Inequalities and Their Graphs

Determine whether each number is a solution of the given inequality.

1.
$$x \le -8$$

a.
$$-10$$

c.
$$-8$$

2.
$$-1 > x$$

3.
$$w < \frac{18}{7}$$

c.
$$3\frac{1}{2}$$

4.
$$0.65 \ge y$$

5.
$$2y + 1 > -5$$
 a. -4

$$a. -4$$

6.
$$7x - 14 \le 6x - 16$$
 a. 0

7.
$$n(n-6) \ge -4$$

Write an inequality for each graph.

Write each inequality in words and then graph.

12.
$$x > 6$$

13.
$$v \leq -10$$

14.
$$8 \ge b$$

15.
$$-4 < w$$

16.
$$x < -7$$

17.
$$x \ge 12$$

Define a variable and write an inequality to model each situation.

18. The temperature in a refrigerated truck must be kept at or below 38°F.

19. The maximum weight on an elevator is 2000 pounds.

20. A least 20 students were sick with the flu.

21. The maximum occupancy in an auditorium is 250 people.

22. The maximum speed on the highway is 55 mi/h.

23. A student must have at least 450 out of 500 points to earn an A.

24. The circumference of an official major league baseball is at least 9.00 inches.

Match the inequality with its graph.

25.
$$6 < x$$

26.
$$-6 \ge x$$

27.
$$4 > x$$

28.
$$x \le -4$$

Solving Inequalities Using Addition and Subtraction

Solve each inequality. Graph and check the solution.

1.
$$n - 7 \ge 2$$

2.
$$10 + y > 12$$

3.
$$3.2 < r + 4.7$$

4.
$$7 + b > 13$$

5.
$$n + \frac{3}{4} > \frac{1}{2}$$
 6. $-\frac{5}{7} \ge c + \frac{2}{7}$ **7.** $g + 4.6 < 5.9$ **8.** $0 > d - 2.7$

6.
$$-\frac{5}{7} \ge c + \frac{2}{5}$$

7.
$$g + 4.6 < 5.9$$

8.
$$0 > d - 2$$

9.
$$f + 4 \ge 14$$

10.
$$x + 1 \le -3$$

11.
$$d - 13 \le -8$$

12.
$$m-7 \ge -8$$

13.
$$12 + \nu < 19$$

14.
$$-4 \le t + 9$$

15.
$$6 < y - 3$$
 16. $a + 15 > 19$

17.
$$8 + d < 9$$

18.
$$s + 3 \le 3$$

19.
$$9 + h \le 5$$

20.
$$7.6 \ge t - 2.4$$

Write and solve an inequality that models each situation.

- 21. It will take at least 360 points for Kiko's team to win the math contest. The scores for Kiko's teammates were 94, 82, and 87, but one of Kiko's teammates lost 2 of those points for an incomplete answer. How many points must Kiko earn for her team to win the contest?
- 22. This season, Nora has 125 at-bats in softball. By the end of the season she wants to have at least 140 at-bats. How many more at-bats does Nora need to reach her goal?
- 23. The average wind speed increased 19 mi/h from 8 A.M. to noon. The average wind speed decreased 5 mi/h from noon to 4 P.M. At 4 P.M., the average wind speed was at least 32 mi/h. What is the minimum value of the average wind speed at 8 A.M.?
- 24. Suppose it takes no more than 25 min for you to get to school. If you have traveled for 13.5 min already, how much longer, at most, might you take to get to school?
- 25. Joan has started a physical fitness program. One of her goals is to be able to run at least 5 mi without stopping. She can now run 3.5 mi without stopping. How many more miles must she run non-stop to achieve her goal?
- 26. Suppose you can get a higher interest rate on your savings if you maintain a balance of at least \$1000 in your savings account. The balance in your savings account is now \$1058. You deposit \$44.50 into your account. What is the greatest amount that you can withdraw and still get the higher interest rate?

Solve each inequality. Graph and check the solution.

27.
$$\frac{3}{4} + z \ge -\frac{3}{4}$$
 28. $12 + d + 3 \le 10$ **29.** $\nu - \frac{3}{4} > 1\frac{1}{4}$ **30.** $8 + m > 4$

28.
$$12 + d + 3 \le 1$$

29.
$$v - \frac{3}{4} > 1$$

30.
$$8 + m > 4$$

31.
$$2 + f > -3$$

32.
$$-27 \ge w - 24$$

33.
$$b + \frac{1}{2} > \frac{3}{4}$$

31.
$$2 + f > -3$$
 32. $-27 \ge w - 24$ 33. $b + \frac{1}{2} > \frac{3}{4}$ 34. $12 + t < 4 - 15$

35.
$$-14 > -16 + u$$
 36. $-7 \le -11 + z$ **37.** $38 \ge 33 + b$ **38.** $k - 27 < -29$

36.
$$-7 \le -11 + 3$$

37.
$$38 \ge 33 + t$$

38.
$$k - 27 < -29$$

39.
$$a + 8 \le 10$$
 40. $b + 6 > 17$

40.
$$b + 6 > 17$$

41.
$$13 < 8 + k - 6$$
 42. $j + 1.3 > 2.8$

42.
$$i + 1.3 > 2.8$$

Solving Inequalities Using Multiplication and Division

Solve each inequality. Graph and check the solution.

