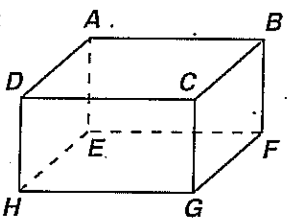
Parallel and Perpendicular Lines

DIAGNOSING READINESS — page 112

1. $\frac{360}{n} = \frac{360}{5} = 72$ **2.** $(n-2)180 = (9-2)180 = (7)180 = 1260$ **3.** $(n-2)180 = (17-2)180 = (15)180 = 2700$ **4.** $3x + 11 = 7x - 5; -4x + 11 = -5; -4x = -16; x = 4$ **5.** $(2x+5) + (3x - 10) = 70; 5x - 5 = 70; 5x = 75; x = 15$ **6.** $(3x+2) - (2x-3) = -19; 3x + 2 - 2x + 3 = -19; x + 5 = -19; x = -24$ **7.** $2x + x + x = 180$; $4x = 180$; $x = 45$. The $\angle$s have measures 45, 45, and 90. **8.** $\frac{1}{2}x + x + x = 180$; $\frac{5}{2}x = 180$; $x = 72$. The $\angle$s have measures 72, 72, and 36.

9. Drawings may vary. Sample: $\overleftrightarrow{AB}$ and $\overleftrightarrow{HG}$ appear to be parallel. $\overleftrightarrow{AE}$ and $\overleftrightarrow{EF}$ appear to be perpendicular.



10–12. Check students' work.

TECHNOLOGY — page 114

1. $\angle 2 \cong \angle 4 \cong \angle 6 \cong \angle 8$; $\angle 1 \cong \angle 3 \cong \angle 5 \cong \angle 7$; when a transversal intersects two parallel lines, the $\angle$s formed have one of two measures and the sum of the two different measures is 180. Angles between the $\parallel$ lines on opposite sides of the transversal are $\cong$. Angles between the $\parallel$ lines on the same side of the transversal are suppl. **2a.** Half of the $\angle$s are one measure and the other half are a different measure. The sum of the two measures is 180. **2b.** There are exactly 2 different $\angle$ measures for all the $\angle$s formed. **3.** The transversal is also $\perp$ to the other line. It makes right $\angle$s with both lines. **4a.** If the $\angle$s between the lines on alt. sides of the transversal are $\cong$, then the lines are $\parallel$. **4b.** The hypothesis and conclusion are switched, so the other conjecture is the converse. **5a.** If the same-side interior $\angle$s are suppl., then the two lines are $\parallel$. **5b.** Since the hypothesis and conclusion are switched, the other conjecture is its converse.

3-1 Properties of Parallel Lines — pages 115–121

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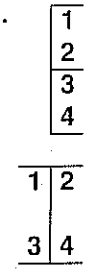
. 30 **2.** 30 **3.** 60 **4.** 9 **5.** $m\angle 1 + 2(90 - m\angle 1) = 146$; $m\angle 1 = 34$ **6.** 72 and 108

Check Understanding **1.** Corresp. $\angle$s are on the same side of the transversal. One $\angle$ is between the $\parallel$ lines and the other is not. $\angle 5$ and $\angle 4$, $\angle 6$ and $\angle 2$, $\angle 3$ and $\angle 8$. They are both interior $\angle$s and on the same side of the transversal, so they are same-side int. $\angle$s. **3.** ① $a \parallel b$ (Given) ② $m\angle 3 + m\angle 2 = 180$ (Angle Add. Post.) ③ $m\angle 1 = m\angle 3$ (Corr. Angles Post.) ④ $m\angle 1 + m\angle 2 = 180$ (Substitute $m\angle 1$ for $m\angle 3$ in the equation in Step 2.) ⑤ $\angle 1$ and $\angle 2$ are suppl. (Def. of Suppl. Angles) **4a.** Since $a \parallel b$ with transversal c and since corr. $\angle$s are $\cong$, $m\angle 1 = 50$. Since $c \parallel d$ with transversal b and since same-side interior $\angle$s are suppl., $m\angle 1 + m\angle 3 = 180$, so $m\angle 3 = 180 - 50 = 130$. **4b.** From 4a, $m\angle 3 = 130$. Since vertical $\angle$s are $\cong$, $m\angle 4 = m\angle 3 = 130$. **4c.** Since $a \parallel b$ and since alt. int. $\angle$s are $\cong$, $m\angle 5 = 50$. **4d.** Since $c \parallel d$ and since alt. int. $\angle$s are $\cong$, $m\angle 6 = 50$. **4e.** Answers may vary. Sample: $\angle 7$ and the named $50°$ $\angle$ are supplementary, so $m\angle 7 = 180 - 50 = 130$. Or, from 4d, $m\angle 6 = 50$, and since $c \parallel d$ with transversal a and since same-side interior $\angle$s are suppl., $m\angle 7 = 180 - 50 = 130$. **4f.** Answers may vary. Sample: From 4d, $m\angle 6 = 50$, and since $c \parallel d$ with transversal a and since corr. $\angle$s are $\cong$, $m\angle 8 = m\angle 6 = 50$. Or since vert. $\angle$s are $\cong$, $m\angle 8 = 50$. **5.** Since same-side int. $\angle$s are suppl., $2x + 90 = 180$; $2x = 90$; $x = 45$. Since same-side int. $\angle$s are suppl., $y + (y - 50) = 180$; $2y - 50 = 180$; $2y = 230$; $y = 115$. Solve for $\angle$ measures: $2x = 2(45) = 90$, $y = 115$, and $y - 50 = 115 - 50 = 65$.

Exercises **1.** $\overleftrightarrow{PQ} \parallel \overleftrightarrow{SR}$ with transversal $\overleftrightarrow{SQ}$. Since the $\angle$s are on alt. sides of the transversal and between the $\parallel$ lines, they are alt. int. $\angle$s. **2.** $\overleftrightarrow{PS} \parallel \overleftrightarrow{QR}$ with transversal $\overleftrightarrow{SQ}$. Since the $\angle$s are on alt. sides of the transversal and between the $\parallel$ lines, they are alt. int. $\angle$s. **3.** $\overleftrightarrow{PS} \parallel \overleftrightarrow{QR}$ with transversal $\overleftrightarrow{PQ}$. Since the $\angle$s are on the same side of the transversal and between the $\parallel$ lines, they are same-side int. $\angle$s. **4.** $\overleftrightarrow{PS} \parallel \overleftrightarrow{QR}$ with transversal $\overleftrightarrow{SR}$. Since the $\angle$s are on the same side of the transversal and since one $\angle$ is between the $\parallel$ lines and the other is not, they are corr. $\angle$s. **5.** Since $\angle 1$ and $\angle 2$ are on the same side of the transversal and since one $\angle$ is between the other two lines and the other is not, they are corr. $\angle$s. Since $\angle 3$ and $\angle 4$ are on alt. sides of the transversal and since both are between the other two lines, they are alt. int. $\angle$s. Since $\angle 5$ and $\angle 6$ are on the same side of the transversal and since one of the $\angle$s is between the other two lines and the other is not, they are corr. $\angle$s. **6.** Since $\angle 1$ and $\angle 2$ are on the same side of the transversal and since both are between the other two lines, they are same-side int. $\angle$s. Since $\angle 3$ and $\angle 4$ are on the same side of the transversal and since one $\angle$ is between the other two lines and the other is not, they are corr. $\angle$s. Since $\angle 5$ and $\angle 6$ are on the same side of the transversal and since one $\angle$ is between the other two lines and the other is not, they are corr. $\angle$s. **7.** Since $\angle 1$ and $\angle 2$ are on the same side of the transversal and since one $\angle$ is between the other two lines and the other is not, they are corr. $\angle$s. Since $\angle 2$ and $\angle 4$ are on the same side of the transversal and since both $\angle$s are between the other two lines, they are same-side int. $\angle$s. Since $\angle 5$ and $\angle 6$ are on alt. sides of the transversal and since they are both between the other two lines, they are alt. int. $\angle$s. **8.** Since the $\angle$s are both between the $\parallel$ lines and on alt. sides of the transversal, they are alt. int. $\angle$s. **9a.** Working backwards, show that $m \perp t$ by showing that $\angle 2$ is a right $\angle$: 2. **9b.** 1 **9c.** Since the two $\angle$s are on the same side of the transversal, t, and since one is in the interior of the $\parallel$ lines and the other is in their exterior, they are corr. $\angle$s: corresponding. **10a.** Def. of $\perp$ **10b.** Def. of right $\angle$ **10c.** $\angle 1$ and $\angle 2$ are on the same side of the transversal, k, and one is in the interior and the other is in the exterior region of the $\parallel$ lines, so they are corr. $\angle$s: Corr. $\angle$s of $\parallel$ lines are $\cong$. **10d.** From step 3 and step 6, substitute 90 for $m\angle 1$: Substitution. **10e.** Def. of right $\angle$ **10f.** Def. of $\perp$ **11.** Since corr. $\angle$s of $\parallel$ lines are $\cong$, $m\angle 1 = 75$. Since same-side interior $\angle$s of $\parallel$ lines are suppl., $m\angle 2 = 180 - 75 = 105$. **12.** Since corr. $\angle$s of $\parallel$ lines are $\cong$, $m\angle 1 = 120$. Since same-side interior $\angle$s of $\parallel$ lines are suppl., $m\angle 2 = 180 - 120 = 60$. **13.** Since same-side interior $\angle$s of $\parallel$ lines are suppl., $m\angle 1 = 180 - 100 = 80$. Since there is no information showing $\overline{AD} \parallel \overline{BC}$, use alt. int. $\angle$s of $\parallel$ lines have = measure, so $m\angle 2 = 70$. **14.** Since same-side int. $\angle$s are suppl., $x + (x + 40) = 180$; $2x + 40 = 180$; $x + 20 = 90$; $x = 70$. The $\angle$ measures are $x = 70$, and $x + 40 = 70 + 40 = 110$. **15.** Since corr. $\angle$s of $\parallel$ lines are $\cong$, $3x - 10 = x + 40$; $2x - 10 = 40$; $x - 5 = 20$; $x = 25$. The $\angle$ measures are $\cong$, so both their measures are $x + 40 = 25 + 40 = 65$. **16.** Since same-side int. $\angle$s of $\parallel$ lines are suppl., $5x + 4x = 180$; $9x = 180$; $x = 20$. The $\angle$ measures are $5x = 5(20) = 100$, and $4x = 4(20) = 80$. **17.** Since $f \parallel g$, all $\angle$s formed by their transversals will measure 128 or $180 - 128$, which is 52. Similarly, since $m \parallel n$, all $\angle$s formed by their transversals will measure 128 or 52. So, $m\angle 1 = m\angle 3 = m\angle 6 = m\angle 8 = m\angle 9 = m\angle 11 = m\angle 13 = m\angle 15 = 52$; $m\angle 2 = m\angle 4 = m\angle 5 = m\angle 7 = m\angle 10 = m\angle 12 = m\angle 14 = 128$. **18.** You must find the measure of one $\angle$. All $\angle$s that are vert., corr., or alt. int. to that $\angle$ will have that measure. All other $\angle$s will be the suppl. of that measure. **19.** There are 4 interior $\angle$s, so there are *two* pairs of alternate interior $\angle$s. **20.** There are 4 interior $\angle$s and 4 exterior $\angle$s. Since corr. $\angle$s have one $\angle$ in the interior and the other in the exterior regions, there are *four* pairs of corr. $\angle$s. **21.** There are four interior $\angle$s, so there are *two* pairs of same-side interior $\angle$s. **22.** There are two pairs of vert. $\angle$s at each line intersection and there are two intersections, so there are *four* pairs of vertical $\angle$s formed. **23.** Since corr. $\angle$s of $\parallel$ lines are $\cong$, $3p - 6 = 90$; $p - 2 = 30$; $p = 32$. **24.** Since alt. int. $\angle$s of $\parallel$ lines are $\cong$, $x = 76$. Since the sum of the $\angle$s of a $\triangle$ is 180, $42 + 25 + x + y = 180$; $42 + 25 + (76) + y = 180$; $143 + y = 180$; $y = 37$. Since alt. int. $\angle$s of $\parallel$ lines are $\cong$, $v = 42$. Since alt. int. $\angle$s of $\parallel$ lines are $\cong$, $w = 25$. **25.** $x + y = 180$, and since corr. $\angle$s of $\parallel$ lines are $\cong$, $x = 3y$. By substitution, $(3y) + y = 180$; $4y = 180$; $y = 45$. $x = 180 - 45 = 135$ **26.** If the lines are $\parallel$, then the labeled $\angle$s are corr. $\angle$s and should be $\cong$. So, $2x - 60 = 60 - 2x$; $4x = 120$; $x = 30$. Substitute 30 for x in $2x - 60 = 60 - 2x$: $60 - 2(30) = 60 - 60 = 0$. If the $\angle$ measured 0, it would not show as an $\angle$ in the diagram. **27.** *Trans* means across or over. A transversal cuts across other lines.

28. 

Answers may vary. Samples: E illustrates corr. $\angle$s ($\angle 1$ and $\angle 3$, $\angle 2$ and $\angle 4$) and same-side int. $\angle$s ($\angle 1$ and $\angle 2$, $\angle 3$ and $\angle 4$); I illustrates alt. int. $\angle$s ($\angle 1$ and $\angle 4$, $\angle 2$ and $\angle 3$) and same-side int. $\angle$s ($\angle 1$ and $\angle 3$, $\angle 2$ and $\angle 4$).

29a. They are on alt. sides of the radius transversal and between the $\parallel$ lines of the sun's rays, so they are alt. int. $\angle$s. **29b.** He knew that alt. int. $\angle$s of $\parallel$ lines are $\cong$. **30a.** The $\angle$s are same-side interior $\angle$s, so their sum should be 180. $m\angle 1 + 123 = 180$, so $m\angle 1 = 57$. **30b.** They are in the interior of the $\parallel$ lines and on the same side of the transversal (pipe), so they are same-side interior $\angle$s. **31a.** If two lines are $\parallel$ and cut by a transversal, then the same-side ext. $\angle$s are suppl. **31b.** Given: $a \parallel b$; prove: $\angle 4$ and $\angle 5$ are suppl.: ① $a \parallel b$ (Given) ② $m\angle 5 + m\angle 6 = 180$ ($\angle$ Add. Post.) ③ $\angle 4 \cong \angle 6$ (Corr. $\angle$s of $\parallel$ lines are $\cong$.) ④ $m\angle 4 = m\angle 6$ (Def. of $\cong$) ⑤ $m\angle 5 + m\angle 4 = 180$ (Steps 2 and 4: Substitution) ⑥ $\angle 4$ and $\angle 5$ are suppl. (Def. of suppl.) **32.** ① $a \parallel b$ (Given) ② $\angle 1 \cong \angle 2$ (Vert. $\angle$s are $\cong$.) ③ $\angle 2 \cong \angle 3$ (Corr. $\angle$s are $\cong$.) ④ $\angle 1 \cong \angle 3$ (Trans. Prop.) **33.** Since the planes are $\parallel$, they do not intersect, so they have no points in common. Lines m and n *never* intersect. **34.** If lines m and n are parallel, then they are coplanar. Lines m and n are *sometimes* coplanar. **35.** The lines will never intersect, so they will be either parallel or skew. Lines m and n are *sometimes* parallel. **36.** The lines will never intersect, so they will be either parallel or skew. Lines m and n are *sometimes* skew. **37.** The rail that is a side for each $\angle$ is P. The answer is choice D. **38.** Since corr. $\angle$s are $\cong$, $\angle 1 \cong \angle 9$, so $m\angle 9 = 115$. Since $\angle 9$ and $\angle 16$ are suppl., $m\angle 16 = 180 - 115 = 65$. The answer is choice G. **39.** Since the two $\angle$s are same-side int. $\angle$s, they are suppl., $m\angle 10 + m\angle 7 = 180$; $x - 24 + m\angle 7 = 180$; $m\angle 7 = 180 - (x - 24) = 180 - x + 24 = 204 - x$. The answer is choice D. **40.** Since vert. $\angle$s are $\cong$, $\angle 1 \cong \angle 7$. Since corr. $\angle$s of $\parallel$ lines are $\cong$, $\angle 7 \cong \angle 5$, so $m\angle 5 = m\angle 1 = 6x$. Since $\angle 5$ and $\angle 12$ are same-side int. $\angle$s, $m\angle 5 + m\angle 12 = 180$; $6x + 4x = 180$; $10x = 180$; $x = 18$. So, $m\angle 5 = 6x = 6(18) = 108$. The answer is choice I. **41.** [2] **a.** First show that $\angle 1 \cong \angle 7$. Then show that $\angle 7 \cong \angle 5$. Finally, show that $\angle 1 \cong \angle 5$ (OR other valid solution plan). **b.** $\angle 1 \cong \angle 7$ because vert. $\angle$s are $\cong$. $\angle 7 \cong \angle 5$ because corr. $\angle$s of $\parallel$ lines are $\cong$. Finally, by the Transitive Property of Congruence, $\angle 1 \cong \angle 5$. [1] incorrect sequence of steps OR incorrect logical argument **42.** Since vert. $\angle$s are $\cong$, $m\angle IDA = m\angle YDF = 121$. **43.** $\angle YDA$ and $\angle YDF$ are suppl., so $m\angle YDA = 180 - 121 = 59$. **44.** From Exercise 43, $m\angle YDA = 59$. Since vert. $\angle$s are $\cong$, $m\angle FDI = m\angle YDA = 59$. By def. of $\angle$ bis., $m\angle RDI = \frac{1}{2}m\angle YDA = \frac{1}{2}(59) = 29.5$.

45. The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0 + 1}{2}, \frac{9 + 5}{2}\right) = \left(\frac{1}{2}, \frac{14}{2}\right) = (0.5, 7)$. **46.** The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3 + 2}{2}, \frac{8 + (-1)}{2}\right) = \left(\frac{-1}{2}, \frac{7}{2}\right) = (-0.5, 3.5)$. **47.** The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{10 + (-4)}{2}, \frac{-1 + 7}{2}\right) = \left(\frac{6}{2}, \frac{6}{2}\right) = (3, 3)$. **48.** Each term is 4 more than the previous term, so the next two terms after 16 are 16 + 4, or 20, and 20 + 4, or 24. **49.** Each term is the product of −2 and its previous term, so the next two terms after −8 are −2(−8), or 16, and −2(16), or −32. **50.** Each term is 7 less than the previous term, so the next two terms after 2 are 2 − 7, or −5, and −5 −7, or −12.

3-2 Proving Lines Parallel pages 122–129

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*
1. 11 **2.** 4 **3.** 26 **4.** 6 **5.** If a △ has a 90° ∠, then it is a right △; true. **6.** If two ∠s are ≅, then they are vert. ∠s; false. **7.** If two ∠s are suppl., then they are same-side interior ∠s; false.

Check Understanding **1.** Since 2 lines and a transversal form ≅ corr. ∠s, use the Converse of the Corr. Angles Post.: If corr. ∠s are ≅, then the lines are ∥. **2.** ∠3 and ∠4 are ≅ corr. ∠s with transversal $\overleftrightarrow{CK}$, so $\overleftrightarrow{EC} \cong \overleftrightarrow{DK}$ by the Conv. of Corr. Angles Post. **3.** If same-side exterior ∠s are ≅ or if same side interior ∠s are ≅, and if the transversal does not form 90° ∠s with either of the other two lines, then the lines are not ∥. So, the answer is no. **4.** If $a \parallel b$, then $(7x - 8) + 62 = 180$; $7x + 54 = 180$; $7x = 126$; $x = 18$. Substitute 18 for x in $7x - 8$ and see if it is the suppl. of 62: $7(18) - 8 = 126 - 8 = 118$ and $118 + 62 = 180$, so the answer checks. **5.** By def. of ⊥, all ∠s formed are 90°. So, the sum of each pair of same-side int. ∠s is 180°, making them suppl. By the Converse of the Same-Side Int. Angles Thm., the lines are ∥.

