\triangle GAT \cong \triangle TSG$ by SAS. **14.** By def. of bis., $\angle OLN \cong \angle MLN$ and $\angle ONL \cong \angle MNL$. By the Refl. Prop. of $\cong$, $\overline{LN} \cong \overline{LN}$. Since two corresp. $\angle$s and their included sides are $\cong$, $\triangle OLN \cong \triangle MLN$ by ASA. **15.** $\overline{FE} \cong \overline{EF}$ by the Refl. Prop. of $\cong$. Since 3 pairs of sides are $\cong$, $\triangle CFE \cong \triangle DEF$ by SSS. **16.** $\angle T \cong \angle T$ by the Refl. Prop. of $\cong$. Since two corresp. sides and their included $\angle$s are $\cong$, $\triangle TQS \cong \triangle TRA$ by SAS.

17.

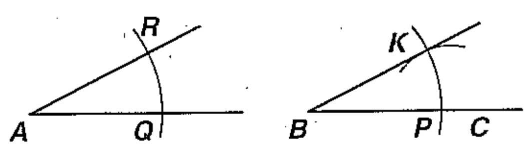


Answers may vary. Sample: $\overline{AC} \cong \overline{CA}$ by the Refl. Prop. of $\cong$. $\angle BAC \cong \angle DCA$ and $\angle ACB \cong \angle CAD$ because if $\parallel$ lines, then alt. int. $\angle$s are $\cong$. Since 2 corresp. $\angle$s and their included side are $\cong$, $\triangle ABC \cong \triangle CDA$ by ASA.

STANDARDIZED TEST PREP *page 237*

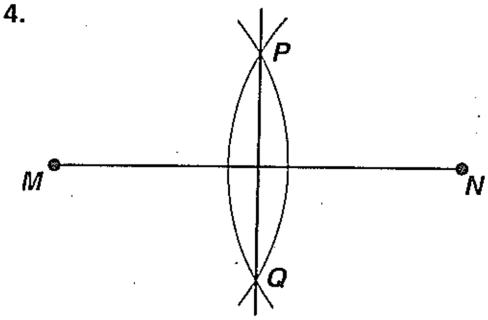
1. By the Angle Addition Postulate, $m\angle CDF = 42 + 34 = 86$. The answer is choice B. **2.** Vertical $\angle$s are not found in a single $\triangle$, so eliminate IV. An obtuse $\triangle$ always has one obtuse $\angle$ measuring between 90 and 180. Since the sum of the measures of the $\angle$s of a $\triangle$ is 180, the sum of the remaining 2 $\angle$s must be between 0 and 90, so they must both be acute. The answer is choice G. **3.** The length of one side is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(-2-3)^2 + (5-5)^2} = \sqrt{(-5)^2 + (0)^2} = \sqrt{25} = 5$. The length of the other side is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(3-3)^2 + (5-(-1))^2} = \sqrt{(0)^2 + (6)^2} = 6$. So, the area is $(5)(6) = 30$. The answer is choice B. **4.** T is the last letter and S is the third letter of $QRST$, so they corresp. to the last and third letters of $ABCD$. $\overline{TS} \cong \overline{DC}$. The answer is choice I. **5.** Two corresp. $\angle$s and their included sides are $\cong$, so the $\triangle$s are $\cong$ by SAS. The answer is choice A. **6.** Two lines are $\parallel$ if corresp. $\angle$s are $\cong$, alt. int. $\angle$s are $\cong$, interior $\angle$s on the same side of the transversal are $\cong$, or exterior $\angle$s on the same side of the transversal are $\cong$. Angles 1 and 4 are corresp. $\angle$s. Angles 2 and 5, and 3 and 4 are alt. int. $\angle$s. Angles 2 and 4 are same-side interior $\angle$s. The answer is choice I. **7.** No information is given to indicate the slope of either line, only the relationship between the two lines. The answer is choice D. **8.** If two lines are $\perp$ and neither is vertical, then their product is -1. The slope of a horizontal line is 0. Since $-1 < 0$, the answer is choice B. **9.** The sum of the $\angle$s is 180, so the missing $\angle$ is $180 - (54.5 + 71) = 180 - 125.5 = 54.5$. **10.** $C = \pi d \approx (3.14)(10) = 31.4$. **11.** The sum of the measures of an $\angle$ and its complement is 90, so if the measure of the $\angle = x$, then $56 + x = 90$. Thus, $x = 90 - 56$, or 34. **12.** The sum of the measures of an $\angle$ and its supplement is 180, so if the measure of the $\angle = k$, then $35 + k = 180$. Thus, $k = 180 - 35 = 145$.

13.



[2] Draw $\angle A$ and draw $\overleftrightarrow{BC}$. Draw an arc with center A, and copy that arc with center B. Open the compass to length RQ to draw an arc with center P crossing the other arc at K. [1] diagram or explanation is not complete OR contains an error

14.



[2] Draw arcs with the same radius using centers M and N. Label the intersections of the arcs P and Q. Draw $\overleftrightarrow{PQ}$, which is the $\perp$ bis. of $\overline{MN}$. [1] diagram or explanation is not complete OR contains an error

15. [4] $CD = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(5 - 10)^2 + (7 - (-5))^2} = \sqrt{(5 - 10)^2 + (7 + 5)^2} = \sqrt{(-5)^2 + (12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13$. The midpt. of $\overline{CD}$ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{5 + 10}{2}, \frac{7 + (-5)}{2}\right) = \left(\frac{15}{2}, \frac{2}{2}\right) = (7.5, 1)$.
[3] appropriate methods, but with one computational error [2] appropriate methods, but computational errors for both quantities [1] no work shown

REAL-WORLD SNAPSHOTS *pages 238–239*

Activity 1 The mirrors are $\perp$, so the dashed line $\perp$ to the top mirror is $\|$ to the bottom mirror. Since alt. int. $\angle$s of $\|$ lines are $\cong$, the first $\angle$ of reflection is $\cong$ to an acute $\angle$ in the rt. $\triangle$ formed by the 2 mirrors and blue ray. Similarly, the dashed line $\perp$ to the bottom mirror is $\|$ to the top mirror. Since alt. int. $\angle$s of $\|$ lines are $\cong$, the second $\angle$ of incidence is $\cong$ to the other acute $\angle$ in the rt. $\triangle$. So, the sum of the measures of the first $\angle$ of reflection and the second $\angle$ of incidence is 90. Also, each $\angle$ of incidence is $\cong$ to its corr. $\angle$ of reflection, so the incident ray and the hyp. of the rt. $\triangle$ form an $\angle$ that is suppl. to the $\angle$ formed by the hyp. of the rt. $\triangle$ and the reflected ray. Since same-side int. $\angle$s are $\cong$, the incident ray is $\|$ to the reflected ray.

Activity 2 If I wink an eye in the single mirror, it appears that my opposite eye is winking. However, if I wink one eye in the double mirror, it appears that the same eye is winking. The image appears opposite in the single mirror, but the same in the double mirror.