1.
$$\frac{15}{8} \le \frac{5}{2}s$$

2.
$$60 \le 12b$$

3.
$$-\frac{4}{5}r < 8$$

4.
$$\frac{5}{2} < \frac{n}{8}$$

5.
$$-9n \ge -36$$
 6. $\frac{n}{7} \ge -6$

6.
$$\frac{n}{7} \ge -6$$

7.
$$-7c < 28$$

8.
$$16d > -64$$

9.
$$-\frac{t}{3} < -5$$
 10. $54 < -6k$

10.
$$54 < -6k$$

11.
$$\frac{w}{7} > 0$$

12.
$$2.6v > 6.5$$

13.
$$-4 < -\frac{2}{5}m$$
 14. $17 < \frac{p}{2}$

14.
$$17 < \frac{p}{2}$$

15.
$$0.9 \le -1.8v$$

16.
$$-5 \le -\frac{x}{9}$$

17.
$$-1 \ge \frac{d}{7}$$

18.
$$-3x \ge 21$$

19.
$$\frac{c}{12} < \frac{3}{4}$$

20.
$$\frac{a}{4} \le -1$$

Write and solve an inequality that models each situation.

- Suppose you and a friend are working for a nursery planting trees. Together you can plant 8 trees per hour. What is the greatest number of hours that you and your friend would need to plant at most 40 trees?
- 22. Suppose the physics club is going on a field trip. Members will be riding in vans that will hold 7 people each including the driver. At least 28 people will be going on the field trip. What is the least number of vans needed to make the trip?
- 23. You need to buy stamps to mail some letters. The stamps cost \$.34 each. What is the maximum number of stamps that you can buy with \$3.84?
- 24. The Garcias are putting a brick border along one edge of their flower garden. The flower garden is no more than 31 ft long. If each brick is 6 in. long, what is the greatest number of bricks needed?
- 25. Janet needs to travel 275 mi for a conference. She needs to be at the conference in no more than 5.5 h. What is the slowest average speed that she can drive and still arrive at the conference on time?

Solve each inequality. Graph and check the solution.

26.
$$\frac{1}{4}h < 4.9$$

27.
$$\frac{7}{3}x < 21$$

28.
$$-\frac{1}{9}a > 9$$

29.
$$\frac{b}{6} \le 2.5$$

30.
$$-\frac{3}{5}q > 15$$

30.
$$-\frac{3}{5}q > 15$$
 31. $84 \le 21b$ **32.** $\frac{c}{12} > -\frac{5}{6}$

33.
$$80.6 \le -6.5b$$

34.
$$-\frac{1}{9}p > \frac{1}{3}$$

35.
$$-9z > 45$$

36.
$$\frac{1}{7}y \le 6$$

37.
$$-\frac{5}{7} > -\frac{k}{14}$$

38.
$$6.8 > \frac{y}{5}$$

39.
$$75 \le 15b$$

40.
$$39 < -13k$$

42.
$$8.5v > 61.2$$

43.
$$-11n \ge -55$$
 44. $\frac{1}{4}y < 17$

44.
$$\frac{1}{4}y < 17$$

45.
$$92 < -23k$$

)

Solving Multi-Step Inequalities

Solve each inequality. Graph and check the solution.

1.
$$2z + 7 < z + 10$$

2.
$$4(k-1) > 4$$

3.
$$1.5 + 2.1y < 1.1y + 4.5$$

4.
$$h + 2(3h + 4) \ge 1$$

5.
$$r + 4 > 13 - 2r$$

6.
$$6u - 18 - 4u < 22$$

7.
$$2(3 + 3g) \ge 2g + 14$$

8.
$$2h - 13 < -3$$

9.
$$-4p + 28 > 8$$

10.
$$8m - 8 \ge 12 + 4m$$

11.
$$5 + 6a > -1$$

12.
$$\frac{1}{2}(2t+8) \ge 4+6t$$

13.
$$-5x + 12 < -18$$

14.
$$2(3f + 2) > 4f + 12$$

15.
$$13t - 8t > -45$$

16.
$$2(c-4) \le 10-c$$

17.
$$\frac{1}{2}t - \frac{1}{3}t > -1$$

18.
$$3.4 + 1.6\nu < 5.9 - 0.9\nu$$

Write and solve an inequality that models each situation.