Exercises **1.** The ∠s are corr. ∠s with transversal $\overleftrightarrow{EG}$ so $\overleftrightarrow{BE} \parallel \overleftrightarrow{CG}$ by the Conv. of Corr. Angles Post. **2.** The ∠s are corr. ∠s with transversal $\overline{MR}$, so $\overline{CA} \parallel \overline{HR}$ by the Conv. of the Corr. Angles Post. **3.** The ∠s are same-side int. ∠s with transversal $\overline{JL}$, so $\overline{JO} \parallel \overline{LM}$, because, if two lines and a transversal form same-side int. ∠s that are suppl., then the lines are ∥. **4.** The ∠s are same-side int. ∠s with transversal ℓ, so $a \parallel b$. If two lines and a transversal form same-side int. ∠s that are suppl., then the lines are ∥. **5.** The ∠s are same-side int. ∠s with transversal ℓ, so $a \parallel b$. If two lines and a transversal form same-side int. ∠s that are suppl., then the lines are ∥. **6.** Only two lines intersect, so no conclusion regarding ∥ lines can be made. **7.** Since there is no information about ∥ lines, information about ∠ measures at one intersection point cannot be transferred to ∠s at other intersections. So, no conclusions about ∥ lines can be made. **8.** Since the ∠s are ≅ corr. ∠s with transversal ℓ, $a \parallel b$ by the Conv. of the Corr. Angles Post. **9.** These are vert. ∠s. No information can be transferred to ∠s surrounding another intersection point, so no conclusions about ∥ lines can be made. **10.** These are alt. int. ∠s with transversal ℓ, so $a \parallel b$ by the Conv. of the Alt. Int. Angles Post. **11.** These are corr. ∠s with transversal a, so $\ell \parallel m$ by the Conv. of the Corr. Angles Post. **12.** These are vert. ∠s. No information can be transferred to ∠s surrounding another intersection point, so no conclusions about ∥ lines can be made. **13.** These are corr. ∠s with transversal ℓ, so $a \parallel b$ by the Conv. of the Corr. Angles Post. **14.** Since there is no information about ∥ lines, information about ∠ measures at one intersection point cannot be transferred to ∠s at other intersections. So, no conclusions about ∥ lines can be made. **15.** These are alt. int. ∠s with transversal a, so $\ell \parallel m$ by the Conv. of the Alt. Int. Angles Thm. **16a.** Def. of ⊥ **16b.** Given **16c.** All right ∠s are ≅. **16d.** The ≅ ∠s are corr. ∠s with transversal t: Conv. of the Corr. Angles Post. **17a.** It's given that ∠1 and ∠2 are suppl.: ∠1. **17b.** Since ∠3 and ∠1 form a straight line, they are suppl. of each other: ∠1. **17c.** ∠2 **17d.** ∠3 **17e.** Conv. of the Corr. Angles Post. **18.** By the Corr. Angles Post., $x + 25 = 55$; $x = 30$. **19.** By the Alt. Int. Angles Thm., $2x - 5 = 95$; $2x = 100$; $x = 50$. **20.** The two ∠s are either ≅ or they are suppl. of each other. Since vert. ∠s are ≅ and since corr. ∠s of ∥ lines are ≅, the alt. ext. ∠s are ≅. So, $3x - 33 = 2x + 26$; $x - 33 = 26$; $x = 59$. **21.** Each vertical ∠ formed by ℓ and the transversal is 105°. By the Same-Side Int. Angles Thm., $(3x - 18) + 105 = 180$; $3x + 87 = 180$; $3x = 93$; $x = 31$. **22.** The ext. ∠ adjacent to the one marked $19x$ is $180 - 19x$. Since corr. ∠s are ≅, $17x = 180 - 19x$; $36x = 180$; $x = 5$. **23.** The interior ∠ adjacent to the one marked $(5x + 40)$ measures $180 - (5x + 40)$, or $140 - 5x$. Since alt. int. ∠s of ∥ lines are ≅, $2x = 140 - 5x$; $7x = 140$; $x = 20$. **24.** When the frame is put together, each corner ∠ of the frame is a right ∠. Two right ∠s are suppl. By the Conv. of the Same-Side Int. Angles Thm., opp. sides of the frame are ∥. **25.** The corr. ∠s are ≅, so the lines are ∥ by the Conv. of the Corr. Angles Post. **26a.** since the ∠s are corr. ∠s: Corr. Angles **26b and c.** ∠1, ∠3 (any order) **26d.** Corr. ∠s are ≅: Converse of the Corr. Angles. **27.** $r \parallel s$ if and only if $m\angle 1 = m\angle 2$: $80 - x = 90 - 2x$; $80 + x = 90$; $x = 10$. $m\angle 1 = m\angle 2 = 80 - x = 80 - 10 = 70$ **28.** $r \parallel s$ if and only if $m\angle 1 = m\angle 2$: $60 - 2x = 70 - 4x$; $60 + 2x = 70$; $2x = 10$; $x = 5$. $m\angle 1 = m\angle 2 = 60 - 2x = 60 - 2(5) = 60 - 10 = 50$ **29.** $r \parallel s$ if and only if $m\angle 1 = m\angle 2$: $40 - 4x = 50 - 8x$; $40 + 4x = 50$; $4x = 10$; $x = 2.5$. $m\angle 1 = m\angle 2 = 40 - 4x = 40 - 4(2.5) = 40 - 10 = 30$ **30.** $r \parallel s$ if and only if $m\angle 1 = m\angle 2$: $20 - 8x = 30 - 16x$; $20 + 16x = 30$; $16x = 10$; $x = 1.25$. $m\angle 1 = m\angle 2 = 20 - 8x = 20 - 8(1.25) = 20 - 10 = 10$ **31.** The corr. ∠s he draws are ≅. **32.** All pairs of ∠s are either ≅ or suppl. to one another. With transversal $\overline{LA}$, $\overline{PL} \parallel \overline{NA}$. With transversal $\overline{PL}$, $\overline{PN} \parallel \overline{LA}$. Both conclusions rely on the Conv. of the Same-Side Int. Angles Thm. **33.** Only two of the four same-side int. pairs of ∠s are suppl. to one another. With transversal $\overline{LA}$, $\overline{PL} \parallel \overline{NA}$ by the Conv. of the Same-Side Int. Angles Thm. **34.** No same-side int. pairs of $\angle$s are suppl. to one another, so no pairs of lines are parallel. **35.** Only two of the four same-side int. pairs of $\angle$s are suppl. to one another. With transversal $\overline{PL}$, $\overline{PN} \parallel \overline{LA}$ by the Conv. of the Same-Side Int. Angles Thm. **36.** Answers may vary. Sample: In the diagram $\overline{AB} \perp \overline{BH}$, and $\overline{AB} \perp \overline{BD}$. Since $\overline{BH}$ and $\overline{BD}$ intersect, they cannot be $\parallel$. Sample: $\overline{AB} \perp \overline{BH}$ and $\overline{AB} \perp \overline{AC}$. However, $\overline{AC}$ and $\overline{BH}$ are skew, not $\parallel$. **37.** Reflexive: $a \parallel a$ is false, since any line has all the same points as itself, so it intersects itself. Symmetric: If $a \parallel b$, then $b \parallel a$ is true, since, by def. of $\parallel$, a and b are coplanar and do not intersect. Transitive: In general, if $a \parallel b$ and if $b \parallel c$, then $a \parallel c$ is true since, if two lines are $\parallel$ to the same line, then they are $\parallel$ to each other. However, when $a \parallel b$ and $b \parallel a$, it does not follow that $a \parallel a$. **38.** Reflexive: $a \perp a$ is false since a cannot intersect itself in right $\angle$s. Symmetric: If $a \perp b$, then $b \perp a$ is true since b and a intersect to form right $\angle$s. Transitive: If $a \perp b$ and if $b \perp c$, then $a \perp c$ is false since two lines in a plane $\perp$ to a third line are $\parallel$, not $\perp$. **39.** The corr. $\angle$s are $\cong$, so the oars are $\parallel$ by the Conv. of the Corr. Angles Post. **40.** Angle must relate congruence or suppl. relations with an $\angle$ whose vertex is at another intersection. Answers may vary. Sample: $\angle 3 \cong \angle 9$; $j \parallel k$ by the Conv. of the Alt. Int. Angles Thm. **41.** Angle must relate congruence or suppl. relations with an $\angle$ whose vertex is at another intersection. Answers may vary. Sample: $\angle 3 \cong \angle 9$; $j \parallel k$ by the Conv. of the Alt. Int. Angles Thm. and $\ell \parallel m$ by the Conv. of the Same-Side Int. Angles Thm. **42.** Angle must relate congruence or suppl. relations with an $\angle$ whose vertex is at another intersection. Answers may vary. Sample: $\angle 3 \cong \angle 11$; $\ell \parallel m$ by the Conv. of the Alt. Int. Angles Thm., and $j \parallel k$ by the Conv. of the Corr. Angles Post. **43.** Angle must relate congruence or suppl. relations with an $\angle$ whose vertex is at another intersection. Answers may vary. Sample: $\angle 3$ and $\angle 12$ are suppl.; $j \parallel k$ by the Conv. of the Corr. Angles Post. **44.** Answers may vary. Sample: By the Vert. Angles Thm., $\angle 1 \cong \angle 3$. By Transitive Prop., $\angle 3 \cong \angle 7$. By the Conv. of the Corr. Angles Post., the lines are $\parallel$. **45.** It is given that $\ell \parallel m$, so $\angle 4 \cong \angle 8$ by Corr. Angles Post. It is also given that $\angle 12 \cong \angle 8$, so $\angle 4 \cong \angle 12$ by Trans. Prop. of $\cong$. So, $j \parallel k$ by the Conv. of the Corr. Angles Post.

46.

$\boxed{\ell \parallel m}$ Given → $\boxed{\angle 4 \cong \angle 8}$ Corr. $\angle$s Post.

$\boxed{\angle 12 \cong \angle 8}$ Given

→ $\boxed{\angle 4 \cong \angle 12}$ Trans. Prop. of $\cong$

→ $\boxed{j \parallel k}$ Conv. of Corr. $\angle$s Post.

47.

$\boxed{m\angle 1 + m\angle 3 = 180}$ $\angle$ Add. Post. → $\boxed{\angle 1 \text{ and } \angle 3 \text{ are supp.}}$ Def. of suppl. $\angle$s

$\boxed{\angle 1 \text{ and } \angle 2 \text{ are supp.}}$ Given

→ $\boxed{\angle 2 \cong \angle 3}$ $\cong$ Supp. Thm.

→ $\boxed{\ell \parallel m}$ Conv. of Corr. $\angle$s Post.

48.

$\boxed{a \parallel b}$ Given → $\boxed{\angle 2 \cong \angle 4}$ Alt. Int. $\angle$s Theorem

$\boxed{\angle 1 \cong \angle 2}$ Given

→ $\boxed{\angle 1 \cong \angle 4}$ Trans. Prop. of $\cong$

→ $\boxed{\ell \parallel m}$ Conv. of Corr. $\angle$s Post.

49. $\boxed{\angle 1 \text{ is supp. to } \angle 3.}$ Given → $\boxed{m\angle 1 + m\angle 3 = 180}$ Def. of suppl. ∡s

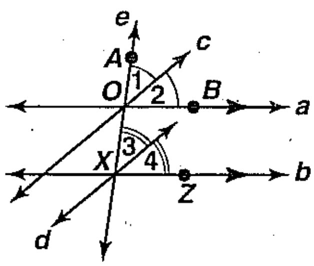
$\boxed{\ell \parallel m}$ Given → $\boxed{\angle 1 \cong \angle 4}$ Corr. ∡s Post.

→ $\boxed{m\angle 4 + m\angle 3 = 180}$ Subst.

→ $\boxed{\angle 3 \text{ is supp. to } \angle 4.}$ Def. of suppl. ∡s

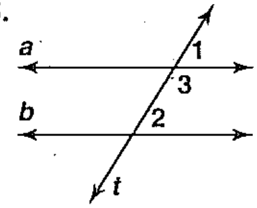
→ $\boxed{a \parallel b}$ Conv. of Same-Side Int. ∡s Thm.

50a. Draw two ∥ lines with a transversal. Then choose two corr. ∡s and bisect each of those ∡s with a ray. Answers may vary. Sample:



50b. Answers may vary. Sample: Given: $a \parallel b$ with transversal e, c bisects $\angle AOB$, d bisects $\angle AXZ$. **50c.** According to the labels in the diagram in Exercise 50a, prove: $c \parallel d$. **50d.** To prove that $c \parallel d$, show that $\angle 1 \cong \angle 3$. $\angle 1 \cong \angle 3$ if $\angle AOB \cong \angle AXZ$. $\angle AOB \cong \angle AXZ$ by the Corr. Angles Post. **50e.** ① $a \parallel b$ (Given) ② $\angle AOB \cong \angle AXZ$ (Corr. Angles Post.) ③ $m\angle AOB = m\angle AXZ$ (Def. of ≅ ∡s) ④ $m\angle AOB = m\angle 1 + m\angle 2$; $m\angle AXZ = m\angle 3 + m\angle 4$ (∠ Add. Prop.) ⑤ c bis. $\angle AOB$; d bis. $\angle AXZ$ (Given) ⑥ $m\angle 1 = m\angle 2$; $m\angle 3 = m\angle 4$ (Def. of ∠ bis.) ⑦ $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$ (Trans. Prop. of ≅) ⑧ $m\angle 1 + m\angle 1 = m\angle 3 + m\angle 3$ (Subst.) ⑨ $2m\angle 1 = 2m\angle 3$ (Add. Prop.) ⑩ $m\angle 1 = m\angle 3$ (Div. Prop.) ⑪ $c \parallel d$ (Conv. of Corr. Angles Post.) **51.** Since $b \perp c$, all the ∥ lines form right ∡s which are both ≅ and suppl. So, $d \perp a$. The answer is choice C. **52.** By the Conv. of the Corr. Angles Post., $x + 21 = 2x$; $21 = x$. The answer is choice F. **53.** The ∠ in the interior region and adjacent to the ∠ measured 136 has measure $180 - 136$, or 44. By the Alt. Int. Angles Thm., $m\angle 1 = 44$. **54.** [2] **a.** $136 + (x + 21) = 180$; $x + 157 = 180$; $x = 23$ (OR equivalent equation resulting in $x = 23$) **b.** If the corr. ∡s are =, then $c \parallel d$. $x + 21 = 2x$; $x = 21$; however, x cannot = both 21 and 23, so c is not ∥ to d. (OR equivalent explanation) [1] incorrect equations OR incorrect solutions

55.



[4] **a.** Answers may vary. Sample: **b.** $m\angle 1 + m\angle 3 = 180$, so $(2x - 38) + (6x + 18) = 180$; $8x - 20 = 180$; $8x = 200$; $x = 25$. Substituting 25 for x, the ∠ measures are $2x - 38 = 2(25) - 38 = 50 - 38 = 12$, and $x = 25$. Since $12 \neq 25$, corr. ∡s are not congruent, so a is not ∥ to b. [3] appropriate methods, but with one computational error [2] incorrect diagram solved correctly OR correct diagram solved incorrectly [1] correct answer (lines a and b are not ∥), without work shown **56.** By the Same-Side Int. Angles Thm., $m\angle 1 + 110 = 180$; $m\angle 1 = 70$. By the Same-Side Int. Angles Thm., $m\angle 2 + m\angle 1 = 180$; $m\angle 2 + 70 = 180$; $m\angle 2 = 110$. **57.** By the Alt. Int. Angles Thm., $m\angle 1 = 66$. By the Same-Side Int. Angles Thm., $m\angle 2 + 94 = 180$; $m\angle 2 = 86$. **58.** The conditional is true. To write the converse, switch the hypothesis and conclusion: If you are west of the Mississippi River, then you are in Nebraska. The converse is false, since a counterexample is that you are in Missouri. **59.** The conditional is true by def. of radius and diameter. To write the converse, switch the hypothesis and conclusion: If a circle has a radius of 4 cm, then it has a diameter of 8 cm. The converse is true. **60.** The conditional is true by the Same-Side Int. Angles Thm. To write the converse, switch the hypothesis and conclusion: If same-side int. ∡s are suppl., then a line intersects a pair of ∥ lines. The converse is also true by the Conv. of the Same-Side Int. Angles Thm. **61.** The conditional is false, since a counterexample is the verb *close*; adding *ed* to *close* creates *closeed*, which is not the past tense of *close*. To write the converse, switch the hypothesis and conclusion: If you form the past tense of a verb, then you add *ed* to a verb. The converse is false, since a counterexample is the verb *is*; its past tense is *was* and not *ised*. **62.** The conditional is true, since rain comes from clouds. To write the converse, switch the hypothesis and conclusion: If there are clouds in the sky, then it is raining. The converse is false, since it can be cloudy and not be raining. **63.** $A = \pi r^2 = \pi(8)^2 = 64\pi$. The area is about 201.1 in.^{2}. **64.** $r = \frac{d}{2} = \frac{6}{2} = 3$; $A = \pi r^2 = \pi(3)^2 = 9\pi$. The area is about 28.3 cm^2. **65.** $r = \frac{d}{2} = \frac{9}{2} = 4.5$; $A = \pi r^2 = \pi(4.5)^2 = 20.25\pi$. The area is about 63.6 ft^2. **66.** $A = \pi r^2 = \pi(5)^2 = 25\pi$. The area is about 78.5 in.^{2}. **67.** $r = \frac{d}{2} = \frac{2.8}{2} = 1.4$ m; $A = \pi r^2 = \pi(1.4)^2 = 1.96\pi$. The area is about 6.2 m^2. **68.** $A = \pi r^2 = \pi(1.2)^2 = 1.44\pi$. The area is about 4.5 m^2. **69.** $r = \frac{d}{2} = \frac{4.75}{2} = 2.375$ ft; $A = \pi r^2 = \pi(2.375)^2 = 5.640625\pi$. The area is about 17.7 ft^2. **70.** $A = \pi r^2 = \pi(0.6)^2 = 0.36\pi$. The area is about 1.1 m^2.

READING MATH *page 130*

a. The ∡s are alt. int. ∡s of ∥ lines: Alt. Int. Angles.
b and c. corr. ∡s for p and r: ∠2, ∠3 (any order)
d. ∠1 and ∠3 are alt. int. ∡s: Conv. of the Alt. Int. Angles.

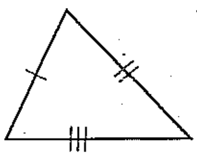
3-3 Parallel Lines and the Triangle Angle-Sum Theorem pages 131–139

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*
1. right **2.** acute **3.** acute **4.** 60 **5.** 20 **6.** 32 **7.** 58

Investigation **1.** The ∠ formed by the three ⦣ is a straight ∠. **2.** The sum of the measures of the ⦣ of a △ is 180.

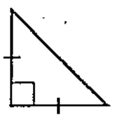
Check Understanding **1a.** $m\angle P + 58 + 90 = 180$; $m\angle P + 148 = 180$; $m\angle P = 32$ **1b.** The sum of the measures of the ⦣ of a △ is 180. If you subtract the measure of the right ∠ from 180, the result is 90. The sum of the remaining two ⦣ is 90, so they are compl.
2. For △GFH, $65 + (39 + 21) + z = 180$. Then, simplifying, $125 + z = 180$. Subtracting 125 from both sides results in $z = 55$.

3a.



The △ will have 3 acute ⦣ and no sides ≅.

3b.

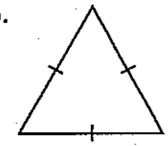


The △ will have at least 2 ≅ sides and they will also be the sides of the right ∠.

3c. Not possible; in an equilateral △, all ⦣ are acute.
4a. $m\angle 3 = 45 + 45 = 90$ **4b.** To write the converse, switch the hypothesis and conclusion: If the acute ⦣ of a △ are compl., then the △ is a right △. The statement is true since the sum of 2 compl. ⦣ is 90, leaving 90 for the measure of the third ∠. **5a.** $m\angle 1 = m\angle 2 + m\angle 3 = 33 + 97 = 130$ **5b.** Answers may vary. Sample: Find the measure of the third ∠ of the △. Subtract that measure from 180 to find $m\angle 1$.

Exercises **1.** $m\angle 1 + 117 + 33 = 180$; $m\angle 3 + 150 = 180$; $m\angle 3 = 30$ **2.** $m\angle 1 + 52.2 + 44.7 = 180$; $m\angle 1 + 96.9 = 180$; $m\angle 1 = 83.1$ **3.** $m\angle 1 + 33 + 57 = 180$; $m\angle 1 + 90 = 180$; $m\angle 1 = 90$ **4.** $m\angle T + m\angle R + m\angle G = 180$; $m\angle T + 19 + 90 = 180$; $m\angle T + 109 = 180$; $m\angle T = 71$
5. $m\angle T + m\angle L + m\angle N = 180$; $m\angle T + m\angle L + 90 = 180$; $m\angle T + m\angle L = 90$ **6.** Solve for x: $x + 30 + 80 = 180$; $x + 110 = 180$; $x = 70$. Solve for y: $y = 30 + 80 = 110$. Solve for z: $110 + 40 + z = 180$; $z + 150 = 180$; $z = 30$.
7. Solve for t: $t + 30 + 90 = 180$; $t + 120 = 180$; $t = 60$. Solve for w: $w = t = 60$. **8.** Solve for x: $x + 70 + 30 = 180$; $x + 100 = 180$; $x = 80$. Solve for y: $y = x = 80$. **9.** One of the two congruent ⦣ is one of a pair of corr. ⦣ with the ∠ labeled a. Since the lines are ∥, the 2 ≅ ⦣ measure a. $2a + 40 = 180$; $a + 20 = 90$; $a = 70$ **10.** Since alt. int. ⦣ of ∥ lines are ≅, the missing measure of the third ∠ of the bottom △ is x. $x + 2x + 90 = 180$; $3x + 90 = 180$; $x + 30 = 60$; $x = 30$ **11.** Since corr. ⦣ of ∥ lines are ≅, the missing measure of the third ∠ of the top △ is c. $c + 30 + 90 = 180$; $c + 120 = 180$; $c = 60$ **12.** The ∠ measures are about 50, 65, and 65, so, since they are all acute, the △ is acute. The side measures are 1.8 cm, 2.1 cm, and 2.1 cm, so, since 2 sides are =, the △ is isosceles. **13.** All the ⦣ measure 60, so the △ is equiangular and acute. All the sides measure 2.25 cm, so the △ is equilateral. **14.** The ⦣ measure 30, 60, and 90, so, since there is a right ∠, the △ is right. The sides measure 1.3 cm, 2.6 cm, and 2.25 cm, so, since none of the measures are =, the △ is scalene. **15.** The ⦣ measure 35, 35 and 110, so, since there is one obtuse ∠, the △ is obtuse.
The sides measure 1.8 cm, 1.8 cm, and 3 cm, so, since two sides are =, the △ is isosceles.

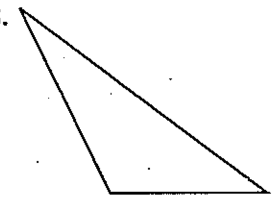
16.



All ⦣ must be acute and all sides must be =.

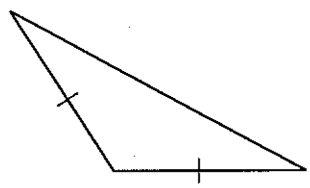
17. not possible, since every right △ has one longest side opp. the right ∠

18.



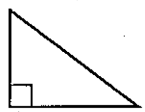
One ∠ must be obtuse and no sides can be =.

19.



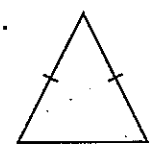
One ∠ must be obtuse, and the sides of that ∠ are the sides of the obtuse ∠.

20.



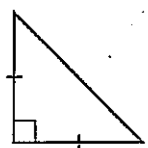
One ∠ must be right and no sides are =.

21.



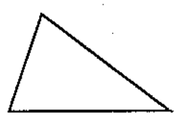
All ⦣ are acute and at least 2 sides are =.

22.



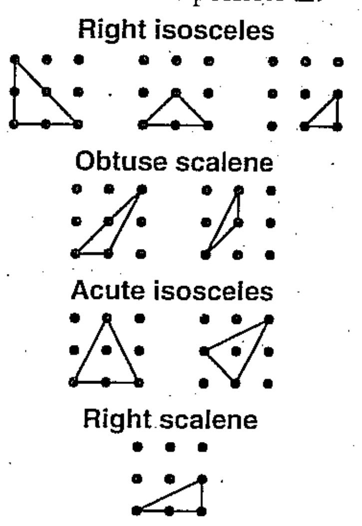
One ∠ is right and the sides of the right ∠ are =.

23.



All ⦣ are acute and no sides are =. **24a.** An exterior ∠ is an ∠ formed by a side and an extension of an adjacent side: ∠5, ∠6, ∠8.
24b. The remote interior ⦣ are the two nonadjacent interior ⦣: ∠1 and ∠3 for ∠5, ∠1 and ∠2 for ∠6, and ∠1 and ∠2 for ∠8. **24c.** They are ≅ vert. ⦣. **25a.** Each vertex has 2 exterior ⦣. The other two ⦣ are an interior ∠ and its vert. ∠. **25b.** There are 2 at each of its 3 vertices, so there are a total of 2(3), or 6 ext. ⦣.
26. $m\angle 1 = 60 + 63 = 123$ **27.** $m\angle 2 + 13 = 128.5$; $m\angle 2 = 115.5$ **28.** $m\angle 3 = 45 + 47 = 92$; $m\angle 4 = 180 - 92 = 88$ **29.** $x = 90 + 57 = 147$; $y = 180 - x = 180 - 147 = 33$ **30.** $a = 90 + 72 = 162$; $b = 180 - a = 180 - 162 = 18$ **31.** $x + x + 75 = 180$; $2x + 75 = 180$;

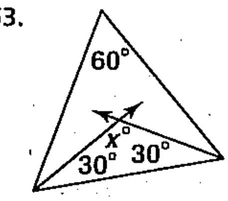
$2x = 105; x = 52.5$. The $\angle$ measures are 52.5, 52.5, and 75. Since all $\angle$s are acute, the $\triangle$ is acute. **32.** Solve for x: $(8x - 1) + (4x + 7) + 90 = 180; 12x + 96 = 180;$ $12x = 84; x = 7$. Solve for the $\angle$ measures: $8x - 1 = 8(7) - 1 = 56 - 1 = 55; 4x + 7 = 4(7) + 7 = 28 + 7 = 35$; so the $\angle$ measures are 55, 35, and 90. Since one $\angle$ is right, the $\triangle$ is a right $\triangle$. **33.** Solve for x: $(x) + (2x + 4) + (2x - 9) = 180; 5x - 5 = 180; 5x = 185; x = 37$. Solve for the $\angle$ measures: $x = 37; 2x + 4 = 2(37) + 4 = 74 + 4 = 78; 2x - 9 = 2(37) - 9 = 74 - 9 = 65$; so the $\angle$ measures are 37, 65, and 78. Since all $\angle$s are acute, the $\triangle$ is acute. **34.** $\angle BDA$ is an exterior $\angle$ of $\triangle BDC$, so $x + 52 = 90; x = 38$. In $\triangle BDA$ $y + 90 + 54 = 180; y + 144 = 180; y = 36$. $\angle BDA$ and $\angle BDC$ are suppl., so $z + 90 = 180; z = 90$. $\triangle ABD$ $\angle$ measures are 36, 90, and 54. Since there is a right $\angle$, the $\triangle$ is right. $\triangle BDC$ $\angle$ measures are 90, 52, and 38. Since there is a right $\angle$, the $\triangle$ is right. $\triangle ABC$ $\angle$ measures are $y + x = 36 + 38 = 74$, 52, and 54. Since all $\angle$s are acute, the $\triangle$ is acute. **35.** Solve for b: $b + 32 = 90; b = 58$. In $\triangle FGH$, $55 + b + a = 180; 55 + 58 + a = 180; 113 + a = 180;$ $a = 67$. Solve for c: $c + 55 = 180; c = 125$. Solve for d: $d + 32 + c = 180; d + 32 + 125 = 180; d + 157 = 180;$ $d = 23$. Solve for e: $e + 90 = 180; e = 90$. $\triangle FGH$ $\angle$ measures are 58, 67, and 55. Since all $\angle$s are acute, the $\triangle$ is acute. $\triangle FEH$ $\angle$ measures are 125, 32, and 23. Since it has an obtuse $\angle$, the $\triangle$ is obtuse. $\triangle EFG$ $\angle$ measures are 67, 23, and 90. Since it has a right $\angle$, the $\triangle$ is right. **36.** By the Angle Add. Post., $(x) + (30) + (2y - 6) = 180$, and by the Alt. Int. Angles Thm., $y = x + 30$. Substitute $x + 30$ for y into the first equation: $x + 30 + 2(x + 30) - 6 = 180; x + 30 + 2x + 60 - 6 = 180; 3x + 84 = 180; 3x = 96; x = 32$, and $y = x + 30 = 32 + 30 = 62$. By the Alt. Int. Angles Thm., $z = x = 32$. By the Angle Add. Post., $w + y = 180; w + 62 = 180;$ $w = 118$. $\triangle ILK$ $\angle$ measures are 118, 32, and 30. Since it has an obtuse $\angle$, the $\triangle$ is obtuse. **37.** If each $\angle$ measures x, then $3x = 180$, so $x = 180 \div 3 = 60$. **38.** Yes; by def., an isosc. $\triangle$ has at least 2 sides $\cong$, so if three sides are $\cong$, then at least 2 sides are $\cong$. Not every isosc. $\triangle$ is equilateral because the third side of an isosc. $\triangle$ does not need to be $\cong$ to the other two sides.
39. There are 8 possible $\triangle$s.



40. $x + x + 115 = 180;$ $2x + 115 = 180; 2x = 65;$ $x = 32.5$ **41.** Check students' work. Answers may vary. Sample: The two exterior $\angle$s formed at vertex A are vertical $\angle$s, so they have the same measure. **42.** Let n be the measure of one $\angle$, then $2n$ is the measure of the other $\angle$. $n + 2n + 90 = 180;$ $3n + 90 = 180;$ $n + 30 = 60; n = 30$. So, one $\angle$ measures 30 and the other measures 60.

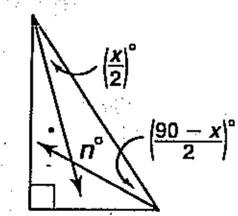
43a. Let $2x$, $3x$, and $4x$ represent the measures of the three $\angle$s. Then, $2x + 3x + 4x = 180; 9x = 180; x = 20$. Since $2x = 2(20) = 40, 3x = 3(20) = 60$, and $4x = 4(20) = 80$, the $\angle$ measures are 40, 60, and 80. **43b.** Since the $\angle$s are all acute, the $\triangle$ is acute. **44.** By the Ext. Angle Thm., $m\angle 5 = 130 + 30 = 160$. **45.** By the Ext. Angle Thm., $m\angle 5 = m\angle 3 + m\angle 4; 130 = m\angle 3 + 30; m\angle 3 = 100$. **46.** By the Angle Add. Post., $m\angle 3 = 180 - m\angle 1$. By the Ext. Angle Thm., $m\angle 5 = m\angle 3 + m\angle 4$. By substitution, $m\angle 5 = (180 - m\angle 1) + m\angle 4; 142 = 180 - m\angle 1 + 65;$ $142 = 245 - m\angle 1; m\angle 1 + 142 = 245; m\angle 1 = 103$. **47.** By the Triangle Angle-Sum Thm., $m\angle 2 + 125 + 23 = 180; m\angle 2 + 148 = 180; m\angle 2 = 32$. **48a.** 90 **48b.** 180 **48c.** Substitute 90 for $m\angle C$ and then subtract 90 from both sides: 90. **48d.** By def. of compl. $\angle$s, their sum is 90: complementary. **48e.** complementary **49a.** $\angle$ Add. **49b.** Triangle Angle-Sum **49c.** Answers may vary. Possible answers: substitution or transitive **49d.** Subtract $m\angle 4$ from both sides: subtraction. **50.** By the Triangle Angle-Sum Thm., $x + 64 + 48 = 180;$ $x + 108 = 180; x = 72$. So, the $\triangle$ $\angle$ measures are 72, 64, and 48. Their ext. $\angle$s are their supplements which are $180 - 72$, or 108, $180 - 64$, or 116, and $180 - 48$, or 132. Since $108 < 116 < 132$, the largest exterior $\angle$ is 132. **51.** Check students' work. **52a.** By the Triangle Angle-Sum Thm., $5\sqrt{x} + 7\sqrt{x} + 8\sqrt{x} = 180; 20\sqrt{x} = 180;$ $\sqrt{x} = 9; (\sqrt{x})^2 = 9^2; x = 81$. **52b.** $5\sqrt{x} = 5\sqrt{81} = 5(9) = 45; 7\sqrt{x} = 7\sqrt{81} = 7(9) = 63; 8\sqrt{x} = 8\sqrt{81} = 8(9) = 72$ **52c.** Since all $\angle$s are acute, the $\triangle$ is acute.

53.



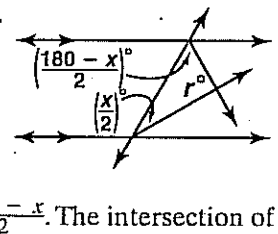
From Exercise 37, the measure of each $\angle$ of an equiangular $\triangle$ is 60. The two $\angle$ bisectors create $\angle$s measuring 30. The two rays form a $\triangle$ inside the equiangular $\triangle$. By the Triangle Angle-Sum Thm., $30 + 30 + x = 180; x + 60 = 180;$ $x = 120$. So, the $\angle$s formed measure 120 and 60.

54.



If one of the acute $\angle$s of the right $\triangle$ measures x, then the other acute $\angle$ measures $90 - x$. The $\angle$ bisectors form $\angle$s measuring $\frac{x}{2}$ and $\frac{90 - x}{2}$. The $\angle$ bisectors intersect to form a $\triangle$ inside the right $\triangle$. If the rays form an $\angle$ that measures n, then $n + \frac{x}{2} + \frac{90 - x}{2} = 180; n + \frac{x + 90 - x}{2} = 180; n + \frac{90}{2} = 180; n + 45 = 180;$ $n = 135$. So, the $\angle$s formed measure 135 and 45.

55.



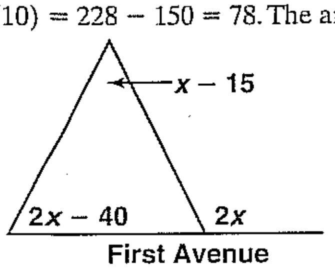
If the measure of one of the $\angle$s is x, then the other $\angle$ on the same side of the transversal measures $180 - x$. The $\angle$ bisectors form $\angle$s measuring $\frac{x}{2}$ and $\frac{180 - x}{2}$. The intersection of the $\angle$ bisectors form a $\triangle$ and the measure of the $\angle$ they form is r. By the Triangle Angle-Sum Thm., $r + \frac{x}{2} + \frac{180 - x}{2} = 180;$

$r + \frac{x + 180 - x}{2} = 180; r + \frac{180}{2} = 180; r + 90 = 180;$ $r = 90$. **56.** There are two lines ⊥ to the equator forming right ∠s. If the ∠ at one of the poles measures p, then the sum of the ∠s is $p + 90 + 90 = p + 180$. Since p has positive measure, the sum of the ∠s is greater than 180. **57.** Possible △s have ∠ measures of: 30-30-120, 30-60-90, and 60-60-60. There are 3 possible △s and only 1 is equiangular, so the probability is $\frac{1}{3}$. **58.** Possible △s have ∠ measures of: 20-20-140, 20-40-120, 20-60-100, 20-80-80, 40-40-100, 40-60-80, 60-60-60. There are 7 possible triangles and only 1 is equiangular, so the probability is $\frac{1}{7}$. **59.** There is only one possible △ having ∠ measures 60-60-60. So, the probability is $\frac{1}{1}$, or 1.

60. Possible △s have ∠ measures of 12-12-156, 12-24-144, 12-36-132, 12-48-120, 12-60-108, 12-72-96, 12-84-84, 24-24-132, 24-36-120, 24-48-108, 24-60-96, 24-72-84, 36-36-108, 36-48-96, 36-60-84, 36-72-72, 48-48-84, 48-60-72, 60-60-60. There are 19 possible △s and only one equiangular △, so the probability is $\frac{1}{19}$.

61. Each ∠ in an equiangular △ measures 60, so the greatest possible measure of an ∠ in an equiangular △ is 60. Obtuse ∠s measure more than 90, which is greater than 60. An equiangular △ with an obtuse ∠ is impossible, so the probability is 0. **62.** $3x - 2 = 5x - 20$; $-2x - 2 = -20$; $-2x = -18$; $x = 9$. $5x - 20 = 5(9) - 20 = 45 - 20 = 25$. By the Exterior Angle Thm., $m\angle DBF = 90 + 25 = 115$. **63.** Answers may vary. Sample: By the Triangle Ext. Angle Thm., the measure of the ext. ∠ is = to the sum of the measures of the two remote int. ∠s. If each of the remote int. ∠s measures x, then the ext. ∠ measures $2x$, and its bisector divides it into 2 ∠s each measuring x. Now, alt. int. ∠s are ≅. (Also, corr. ∠s are ≅.) So, the bisector is ∥ to the included side of the remote interior ∠s. **64.** The sum of the ∠s for A are $6a$ and 180 is evenly divisible by 6, so they are all whole numbers. The sum of the ∠s for B are $8b$; $180 \div 8 = 22.5$; one of the ∠s is $b = 22.5$, which is not a whole number. The ∠ measures for choices C and D also produce whole-number measures. The answer is choice B. **65.** By the Ext. Angle Thm., $m\angle JKM = 25 + 43 = 68$. The answer is choice G. **66.** By the Triangle Angle-Sum Thm., $4x + 5x + 6x = 180$; $15x = 180$; $x = 12$. $m\angle JKM = 180 - 6x = 180 - 6(12) = 180 - 72 = 108$. The answer is choice B. **67.** By the Ext. Angle Thm., $15x - 48 = 40 + (5x + 12)$; $15x - 48 = 5x + 52$; $10x - 48 = 52$; $10x = 100$; $x = 10$. $m\angle JKL = 180 - (15x - 48) = 180 - 15x + 48 = 228 - 15x = 228 - 15(10) = 228 - 150 = 78$. The answer is choice H.

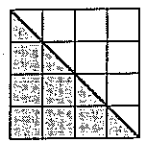
68. [2] **a.**



b. By the Ext. Angle Thm., $2x = (x - 15) + (2x - 40)$; $2x = 3x - 55$; $-x = -55$; $x = 55$. The three interior ∠s measure $180 - 2x = 180 - 2(55) = 180 - 110 = 70$; $x - 15 = 55 - 15 = 40$; $2x - 40 = 2(55) - 40 = 110 - 40 = 70$.

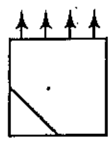
[1] incorrect sketch, equation, OR solution **69.** [2] **a.** By the Triangle Angle-Sum Thm., $m\angle Y + m\angle M + m\angle F = 180$. Substituting 21 for $m\angle F$ results in $m\angle Y + m\angle M + 21 = 180$, so, subtracting 21 from both sides results in $m\angle Y + m\angle M = 159$. **b.** From part ⓐ, $m\angle Y + m\angle M = 159$. Since $\angle Y$ is obtuse, its whole-number range is from 91 to 158, allowing the measure of 1 for $m\angle M$ when $m\angle Y = 158$. When $m\angle Y = 91$, then $m\angle M = 68$. So, the range for $m\angle M$ is from 1 to 68. [1] incorrect answer to part ⓐ or ⓑ OR incorrect computation in either part **70.** By the Alt. Int. Angles Thm., $2x + 18 = 124$; $x + 9 = 62$; $x = 53$. **71.** The interior ∠ adjacent to the 44° ∠ measures 180 − 44, or 136. By the Corr. Angles Post., $3x - 2 = 136$; $3x = 138$; $x = 46$. **72.** By the Angle Addition Post., $(3x + 20) + (x + 32) = 80$; $4x + 52 = 80$; $4x = 28$; $x = 7$.

73.



The side of each subsequent square measures 1 more than the previous square. The left bottom △ is tinted.

74.



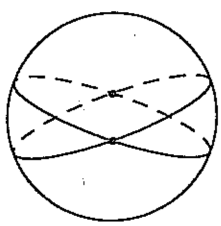
Each square is rotated 90° clockwise and an arrow is added.

CHECKPOINT QUIZ 1 *page 139*

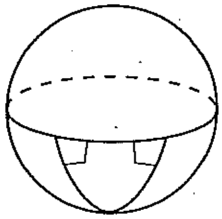
1. The two ∠s are corr. ∠s: Corr. Angles Post. **2.** Use a converse thm. or post. to prove ∥ lines; the two ∠s are corr. ∠s: Conv. of Corr. Angles Post. **3.** The ∠s are on the same side of the transversal to ∥ lines: Same-Side Int. Angles Thm. **4.** Use a converse thm. or post. to prove ∥ lines; the two ∠s are alt. int. ∠s: Conv. of the Alt. Int. Angles Thm. **5.** The two ∠s are vertical ∠s: Vertical Angles Thm. **6.** The two ∠s are alt. int. ∠s of ∥ lines: Alt. Int. Angles Thm. **7.** Use a converse thm. or post. to prove ∥ lines; the two ∠s are corr. ∠s: Converse of Corr. Angles Post. **8.** The two ∠s are corr. ∠s of ∥ lines: Corr. Angles Post. **9.** Use a converse thm. or post. to prove ∥ lines; the two ∠s are same-side int. ∠s: Conv. of Same-Side Int. Angles Thm. **10.** Use the Triangle Angle-Sum Thm. on the left △: $38 + (2x - 23) + x = 180$; $3x + 15 = 180$; $x + 5 = 60$; $x = 55$; $2x - 23 = 2(55) - 23 = 110 - 23 = 87$; the ∠ measures of the left △ are 38, 55, 87, and since all ∠s are acute, the △ is acute. By the Vert. Angles Thm., $y = x = 55$. Use the Triangle Angle-Sum Thm. on the △ on the right: $55 + x + (4w - 5) = 180$; $5w + 50 = 180$; $5w = 130$; $w = 26$; $4w - 5 = 4(26) - 5 = 104 - 5 = 99$; the ∠ measures of the △ on the right are 55, 26, 99, and since one ∠ is obtuse, the △ is obtuse.

EXTENSION *pages 140–141*

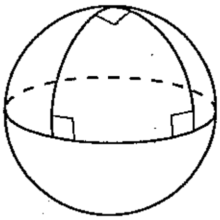
1.



Answers may vary. Sample: In spherical geometry, more than one line can be drawn through any two points, and in Euclidean geometry only one line can be drawn through any two points.

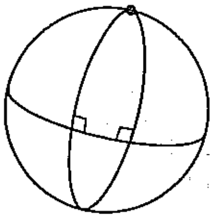
2. 

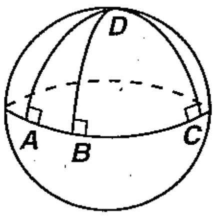
Answers may vary. Sample: In spherical geometry, a $\triangle$ can have more than 1 right $\angle$, and in Euclidean geometry, a $\triangle$ can have no more than 1 right $\angle$.

3. 

Answers may vary. Sample: In spherical geometry, the measure of an $\angle$ in an equiangular $\triangle$ can be a measure other than 60, and in Euclidean geometry, the measure of an $\angle$ in an equiangular $\triangle$ always measures 60.

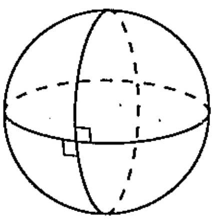
4. Show spherical lines that intersect at "poles." Show another circle that intersects the first two circles half way between the poles. Answers may vary. Sample:



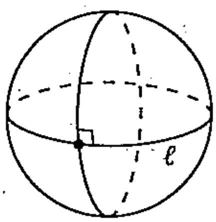
5. 

Lines of longitude form several $\triangle$s with the equator whose base $\angle$s are congruent right $\angle$s, but the vertex $\angle$ varies. Answers may vary. Sample: $\angle ADB$ is not congruent to $\angle ADC$.

6. At least one of the 2 curves is not a great circle, so it is not a line. **7.** In Euclidean geometry, a line segment is part of a line. The same is true in spherical geometry. Since the top circle is not a line, a piece of the top circle can not be a line segment.

8. 

Vertical $\angle$s are also congruent in spherical geometry.

9. 

True, only one line is $\perp$ to a line through a point on the line.

TECHNOLOGY *page 142*

1. The sum of the measures of the exterior $\angle$s of a convex polygon is always 360. **2.** Check students' work. **3.** The sum of the measures of the five $\angle$s meeting at one point is 360. **4a.** Check students' work. **4b.** Polygons with 3 and 6 sides can tile a plane. Polygons with 5 and 8 sides do not. **4c.** 3 sides: 120°; 4 sides: 90°; 5 sides: 72°; 6 sides: 60°; 8 sides: 45° **4d.** Since the exterior $\angle$s of the shapes that tile a plane are 120°, 90°, and 60°, a conjecture is: To tile a plane, the exterior $\angle$ measure must be divisible by 30. **4e.** Check students' work.

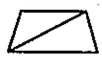
3-4 The Polygon Angle-Sum Theorems *pages 143–150*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*
1. $m\angle DAB = 77; m\angle B = 65; m\angle BCD = 131; m\angle D = 87$ **2.** $m\angle D = m\angle B = 60; m\angle DAB = m\angle DCB = 120$ **3.** $m\angle A = 70; m\angle ABC = 85; m\angle C = 125; m\angle ADC = 80$

Check Understanding **1.** ABE; sides: $\overline{AB}, \overline{BE}, \overline{EA}$; $\angle$s: $\angle A, \angle ABE, \angle BEA$. $BCDE$; sides: $\overline{BC}, \overline{CD}, \overline{DE}, \overline{EB}$; $\angle$s: $\angle EBC, \angle C, \angle D, \angle DEB$. $ABCDE$; sides: $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DE}, \overline{EA}$; $\angle$s: $\angle A, \angle ABC, \angle C, \angle D, \angle AED$ **2a.** There are 6 sides, so it is a hexagon. All diagonals are in the interior of the figure, so it is convex. **2b.** There are 8 sides, so it is an octagon. At least one diagonal has points outside the figure, so it is concave. **2c.** Each point has 2 sides, so it is a 24-gon. Diagonals from one star point to an adjacent point are outside the figure, so it is concave.

Investigation

1.

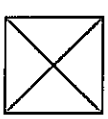
Polygon	No. of Sides	No. of Triangles	Sum of Int. $\angle$ Measures
	4	2	2·180 = 360
	5	3	3·180 = 540
	6	4	4·180 = 720
	7	5	5·180 = 900
	8	6	6·180 = 1080

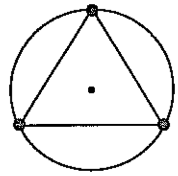
Answers may vary. Sample: The number of $\triangle$s formed is two less than the number of sides. The sum of the $\angle$s increases by 180 from figure to figure. **2.** The sum of the measures of the $\angle$s of an n-gon is $(n - 2) \cdot 180$.

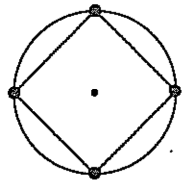
Check Understanding **3a.** $(n - 2)180 = (13 - 2)180 = (11)180 = 1980$ **3b.** You can solve the equation $(n - 2)180 = 720$. **4.** Divide the sum of all the $\angle$s by 5: $\frac{(n-2)180}{5} = \frac{(5-2)180}{5} = \frac{(3)180}{5} = (3)(36) = 108$. **5.** From Example 5, the measure of one interior $\angle$ is 120. By the Angle Addition Post., $2m\angle 2 + 120 = 180$; $2m\angle 2 = 60$; $m\angle 2 = 30$. $\angle 2$ is not an exterior $\angle$ because it is not formed by extending one side of the polygon.

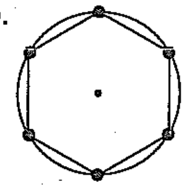
Exercises **1.** All sides are connected and do not intersect between endpoints, so the figure is a polygon. **2.** The figure is not a polygon because it does not have sides made of segments. **3.** The figure is not a polygon because it is not a plane figure. **4.** The figure is not a polygon because two of its sides intersect between endpoints. **5.** $MWBFX$; sides: $\overline{MW}, \overline{WB}, \overline{BF}, \overline{FX}, \overline{XM}$; $\angle$s: $\angle M, \angle W, \angle B, \angle F, \angle X$ **6.** $KCLP$; sides: $\overline{KC}, \overline{CL}, \overline{LP}, \overline{PK}$; $\angle$s: $\angle K, \angle C, \angle L, \angle P$ **7.** $HEPTAGN$; sides: $\overline{HE}, \overline{EP}, \overline{PT}, \overline{TA}, \overline{AG}, \overline{GN}, \overline{NH}$; $\angle$s: $\angle H, \angle E, \angle P, \angle T, \angle A, \angle G, \angle N$ **8.** The hole in the middle is a 5-sided figure: pentagon. No diagonals are outside the figure, so the pentagon is convex. **9.** The star has 5 points, two sides to each point, so it has 10 sides. It is a decagon. Since diagonals from one point to an adjacent point are outside the figure, the decagon is concave. **10.** The flag has 5 sides, so it is a pentagon. Since one of the diagonals is outside the figure, the pentagon is concave. **11.** The figure has 8 sides, so $n = 8$: $(n - 2)180 = (8 - 2)180 = (6)180 = 1080$. **12.** A dodecagon has 12 sides, so $n = 12$: $(n - 2)180 = (12 - 2)180 = (10)180 = 1800$. **13.** A decagon has 10 sides, so $n = 10$: $(n - 2)180 = (10 - 2)180 = (8)180 = 1440$. **14.** The figure has 20 sides, so $n = 20$: $(n - 2)180 = (20 - 2)180 = (18)180 = 3240$. **15.** The figure has 1002 sides, so $n = 1002$: $(n - 2)180 = (1002 - 2)180 = (1000)180 = 180{,}000$. **16.** $(n - 2)180 = (4 - 2)180 = (2)180 = 360$; $y + 120 + 85 + 53 = 360$; $y + 258 = 360$; $y = 102$ **17.** $(n - 2)180 = (5 - 2)180 = (3)180 = 540$; $x + 100 + 117 + 105 + 115 = 540$; $x + 437 = 540$; $x = 103$ **18.** $(n - 2)180 = (7 - 2)180 = (5)180 = 900$; $y + 120 + 116 + 129 + 130 + 135 + 125 = 900$; $y + 755 = 900$; $y = 145$ **19.** $(n - 2)180 = (3 - 2)180 = (1)180 = 180$; $a + 62 + 81 = 180$; $a + 143 = 180$; $a = 37$ **20.** $(n - 2)180 = (4 - 2)180 = (2)180 = 360$; $h + h + 2h + 2h = 360$; $6h = 360$; $h = 60$; $2h = 2(60) = 120$. The $\angle$s are 60, 60, 120, and 120. **21.** $(n - 2)180 = (6 - 2)180 = (4)180 = 720$; $n + (n + 6) + 135 + 62 + 151 + 140 = 720$; $2n + 494 = 720$; $2n = 226$; $n = 113$; $n + 6 = 113 + 6 = 119$. The missing $\angle$s are 113 and 119. **22.** The measure of one interior $\angle$ is $\frac{(n-2)180}{n} = \frac{(5-2)180}{5} = \frac{(3)180}{5} = (3)(36) = 108$. The measure of one exterior $\angle$ is $\frac{360}{n} = \frac{360}{5} = 72$. **23.** The measure of one interior $\angle$ is $\frac{(n-2)180}{n} = \frac{(12-2)180}{12} = \frac{(10)180}{12} = \frac{1800}{12} = 150$. The measure of one exterior $\angle$ is $\frac{360}{n} = \frac{360}{12} = 30$.

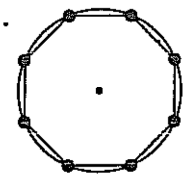
24. The measure of one $\angle$ is $\frac{(n-2)180}{n} = \frac{(18-2)180}{18} = \frac{(16)180}{18} = (16)10 = 160$. The measure of one exterior $\angle$ is $\frac{360}{n} = \frac{360}{18} = 20$. **25.** The measure of one interior $\angle$ is $\frac{(n-2)180}{n} = \frac{(100-2)180}{100} = \frac{(98)180}{100} = \frac{17{,}640}{100} = 176.4$. The measure of each exterior $\angle$ is $180 - 176.4 = 3.6$.
26. Each acute $\angle$ of the right $\triangle$ in the corner is an exterior $\angle$ of the octagon with measure $\frac{360}{8}$, or 45. So, the measure of each $\angle$ of a cheese wedge is 45, 45, and 90.