- 19. Ernest works in the shipping department loading shipping crates with boxes. Each empty crate weighs 150 lb. How many boxes, each weighing 35 lb, can Ernest put in the crate if the total weight is to be no more than 850 lb?
- 20. Beatriz is in charge of setting up a banquet hall. She has five tables that will seat six people each. If no more than 62 people will attend, how many tables seating four people each will she need?
- 21. Suppose it costs \$5 to enter a carnival. Each ride costs \$1.25. You have \$15 to spend at the carnival. What is the greatest number of rides that you can go on?
- 22. The cost to rent a car is \$19.50 plus \$.25 per mile. If you have \$44 to rent a car, what is the greatest number of miles that you can drive?
- 23. The student council is sponsoring a concert as a fund raiser. Tickets are \$3 for students and \$5 for adults. The student council wants to raise at least \$1000. If 200 students attend, how many adults must attend?

Solve each inequality. Check the solution.

24.
$$-18 < 2(12 - 3b)$$

25.
$$5n + 3 - 4n < -5 - 3n$$

26.
$$36 > 4(2d + 10)$$

27.
$$2(5t-25)+5t<-80$$

28.
$$3j + 2 - 2j < -10$$

29.
$$\frac{2}{5}(5x-15) \ge 4$$

30.
$$7(2z + 3) > 35$$

31.
$$2(3b-2) < 4b+8$$

$$32. \ \frac{1}{2}y + \frac{1}{4}y \ge -6$$

33.
$$8(3f-6) < -24$$

$$34. \ \ \frac{3}{4}k < \frac{3}{4} - \frac{1}{4}k$$

35.
$$3(4g-6) \ge 6(g+2)$$

36.
$$\frac{1}{2}(2g+4) > -7$$

37.
$$4(1.25y + 4.2) < 16.8$$

38.
$$38 + 7t > -3(t + 4)$$

39.
$$4(2d + 1) > 28$$

40.
$$4(n-3) < 2-3n$$

41.
$$\frac{3}{4}d - \frac{1}{2} \le 2\frac{1}{2}$$

Compound Inequalities

Solve each compound inequality and graph the solution.

1.
$$-5 < s + 5 < 5$$

3.
$$k-3 > 1$$
 or $k-3 < -1$

5.
$$-4d > 8$$
 and $2d > -6$

7.
$$-3 < 3 + s < 7$$

9.
$$-1 < \frac{1}{2}x < 1$$

11.
$$-6 < 9 + 3v < 6$$

13.
$$d-3 > 4$$
 or $d-3 < -4$

15.
$$7 + 2a > 9$$
 or $-4a > 8$

17.
$$c-1 \ge 2$$
 or $c-1 \le -2$

2.
$$1 < 3x + 4 < 10$$

4.
$$b - 2 > 18$$
 or $3b < 54$

6.
$$-4 < t + 2 < 4$$

8.
$$3i \ge 6$$
 or $3i \le -6$

10.
$$g + 2 > -1$$
 or $g - 6 < -9$

12.
$$3f > 15$$
 or $2f < -4$

14.
$$1 > 2h + 3 > -1$$

16.
$$2z > 2.1$$
 or $3z < -5.85$

18.
$$h + 2.8 < 1.8$$
 or $h + 2.8 > 4.8$

Write and solve a compound inequality that represents each situation. Graph your solution.

- 19. The crowd that heard the President speak was estimated to be 10,000 people. The actual crowd could be 750 people more or less than this. What are the possible values for the actual crowd size?
- 20. Susie has designed an exercise program for herself. One part of the program requires her to walk between 25 and 30 miles each week. She plans to walk the same distance each day five days a week. What is the range of miles that she should walk each day?
- **21.** A box of cereal must weigh more than 629.4 g and less than 630.6 g to pass inspection. The box in which the cereal is packaged weighs 5.5 g. What are the possible weights for the cereal?
- 22. Carmen works in a sporting goods store. Her goal is to sell between \$500 and \$600 worth of sporting equipment every week. So far this week, she has sold \$395 worth of equipment. During the rest of the week, what dollar amount must Carmen sell in order to reach her goal?

Solve each compound inequality and graph the solution.