27.  To be regular, the sides must all be $\cong$ and the $\angle$s must all be $\cong$.

28.  $360 \div 3 = 120$, so mark a point at 0, 120, and 240.

29.  $360 \div 4 = 90$, so mark a point at 0, 90, 180, and 270.

30.  $360 \div 6 = 60$, so mark a point at 0, 60, 120, 180, 240, and 300.

31.  $360 \div 8 = 45$, so mark a point at 0, 45, 90, 135, 180, 225, 270, and 315.

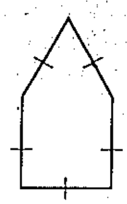
32. $(n - 2)180 = 180$; $n - 2 = 1$; $n = 3$ **33.** $(n - 2)180 = 1080$; $n - 2 = 6$; $n = 8$ **34.** $(n - 2)180 = 1980$; $n - 2 = 11$; $n = 13$ **35.** $(n - 2)180 = 2880$; $n - 2 = 16$; $n = 18$ **36a.** A $\triangle$ has 3 vertices, so 3 letters are needed. The possible names are $ABC, BCA, CAB, CBA, ACB, BAC$, so there are 6 ways to name the $\triangle$. **36b.** A quadrilateral has 4 vertices, so 4 letters are needed. The possible names are $ABCD, BCDA, CDAB, DABC, DCBA, ADCB, BADC, CBAD$, so there are 8 ways to name the quadrilateral. **36c.** A pentagon has 5 vertices, so 5 letters are needed. The possible names are $ABCDE, BCDEA, CDEAB, DEABC, EABCD, EDCBA, AEDCB, BAEDC, CBAED, DCBAE$, so there are 10 ways to name the pentagon. **37.** It has 8 sides, so it is an octagon. The measure of each interior $\angle$ is $\frac{(n-2)180}{n} = \frac{(8-2)180}{8} = \frac{(6)180}{8} = (6)(22.5) = 135$, so $m\angle 1 = 135$. $m\angle 2 = m\angle 1 - 90 = 135 - 90 = 45$
38. If you solve $\frac{(n-2)180}{n} = 130$, $n = 7.2$. The number of sides of a polygon must be a positive integer, and 7.2 is not an integer. **39.** If two of the exterior $\angle$s are 100, then two interior $\angle$s are 80 and 80, so the remaining $\angle$ must be $180 - 80 - 80 = 180 - 160 = 20$. The $\angle$s of one of the possible $\triangle$s are 20-80-80. If only one of the exterior $\angle$s is 100, then one $\angle$ is 80 and the other two $\angle$s total 100; since they are $\cong$, they must each be 50. The $\angle$s of the other possible $\triangle$ are 50-50-80. **40.** The interior $\angle$ measures $180 - 72 = 108$. Use the exterior measure to find the number of sides: $\frac{360}{n} = 72$; $72n = 360$; $n = 5$.
41. The interior $\angle$ measures $180 - 36 = 144$. Use the exterior measure to find the number of sides: $\frac{360}{n} = 36$; $36n = 360$; $n = 10$. **42.** The interior $\angle$ measures $180 - 18 = 162$. Use the exterior measure to find the number of sides: $\frac{360}{n} = 18$; $18n = 360$; $n = 20$. **43.** The

interior $\angle$ measures $180 - 30 = 150$. Use the exterior measure to find the number of sides: $\frac{360}{n} = 30$; $30n = 360$; $n = 12$. **44.** The interior $\angle$ measures $180 - x$. Use the exterior measure to find the number of sides: $\frac{360}{n} = x$; $nx = 360$; $n = \frac{360}{x}$. **45.** Using the formula $\frac{(n-2)180}{n}$ for $n = 3, 4, 5, 6, 7, 8, 9, 10, 11$, and 12 results in the interior $\angle$ measures of 60, 90, 108, 120, $128\frac{4}{7}$, 135, 140, 144, $147\frac{3}{11}$, and 150, respectively. Of the 10 possible measures, 8 are integers, so the probability is $\frac{8}{10}$, or $\frac{4}{5}$. **46a.** The measure of a straight $\angle$ is 180, so n straight $\measuredangle$s measure $n \cdot 180$, or $180n$. **46b.** According to the formula, the sum of the measures is $(n - 2)180$. **46c.** The sum of all of the straight $\measuredangle$s minus the sum of all of the interior $\measuredangle$s is the sum of the exterior $\measuredangle$s: $180n - 180(n - 2) = 180n = 180n + 360 = 360$. **46d.** Polygon Exterior Angle-Sum Thm. **47.** Solve for z: $z + 110 = 180$; $z = 70$. Solve for y: $y + z + 100 + 87 = (n - 2)180$; $y + 70 + 100 + 87 = (4 - 2)180$; $y + 257 = 360$; $y = 103$. The polygon has 4 sides, so it is a quadrilateral. **48.** By the Exterior Angle-Sum Thm., $z + (z - 13) + (z + 10) = 360$; $3z - 3 = 360$; $z - 1 = 120$; $z = 121$. Solve for w: $w = 180 - (z - 13) = 180 - (121 - 13) = 180 - 108 = 72$. Solve for x: $x = 180 - z = 180 - 121 = 59$. Solve for y: $y = 180 - (z + 10) = 180 - (121 + 10) = 180 - 131 = 49$. The figure has 3 sides, so it is a $\triangle$. **49.** There is one exterior $\angle$ at each vertex, so $x + 2x + 3x + 4x = 360$; $10x = 360$; $x = 36$. Solve for $2x$: $2(36) = 72$. Solve for $3x$: $3(36) = 108$. Solve for $4x$: $4(36) = 144$. The figure has 4 sides, so it is a quadrilateral.

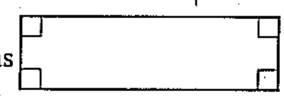
50. Sketch a quadrilateral that is not a square or rectangle. Answers may vary. Sample:

51. Sketch a rectangle that is not a square. Answers may vary. Sample:

52. Sketch a squished polygon. Answers may vary. Sample:

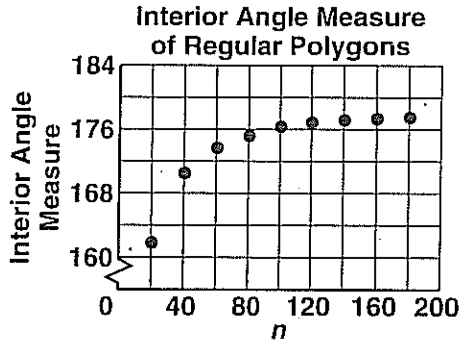


53. Answers may vary. Sample: Sketch a regular polygon that has opp. sides $\parallel$, and then stretch the $\parallel$ sides equally.



54. Yes; the sum of the measures of the $\measuredangle$s at the interior pt. is 360. The sum of the measures of all the $\measuredangle$s is $180n$. $180n - 360 = 180(n - 2) = (n - 2)180$ **55.** Answers may vary. Sample: The figure is a convex equilateral quadrilateral. The sum of its $\measuredangle$s is $(4 - 2)180 = (2)180 = 360$. The sum of the exterior angles is 360. **56.** $\frac{(n-2)180}{n} = 3\left(\frac{360}{n}\right)$; $\frac{(n-2)180}{n} = \frac{3(360)}{n}$; $(n - 2)180 = 3(360)$; $n - 2 = 3(2)$; $n - 2 = 6$; $n = 8$. The figure has 8 sides, so it is an octagon. **57a.** $\left(n, \frac{(n-2)180}{n}\right)$ for each n listed is (20, 162), (40, 171), (60, 174), (80, 175.5), (100, 176.4), (120, 177), (140, $177\frac{3}{7}$), (160, 177.75), (180, 178), (200, 178.2).

57b.



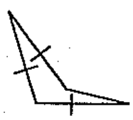
57c. The measure of one interior $\angle$ is very close to 180. **57d.** An $\angle$ that measures 180 is a straight $\angle$, and the rays that make up its sides are collinear. Since two sides cannot be collinear, no polygon has an $\angle$ of 180. **58a.** $[180(n - 2) \div n = \frac{(n-2)180}{n} = \frac{180n - 360}{n} = \frac{180n}{n} - \frac{360}{n} = 180 - \frac{360}{n}$, so the two expressions are equivalent. **58b.** As n gets larger, $\frac{360}{n}$ becomes very small, so the size of one interior $\angle$ gets closer to 180. The more sides it has, the closer the polygon is to a circle.

59. One interior $\angle$ of a decagon is $\frac{(n-2)180}{n} = \frac{(10-2)180}{10} = (10 - 2)18 = (8)18 = 144$. When it is bisected, it forms 2 $\measuredangle$s measuring $144 \div 2$, or 72. The bisecting rays intersect to form a $\triangle$ with two of its $\measuredangle$s measuring 72. By the Triangle Angle-Sum Thm., $x + 2(72) = 180$; $x + 144 = 180$; $x = 36$. So, the measure of the $\angle$ formed by the intersecting rays is 36.

60. Answers may vary. Sample: A concave quadrilateral is a 4-sided figure with one $\angle$ "pointed inward."

61. If the opp. sides are $\cong$, they would overlap. So, it is not possible.

62. Answers may vary. Sample: A concave quadrilateral is a 4-sided figure with one $\angle$ "pointed inward."



63. If opp. sides and adjacent sides are $\cong$, the sides would overlap. So, the figure is not possible. **64.** $(n - 2)180 = (25 - 2)180 = (23)180 = 4140$ **65.** $\frac{(n-2)180}{n} = 162$; $162n = (n - 2)180$; $162n = 180n - 360$; $-18n = -360$; $n = 20$ **66.** $(n - 2)180 = 7740$; $n - 2 = 43$; $n = 45$. So, there are 45 cars that can hold 5 people each. The total number of people is $(45)(5) = 225$. **67.** By the Polygon Ext. Angle Thm., the sum is 360. **68.** Let x represent the measure of one of the 4 $\cong$ $\measuredangle$s. Then the complementary $\measuredangle$s measure y and $90 - y$: $4x + y + 90 - y = (6 - 2)180$; $4x + 90 = 720$; $4x = 630$; $x = 157.5$. **69.** $(n - 2)180 = 4500$; $n - 2 = 25$; $n = 27$ **70.** $\frac{360}{n} = \frac{360}{36} = 10$ **71.** By the Triangle Angle-Sum Thm., $3y + (y - 15) + 35 = 180$; $4y + 20 = 180$; $4y = 160$; $y = 40$. The $\angle$ measures are $3y = 3(40) = 120$ and $y - 15 = 40 - 15 = 25$. **72.** By the Triangle Angle-Sum Thm., $(x + 23) + (x + 13) + 90 = 180$; $2x + 126 = 180$; $2x = 54$; $x = 27$. The $\angle$ measures are $x + 23 = 27 + 23 = 50$ and $x + 13 = 27 + 13 = 40$. **73.** Solve for x: $x + (x - 28) = 180$; $2x - 28 = 180$; $x - 14 = 90$; $x = 104$. So, its suppl. is $x - 28 =$

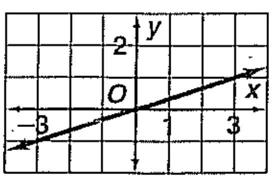
$104 - 28 = 76$. By the Ext. Angle Thm., $(2y - 1) + y = 104$; $3y - 1 = 104$; $3y = 105$; $y = 35$. So, $2y - 1 = 2(35) - 1 = 70 - 1 = 69$. The $\angle$ measures are 104, 76, 35, and 69. **74.** 4 is distributed over $(2a - 3)$: Distributive Prop. **75.** 2 is substituted for b in $b + c = 7$: Subst. Prop. **76.** Both sides of the $\cong$ symbol are the same: Reflexive Prop. of $\cong$. **77.** The information on each side of the $\cong$ symbol is reversed: Symm. Prop. of $\cong$. **78.** Both sides are divided by 2: Div. Prop. **79.** Trans. Prop. **80.** Opp. rays share an endpoint and form a line: $\overrightarrow{RT}$, $\overrightarrow{RK}$. **81.** by the markings on the diagram, $\angle BRT$ and its suppl. $\angle BRK$ **82.** Answers may vary. Samples: $\overline{MR}, \overline{RK}, \overline{TR}, \overline{BR}, \overline{TK}$ **83.** Answers may vary. Samples: $\angle BRM$, $\angle MRK$ **84.** Any $\angle$ measuring between 90 and 180 is obtuse: $\angle TRM$. **85.** A straight $\angle$ forms a line and measures 180: $\angle TRK$. **86.** Since $TR = RK$, R is the midpt. of $\overline{TK}$.

ALGEBRA 1 REVIEW *page 151*

1. Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-6)}{7 - 4} = \frac{2 + 6}{7 - 4} = \frac{8}{3}$. The slope is positive, so it rises from left to right, the same tilt as line ℓ. Since $|\frac{8}{3}| > |1|$, it is steeper than line ℓ. **2.** Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-8 - (-6)}{-5 - 7} = \frac{-8 + 6}{-5 - 7} = \frac{-2}{-12} = \frac{1}{6}$. The slope is positive, so it rises from left to right, the same tilt as line ℓ. Since $|\frac{1}{6}| < |1|$, it is not as steep as line ℓ. **3.** Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 7}{-1 - (-3)} = \frac{4 - 7}{-1 + 3} = \frac{-3}{2} = -\frac{3}{2}$. The slope is negative, so it falls from left to right, the same tilt as r. Since $|-\frac{3}{2}| > |1|$, it is steeper than line r. **4.** Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - (-5)}{1 - (-2)} = \frac{-7 + 5}{1 + 2} = \frac{-2}{3} = -\frac{2}{3}$. The slope is negative, so it falls from left to right, the same tilt as r. Since $|-\frac{2}{3}| < |1|$, it is not as steep as line r. **5.** Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 4}{4 - 0} = \frac{-4}{4} = -1$. The slope is negative, so it falls from left to right, the same tilt as line r. Since the slopes are the same, it has the same steepness as line r.

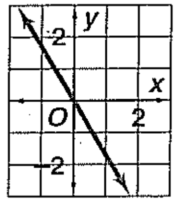
6. Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{2\frac{1}{2} - 3}{-7 - \left(-3\frac{1}{2}\right)} = \frac{2\frac{1}{2} - 3}{-7 + 3\frac{1}{2}} = \frac{-\frac{1}{2}}{-3\frac{1}{2}} = \frac{-\frac{1}{2}}{-3\frac{1}{2}} \cdot \frac{-2}{-2} = \frac{1}{7}$. The slope is positive, so it rises from left to right, the same tilt as line ℓ. Since $|\frac{1}{7}| < |1|$, it is not as steep as line ℓ. **7.** Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{1.3 - (-3.7)}{-2.4 - (-1.4)} = \frac{1.3 + 3.7}{-2.4 + 1.4} = \frac{5}{-1} = -5$. The slope is negative, so it falls from left to right, the same tilt as line r. Since $|-5| > |1|$, it is steeper than line r. **8.** Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-2)}{-6 - 3} = \frac{-2 + 2}{-6 - 3} = \frac{0}{-9} = 0$. The line is horizontal, so it has no steepness; it is less steep than either line ℓ or line r. **9.** Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 9}{5 - 5} = \frac{-15}{0}$. The slope is undefined, so the line is vertical and steeper than either line ℓ or line r.

10.



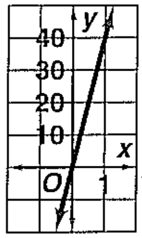
The slope is positive, so it rises from left to right, the same tilt as line ℓ. Since $|\frac{1}{3}| < |1|$, it is not as steep as line ℓ.

11.



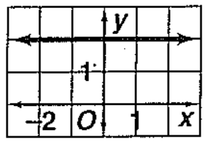
The slope is negative, so it falls from left to right, the same tilt as line r. Since $|-1.7| > |-1|$, it is steeper than line r.

12.



The slope is positive, so it rises from left to right, the same tilt as line ℓ. Since $|37| > |1|$, it is steeper than line ℓ.

13.



The slope is neither positive nor negative, so it is horizontal. Since $|0| < |-1|$ and $|0| < |1|$, it is not as steep as either line r or line ℓ.

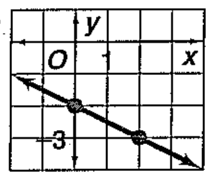
3-5 Lines in the Coordinate Plane *pages 152–157*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1. $-\frac{2}{3}$ **2.** $\frac{5}{3}$ **3.** 0 **4.** undefined, or no slope **5.** $\frac{2}{3}$ **6.** $-\frac{3}{2}$ **7.** $\frac{4}{5}$ **8.** -1

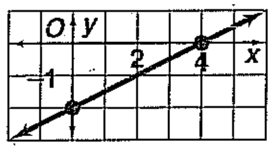
Check Understanding

1.



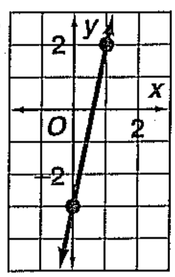
The y-intercept is -2, so plot $(0, -2)$. The slope is $-\frac{1}{2}$, so plot the second point 1 unit down and 2 units right from $(0, -2)$ at $(2, -3)$. Draw a line through the two points.

2.



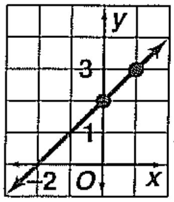
When $x = 0$, $-2x + 4y = -8$; $-2(0) + 4y = -8$; $y = -2$, so $(0, -2)$ is one of the points. When $y = 0$, $-2x + 4(0) = -8$; $x = 4$, so $(4, 0)$ is another point. Plot the two points and draw a line through them.

3.

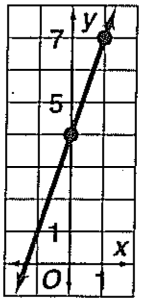


Transform $-5x + y = -3$ to slope-intercept form: $y = 5x - 3$. The y-intercept is -3, so plot $(0, -3)$. The slope is 5 or $\frac{5}{1}$, so plot the second point 5 units up and 1 unit right of $(0, -3)$ at $(1, 2)$. **4.** $y - y_1 = m(x - x_1)$; $y - (-4) = 3(x - 2)$; $y + 4 = 3(x - 2)$ **5.** $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 0}{7 - 5} = \frac{-3}{2} = -\frac{3}{2}$; $y - y_1 = m(x - x_1)$; $y - 0 = -\frac{3}{2}(x - 5)$; $y = -\frac{3}{2}(x - 5)$ or $y - (-3) = -\frac{3}{2}(x - 7)$; $y + 3 = -\frac{3}{2}(x - 7)$ **6.** For the horizontal line, for all values of x, $y = -1$. For the vertical line, for all values of y, $x = 5$.

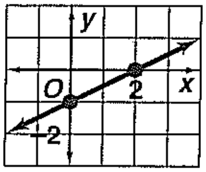
Exercises

1. 

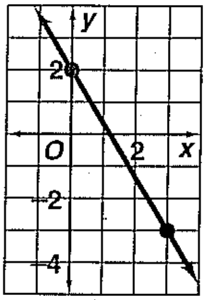
The y-intercept is 2, so plot $(0, 2)$. The slope is 1, or $\frac{1}{1}$, so also plot the point 1 up and 1 to the right of $(0, 2)$ at $(1, 3)$.

2. 

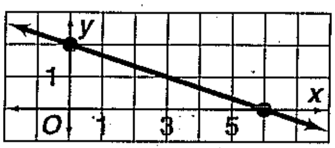
The y-intercept is 4, so plot $(0, 4)$. The slope is 3, or $\frac{3}{1}$, so also plot the point 3 up and 1 to the right of $(0, 4)$ at $(1, 7)$.

3. 

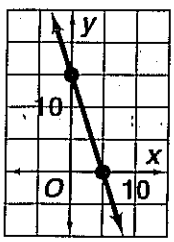
The y-intercept is -1, so plot $(0, -1)$. The slope is $\frac{1}{2}$, so also plot the point 1 up and 2 right of $(0, -1)$ at $(2, 0)$.

4. 

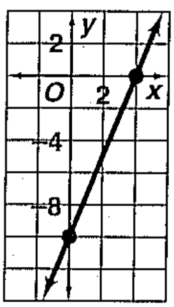
The y-intercept is 2, so plot the point $(0, 2)$. The slope is $-\frac{5}{3}$, so also plot the point 5 down and 3 right of $(0, 2)$ at $(3, -3)$.

5. 

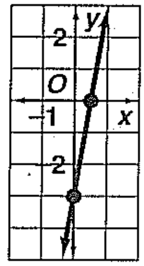
When $x = 0, 2x + 6y = 12; 2(0) + 6y = 12$, so $y = 2$. Plot $(0, 2)$. When $y = 0, 2x + 6(0) = 12$, so $x = 6$. Plot $(6, 0)$.

6. 

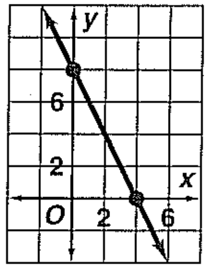
When $x = 0, 3x + y = 15; 3(0) + y = 15$, so $y = 15$. Plot $(0, 15)$. When $y = 0$, $3x + 0 = 15$, so $x = 5$. Plot $(5, 0)$.

7. 

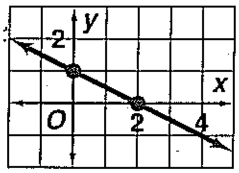
When $x = 0, 5x - 2y = 20; 5(0) - 2y = 20$, so $y = -10$. Plot $(0, -10)$. When $y = 0, 5x - 2(0) = 20$, so $x = 4$. Plot $(4, 0)$.

8. 

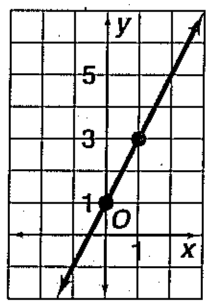
When $x = 0, 6x - y = 3; 6(0) - y = 3$, so $y = -3$. Plot $(0, -3)$. When $y = 0$, $6x - 0 = 3$, so $x = \frac{1}{2}$. Plot $\left(\frac{1}{2}, 0\right)$.

9. 

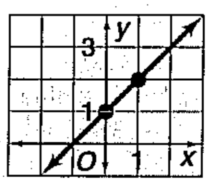
When $x = 0, 10x + 5y = 40$; $10(0) + 5y = 40$, so $y = 8$. Plot $(0, 8)$. When $y = 0$, $10x + 5(0) = 40$, so $x = 4$. Plot $(4, 0)$.

10. 

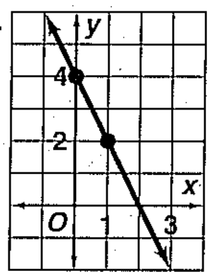
Multiplying both sides of $1.2x + 2.4y = 2.4$ by 5 results in the equivalent equation $6x + 12y = 12$. When $x = 0$, $6x + 12y = 12; 6(0) + 12y = 12$, so $y = 1$. Plot $(0, 1)$. When $y = 0, 6x + 12(0) = 12$, so $x = 2$. Plot $(2, 0)$.

11. 

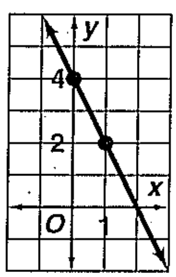
The equation is already in the slope-intercept form of $y = mx + b$: $y = 2x + 1$.

12. 

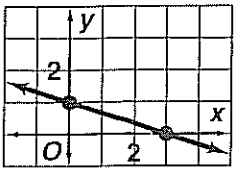
$y - 1 = x; y = x + 1$. Plot $(0, 1)$ from the y-intercept and, since the slope is 1, or $\frac{1}{1}$, also plot $(0 + 1, 1 + 1)$, or $(1, 2)$.

13. 

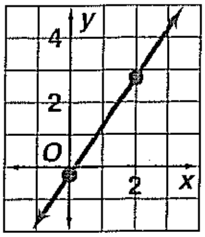
$y + 2x = 4; y = -2x + 4$. Plot $(0, 4)$ from the y-intercept and, since the slope is -2, or $-\frac{2}{1}$, also plot $(0 + 1, 4 - 2)$, or $(1, 2)$.

14. 

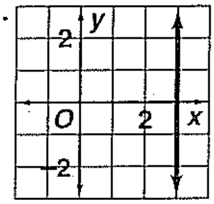
$8x + 4y = 16; 4y = -8x + 16$; $y = -2x + 4$. Plot $(0, 4)$ from the y-intercept and, since the slope is -2, or $-\frac{2}{1}$, also plot $(0 + 1, 4 - 2)$, or $(1, 2)$.

15. 

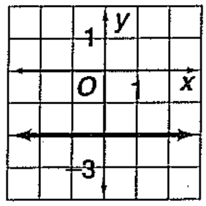
$2x + 6y = 6; 6y = -2x + 6;$ $y = -\frac{1}{3}x + 1$. Plot (0, 1) from the y-intercept and, since the slope is $-\frac{1}{3}$, also plot $(0 + 3, 1 - 1)$, or $(3, 0)$.

16. 

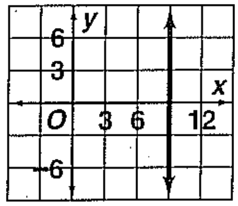
Multiply both sides of $\frac{3}{4}x - \frac{1}{2}y = \frac{1}{8}$ to get the equivalent equation $6x - 4y = 1; -4y = -6x + 1;$ $y = \frac{3}{2}x - \frac{1}{4}$. Plot $(0, -\frac{1}{4})$ from the y-intercept and, since the slope is $\frac{3}{2}$, also plot $(0 + 2, -\frac{1}{4} + 3)$, or $(2, 2\frac{3}{4})$. **17.** $y - y_1 = m(x - x_1);$ $y - 3 = 2(x - 2)$ **18.** $y - y_1 = m(x - x_1); y - (-1) = 3(x - 4); y + 1 = 3(x - 4)$ **19.** $y - y_1 = m(x - x_1);$ $y - 5 = -1(x - (-3)); y - 5 = -1(x + 3)$ **20.** $y - y_1 = m(x - x_1); y - (-6) = -4(x - (-2)); y + 6 = -4(x + 2)$ **21.** $y - y_1 = m(x - x_1); y - 1 = \frac{1}{2}(x - 6)$
22. $y - y_1 = m(x - x_1); y - 4 = 1(x - 0); y - 4 = x$
23. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 5}{5 - 0} = \frac{3}{5}; y - y_1 = m(x - x_1)$.
For (0, 5), $y - 5 = \frac{3}{5}(x - 0)$ or $y - 5 = \frac{3}{5}x$.
If (5, 8) is used, the equation is $y - 8 = \frac{3}{5}(x - 5)$.
24. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{2 - 6} = \frac{2}{-4} = -\frac{1}{2}; y - y_1 = m(x - x_1)$.
For (6, 2), $y - 2 = -\frac{1}{2}(x - 6)$. If (2, 4) is used, the equation is $y - 4 = -\frac{1}{2}(x - 2)$. **25.** $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 6}{-1 - 2} = \frac{-3}{-3} = 1; y - y_1 = m(x - x_1)$. For (2, 6), $y - 6 = 1(x - 2)$ or $y - 6 = x - 2$. If (−1, 3) is used, the equation is $y - 3 = 1(x - (-1)); y - 3 = 1(x + 1)$ or $y - 3 = x + 1$. **26.** $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - (-4)}{2 - (-4)} = \frac{10 + 4}{2 + 4} = \frac{14}{6} = \frac{7}{3}; y - y_1 = m(x - x_1)$. For (−4, 4), $y - 4 = \frac{7}{3}(x - (-4))$ or $y - 4 = \frac{7}{3}(x + 4)$. If (2, 10) is used, the equation is $y - 10 = \frac{7}{3}(x - 2)$. **27.** $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 0}{-3 - (-1)} = \frac{-1 - 0}{-3 + 1} = \frac{-1}{-2} = \frac{1}{2}; y - y_1 = m(x - x_1)$.
For (−1, 0), $y - 0 = \frac{1}{2}(x - (-1)); y - 0 = \frac{1}{2}(x + 1)$ or $y = \frac{1}{2}(x + 1)$. If (−3, −1) is used, the equation is $y - (-1) = \frac{1}{2}(x - (-3)); y + 1 = \frac{1}{2}(x + 3)$.
28. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 10}{-4 - 8} = \frac{-8}{-12} = \frac{2}{3}; y - y_1 = m(x - x_1)$. For (8, 10), $y - 10 = \frac{2}{3}(x - 8)$. If (−4, 2) is used, the equation is $y - 2 = \frac{2}{3}(x - (-4)); y - 2 = \frac{2}{3}(x + 4)$. **29.** For the horizontal line, for all values of x, $y = 7$. For the vertical line, for all values of y, $x = 4$. **30.** For the horizontal line, for all values of x, $y = -2$. For the vertical line, for all values of y, $x = 3$. **31.** For the horizontal line, for all values of x, $y = -1$. For the vertical line, for all values of y, $x = 0$. **32.** For the horizontal line, for all values of x, $y = 4$. For the vertical line, for all values of y, $x = 6$.

33. 

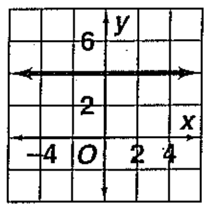
For all y-values, $x = 3$, so the line is vertical through (3, 0).

34. 

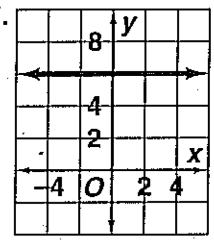
For all x-values, $y = -2$, so the line is horizontal through (0, −2).

35. 

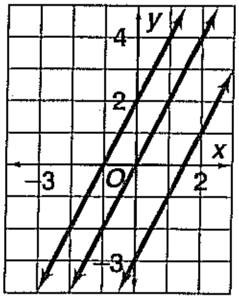
For all y-values, $x = 9$, so the line is vertical through (9, 0).

36. 

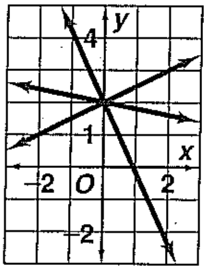
For all x-values, $y = 4$, so the line is horizontal through (0, 4).

37. 

For all x-values, $y = 6$, so the line is horizontal through (0, 6). **38a.** The equation is in slope-intercept form, so the slope is 0.05. **38b.** Since the variable m in this equation is the number of minutes, \$0.05 is the cost per minute. **38c.** The number in the b, or y-intercept, position of $y = mx + b$ is 4.95. **38d.** It is the initial charge for a call when $m = 0$. **39.** No; a line with no slope is a vertical line. A line with 0 slope is a horizontal line. Vertical lines are $\perp$ to horizontal lines, so they cannot be the same. **40a.** The x-axis is a horizontal line, so $m = 0$. **40b.** Since it is horizontal and passes through (0, 0), the equation is $y = 0$. **41a.** The y-axis is a vertical line, so the slope is undefined. **41b.** Since it is vertical and passes through (0, 0), the equation is $x = 0$. **42.** The equation is in $Ax + By = C$, or standard form. Answers may vary. Sample: Change to slope-intercept form, because it is easy to graph the eq. from that form. **43.** The eq. is in $y = mx + b$, or slope-intercept form. Answers may vary. Sample: Use slope-intercept form because it is already in that form. **44.** The equation is in $y - y_1 = m(x - x_1)$, or point-slope form. Answers may vary. Sample: Use point-slope form, because the eq. is already in that form.

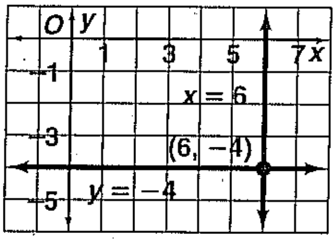
45. 

The slopes are the same, but their y-intercepts are different.

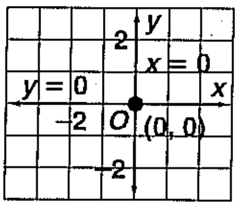
46. 

The slopes are all different, but their y-intercepts are the same.

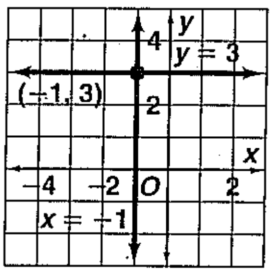
47. Check students' work.

48. 

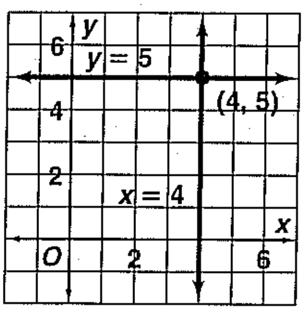
The graph of $y = -4$ has 0 slope and passes through $(0, -4)$. The graph of $x = 6$ has no slope and passes through $(6, 0)$. The point of intersection is $(6, -4)$.

49. 

The graph of $x = 0$ has no slope and passes through $(0, 0)$, so it is the y-axis. The graph of $y = 0$ has 0 slope and passes through $(0, 0)$, so it is the x-axis. The point of intersection is $(0, 0)$.

50. 

The graph of $x = -1$ has no slope and passes through $(-1, 0)$. The graph of $y = 3$ has 0 slope and passes through $(0, 3)$. The point of intersection is $(-1, 3)$.

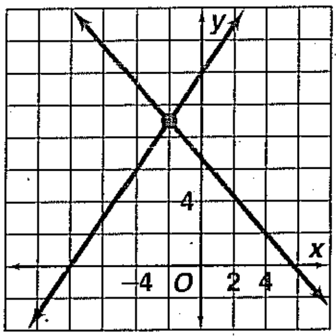
51. 

The graph of $y = 5$ has 0 slope and passes through $(0, 5)$. The graph of $x = 4$ has no slope and passes through $(4, 0)$. The point of intersection is $(4, 5)$. **52.** $\frac{3}{10} = 0.3, \frac{1}{12} = 0.08\overline{3}$; $\frac{3}{10} > \frac{1}{12}$. It is not possible if the ramp is straight; $\frac{1}{12} = \frac{3}{x}$, so $x = 36$. It is possible if the ramp zigzags so its total length is 36 ft. **53.** They are both in the form of $y = mx + b$, so it's easy to compare their y-intercepts, b, and their slopes, m. The y-intercepts are both -2 and, since $|-5| = |5|$, the lines have the same steepness. One line rises from the left to right while the other falls from left to right. **54.** Make certain that the slopes of the equations are all different. Answers may vary. Sample: $x = 5, y - 6 = 2(x - 5), y = x + 1$ **55.** The x-intercept is $(2, 0)$ and the y-intercept is $(0, 4)$. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 4}{2 - 0} = \frac{-4}{2} = -2$; $y - 0 = -2(x - 2)$, or another equivalent equation such as $y = -2x + 4$, $2x + y = 4$, or $y = -2(x - 2)$. **56a.** $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 0}{2 - 0} = \frac{5}{2}$; $y - 0 = \frac{5}{2}(x - 0)$; $y = \frac{5}{2}x$ **56b.** $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 5}{4 - 2} = \frac{-5}{2} = -\frac{5}{2}$; $y - 5 = -\frac{5}{2}(x - 2)$ or $y = -\frac{5}{2}x + 10$ **56c.** The abs. values of the slopes are the same, but one slope is pos. and the other is neg. For $y = \frac{5}{2}x$, the y-int. is at $(0, 0)$ and for $y = -\frac{5}{2}x + 10$, the y-intercept is at $(0, 10)$. **57.** If the slopes of the segments are $=$, then the three points are collinear. The slope of $\overline{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 6}{3 - 5} = \frac{-4}{-2} = 2$. The slope of $\overline{BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 2}{6 - 3} = \frac{6}{3} = 2$. The slopes are $=$, so the points are collinear. **58.** If the slopes of the segments are $=$, then the three pts. are collinear. The slope of $\overline{DE} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - (-2)}{4 - (-2)} = \frac{-4 + 2}{4 + 2} = \frac{-2}{6} = -\frac{1}{3}$. The slope of $\overline{EF} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 0}{4 - 0} = \frac{-4}{4} = -1$. The slopes are not $=$, so the points are not collinear. **59.** If the slopes of the segments are $=$, then the three pts. are collinear. The slope of $\overline{GH} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 3}{5 - 2} = \frac{-7}{3} = -\frac{7}{3}$. The slope of $\overline{HI} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 3}{-1 - 2} = \frac{7}{-3} = -\frac{7}{3}$. The slopes are $=$, so the pts. are collinear. **60.** If the slopes of the segments are $=$, then the three pts. are collinear. The slope of $\overline{JK} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 9}{1 - (-2)} = \frac{-1 - 9}{1 + 2} = \frac{-10}{3} = -\frac{10}{3}$. The slope of $\overline{KL} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-11 - (-1)}{4 - 1} = \frac{-11 + 1}{4 - 1} = \frac{-10}{3} = -\frac{10}{3}$. The slopes are $=$, so the pts. are collinear. **61.** $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 8}{-2 - 0} = \frac{-6}{-2} = 3$. The equation in point-slope form using the point $(-2, 2)$ is $y - 2 = 3(x + 2)$. The equation in point-slope form using the point $(0, 8)$ is $y - 8 = 3(x - 0)$, or $y - 8 = 3x$. The equation in standard form of $Ax + By = C$ is $3x - y = -8$. **62.** $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 5}{7 - 5} = \frac{1}{2}$. The equation in point-slope form using the point $(5, 5)$ is $y - 5 = \frac{1}{2}(x - 5)$. The equation in point-slope form using the point $(7, 6)$ is $y - 6 = \frac{1}{2}(x - 7)$. The equation in standard form of $Ax + By = C$ is $x - 2y = -5$. **63.** $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 6}{5 - 2} = \frac{2}{3}$. The equation in point-slope form using the point $(2, 6)$ is $y - 6 = \frac{2}{3}(x - 2)$. The equation in point-slope form using the point $(5, 8)$ is $y - 8 = \frac{2}{3}(x - 5)$. The equation in standard form of $Ax + By = C$ is $2x - 3y = -14$. **64.** All of the choices are in slope-intercept form, so rewrite $15x + 3y = 10$ in slope-intercept form: $3y = -15x + 10$; $y = -5x + \frac{10}{3}$. The answer is choice D. **65.** The equation for $\overleftrightarrow{AB}$ is $y = -7x - 9$. The equation for $\overleftrightarrow{AC}$ is $y = -\frac{5}{3}x + \frac{5}{3}$. The equation for $\overleftrightarrow{BC}$ is $y = -\frac{3}{5}x - \frac{13}{5}$. The equation for $\overleftrightarrow{BD}$ is $y = \frac{1}{4}x - \frac{7}{4}$. The distance the y-intercept of an equation in the form of $y = mx + b$ is from the origin is

$|b|$. $|-9| > \left|-\frac{13}{5}\right| > \left|-\frac{7}{4}\right| > \left|\frac{5}{3}\right|$, so $\overleftrightarrow{AC}$ is closest to the origin. The answer is choice G. **66.** For Col. A, rewrite $3x - 8y = 60$ in slope-intercept form: $y = \frac{3}{8}x - \frac{60}{8}$. For Col. B, rewrite $3x + 60 = 8y$ in slope-intercept form: $y = \frac{3}{8}x + \frac{60}{8}$. Since $-\frac{60}{8} < \frac{60}{8}$, the y-intercept in Col. B is greater. The answer is choice B. **67.** The slope of $4y = -10$ is 0. The slope of $y = 5$ is 0. The slopes are =. The answer is choice C. **68.** For Col. A, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-11 - (-9)}{-4 - 0} = \frac{-11 + 9}{-4 - 0} = \frac{-2}{-4} = \frac{1}{2}$. For Col. B, $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - 0}{4 - 18} = \frac{-7}{-14} = \frac{1}{2}$. The slopes are =. The answer is choice C.

69a. Line a in $y = mx + b$ form: $y = \frac{3}{2}x + 12$, or equivalent equation; Line b: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 1}{7 - 4} = \frac{-4}{3} = -\frac{4}{3}$; $(y - 1) = -\frac{4}{3}(x - 4)$; $3y - 3 = -4x + 16$; $4x + 3y = 19$ or equivalent equation.

69b.



The lines intersect at $(-2, 9)$.

70. A nonagon has 9 sides: $(n - 2)180 = (9 - 2)180 = 1260$. **71.** A pentagon has 5 sides: $(n - 2)180 = (5 - 2)180 = (3)180 = 540$. **72.** $(n - 2)180 = (11 - 2)180 = (9)180 = 1620$ **73.** $(n - 2)180 = (14 - 2)180 = (12)180 = 2160$ **74.** Yes; it is the def. of a quadrilateral. **75.** No; a counterexample is parallel lines, which do not intersect but are not skew. **76.** No; all △ have at least 2 acute ∠; all obtuse △ have 2 acute ∠. **77.** By the def. of ∠ bis., $3a = 2a + 5$; $a = 5$. By the Angle Add. Post., $m\angle MPR = 3a + 2a + 5 = 5a + 5 = 5(5) + 5 = 25 + 5 = 30$. **78.** By the def. of ∠ bis., $7a = 4a + 12$; $3a = 12$; $a = 4$. By the Angle Add. Post., $m\angle MPR = 7a + 4a + 12 = 11a + 12 = 11(4) + 12 = 44 + 12 = 56$. **79.** By the def. of ∠ bis., $8a - 8 = 5a - 2$; $3a = 6$; $a = 2$. By substitution, $m\angle QPR = 5a - 2 = 5(2) - 2 = 10 - 2 = 8$. **80.** By the def. of ∠ bis., $2a + 9 = 4a - 3$; $-2a = -12$; $a = 6$. By substitution, $m\angle MPQ = 2a + 9 = 2(6) + 9 = 12 + 9 = 21$.

3-6 Slopes of Parallel and Perpendicular Lines *pages 158–164*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM*.

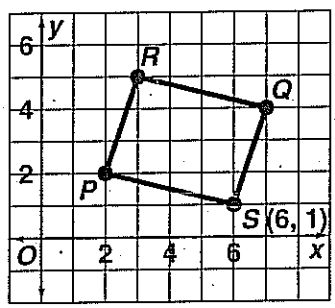
1. $\frac{1}{2}$ **2.** $\frac{5}{2}$ **3.** -5 **4.** 2 **5.** -1 **6.** $\frac{2}{3}$ **7.** 1 **8.** 0 **9.** $\frac{2}{3}$

Check Understanding **1.** If the slopes are = and the points are not collinear, the lines are ∥. The slope of $\ell_3 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 1}{-4 - 3} = \frac{1}{-7} = -\frac{1}{7}$. The slope of $\ell_4 = \frac{0 - (-2)}{-4 - 8} = \frac{2}{-12} = -\frac{1}{6}$. Since $-\frac{1}{7} \neq -\frac{1}{6}$, the slopes are not =, so the lines are not parallel. **2a.** Write $2x + 4y = 9$ in slope-intercept form: $4y = -2x + 9$; $y = -\frac{1}{2}x + \frac{9}{4}$. Each line has a slope of $-\frac{1}{2}$, and, since the y-intercepts are not =, the lines are ∥. **2b.** Write $2x + 4y = 20$ in slope-intercept form: $4y = -2x + 20$; $y = -\frac{1}{2}x + 5$. The lines show the same slope and y-intercept, so they are the same line, not ∥ lines. **3.** The slope is -1. Using point-slope form, $y - 5 = -1(x - (-2))$; $y - 5 = -1(x + 2)$. **4.** The lines are ⊥ if the product of their slopes is -1. The slope of $\ell_3 = \frac{1 - 5}{-4 - 5} = \frac{-4}{-9} = \frac{4}{9}$. The slope of $\ell_4 = \frac{5 - (-2)}{0 - 3} = \frac{5 + 2}{0 - 3} = -\frac{7}{3}$. $\frac{4}{9}(-\frac{7}{3}) = -\frac{28}{27} \neq -1$, so the lines are not ⊥. **5.** To identify the slope of the given line, write $5y - x = 10$ in slope-intercept form: $5y = x + 10$; $y = \frac{1}{5}x + 2$. So, the slope of the given line is $\frac{1}{5}$. If m is the slope of the ⊥ line, then $\frac{1}{5}m = -1$, so $m = -5$. Write the equation of the ⊥ line in point-slope form: $y - (-4) = -5(x - 15)$; $y + 4 = -5(x - 15)$. **6.** If m = the slope of the ⊥ line, then $\left(-\frac{2}{3}\right)m = -1$, so $m = \frac{3}{2}$. In point-slope form, the eq. of the ⊥ line is $y - 8 = \frac{3}{2}(x - 2)$, or, in slope-intercept form it is $y - 8 = \frac{3}{2}x - 3 \rightarrow y = \frac{3}{2}x + 5$.