23.
$$2n-1 \ge 1$$
 or $2n-1 \le -1$

25.
$$-1 < h - 2 < 1$$

27.
$$9 < x + 2 < 11$$

29.
$$-3 \le \frac{3}{2}x + 6 \le 3$$

31.
$$\frac{1}{2}x + 1 > 1$$
 or $\frac{1}{2}x + 1 < -1$

33.
$$w - 3 > 4$$
 or $w - 3 < -4$

35.
$$t + 5 < 2$$
 or $3t + 1 > 10$

37.
$$6x - 3 \ge 3$$
 or $6x - 3 \le -3$

24.
$$2k - 3 > 3$$
 or $2k - 3 < -3$

26.
$$2.2 + p > 1$$
 and $1.5p < -0.3$

28.
$$5m + 8 < 23$$
 or $6m > 48$

30.
$$7 > 5 - x > 6$$

32.
$$-2 \le s - 4 \le 2$$

34.
$$6 > 4x - 2 > -6$$

36.
$$2g > 12$$
 and $3g < 24$

38.
$$2y - 3 > -1$$
 or $5 - y > 4$

Solve each inequality. Graph the solution.

1.
$$|d| > 2$$

2.
$$|h| > 6$$

3.
$$|2k| > 1$$

2.
$$|h| > 6$$
 3. $|2k| > 8$ **4.** $|s+4| > 2$

5.
$$|3c - 6| \ge 3$$

6.
$$|2n+3| \le 5$$

7.
$$|3.5z| > |7$$

5.
$$|3c - 6| \ge 3$$
 6. $|2n + 3| \le 5$ 7. $|3.5z| > |7|$ 8. $\left|\frac{2}{3}x\right| \le 4$

9.
$$9 > |6 + 3t|$$

10.
$$|i| - 2 \ge 6$$

11.
$$5 > |\nu + 2| + 3$$

9.
$$9 > |6 + 3t|$$
 10. $|j| - 2 \ge 6$ **11.** $5 > |\nu + 2| + 3$ **12.** $|4y + 11| < 7$

13.
$$|2n-1| \ge 1$$

13.
$$|2n-1| \ge 1$$
 14. $\left|\frac{1}{2}x+1\right| > 1$ **15.** $-2|h-2| > -2$ **16.** $3|2x| \le 12$

15.
$$-2|h-2| > -2$$

16.
$$3|2x| \le 12$$

17.
$$3|s-4|+21 \le 27$$
 18. $-6|w-3|<-24$ **19.** $-\frac{1}{2}|6x-3| \le -\frac{3}{2}$ **20.** $-2|3j|-8 \le -20$

19.
$$-\frac{1}{2}|6x-3| \le -\frac{3}{2}$$

20.
$$-2|3j| - 8 \le -20$$

Solve each equation. If there is no solution, write no solution.

21.
$$|a| = 9.5$$

22.
$$|b| = -2$$

22.
$$|b| = -2$$
 23. $|d| - 25 = -13$ **24.** $|6z| + 3 = 21$

24.
$$|6z| + 3 = 21$$

25.
$$|3c| - 45 = -18$$
 26. $-2 = -\frac{|z|}{7}$ **27.** $|x| = -0.8$ **28.** $-4|7 + d| = -44$

26.
$$-2 = -\frac{|z|}{7}$$

27.
$$|x| = -0.8$$

28.
$$-4|7+d|=-44$$

Write and solve an absolute value equation or inequality that represents each situation.

- 29. The average number of cucumber seeds in a package is 25. The number of seeds in the package can vary by three. Find the range of acceptable numbers of seeds in each package.
- 30. The mean distance of the earth from the sun is 93 million miles. The distance varies by 1.6 million miles. Find the range of distances of the earth from the sun.
- 31. Leona was in a golf tournament last week. All four of her rounds of golf were within 2 strokes of par. If par was 72, find the range of scores that Leona could have shot for each round of the golf tournament.
- 32. Victor's goal is to earn \$75 per week at his after-school job. Last month he was within \$6.50 of his goal. Find the range of amounts that Victor might have earned last month.
- 33. Members of the track team can run 400 m in an average time of 58.2 s. The fastest and slowest times vary from the average by 6.4 s. Find the range of times for the track team.
- 34. The ideal length of a particular metal rod is 25.5 cm. The measured length may vary from the ideal length by at most 0.025 cm. Find the range of acceptable lengths for the rod.
- 35. When measured on a particular scale, the weight of an object may vary from its actual weight by at most 0.4 lb. If the reading on the scale is 125.2 lb, find the range of actual weights of the object.
- 36. One poll reported that the approval rating of the job performance of the President of the United States was 63%. The poll was said to be accurate to within 3.8%. What is the range of actual approval ratings?

Inequalities and Their Graphs

OBJECTIVE: Identifying solutions of inequalities

MATERIALS: None

A sentence that contains the symbol >, <, \ge , or \le is called an **inequality**. An inequality expresses the relative order of two mathematical expressions. The sentence can be either numerical or variable.