Exercises **1.** The lines are ∥ if the slopes are =. The slope of $\ell_1 = \frac{1 - (-2)}{-6 - 0} = \frac{1 + 2}{-6 - 0} = \frac{3}{-6} = -\frac{1}{2}$. The slope of $\ell_2 = \frac{3 - 1}{0 - 4} = \frac{2}{-4} = -\frac{1}{2}$. Since the slopes are =, the lines are ∥. **2.** The lines are ∥ if the slopes are =. The slope of $\ell_1 = \frac{1 - 3}{-5 - 1} = \frac{-2}{-6} = \frac{1}{3}$. The slope of $\ell_2 = \frac{-2 - 2}{-4 - 4} = \frac{-4}{-8} = \frac{1}{2}$. Since $\frac{1}{3} \neq \frac{1}{2}$, the lines are not ∥. **3.** The lines are ∥ if the slopes are =. The slope of $\ell_1 = \frac{2 - (-4)}{0 - (-4)} = \frac{2 + 4}{0 + 4} = \frac{6}{4} = \frac{3}{2}$. The slope of $\ell_2 = \frac{3 - (-3)}{5 - 2} = \frac{3 + 3}{5 - 2} = \frac{6}{3} = 2$. Since $\frac{3}{2} \neq 2$, the lines are not ∥. **4.** The lines are ∥ if the slopes are =. The slope of $\ell_1 = \frac{5 - 1}{-2 - (-3)} = \frac{5 - 1}{-2 + 3} = \frac{4}{1} = 4$. The slope of $\ell_2 = \frac{2 - (-2)}{1 - 0} = \frac{2 + 2}{1 - 0} = \frac{4}{1} = 4$. Since the slopes are =, the lines are ∥. **5.** The lines are ∥ if the slopes are =. The slope of $\ell_1 = \frac{6 - 6}{-3 - 2} = \frac{0}{-5} = 0$. The slope of $\ell_2 = \frac{0 - 0}{7 - 0} = \frac{0}{7} = 0$. Since the slopes are =, the lines are ∥. **6.** The equations are both in the form of $y = mx + b$, where m is the slope and b is the y-intercept. The lines are ∥ if the slopes are = and the y-intercepts are different. The slope of both lines is 2 and the y-intercept of one line is 5 and the other line is 0, so the lines are ∥. **7.** The equations are both in $y = mx + b$ form, where m is the slope and b is the y-intercept. The lines are ∥ if the slopes are = and the y-intercepts are different. The slope of both lines is $\frac{3}{4}$, and the y-intercepts are -10 and 2, so the lines are ∥. **8.** The lines are ∥ if the slopes are the same and the y-intercepts are different. Rewrite $x + y = 20$ in $y = mx + b$ form: $y = -x + 20$. The slope of both lines is -1 and the y-intercepts are 6 and 20, so the lines are ∥. **9.** The lines are ∥ if the slopes are the same and the y-intercepts are different. Rewrite $y - 7x = 6$ in $y = mx + b$ form: $y = 7x + 6$. Rewrite $y + 7x = 8$ in $y = mx + b$ form: $y = -7x + 8$. Since the slopes are 7 and -7, the slopes are different, so the lines are not $\parallel$. **10.** The lines are $\parallel$ if the slopes are the same and the y-intercepts are different. Rewrite $3x + 4y = 12$ in $y = mx + b$ form: $y = -\frac{3}{4}x + 3$. Rewrite $6x + 2y = 6$ in $y = mx + b$ form: $y = -3x + 3$. Since the slopes are $-\frac{3}{4}$ and -3, the slopes are different, so the lines are not $\parallel$. **11.** The lines are $\parallel$ if the slopes are the same and the y-intercepts are different. Rewrite $2x + 5y = -1$ in $y = mx + b$ form: $y = -\frac{2}{5}x - \frac{1}{5}$. Rewrite $10y = -4x - 20$ in $y = mx + b$ form: $y = -\frac{2}{5}x - 2$. The slope of both lines is $-\frac{2}{5}$ and the y-intercepts are $-\frac{1}{5}$ and 2, so the lines are $\parallel$. **12.** The slope is -2. Using point-slope form, $y - 3 = -2(x - 0)$ or $y - 3 = -2x$. **13.** The slope is $\frac{1}{3}$. Using point-slope form, $y - 0 = \frac{1}{3}(x - 6)$ or $y = \frac{1}{3}(x - 6)$. **14.** Rewrite $-x + 2y = 4$ in slope-intercept form: $y = \frac{1}{2}x + 2$. The slope is $\frac{1}{2}$. Using point-slope form, $y - 4 = \frac{1}{2}(x - (-2))$; $y - 4 = \frac{1}{2}(x + 2)$. **15.** Rewrite $3x + 2y = 12$ in slope-intercept form: $y = -\frac{3}{2}x + 6$. The slope is $-\frac{3}{2}$. Using point-slope form, $y - (-2) = -\frac{3}{2}(x - 6)$; $y + 2 = -\frac{3}{2}(x - 6)$. **16.** The lines are $\perp$ if the product of their slopes is -1. The slope of $\ell_1 = \frac{-2 - (-4)}{0 - 4} = \frac{-2 + 4}{0 - 4} = \frac{2}{-4} = -\frac{1}{2}$. The slope of $\ell_2 = \frac{1 - (-5)}{4 - 1} = \frac{1 + 5}{4 - 1} = \frac{6}{3} = 2$. Since $-\frac{1}{2} \cdot 2 = -1$, the lines are $\perp$. **17.** The lines are $\perp$ if the product of their slopes is -1. The slope of $\ell_1 = \frac{-4 - 2}{0 - (-4)} = \frac{-6}{4} = -\frac{3}{2}$. The slope of $\ell_2 = \frac{3 - (-3)}{4 - (-5)} = \frac{3 + 3}{4 + 5} = \frac{6}{9} = \frac{2}{3}$. Since $-\frac{3}{2} \cdot \frac{2}{3} = -1$, the lines are $\perp$. **18.** The lines are $\perp$ if the product of their slopes is -1. The slope of $\ell_1 = \frac{4 - (-3)}{-4 - 3} = \frac{4 + 3}{-4 - 3} = -1$. The slope of $\ell_2 = \frac{3 - (-1)}{6 - 1} = \frac{3 + 1}{6 - 1} = \frac{4}{5}$. Since $-1 \cdot \frac{4}{5} = -\frac{4}{5}$ and not -1, the lines are not $\perp$. **19.** The lines are $\perp$ if the product of their slopes is -1. The slope of $\ell_1 = \frac{4 - 0}{-2 - 2} = \frac{4}{-4} = -1$. The slope of $\ell_2 = \frac{4 - (-2)}{3 - (-3)} = \frac{4 + 2}{3 + 3} = \frac{6}{6} = 1$. Since $-1 \cdot 1 = -1$, the lines are $\perp$. **20.** Answers may vary. Sample: The equation is in slope-intercept form, so the slope is $\frac{2}{3}$. If m is the slope of the $\perp$ line, then $\frac{2}{3}m = -1$, so $m = -\frac{3}{2}$. In point-slope form, the eq. of the $\perp$ line through $(6, 6)$ is $y - 6 = -\frac{3}{2}(x - 6)$. Other equivalent equations are possible. **21.** Answers may vary. Sample: The equation is in slope-intercept form, so the slope is $\frac{1}{2}$. If m is the slope of the $\perp$ line, then $\frac{1}{2}m = -1$, so $m = -2$. In point-slope form, the eq. of the $\perp$ line through $(4, 0)$ is $y - 0 = -2(x - 4)$; $y = -2(x - 4)$. Other equivalent equations are possible. **22.** Answers may vary. Sample: To identify the slope of the given line, write $y + 2x = -8$ in slope-intercept form: $y = -2x - 8$. So, the slope of the given line is -2. If m is the slope of the $\perp$ line, then $-2m = -1$, so $m = \frac{1}{2}$. In point-slope form, the eq. of the $\perp$ line through $(4, 4)$ is $y - 4 = \frac{1}{2}(x - 4)$. Other equivalent equations are possible. **23.** Answers may vary. Sample: To identify the slope of the given line, write $4y + 5x = 20$ in slope-intercept form: $y = -\frac{5}{4}x + 5$. So, the slope of the given line is $-\frac{5}{4}$. If m is the slope of the $\perp$ line, then $-\frac{5}{4}m = -1$, so $m = \frac{4}{5}$. In point-slope form, the eq. of the $\perp$ line through $(0, 0)$ is $y - 0 = \frac{4}{5}(x - 0)$, or $y = \frac{4}{5}x$. Other equivalent equations are possible. **24.** Answers may vary. Sample: The slope of the existing road is $-\frac{3}{2}$. If m is the slope of the $\perp$ road, then $-\frac{3}{2}m = -1$, so $m = \frac{2}{3}$. In point-slope form, the $\perp$ road through $(0, 0)$ is $y - 0 = \frac{2}{3}(x - 0)$, or $y = \frac{2}{3}x$. Other equivalent equations are possible. **25.** The lines are $\perp$ if the product of their slopes is -1. Write $y - \frac{1}{2}x = 0$ in $y = mx + b$ form: $y = \frac{1}{2}x$, so its slope is $\frac{1}{2}$. Write $y - 2x = -1$ in $y = mx + b$ form: $y = 2x - 1$, so its slope is 2. Since $\frac{1}{2} \cdot 2 = 1$ and not -1, the lines are not $\perp$. **26.** The lines are $\perp$ if the product of their slopes is -1. The slope of the first equation is -1. Write $y - x = 20$ in $y = mx + b$ form: $y = x + 20$, so the slope is 1. Since $-1 \cdot 1 = -1$, the lines are $\perp$. **27.** Since $y = 3$ is horizontal and $x = -2$ is vertical, the lines are $\perp$. **28.** The lines are $\perp$ if the product of their slopes is -1. Write $3y + 2x = 12$ in $y = mx + b$ form: $y = -\frac{2}{3}x + 4$, so the slope is $-\frac{2}{3}$. Write $y + 3x = -2$ in $y = mx + b$ form: $y = -3x - 2$, so the slope is -3. Since $-\frac{2}{3} \cdot (-3) = 2$ and not -1, the lines are not $\perp$. **29.** The lines are $\perp$ if the product of their slopes is -1. Write $2x + 3y = 6$ in $y = mx + b$ form: $y = -\frac{2}{3}x + 2$, so the slope is $-\frac{2}{3}$. Write $6x - 4y = 24$ in $y = mx + b$ form: $y = \frac{3}{2}x - 6$, so the slope is $\frac{3}{2}$. Since $-\frac{2}{3} \cdot \frac{3}{2} = -1$, the lines are $\perp$. **30.** The lines are $\perp$ if the product of their slopes is -1. Write $2x - 7y = -42$ in $y = mx + b$ form: $y = \frac{2}{7}x + 6$, so the slope is $\frac{2}{7}$. Write $4y = -7x - 2$ in $y = mx + b$ form: $y = -\frac{7}{4}x - \frac{1}{2}$, so the slope is $-\frac{7}{4}$. Since $\frac{2}{7} \cdot \left(-\frac{7}{4}\right) = -\frac{1}{2}$ and not -1, the lines are not $\perp$. **31.** Opposite sides are $\parallel$ if their slopes are the same. Pairs of opposite sides are $\overline{AB}$ and $\overline{CD}$, and $\overline{BC}$ and $\overline{AD}$. The slope of $\overline{AB} = \frac{4 - 2}{3 - 0} = \frac{2}{3}$. The slope of $\overline{CD} = \frac{7 - 5}{2 - (-1)} = \frac{2}{3}$. Since their slopes are $=$, $\overline{AB} \parallel \overline{CD}$. The slope of $\overline{BC} = \frac{4 - 7}{3 - 2} = -3$. The slope of $\overline{AD} = \frac{2 - 5}{0 - (-1)} = -3$. Since their slopes are $=$, $\overline{BC} \parallel \overline{AD}$. **32.** Opposite sides are $\parallel$ if their slopes are the same. Pairs of opposite sides are $\overline{AB}$ and $\overline{CD}$, and $\overline{BC}$ and $\overline{AD}$. The slope of $\overline{AB}$ is $\frac{-2 - 1}{1 - (-3)} = \frac{-3}{4} = -\frac{3}{4}$. The slope of $\overline{CD}$ is $\frac{-3 - 0}{0 - (-4)} = \frac{-3}{4} = -\frac{3}{4}$. Since their slopes are $=$, $\overline{AB} \parallel \overline{CD}$. The slope of $\overline{BC}$ is $\frac{-3 - (-2)}{0 - 1} = \frac{-1}{-1} = 1$. The slope of $\overline{AD}$ is $\frac{0 - 1}{-4 - (-3)} = \frac{-1}{-1} = 1$. Since their slopes are $=$, $\overline{BC} \parallel \overline{AD}$. **33.** Opposite sides are $\parallel$ if their slopes are the same. Pairs of opposite sides are $\overline{AB}$ and $\overline{CD}$, and $\overline{BC}$ and $\overline{AD}$. The slope of $\overline{AB} = \frac{3 - 1}{5 - 1} = \frac{2}{4} = \frac{1}{2}$. The slope of $\overline{CD} = \frac{1 - 0}{7 - 3} = \frac{1}{4}$. Since $\frac{1}{2} \neq \frac{1}{4}$, $\overline{AB}$ and $\overline{CD}$ are not $\parallel$. The slope of $\overline{BC} = \frac{3 - 1}{5 - 7} = \frac{2}{-2} = -1$. The slope of $\overline{AD} = \frac{1 - 0}{1 - 3} = -\frac{1}{2}$. Since $-1 \neq -\frac{1}{2}$, $\overline{BC}$ and $\overline{AD}$ are not $\parallel$. **34.** Opposite sides are $\parallel$ if their slopes are the same. Pairs of opposite sides are $\overline{AB}$ and $\overline{CD}$, and $\overline{BC}$ and $\overline{AD}$. The slope of $\overline{AB} = \frac{0 - 0}{4 - 1} = \frac{0}{3} = 0$. The slope of $\overline{CD} = \frac{-3 - (-3)}{3 - (-1)} = \frac{0}{4} = 0$. Since the slopes are $=$, $\overline{AB} \parallel \overline{CD}$.

The slope of $\overline{BC} = \frac{0-(-3)}{4-3} = 3$. The slope of $\overline{AD} = \frac{0-(-3)}{1-(-1)} = \frac{3}{2}$. Since $3 \neq \frac{3}{2}$, $\overline{BC}$ and $\overline{AD}$ are not ∥. **35.** The y-intercepts should be =, and, if the slope of one line is m, the slope of the other is $-\frac{1}{m}$. **36.** Since the slopes of ∥ lines must be =, if their y-intercepts were also =, they would be the same line. So, the y-intercepts of 2 ∥ lines cannot be the same. **37.** Since $\overline{RS}$ and $\overline{VU}$ are horizontal, their slope is 0, so $\overline{RS} \parallel \overline{VU}$. The endpoints of $\overline{RW}$ are (2, 4) and (0, 2), so the slope of $\overline{RW} = \frac{4-2}{2-0} = 1$. The endpoints of $\overline{UT}$ are (6, 2) and (4, 0), so the slope of $\overline{UT} = \frac{2-0}{6-4} = 1$. Since the slopes are =, $\overline{RW} \parallel \overline{UT}$. The endpoints of $\overline{WV}$ are (0, 2) and (2, 0), so the slope of $\overline{WV} = \frac{0-2}{2-0} = -1$. The endpoints of $\overline{ST}$ are (4, 4) and (6, 2), so the slope $= \frac{4-2}{4-6} = -1$. Since the slopes are =, $\overline{WV} \parallel \overline{ST}$.
38. If any two slopes have a product of -1, then two sides are ⊥, forming a right ∠. The slope of $\overline{GH} = \frac{5-2}{8-3} = \frac{3}{5}$. The slope of $\overline{GK} = \frac{10-2}{0-3} = -\frac{8}{3}$. The slope of $\overline{HK} = \frac{5-10}{8-0} = -\frac{5}{8}$. Since $\frac{3}{5} \cdot \left(-\frac{8}{3}\right) = -\frac{8}{5}$ and not -1, $\overline{GH}$ and $\overline{GK}$ are not ⊥. Since $\frac{3}{5} \cdot \left(-\frac{5}{8}\right) = -\frac{3}{8}$ and not -1, $\overline{GH}$ and $\overline{HK}$ are not ⊥. Since $\left(-\frac{8}{3}\right)\left(-\frac{5}{8}\right) = \frac{5}{3}$ and not -1, $\overline{GK}$ and $\overline{HK}$ are not ⊥. Since no sides of the △ are ⊥, the △ has no right ∠, so it is not a right △. **39.** If r_1 and r_2 are lines and $r_1 \parallel r_2$, then they both must have the same slope, m. If a third line r_3 is ∥ to r_2, it must have the same slope, m. Since r_1 and r_3 have the same slope, m, $r_1 \parallel r_3$. So, all three lines are ∥ because they all have the same slope.
40. When lines are ⊥, the product of their slopes is -1. If r_1 has slope m and is ⊥ to r_2, then r_2 must have slope $-\frac{1}{m}$. If $r_3 \perp r_2$, then the slope of r_3 is $-\left(-\frac{m}{1}\right)$, or m. Both r_1 and r_3 have slope m, so they are ∥. So, two lines ⊥ to the same line must have the same slope.
41a. The slope of the line Joe wants to run ⊥ to is $-\frac{4}{3}$, so the slope of his running line should be $\frac{3}{4}$. Use the point-slope form of eq.: $y-(-20) = \frac{3}{4}(x-35)$, or $y + 20 = \frac{3}{4}(x-35)$. **41b.** You are given a point and can quickly find the slope. You can just insert them into the point-slope form without other computations. **42.** If the slopes are =, the lines are ∥. If the product of the slopes is -1, the lines are ⊥. The slope of $\overleftrightarrow{AB} = \frac{2-\frac{1}{2}}{-1(-1)} = \frac{2-\frac{1}{2}}{-1+1} = \frac{2-\frac{1}{2}}{0}$, so the slope is undefined and the line is vertical. The slope of $\overleftrightarrow{CD}$ is $\frac{-1-7}{3-3} = \frac{-1-7}{0}$, so the slope is undefined and the line is vertical. Since both lines are vertical and they are two distinct lines at $x = -1$ and $x = 3$, the lines are ∥. **43.** Compare slopes. The slope of $\overleftrightarrow{AB} = \frac{5-3}{-2-(-2)} = \frac{5-3}{0}$, so the slope is undefined and the line is vertical. The slope of $\overleftrightarrow{CD}$ is $\frac{4-4}{2-1} = 0$, so the line is horizontal. Since one line is vertical and the other is horizontal, the lines are ⊥.
44. If the lines are ∥, the slopes are =. If the lines are ⊥, the product of their slopes is -1. The slope of $\overleftrightarrow{AB}$ is $\frac{4-4}{5-2} = 0$. The slope of $\overleftrightarrow{CD}$ is $\frac{8-2}{0-3} = -\frac{6}{3} = -2$. The slopes are not =, so the lines are not ∥. Since $-2(0) = 0$ and not -1, the lines are not ⊥. So, the lines are neither ⊥ nor ∥. **45.** If the slopes are =, the lines are ∥. If the product of the slopes is -1, then the lines are ⊥. The slope of $\overleftrightarrow{AB} = \frac{1-2}{5-(-3)} = -\frac{1}{8}$. The slope of $\overleftrightarrow{CD} = \frac{7-(-1)}{2-1} = 8$. The slopes are not =, so the lines are not ∥. The product of the slopes is $-\frac{1}{8}(8) = -1$, so the lines are ⊥.
46. Slope for $\overleftrightarrow{AB} = -\frac{1}{8}$. Slope for $\overleftrightarrow{CD} = 8$. Since $-\frac{1}{8}(8) = -1$, the lines are ⊥. **47.** $A = (7, 11)$ and $C = (9, 1)$, so $AC = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(7-9)^2 + (11-1)^2} = \sqrt{(-2)^2 + (10)^2} = \sqrt{4+100} = \sqrt{104}$. $B = (13, 7)$ and $D = (3, 5)$, so $BD = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = \sqrt{(13-3)^2 + (7-5)^2} = \sqrt{(10)^2 + (2)^2} = \sqrt{100+4} = \sqrt{104}$. Since $AC = BD = \sqrt{104}$, $\overline{AC} \cong \overline{BD}$.
48. If the segments are ⊥, then the product of their slopes is -1. $A = (7, 11)$ and $C = (9, 1)$, so the slope of $\overline{AC} = \frac{1-11}{9-7} = -\frac{10}{2} = -5$. $B = (13, 7)$ and $D = (3, 5)$, so the slope of $\overline{BD} = \frac{7-5}{13-3} = \frac{1}{5}$. Since $-5\left(\frac{1}{5}\right) = -1$, then $\overline{AC} \perp \overline{BD}$. The midpt. of $\overline{AC}$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{7+9}{2}, \frac{11+1}{2}\right) = (8, 6)$. The midpt. of $\overline{BD}$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{13+3}{2}, \frac{7+5}{2}\right) = (8, 6)$. Since the midpts. are the same, and the segments are ⊥ to each other, they are ⊥ bisectors of each other.

49a-b.



Answers may vary. Sample: By counting the units on the graph, the slope of $\overline{RP}$ is -3. So count down 3 units and left 1 unit from Q to find S and draw $PRQS$.

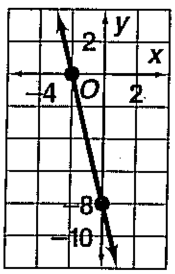
49c. To find the other possible locations, for a side ∥ to $\overline{RP}$, count up 3 and right 1 from Q to S at (8, 7) and draw $PRSQ$. For a side ∥ to $\overline{RQ}$, count units to find the slope of $\overline{RQ}$: $-\frac{1}{4}$. From P, count up 1 unit and left 4 units from P to S at $(-2, 3)$. **50.** The slope of $\overline{LM} = \frac{6-(-3)}{-5-(-2)} = \frac{9}{-3} = -3$, so the slope of the line ⊥ to it is $\frac{1}{3}$. Write an eq. in pt.-slope form: $y - 5 = \frac{1}{3}(x-4)$.
51. Write $6x - 4y = 12$ in slope-intercept form: $y = \frac{3}{2}x - 3$. The slope is $\frac{3}{2}$. The answer is choice B. **52.** If two lines are ⊥, then the product of their slopes is -1: $6m = -1$, so $m = -\frac{1}{6}$. The answer is choice I. **53.** When two lines are ⊥, the product of their slopes is -1. The slope of $f = \frac{-4-(-6)}{5-4} = 2$. The slope of a line ⊥ to $f = -\frac{1}{2}$. The answer is choice C. **54.** [2] **a.** The slope of line $c = \frac{1-(-2)}{-4-2} = \frac{3}{-6} = -\frac{1}{2}$. The product of the slopes of ⊥ lines is -1, so the slope of a line ⊥ to c is 2, since $-\frac{1}{2}(2) = -1$.

b. Using the point-slope form, $y - 2 = 2(x - 1)$; $y - 2 = 2x - 2$; $y = 2x + 0$, so the y-intercept is 0. **55.** The slope is $\frac{3-0}{0-6} = -\frac{1}{2}$. Use the point-slope form of eq.: $y - 3 = -\frac{1}{2}(x - 0)$; $y - 3 = -\frac{1}{2}x$. Other possible equivalent equations include $y - 0 = -\frac{1}{2}(x - 6)$, $y = -\frac{1}{2}(x - 6)$, $y = -\frac{1}{2}x + 3$, $x + 2y = 3$. **56.** The slope is $\frac{7-2}{-1-(-4)} = \frac{5}{3}$. Use the point-slope form of eq.: $y - 2 = \frac{5}{3}(x - (-4))$; $y - 2 = \frac{5}{3}(x + 4)$. Other possible equivalent equations include $y - 7 = \frac{5}{3}(x + 1)$, $y = \frac{5}{3}x + \frac{26}{3}$, $5x - 3y = -26$. **57.** The slope is $\frac{-2-(-8)}{3-(-5)} = \frac{6}{8} = \frac{3}{4}$. Use the point-slope form of eq.: $y - (-2) = \frac{3}{4}(x - 3)$; $y + 2 = \frac{3}{4}(x - 3)$. Other possible equivalent equations include $y + 8 = \frac{3}{4}(x + 5)$, $y = \frac{3}{4}x - \frac{17}{4}$, $3x - 4y = 17$. **58.** Both sides are identical: Refl. Prop. of $\cong$. **59.** Both sides are multiplied by 2: Mult. Prop. of =. **60.** The distributive property reversed: Dist. Prop. **61.** The information on both sides of the $\cong$ symbol are switched. **62.** Eliminate the phrase that is both a hypothesis and conclusion and combine the hypothesis and conclusion that remain: If you are in geometry class, then you are at school. **63.** Eliminate the phrase that is both a hypothesis and conclusion and combine the hypothesis and conclusion that remain: If you travel to Switzerland, then you have a passport.

CHECKPOINT QUIZ 2 *page 164*

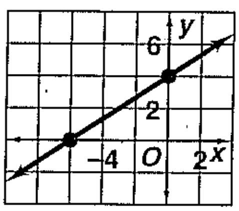
1. The figure has 8 sides, so it is an octagon. The sum of the interior $\angle$s is $(n - 2)180 = (8 - 2)180 = (6)180 = 1080$; $(n) + (134) + (n + 16) + (126) + (n) + (161) + (n - 12) + (n + 30) = 1080$; $5n + 455 = 1080$; $5n = 625$; $n = 125$. **2.** The figure has 6 sides, so it is a hexagon. By the Angle Add. Post., $x + 58 = 180$, so $x = 122$. By the Angle Add. Post., $w + 90 = 180$, so $w = 90$. **3.** The figure has 4 sides, so it is a quadrilateral. By the Angle Add. Post., $m + 64 = 180$; $m = 116$. The sum of the interior $\angle$s is $(n - 2)180 = (4 - 2)180 = 360$; $m + 50 + 89 + a = 360$; $116 + 50 + 89 + a = 360$; $a + 255 = 360$; $a = 105$.

4.



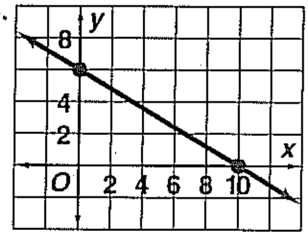
When $x = 0, y = -8$, so plot $(0, -8)$. When $y = 0, x = -2$, so plot $(-2, 0)$. Draw a line through the two points.

5.



When $x = 0, y = 4$, so plot $(0, 4)$. When $y = 0, x = -6$, so plot $(-6, 0)$. Draw a line through the two points.

6.



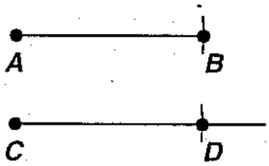
When $x = 0, y = 6$, so plot $(0, 6)$. When $y = 0, x = 10$, so plot $(10, 0)$. Draw a line through the two points.