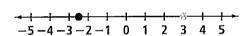
Symbol	Description				
>	is greater than				
<	is less than				
≤	is less than or equal to				
≥	is greater than or equal to				

Example

One way to determine whether a number is a solution of an inequality is to plot the number on a number line.

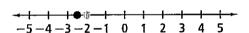
Is each number a solution of x > -2.5?

a. 3



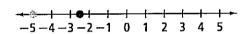
Since 3 is to the right of -2.5, it is greater than -2.5, so it is a solution. 3 > -2.5

c. -2



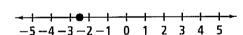
Since -2 is to the right of -2.5, it is greater than -2.5, so it is a solution. -2 > -2.5

b. -5



Since -5 is to the left of -2.5, it is less than -2.5, so it is not a solution. $-5 \ge -2.5$

d. -2.5



This is a special case: -2.5 is not greater than itself. Therefore, it is not a solution. $-2.5 \ge -2.5$

Exercises

Is each number a solution of $x \le 8$?

1. −2

2. 7

3. -7

4. 8

Is each number a solution of $x \le -9$?

5. 9

6. −14

7. −8.5

8. -6

Solving Inequalities Using Addition and Subtraction

OBJECTIVE: Using addition and subtraction to solve one-step inequalities

MATERIALS: Tiles

- To solve one-step inequalities, use the same strategies you use to solve equations. Apply the Addition and Subtraction Properties of Inequality.
- When you add or subtract the same quantity from each side of an inequality, the direction of the inequality symbol stays the same.

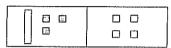
Example

Using tiles, solve the inequality x - 3 < 4.

- a. Model the inequality with tiles.
- **b.** Add or subtract the same quantity on each side to get the variable alone on one side of the inequality symbol.

For this example, add 3 to each side.

- c. Simplify by removing zero pairs.
- **d.** Write the solution to the inequality.



$$x - 3 + 3 < 4 + 3$$

x - 3 < 4

Note that even though you are adding the same quantity to each side of the inequality, the direction of the inequality symbol stays the same.

Exercises

Use tiles and steps a-d to model and solve each inequality.

1.
$$y - 2 < 4$$

2.
$$x - 4 < 1$$

3.
$$7 < w + 2$$

4.
$$x - 6 > 10$$

5.
$$10 \le y + 8$$

6.
$$a - 1 > 3$$

7.
$$4 + h \le 7$$

8.
$$s - 3 > 2$$

Solve.

10.
$$x - 9 < 6$$

11.
$$a - 7 > 5$$

12.
$$b - 4 < 10$$

13.
$$c + 5 > 7$$

14.
$$6 + d < 11$$

15.
$$f - 4 > 15$$

16. The band must earn at least \$75 for a trip. Band members already earned \$35. Write and solve an inequality to find how much money they still need to earn.

Solving Inequalities Using Multiplication and Division

OBJECTIVE: Using multiplication and division to solve one-step inequalities

MATERIALS: Tiles, an index card or piece of paper with < written on it

Example

Use tiles and the card to compare quantities as shown in these steps.

$$-2(2)$$
 $-2(3)$

$$\leftarrow$$
 Multiply each quantity by -2 .



Model the new quantities with tiles and place your card to show how the new quantities compare. Notice that you must rotate the card to make the new statement true.

What happens to the direction of the inequality symbol when you multiply or divide an inequality by a negative number? The direction of the inequality symbol reverses.

Exercises

Model both lines of each exercise with tiles. Use your card to compare the quantities. Then fill in the blanks by writing < or >.

-1(3) ______ -1(-1)

$$\frac{4}{-2}$$
 $\frac{-3}{-2}$

5.
$$-2$$
 ____ 2 $\frac{-2}{-4}$ ____ $\frac{2}{-4}$

Solve and check.

7.
$$2.5a < 15$$

8.
$$-3b > 21$$

9.
$$6c < 24$$

10.
$$\frac{x}{7} < -2$$

11.
$$-\frac{y}{6} < \frac{2}{3}$$

13.
$$-4.2d \ge 10.5$$

14.
$$-\frac{2}{3}m \le 10$$

15.
$$-1.7x > -34$$

16.
$$\frac{n}{8} \le -2.5$$

17.
$$-1.5k < 2.4$$

18.
$$\frac{3}{5}p \ge -9$$

19.
$$4t > -14$$

20.
$$\frac{z}{15} > -\frac{2}{3}$$

21.
$$-\frac{1}{2}w \le -3.6$$

.)

Solving Multi-Step Inequalities

OBJECTIVE: Solving multi-step inequalities and

MATERIALS: None

graphing the solutions on a number line

As you solve multi-step inequalities, keep these strategies in mind.

- Circle all the terms with variables. Then decide on which side of the inequality you are going to collect the variable terms. You may want to select the side that has the variable term with the greatest coefficient.
- Rewrite the inequality by using inverse operations in the same way you solve equations. If you multiply or divide both sides by a negative number, reverse the direction of the inequality symbol.
- Check three values on your graph: the number where the arrow starts, a number to the right of the starting value, and another to the left.

Example

Solve 3x + 2 < 5 + 2x. Graph and check the solution.

$$(3x) + 2 < 5 + 2x$$

$$3x + 2 < 5 + 2x$$

3x + 2 < 5 + 2x — Box all constant terms. Plan your steps to collect variable terms on one side and constant terms on the other.

$$3x + 2 - 2x < 5 + 2x - 2x$$

To get variables on the left side, subtract 2x from each side.

$$x + 2 < 5$$

- Simplify.

$$x + 2 - 2 < 5 - 2$$

To get constants on the right side, subtract 2 from each side.

→ Simplify.

- Graph your solution on a number line. Since 3 is not a solution, use an open circle.

Check three values for the variable: 0, 3 (where the arrow starts), and 4.

$$3(0) + 2 \stackrel{?}{<} 5 + 2(0)$$
 $3(3) + 2 \stackrel{?}{<} 5 + 2(3)$ $3(4) + 2 \stackrel{?}{<} 5 + 2(4)$ $11 < 11 \checkmark$ $14 < 13 \checkmark$

$$3(3) + 2 < 5 + 2(3)$$

 $11 < 11 \checkmark$

$$3(4) + 2 < 5 + 2(4)$$

 $14 < 13 \checkmark$

Exercises

Use circles and boxes to identify the variable and constant terms. Then solve, graph, and check your solution for each inequality.

1.
$$4x + 3 < 11$$

2.
$$3x + 2 < 2x + 5$$

3.
$$5x + 4 < 14$$

4.
$$4x - 3 < 3x - 1$$

5.
$$3x + 4 > 2x + 3$$

6.
$$2x + 5 > -1$$

OBJECTIVE: Solving compound inequalities and graphing the solutions on a number line

MATERIALS: Two highlighting markers in colors that combine to make a third color, for example, blue and yellow or pink and yellow

Before you graph, practice making overlapping lines with your two markers. Notice, for example, that marks from a yellow marker and a blue one overlap to make a green line. A pink line and a yellow line combine to make an orange line. (If you have trouble seeing some colors, ask a partner to help you.)

Example

Solve and graph $-3 < x + 5 \le 2$.

$$-3 < x + 5$$

and

$$x + 5 \le 2$$

- Rewrite the compound inequality as two inequalities joined by and.

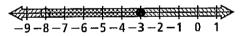
$$-3 - 5 < x + 5 - 5$$
 and $x + 5 - 5 \le 2 - 5$

-8 < x

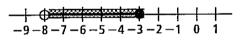
and

$$x \leq -3$$

- Solve each inequality.



Graph the solutions separately on the same number line. Use a blue marker for one arrow () and a yellow marker (22222) for the other. Notice that -8 is colored in only one of the graphs.



Think: The green points are blue AND yellow. This graph shows the solution set for the compound inequality.

$$-8 < x \le -3$$

Write the solution set.

Exercises

Solve each inequality and graph the solution. Hint: The solution set for or statements is all points that are blue or yellow or green.

1.
$$-3 \le x - 5$$
 or $x + 5 \le 2$

2.
$$x + 5 \le 4$$
 or $-2x < -6$

3.
$$x - 2 \ge -6$$
 and $5 + x < 7$

4.
$$x - 2 \le -6 \text{ or } 5 + x > 7$$
 5. $-3 \le x + 1 < 3$

5.
$$-3 \le x + 1 < 3$$

6.
$$3 \ge \frac{1}{2}x > -2$$

7.
$$-5 \le 2x - 1 < 7$$

8.
$$3x < -6$$
 or $4x - 3 \ge 9$

9.
$$-5x < 15$$
 or $x \le -5$

10.
$$-1 \le \frac{1}{2}x + 1 < 0$$

11.
$$1 - 2x \le -5$$
 or $2x < -10$

12.
$$-9 < x - 7 \le 1$$

Absolute Value Equations and Inequalities

OBJECTIVE: Solving absolute value inequalities

MATERIALS: None

The absolute value of a real number x, written |x|, is the distance of x from 0 on the real number line.

An inequality such as |x| < k, where k is a positive real number, is true for values of x that are less than k units from 0 on the number line. These are the numbers between -k and k on the number line. Thus, x is a solution of |x| < k whenever -k < x < k.