7. If the slopes are = and the points noncollinear, then the lines are $\parallel$. If the product of the slopes is -1, then the lines are $\perp$. The slope of $\overleftrightarrow{RS} = \frac{6-4}{-2-3} = -\frac{2}{5}$. The slope of $\overleftrightarrow{TV} = \frac{5}{3}$. The slopes are not =, so the lines are not $\parallel$. Since $-\frac{2}{5}\left(\frac{5}{3}\right) = -\frac{2}{3}$ and not -1, the lines are not $\perp$. So, the lines are neither $\parallel$ nor $\perp$. **8.** If the slopes are = and the points are not collinear, then the lines are $\parallel$. If the product of the slopes $= -1$, then the lines are $\perp$. The slope of $\overleftrightarrow{RS} = \frac{-1-0}{6-7} = 1$. The slope of $\overleftrightarrow{TV} = \frac{-4-(-1)}{3-0} = \frac{-3}{3} = -1$. Since the slopes are not =, the lines are not $\parallel$. Since $(1)(-1) = -1$, the lines are $\perp$. **9.** If the slopes are = and the points are not collinear, then the lines are $\parallel$. If the product of the slopes $= -1$, then the lines are $\perp$. The slope of $\overleftrightarrow{RS} = \frac{1-6}{9-5} = -\frac{5}{4}$. The slope of $\overleftrightarrow{TV} = \frac{8-4}{3-(-2)} = \frac{4}{5}$. Since the slopes are not =, the lines are not $\parallel$. Since $\left(-\frac{5}{4}\right)\left(\frac{4}{5}\right) = -1$, the lines are $\perp$. **10.** If the slopes are = and the points are not collinear, then the lines are $\parallel$. If the product of the slopes is -1, then the lines are $\perp$. The slope of $\overleftrightarrow{RS} = \frac{-7-(-9)}{5-(-4)} = \frac{2}{9}$. The slope of $\overleftrightarrow{TV} = \frac{2-0}{6-(-3)} = \frac{2}{9}$. The slope of $\overleftrightarrow{ST} = \frac{2-(-9)}{6-(-4)} = \frac{11}{10}$, so R, S, and T do not have the same slope, so they are not collinear. Since the slopes are = and not collinear, the lines are $\parallel$.

3-7 Constructing Parallel and Perpendicular Lines *pages 165–170*

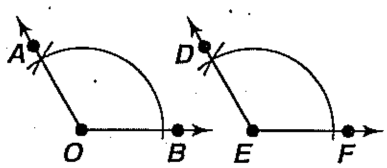
Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM*.

1.



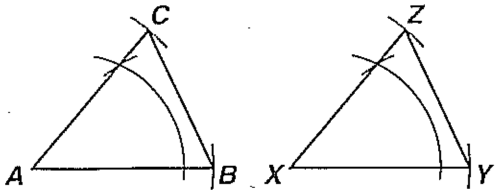
$\overline{AB} \cong \overline{CD}$

2.



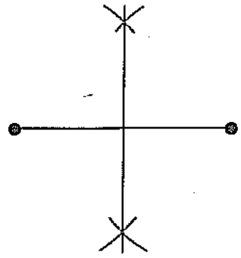
$\angle AOB \cong \angle DEF$

3.

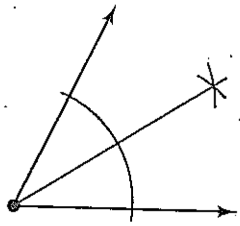


$\triangle ABC \cong \triangle XYZ$

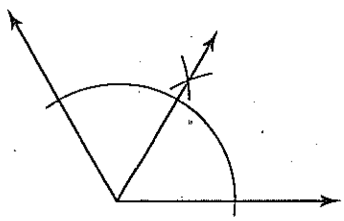
4.



5.

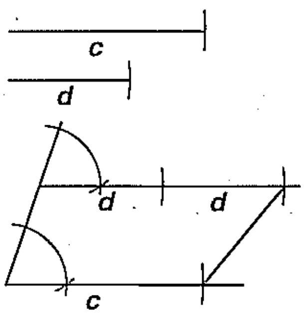


6.



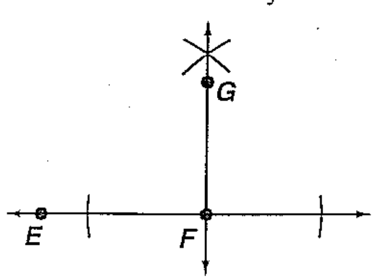
Check Understanding **1.** The constructed ∠s are corr. ∠s: If corr. ∠s are ≅, then the lines are ∥ by the Converse of Corr. Angles Post.

2.



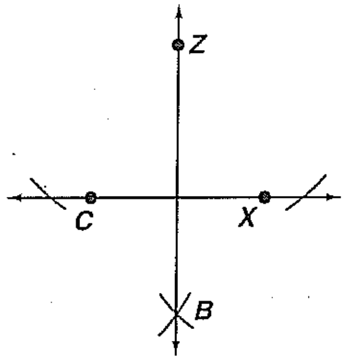
Draw a ray. Open the compass to length c. Keep the setting and swing an arc from the endpt. of the ray to intersect the ray. Draw a new ray whose endpt. is the same as the endpt. of the first ray. Open the compass to length d. Keep the setting and swing an arc from the endpt. to intersect the new ray. With the compass pt. on the last arc intersection, swing a wide arc. Keep the setting and, with the compass pt. on the ray endpt., swing a wide arc that intersects both rays. Place the compass pt. on the intersection of the arc with one of the rays and open it to where it intersects the other ray. Keep that setting and place the compass pt. on the intersection of the previous arc with the new ray, and swing an arc that intersects that arc. Draw a ray that passes through both arc intersections. Measure length c with the compass and swing an arc on the third ray of length c. Connect the arc intersections with the third and first rays to form the fourth side.

3.



Draw $\overleftrightarrow{EF}$ and label points E and F. With the compass pt. on F, swing arcs that intersect the line on each side of F. Make the setting of the compass a little wider. With the compass pt. on one of the arc intersections, swing an arc above F. Keep the same setting. With the compass pt. on the other arc intersection with the line, swing an arc above F that intersects the previous arc. Draw a line through F and the intersection of the two arcs. Label a pt. G.

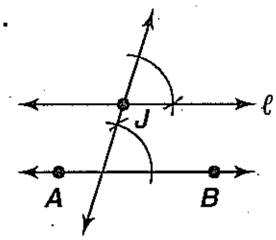
4.



With the compass pt. on Z, swing an arc that intersects $\overleftrightarrow{CX}$ in two places. Place the compass pt. on one of the intersections and swing an arc below Z. Keep the same setting, place the compass pt. on the other intersection, and swing an arc to intersect the previous arc. Call that intersection B. Draw $\overleftrightarrow{ZB}$.

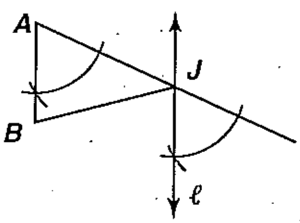
Exercises

1.



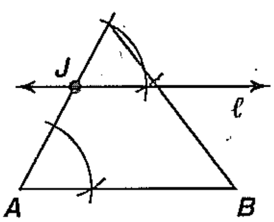
Draw a line through J that intersects $\overleftrightarrow{AB}$. Construct a corr. ∠ with vertex J ≅ to the ∠ just formed by the intersecting lines. $\ell \parallel \overleftrightarrow{AB}$

2.



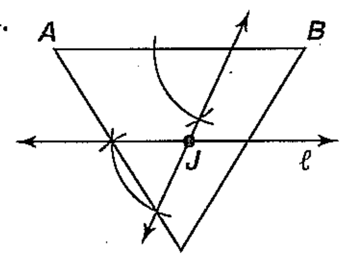
Extend $\overline{AJ}$ beyond J. Construct a corr. ∠ with vertex J congruent to $\angle A$. $\ell \parallel \overline{AB}$

3.



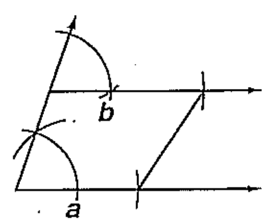
Construct a corr. ∠ with vertex J congruent to $\angle A$. $\ell \parallel \overline{AB}$

4.

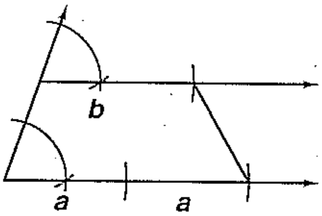


Draw a line through J that intersects $\overline{AB}$. Construct a corr. ∠ with vertex J congruent to the ∠ just formed by the new line and $\overline{AB}$. $\ell \parallel \overline{AB}$

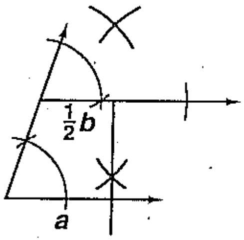
5. Draw 2 rays that share an endpt. Construct congruent corr. ∠s. The third line or ray will be ∥ to one of the other rays. Measure a and b on two separate rays. Draw the fourth side. Constructions may vary. Sample:



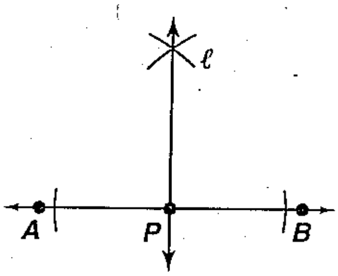
6. Draw 2 rays that share an endpt. Construct congruent corr. $\angle$s. The third line or ray will be $\parallel$ to one of the other rays. Measure a twice on one of the rays, and b on a separate ray. Draw the fourth side. Constructions may vary. Sample:



7. Draw 2 rays that share an endpt. Construct congruent corr. $\angle$s. The third line or ray will be $\parallel$ to one of the other rays. Measure a on one ray and b on a second ray. Construct the $\perp$ bisector of the segment having length b. Draw the fourth side. Constructions may vary. Sample:

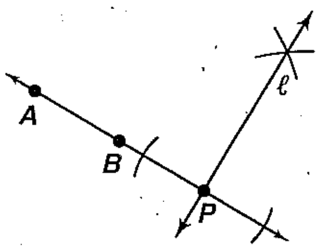


8.



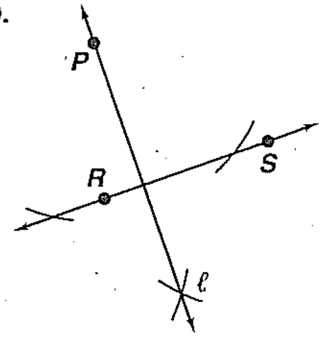
With the compass pt. on P, swing arcs that intersect the line on each side of P. Make the setting of the compass a little wider. With the compass pt. on one of the arc intersections, swing an arc above P. Keep the same setting. With the compass pt. on the other arc intersection with the line, swing an arc above P that intersects the previous arc. Draw a line through P and the intersection of the two arcs. $\ell \perp \overleftrightarrow{AB}$

9.



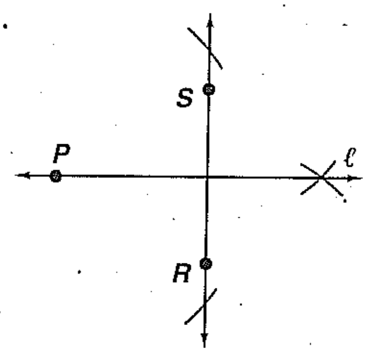
With the compass pt. on P, swing arcs that intersect the line on each side of P. Make the setting of the compass a little wider. With the compass pt. on one of the arc intersections, swing an arc above P. Keep the same setting. With the compass pt. on the other arc intersection with the line, swing an arc above P that intersects the previous arc. Draw a line through P and the intersection of the two arcs. $\ell \perp \overleftrightarrow{AB}$

10.



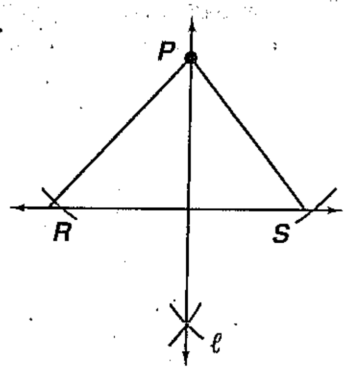
With the compass pt. on P, swing an arc that intersects $\overleftrightarrow{RS}$ in two places. Place the compass pt. on one of the intersections and swing an arc below P. Keep the same setting, place the compass pt. on the other intersection, and swing an arc to intersect the previous arc. Draw a line through the arc intersection and P. $\ell \perp \overleftrightarrow{RS}$

11.



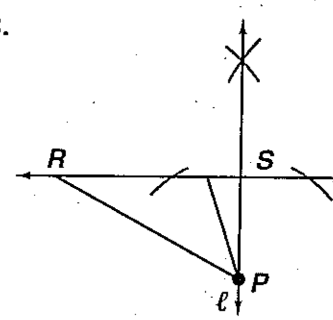
With the compass pt. on P, swing an arc that intersects $\overleftrightarrow{RS}$ in two places. Place the compass pt. on one of the intersections and swing an arc below P. Keep the same setting, place the compass pt. on the other intersection, and swing an arc to intersect the previous arc. Draw a line through the arc intersection and P. $\ell \perp \overleftrightarrow{RS}$

12.



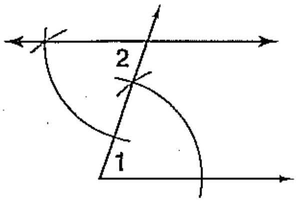
With the compass pt. on P, swing an arc that intersects $\overleftrightarrow{RS}$ in two places. Place the compass pt. on one of the intersections and swing an arc below P. Keep the same setting, place the compass pt. on the other intersection, and swing an arc to intersect the previous arc. Draw a line through the arc intersection and P. $\ell \perp \overleftrightarrow{RS}$

13.



With the compass pt. on P, swing an arc that intersects $\overleftrightarrow{RS}$ in two places. Place the compass pt. on one of the intersections and swing an arc below P. Keep the same setting, place the compass pt. on the other intersection, and swing an arc to intersect the previous arc. Draw a line through the arc intersection and P. $\ell \perp \overleftrightarrow{RS}$

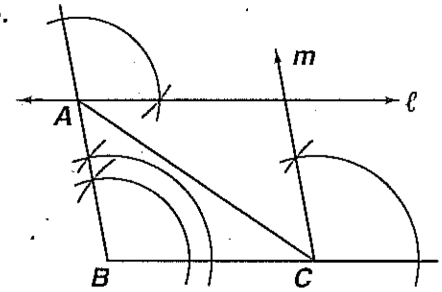
14.



Select a ray as a transversal and a point for the vertex of the new ∠ that is not the same as the vertex for the existing ∠. Construct congruent ∠s on opposite sides of the transversal. $\angle 1 \cong \angle 2$

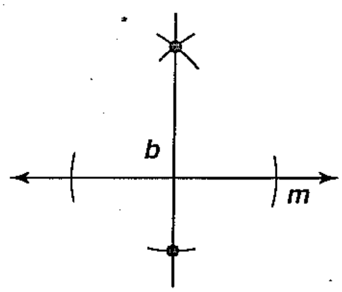
15. Construct a congruent alt. int. ∠, as in Exercise 14, then draw the ∥ line.

16a and b.

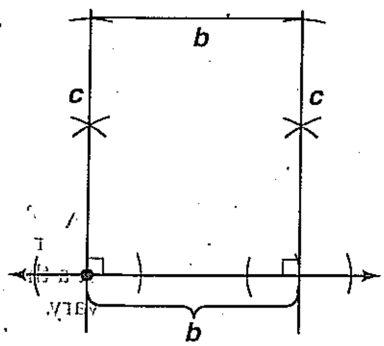


Construct an ∠ with vertex at A, congruent and corr. to $\angle B$. Repeat with a congruent corr. ∠ at C.

17. Choose a point on m and construct a line ⊥ to m through the point. Then construct a segment of length b on the ⊥ line. Constructions may vary. Sample:

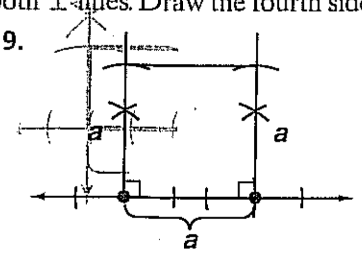


18.



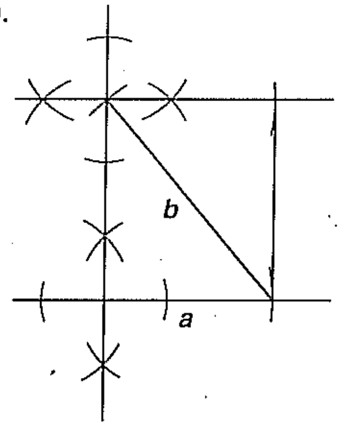
Constructions may vary. Sample: Draw a line. Construct a line ⊥ to the line. Measure b on the line from the intersection of the line and the line ⊥ to it. Construct a second line ⊥ to the original line at the other end of the segment of length b. Measure c on both ⊥ lines. Draw the fourth side.

19.



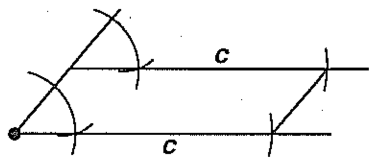
Constructions may vary. Sample: Draw a line. Construct a line ⊥ to the line. Measure a on the line from the intersection of the line and the line ⊥ to it. Construct a second line ⊥ to the original line at the other end of the segment of length a. Measure a on both ⊥ lines. Draw the fourth side.

20.



Draw a line. Construct a line ⊥ to it. Measure distance a, on the original line from the intersection of the two lines. At the other endpt. of the segment of length a, swing an arc whose radius is b to intersect the line ⊥ to the first line. Draw the segment. On the last segment drawn, at its endpt. that lies on the ⊥ line, construct a line ⊥ to the ⊥ line. Measure a on that line. Draw the fourth side.

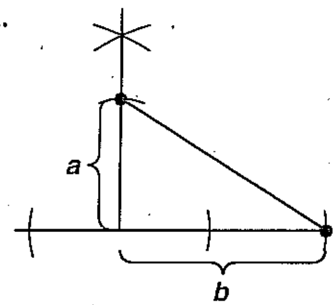
21a.



Draw a ray and measure c from its endpoint. Draw a transversal through the endpt. of the ray. Construct congruent corr. ∠s and draw the ∥ side having length c. Draw the fourth side.

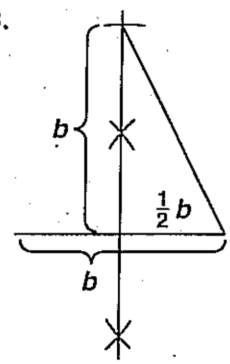
21b. The other pair of sides are ∥ and ≅. **21c.** Check students' work.

22.



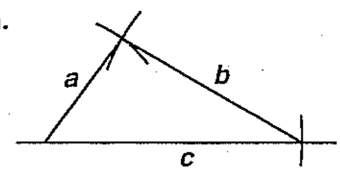
Draw a line and a line ⊥ to it. From the intersection, measure a on one line and b on the other line. Draw the third side.

23.



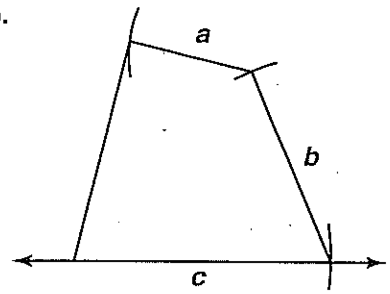
Draw a line and a line ⊥ to it. From the intersection, measure b on both lines. Construct the ⊥ bis. of the last constructed segment. Draw the third side of the △.

24a.



Draw a line and choose a pt. on the line for the vertex of one of the ∠s. Measure c on the line from the vertex pt. With the compass pt. on the vertex pt. swing an arc whose radius is a. With the compass pt. on the other endpt. of the c-length segment, swing an arc whose radius is b to intersect the last arc. Draw segments from the endpts. of the c-length segment to the intersection of the two arcs.

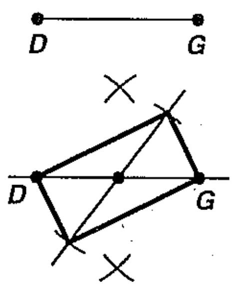
24b.



Answers may vary. Sample: Draw a segment of length c on a line. From the endpt. of the segment, construct a segment of length b. From the open endpt. of that segment, construct a segment of length a. Connect the endpt. of the last segment to the open endpt. of the first segment.

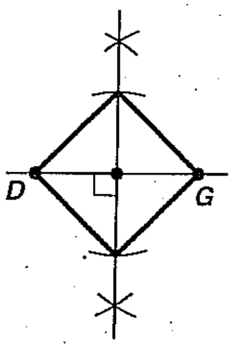
24c. If the lengths for the 3 sides are given, only one △ is possible. All ⚠ will be the same size and shape. However, many different quadrilaterals are possible because the ⚠ formed by the sides can vary. Quadrilaterals can be "squished" while ⚠ cannot.
25a–c. Construct a △ with sides of a, b, and c as in Exercise 24a. Then construct the ⊥ bis. of each side. Connect the bisectors at their intersections with the sides. **25d.** The sides of the smaller △ are ∥ to and half the length of the sides of the larger △. **25e.** Check students' work. **26a and b.** Check students' work.
26c. $p \parallel m$, because in a plane, two lines ⊥ to a third line are ∥ to each other.

27.



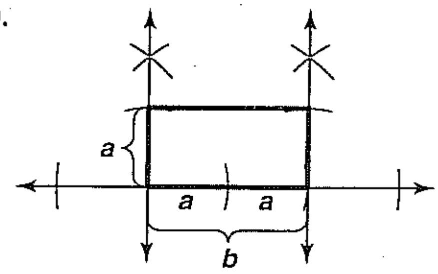
Answers may vary. Sample: Construct $\overline{DG}$. Locate the intersection point of the ⊥ bis. of $\overline{DG}$ and $\overline{DG}$. This is the midpt. of $\overline{DG}$. Draw a line through the midpt. Measure the length of half DG and mark that length on the new line from the midpt. Repeat on the other side of the midpt. Draw segments from the endpts. of $\overline{DG}$ to each of the arc intersections with the new line. The quad. is a rectangle.

28.



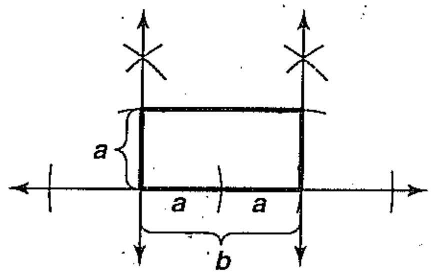
Answers may vary. Sample: Construct $\overline{DG}$. Construct the ⊥ bis. of $\overline{DG}$. The intersection of the two lines is the midpt. of $\overline{DG}$. Measure the length of half DG and mark that length on the new line from the midpt. Repeat on the other side of the midpt. Draw segments from the endpts. of $\overline{DG}$ to each of the arc intersections with the new line. The quad. is a square.

29.



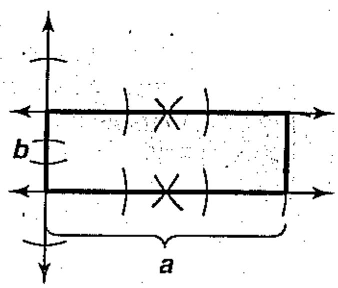
Draw a line. Locate a point for a vertex of the rectangle. Construct a line ⊥ to the line through the vertex. Construct a segment on both lines that measure a from the vertex. Construct a second segment of length a from the endpt. of one of the segments. Construct a line ⊥ to the open endpt. of the $2a$ segment. From the intersection of the two lines, construct a segment of length a. Draw the fourth side.

30.



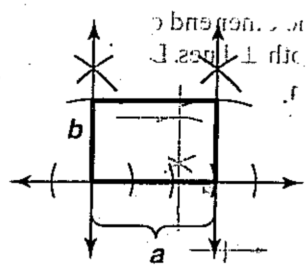
Draw a line and construct a segment of length a. Construct a line ⊥ to the segment at both ends. Then construct the ⊥ bis. of the segment and measure one of the two new segments as $\frac{1}{2}a$, or b. Construct a segment of length b on both of the ⊥ lines at the endpts. of the a-length segment. Draw the fourth side.

31.

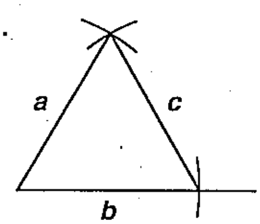


Draw a line. Measure b 3 times on that line to construct a as $3b$. Construct a line ⊥ to the endpt. of the segment. Construct a segment of length b on the ⊥ line. At the open endpt. of that segment, construct another ⊥ line. Measure b 3 times on that line to construct a third side whose measure is $3b$. Constructions may vary. Sample: Draw the fourth side.