An inequality such as |x| > k, where k is a positive real number, is true for values of x that are more than k units from 0 on the number line. These are the numbers to the left of -k and to the right of k on the number line. Thus, x is a solution of |x| > k whenever x < -k or x > k.

Example

Solve each inequality and graph the solution.

a.
$$|t + 3| < 4$$

$$-4 < t + 3 < 4$$

$$\leftarrow$$
 The inequality is in the form $|x| < k$.

Replace the form
$$|x| < k$$
 with the form $-k < x < k$. Here the

$$-7 < t < 1$$

- Subtract 3 from each part of the inequality.

expression t + 3 is in the place of x and k = 4.

b. $|2y + 1| \ge 3$

$$2y + 1 \le -3 \text{ or } 2y + 1 \ge 3$$

8-7-6-5-4-3-2-1 0

$$\leftarrow$$
 The inequality is in the form $|x| \ge k$.

Replace the form
$$|x| \ge k$$
 with the form $x \le -k$ or $x \ge k$.
Here the expression $2y + 1$ is in the place of x and $k = 3$.

$$2y \le -4 \text{ or } 2y \ge 2$$

$$y \le -2 \text{ or } y \ge 1$$
 $-5-4-3-2-1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$

Exercises

Solve each inequality and graph the solution.

1.
$$|c| < 5$$

2.
$$|u| \ge 1$$

3.
$$|a+1| \le 2$$

4.
$$|3m-2|>1$$

4.
$$|3m-2| > 1$$
 5. $\left|\frac{1}{2}y - 3\right| \ge \frac{1}{2}$

6.
$$|2n + 1| < 7$$

7.
$$|4 - 2u| \le 8$$

8.
$$|2g + 5| > 3$$

9.
$$|1-2y| \ge 9$$

Enrichment 3-1

Logical Reasoning

Five girls live in the same apartment building. To determine the age of each girl, write each clue as an equation or an inequality.

- 1. All five girls are older than 10 but younger than 20.
- 2. Karen is older than Angela but younger than Janet.
- 3. Janet's age is halfway between Tamara's age and Angela's age.
- 4. The ages of all of the girls except Janet are even numbers.
- 5. Susan is neither the oldest nor the youngest.
- 6. Karen is as much older than Angela as Tamara is older than Susan.
- 7. No two girls are the same age.

Organize what you know in a table. Place an X in the boxes you eliminate.

11	12	13	14	15	16	17	18	19

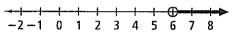
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Chapter 3 Answers

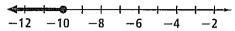
Practice 3-1

1a. yes 1b. no 1c. yes 2a. no 2b. yes 2c. yes 3a. no 3b. yes 3c. no 4a. yes 4b. yes 4c. yes 5a. no 5b. yes 5c. yes 6a. no 6b. yes 6c. no 7a. no 7b. yes 7c. no 8.x > -5 $9.x \le -5$ 10. x < 3 11. $x \ge -8$

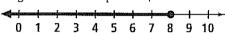
12. x is greater than 6;



13. y is less than or equal to -10;



14. 8 is greater than or equal to b;



15. -4 is less than w;

16. x is less than -7;

17. x is greater than or equal to 12;

18. Let t = temperature (in degrees Fahrenheit); $t \le 38$

19. Let $w = \text{weight (in lb)}; w \le 2000$

20. Let $n = \text{number of students}; n \ge 20$

21. Let $n = \text{number of people}; n \le 250$

22. Let $s = \text{speed (in mi/h)}; s \le 55$

23. Let n = number of points; $450 \le n \le 500$

24. Let $c = \text{circumference (in in.)}; c \ge 9.00$

25. C 26. D 27. B 28. A

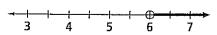
Practice 3-2

1. $n \ge 9$;

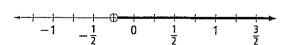
2. y > 2;

3. r > -1.5:

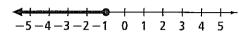
4. b > 6;



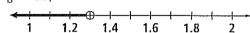
5. $n > -\frac{1}{d}$;



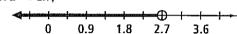
6. $c \leq -1$;



7. g < 1.3;

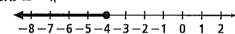


8. d < 2.7;



9. $f \ge 10$;

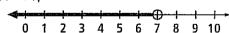
10. $x \le -4$;



11. $d \le 5$;

 $12 m \ge -1$:

13. ν < 7;



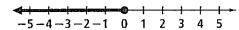
14. $t \ge -13$;

15. y > 9;

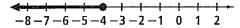
16. a > 4;

17. d < 1;

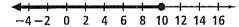




19. $h \leq -4$;



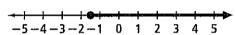
20. $t \le 10$;