32. Construct a segment of length x. Then construct a segment of length $2x$ for b and $3x$ for a. Construct a rectangle using these lengths. Constructions may vary. Sample:



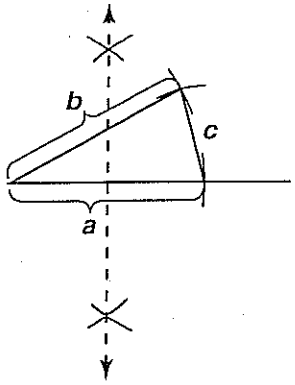
33.



$a = b = c$

Construct an equilateral △.

34.

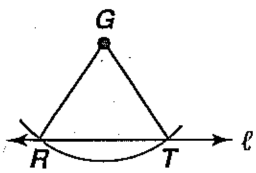


$a = b = 2c$

Construct a segment of length c. Construct two more segments having lengths $2c$ and call their measures a and b. Construct a △.

35. It is not possible to construct a △ having lengths a, b, and c if $a = 2b = 2c$, since $2a = 2b + 2c$ or $a = b + c$; the smaller sides would meet at the midpoint of the longer side, forming a segment. **36.** It is not possible to construct a △ having lengths a, b, and c if $a = b + c$. The smaller sides would meet at the midpoint of the longer side, forming a segment. **37.** The figure shows the construction of a ⊥ bis. The answer is choice A.
38. Eliminate choice H, since $\ell \parallel n$. Eliminate choice G, since $\ell \perp m$. Since ℓ intersects m and $\ell \parallel n$, then n intersects m, also. The answer is choice I.

39. [2] **a.**



b. All points on the arc with center G are the same dist. from G, so $GR = GT$. The △ is isosc. because an isosc. △ must have at least two sides of the same length. [1] incorrect sketch OR incorrect △ classification **40.** [2] **a.** The diagram with the least markings is the first step, and they increase in markings with each step: II, IV, III, I. **b.** (III): location of compass at points C and G; (I): same as III *and* the intersection points of $\overleftrightarrow{CG}$ with arcs drawn in III
[1] incorrect sequence OR incorrect location of compass point **41.** The lines are ∥ if the slopes are =. The slope of the first eq. is -4 and the slope of the second eq. is 4. Since $-4 \neq 4$, the lines are not ∥. **42.** The lines are ∥ if the slopes are =. The slope of the first eq. is $\frac{1}{2}$ and the slope of the second eq. is -2. Since $\frac{1}{2} \neq -2$, the lines are not ∥. **43.** The first eq. in slope-int. form is $y = -\frac{1}{3}x - 2$, so the slope is $-\frac{1}{3}$. The second eq. in slope-int. form is $y = -\frac{1}{3}x - \frac{1}{2}$. Since the slopes are = and the y-intercepts are ≠, the lines are ∥. **44.** $WZ =$
$\sqrt{(8 - 2)^2 + (-2 - 6)^2} = \sqrt{(6)^2 + (-8)^2} =$
$\sqrt{36 + 64} = \sqrt{100} = 10$ **45.** $WZ =$
$\sqrt{(-4.5 - 3.5)^2 + (1.2 - (-2.8))^2} =$
$\sqrt{(-8)^2 + (4)^2} = \sqrt{64 + 16} = \sqrt{80} \approx 8.9$ **46.** Two planes intersect in a line, and the points common to both planes are B and E: $\overleftrightarrow{EB}$. **47.** Two planes intersect in a line, and the points common to both planes are D and F: $\overleftrightarrow{DF}$.

TECHNOLOGY — page 171

1. Check students' work. **2a.** Check students' work. **2b.** The ∠ measures increase by smaller and smaller increments.

2c.

L1	L2
3	60
4	90
5	108
6	120
7	128.57
8	135
9	140
10	144
11	147.27
12	150
13	152.31
14	154.29
15	156
16	157.5
17	158.82
18	160
19	161.05
20	162

2d. The ∠ measures increase by smaller and smaller increments.

3. The ∠ measures decrease and approach 0.

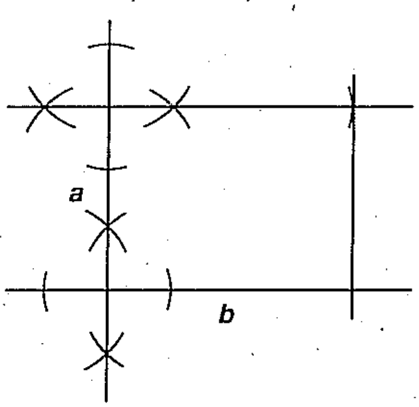
TEST-TAKING STRATEGIES — page 172

1. In the line after the initial equation, 12 should be -12. **2.** Answers may vary. Sample:
[4] **a.** $x + (x + 4) + (x + 8) = 360$ **b.** $3x = 348; x = 116$
c. Since $x + 8$ is the measure of the largest ext. ∠, its suppl. will be the smallest int. ∠. $116 + 8 = 124$ and $180 - 124 = 56$. The smallest interior ∠ has measure 56.

CHAPTER REVIEW — pages 173–175

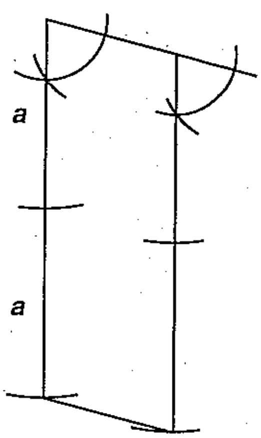
1. The only plane geometric ∠s that do not appear in a △ are straight and 0-degree ∠s. So, in a △, an ∠ is right, obtuse, or acute. **2.** By def. of obtuse, an obtuse ∠ has a measure between 90 and 180. **3.** When two coplanar lines are cut by a transversal, two ∠s that occupy similar positions with the transversal and cut lines are called corresponding ∠s. **4.** By the Triangle Exterior Angle Thm., the measure of an exterior ∠ of a polygon is equal to the sum of the measures of its two remote interior ∠s. **5.** By def. of convex, a polygon is convex if no diagonal contains points outside the polygon. **6.** By def. of equiangular, an equiangular polygon has all ∠s congruent. **7.** By def. of regular, a regular polygon is both equiangular and equilateral. **8.** Since point-slope form of an equation is $y - y_1 = m(x - x_1)$, the linear equation $y - 3 = 4(x + 5)$ is written in point-slope form. **9.** Since the slope-intercept form of a linear equation readily reveals both the slope and the intercept as m and b, respectively, in $y = mx + b$, from the slope-intercept form of a linear equation, you can easily read the value of the slope and the value of the y-intercept. **10.** By def. of alt. int. ∠s, when two coplanar lines are cut by a transversal, the ∠s between the two lines and on opposite sides of the transversal are called alternate interior ∠s. **11.** $\angle 1$ and $\angle 2$ are suppl., so $m\angle 1 + m\angle 2 = 180$; $59 + m\angle 2 = 180$; $m\angle 2 = 121$. $\angle 1$ and $\angle 3$ are vertical ∠s, so $m\angle 3 = m\angle 1 = 59$. $\angle 1$ and $\angle 4$ are corr. ∠s, so $m\angle 4 = m\angle 1 = 59$. **12.** Since corr. ∠s of ∥ lines are ≅, $m\angle 1 = 120$. Since alt. ext. ∠s are ≅, $m\angle 2 = 120$, or, since vert. ∠s are ≅, $m\angle 2 = m\angle 1 = 120$. **13.** Since same-side int. ∠s are suppl., $m\angle 1 = 180 - 105 = 75$. Since alt. int. ∠s are ≅, $m\angle 2 = 105$. Or, since $\angle 1$ and $\angle 2$ are 2 ∠s that form a straight ∠ so they are suppl., $m\angle 2 = 180 - m\angle 1 = 180 - 75 = 105$. **14.** Since same-side int. ∠s are suppl., $m\angle 1 = 180 - 125 = 55$. Since alt. int. ∠s are ≅, $m\angle 2 = 90$. **15.** Pairs of consecutive ∠s are suppl. since the sides of the quad. are transversals and the int. ∠s are on the same side of the transversal. **16.** Since the lines are ∥, alt. int. ∠s are =, so $3x + 5 = 65$; $3x = 60$; $x = 20$. **17.** Since vert. ∠s are ≅, the int. ∠ on the opp. side of the transversal of the 130-degree ∠ measures 130. Since same-side int. ∠s are suppl., $2x + 10 + 130 = 180$; $2x + 140 = 180$; $2x = 40$; $x = 20$. **18.** Since the lines are ∥, corr. ∠s are ≅, so $5x - 32 + 3x + 20 = 180$; $8x - 12 = 180$; $8x = 192$; $x = 24$.

19.



Constructions may vary. Sample: Draw a line. Construct a segment of length b on the line. At one endpt. of the segment, construct a ⊥ line. Measure a on that line. Construct a ⊥ line at one of the two segment's open endpt. Measure b on it if it is adjacent to a, or a if it is adjacent to b.

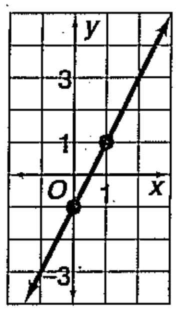
20.



Draw a line and construct a segment having length $2a$. Use congruent corr. ∠s to construct a line ∥ to it, and measure $2a$ on that line. Draw the fourth side.

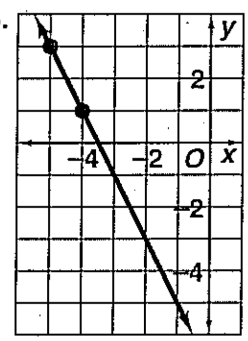
21. Congruent; specifically, they are congruent corr. ∠s. **22.** By the Triangle Angle-Sum Thm., $x + 78 + 41 = 180$; $x = 119 + 180$; $x = 61$. Since no side lengths are =, the △ is scalene. Since all the ∠s are acute, the △ is acute. **23.** By the Triangle Ext. Angle Thm., $x + 60 = 120$, so $x = 60$. By the Triangle Angle-Sum Thm., $x + y + 60 = 180$; $60 + y + 60 = 180$; $y + 120 = 180$; $y = 60$. Since all side lengths are =, the △ is equilateral. Since all ∠ measures are =, the △ is equiangular. **24.** By the Triangle Ext. Angle Thm., $x + 90 = 135$, so $x = 45$. By the Triangle Angle-Sum Thm., $x + y + 90 = 180$; $45 + y + 90 = 180$; $y + 135 = 180$; $y = 45$. Since 2 sides of the △ have = length, the △ is isosceles. Since the △ has a right ∠, it is a right △. **25.** By the Triangle Angle-Sum Thm., $(x + 10) + (x - 20) + (x + 25) = 180$; $3x + 15 = 180$; $x + 5 = 60$; $x = 55$. By substitution, $x + 10 = 55 + 10 = 65$. By substitution, $x - 20 = 55 - 20 = 35$. By substitution, $x + 25 = 55 + 25 = 80$. Since all ∠s are acute, the △ is acute. **26.** By the Triangle Angle-Sum Thm., $x + 2x + 3x = 180$; $6x = 180$; $x = 30$. $2x = 2(30) = 60$; $3x = 3(30) = 90$. Since one of the ∠s is a right ∠, the △ is right. **27.** By the Triangle Angle-Sum Thm., $(20x + 10) + (30x - 2) + (7x + 1) = 180$; $57x + 9 = 180$; $57x = 171$; $x = 3$. By substitution, $20x + 10 = 20(3) + 10 = 60 + 10 = 70$. By substitution, $30x - 2 = 30(3) - 2 = 90 - 2 = 88$. By substitution, $7x + 1 = 7(3) + 1 = 21 + 1 = 22$. Since all ∠s are acute, the △ is acute. **28.** By the Triangle Angle-Sum Thm., $(10x - 3) + (14x - 20) + (x + 3) = 180$; $25x - 20 = 180$; $25x = 200$; $x = 8$. By substitution, $10x - 3 = 10(8) - 3 = 80 - 3 = 77$. By substitution, $14x - 20 = 14(8) - 20 = 112 - 20 = 92$. By substitution, $x + 3 = 8 + 3 = 11$. Since one ∠ is obtuse, the △ is obtuse. **29.** One ∠ is a right ∠, so it measures 90. The sum of the measures of the other two ∠s must be 90, so the other two ∠s are complementary. **30.** A hexagon has 6 sides, so one exterior ∠ measures $\frac{360}{n} = \frac{360}{6} = 60$. An interior ∠ measures $180 - 60$, or 120. **31.** An octagon has 8 sides, so one exterior ∠ measures $\frac{360}{n} = \frac{360}{8} = 45$. An interior ∠ measures $180 - 45 = 135$. **32.** A decagon has 10 sides, so an exterior ∠ measures $\frac{360}{n} = \frac{360}{10} = 36$. An interior ∠ measures $180 - 36 = 144$. **33.** An exterior ∠ of a 24-gon measures $\frac{360}{n} = \frac{360}{24} = 15$. An interior ∠ measures $180 - 15 = 165$. **34.** The sum of the measures of the exterior ∠s of any polygon is always 360.

35.



If an equation is in the form $y = mx + b$, then m is the slope and b is the y-intercept, so in $y = 2x - 1$, the slope is 2 and the y-intercept is -1. To graph, plot $(0, -1)$ and count 2 up and 1 right to plot $(1, 1)$; draw a line through the 2 points.

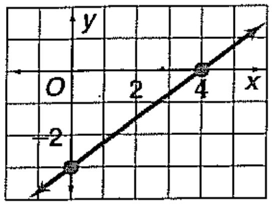
36.



If an equation is in the form $y - y_1 = m(x - x_1)$, then m is the slope and a point on the line is (x_1, y_1). So, for $y - 3 = -2(x + 5)$, -2 is the slope and $(-5, 3)$ is a point on the line. To graph, plot $(-5, 3)$ and then count down 2 units and right 1 unit to plot $(-4, 1)$. Draw a line through the two points.

37. For all x, $y = -\frac{1}{2}$, so draw a horizontal line that passes through $(0, -\frac{1}{2})$.

38.

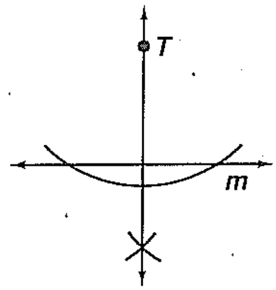


When $x = 0$, $y = -3$, so plot $(0, -3)$. When $y = 0$, then $x = 4$, so plot $(4, 0)$. Draw a line through the two points. **39.** For all y-values, $x = 6$, so the equation is $x = 6$. **40.** The slope of $\overleftrightarrow{AB} = \frac{11 - (-4)}{2 - (-1)} = \frac{15}{3} = 5$. The slope of $\overleftrightarrow{CD} = \frac{10 - 1}{4 - 1} = \frac{9}{3} = 3$. Since $5 \neq 3$, the lines are not $\parallel$. Since $5(3) = 15$ and not -1, the lines are not $\perp$. So the lines are *neither* $\parallel$ nor $\perp$. **41.** The slope of $\overleftrightarrow{AB} = \frac{8 - (-2)}{2 - (-1)} = \frac{10}{3}$. The slope of $\overleftrightarrow{CD} = \frac{7 - (-3)}{3 - 0} = \frac{10}{3}$. The slope of $\overleftrightarrow{BC} = \frac{8 - 0}{4 - (-6)} = \frac{8}{10} = \frac{4}{5}$, so A, B, and C are not collinear. Since the slopes are = and the points are not collinear, the lines are $\parallel$. **42.** The slope of $\overleftrightarrow{AB} = \frac{3 - 2}{-3 - 0} = -\frac{1}{3}$. The slope of $\overleftrightarrow{CD} = \frac{3 - (-6)}{1 - (-2)} = \frac{9}{3} = 3$. Since $\left(-\frac{1}{3}\right)(3) = -1$, the lines are $\perp$. **43.** The slope of $\overleftrightarrow{AB} = \frac{8 - 3}{4 - (-1)} = \frac{5}{5} = 1$. The slope of $\overleftrightarrow{CD} = \frac{8 - 0}{2 - (-6)} = \frac{8}{8} = 1$. The slope of $\overleftrightarrow{BC}$ is $\frac{8 - 0}{4 - (-6)} = \frac{8}{10} = \frac{4}{5}$. Since A, B, and C are not collinear and since the slopes are =, the lines are $\parallel$. **44.** The slope is $\frac{8 - 8}{4 - 2} = \frac{0}{2} = 0$. The y-coordinates are always =, so their difference is always zero.

CHAPTER TEST page 176

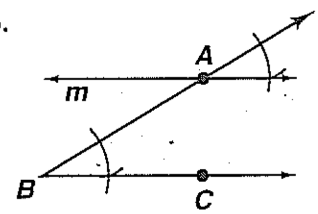
1. The $\angle$s measure 48, 66, and 66, so, since they are all acute, the $\triangle$ is acute. The sides measure 2.2 cm, 2.2 cm, and 1.8 cm, so, since two sides are =, the $\triangle$ is isosceles. **2.** The measure of the largest $\angle$ is 130, so it is obtuse, so also the $\triangle$ is obtuse. The measure of the sides are 2.3, 3.8, and 1.8 cm. Since none of the sides are =, the $\triangle$ is scalene. **3.** Since the corr. $\angle$s to $\parallel$ lines are $\cong$, $m\angle 1 = 65$. Since vert. $\angle$s are $\cong$, $m\angle 2 = m\angle 1 = 65$. **4.** Since alt. int. $\angle$s of $\parallel$ lines are $\cong$, $m\angle 1 = 85$. Since same-side int. $\angle$s of $\parallel$ lines are suppl., $m\angle 2 = 180 - 70 = 110$. **5.** Since corr. $\angle$s of $\parallel$ lines are $\cong$, $m\angle 1 = 85$. Since same-side interior $\angle$s of $\parallel$ lines are suppl., $m\angle 2 = 180 - m\angle 1 = 180 - 85 = 95$. **6.** Since corr. $\angle$s of $\parallel$ lines are $\cong$, $m\angle 1 = 70$. Since same-side int. $\angle$s are suppl., $m\angle 2 = 180 - m\angle 1 = 180 - 70 = 110$. **7.** Since $40 + 140 = 180$, the $\angle$s are suppl., so they can be the measures of a pair of same-side int. $\angle$s. **8.** Since $90 + 90 = 180$, the $\angle$s are suppl., so they can be the measures of a pair of same-side int. $\angle$s. **9.** Since $60 + 60 = 120$ and not 180, the $\angle$s are not suppl., so they cannot be the measures of a pair of same-side int. $\angle$s. **10.** Since $27 + 27 = 54$ and not 180, the $\angle$s are not suppl., so they cannot be the measures of a pair of same-side int. $\angle$s. **11.** Since alt. int. $\angle$s of $\parallel$ lines are $\cong$, $14x - 5 = 13x$; so $x = 5$. **12.** Since same-side int. $\angle$s of $\parallel$ lines are suppl., $(5x - 20) + 3x = 180$; $8x - 20 = 180$; $8x = 200$; $x = 25$. **13.** Since alt. int. $\angle$s of $\parallel$ lines are $\cong$, the other remote int. $\angle$ from 126° is $10x$. By the Triangle Ext. Angle Thm., $10x + 11x = 126$; $21x = 126$; $x = 6$. **14.** Since alt. ext. $\angle$s of $\parallel$ lines are $\cong$, $2x - 30 = x + 45$; $x - 30 = 45$; $x = 75$.

15.



From T swing an arc to intersect the line in 2 places. From each arc intersection with the line, swing congruent arcs to intersect each other. Draw a line through the arc intersection through T.

16.



With $\overrightarrow{AB}$ as the transversal, construct a congruent corr. $\angle$ whose vertex is at A. **17.** Answers may vary. Possible answers include H, I, N, and Z. **18.** Check students' work. **19a.** Given **19b.** They are corr. $\angle$s of $\parallel$ lines: Corr. $\angle$s are $\cong$. **19c.** Given **19d.** from steps 2 and 3: Trans. Prop. of $\cong$ **19e.** Conv. of Corr. Angles Post.: If corr. $\angle$s are $\cong$, then the lines are $\parallel$. **20.** In point-slope form, the eq. is $y - (-1) = -5(x - 3)$, which simplifies to $y + 1 = -5(x - 3)$. Other equivalent forms of the equation include: $5x + y = 14$ and $y = -5x + 14$. **21.** Draw segments connecting each of the vertices. If no points of any of the segments are outside the polygon, then it is convex. Otherwise, the polygon is concave. **22.** Since the figure has 5 sides, the sum of the measures of the interior $\angle$s is $(n - 2)180 = (5 - 2)180 = (3)180 = 540$. So, $x + 120 + 95 + 96 + 120 = 540$; $x + 431 = 540$; $x = 109$. **23.** Solve for x: $x = 180 - 95 = 85$. Solve for y: $y = 180 - 80 = 100$. The sum of the exterior $\angle$s is 360 for all polygons, so the suppl. of $z = 180 - z = 360 - (80 + 62 + x + 53)$; $180 - z = 360 - (80 + 62 + 85 + 53)$; $180 - z = 80$; $-z = -100$; $z = 100$. **24.** The slopes are 4 and $-\frac{1}{4}$. Since $(4)\left(-\frac{1}{4}\right) = -1$, the lines are $\perp$. **25.** The slopes are both 3 and the y-intercepts are both different, so the lines are $\parallel$. **26.** The slopes are 1 and -5. Since they are not =, the lines are not $\parallel$. Since $(1)(-5) = -5$ and not -1, the lines are not $\perp$. So, the lines are *neither* $\parallel$ nor $\perp$. **27.** $y = -3$ has slope 0, so it is horizontal. $x = 10$ has no slope, so it is vertical. Since one is horizontal and the other is vertical, the lines are $\perp$. **28.** $\frac{360}{n} = \frac{360}{12} = 30$

STANDARDIZED TEST PREP page 177

1. From the second sentence in the first paragraph, the present runways are labeled A through G, 7 letters, so there are currently 7 runways. The answer is choice D. **2.** From the diagram, A and C intersect, so eliminate choices F and I. The second sentence indicates that there are 3 pairs of parallel runways, and those listed in choice G are correct. The answer is choice G. **3.** The third sentence in the first paragraph states that Runways B

and D are $\perp$. The answer is choice B. **4.** The first sentence in the second paragraph indicates that the crisscrossing layout and takeoff/landing patterns interfere with full-capacity use. Runways E and G do not crisscross. The answer is choice H. **5.** Runways B, C, and D form a right $\triangle$ and 2 of the $\angle$s measure 90 and 48. They are the remote interior $\angle$s of the obtuse $\angle$, so its measure is, by the Triangle Exterior Angle Thm., $90 + 48 = 138$. The answer is choice D.
6. The 48-degree $\angle$ and the $\angle$ formed by Runways A and D are the remote interior $\angle$s of the 94-degree $\angle$. By the Triangle Ext. Angle Thm., $x + 48 = 94$; $x = 46$. The answer is choice F. **7.** Since Runways A and E are $\parallel$, Runway C is a transversal, forming $\angle$s measuring 94 and $180 - 94$, or 86. Since 86 is acute, the answer is choice C.
8. Since Runways A and E are $\parallel$, Runway C is a transversal, forming $\angle$s measuring 94 and $180 - 94$, or 86. Since Runways C and G are $\parallel$, the 86-degree $\angle$ is a corr. $\angle$ with the acute int. $\angle$ of the intersection of Runways E and G. Since corr. $\angle$s of $\parallel$ lines are $\cong$, the intersection of the two runways is 86° and the obtuse $\angle$ formed is 94°. The answer is choice H. **9.** The two paragraphs and diagram offer no information about the rate of landings and takeoffs. The answer is choice D. **10.** The third sentence in the second paragraph indicates that an additional 292 acres of land from the south and 141 acres from the north are needed, for a total of 433 acres. The answer is choice I. **11.** From the last sentence in the second paragraph, the numbers of real estate units are 240 apartments, more than 300 houses, and 70 businesses for a total of more than 610 real estate units. The total number of acres needed is $292 + 141$, or 433 acres. The average is the number of real estate units divided by the number of new acres. There are more than $\frac{610}{433}$, or more than 1.4 real estate units per acre. **12.** Runways with a positive slope rise from left to right. The answer is choice C. **13.** Runways with a negative slope fall from left to right. The answer is choice I. **14.** The nonvertical and nonhorizontal runways that intersect at about 90° are Runways A and C. Since Runways C and G are $\parallel$, their slopes are $=$. So, Runways A and G have slopes whose product is nearly -1. The answer is choice A.