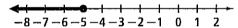
21. $n + 94 + 82 + 87 - 2 \ge 360$, where n = number of points; at least 99 points 22. $n + 125 \ge 140$, where n = number of at-bats; at least 15 at-bats 23. $s + 19 - 5 \ge 32$, where s = average wind speed at 8 A.M.; 18 mi/h 24. $t + 13.5 \le 25$, where t = time in minutes; 11.5 min 25. $n + 3.5 \ge 5$, where n = number of miles; at least 1.5 mi

25. $n + 3.5 \ge 5$, where n = number of miles; at least 1.5 m **26.** 1058 + 44.50 − $w \ge 1000$, where w = amount of money withdrawn; \$102.50

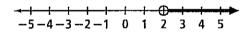
27.
$$z \ge -1\frac{1}{2}$$
;



28. $d \leq -5$;



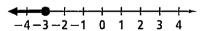
29. v > 2;



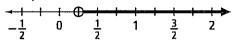
30. m > -4;

31.
$$f > -5$$
;

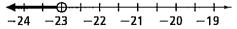
32.
$$w \leq -3$$
;



33.
$$b > \frac{1}{4}$$
;



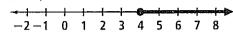
34. t < -23;



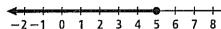
Answers

35.
$$u < 2$$
:

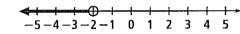
36. $z \ge 4$:



37. $b \le 5$;



38. k < -2:



39. $a \le 2$;

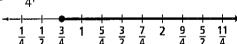
40. b > 11;

41. k > 11;

42. j > 1.5;

Practice 3-3

 $1. s \geq \frac{3}{4};$



2. $b \ge 5$;

3. r > -10;

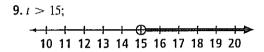
4. n > 20:

 $5. n \leq 4;$

$$6. n \ge -42;$$

7.
$$c > -4$$
;
 $-8-7-6-5-4-3-2-1$ 0 1 2

8.
$$d > -4$$
;



10.
$$k < -9$$
;
 $-10-9-8$

11.
$$w > 0$$
;
 $-2-1$ 0 1 2

15.
$$v \le -0.5$$
;
 $-5-4-3-2-1$ 0 1 2 3 4 5

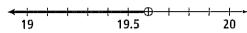
16.
$$x \le 45$$
;
40 41 42 43 44 45 46 47 48 49 50

17.
$$d \le -7$$
;
 $-9 - 8 - 7 - 6$

18.
$$x \le -7$$
;
 $-9 - 8 - 7 - 6$

20.
$$a \le -4$$
;

21. $8h \le 40$, where h = number of hours; 5 hours 22. $7n \ge 28$, where n = number of vans; 4 vans 23. $.34n \le 3.84$, where n = number of stamps; 11 stamps 24. $0.5n \le 31$, where n = number of bricks; 62 bricks 25. $5.5s \ge 275$, where s = speed in mi/h; 50 mi/h 26. h < 19.6;



27. x < 9;

29.
$$b \le 15$$
;
13 14 15 16

30.
$$q < -25$$
;

31.
$$b \ge 4$$
;
3 4 5

32.
$$c > -10$$
;
 $-10 - 8$

33.
$$b \le -12.4$$
;
 $-14 - 13.6 - 13.2 - 12.8 - 12.4 - 12$

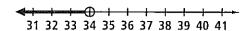
34.
$$p < -3$$
;

35.
$$z < -5$$
;
 $-11-9-7-5-3-1$ 0 1

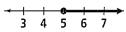
36.
$$y \le 42$$
;
32. 34. 36. 38. 40. 42. 44. 46

37.
$$k > 10$$
;
4 5 6 7 8 9 10 11 12 13 14

38.
$$y < 34$$
;

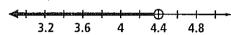


39.
$$b \ge 5$$
;

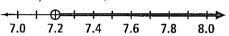


40.
$$k < -3$$
;

41. d < 4.4;



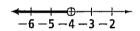
42. $\nu > 7.2$;



43. $n \le 5$:

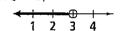
44. y < 68;

45.
$$k < -4$$
;



Practice 3-4

1. z < 3;

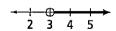


2. k > 2;

3. y < 3;

4.
$$h \ge -1$$
;

5.
$$r > 3$$
:



6.
$$u < 20$$
;

7.
$$g \ge 2$$
;

8.
$$h < 5$$
:

9.
$$p < 5$$
;

10.
$$m \ge 5$$
;

11.
$$a > -1$$
;

12.
$$t \leq 0$$
;

13. x > 6;

14. f > 4;

15.
$$t > -9$$
;

16.
$$c \le 6$$
;

17.
$$t > -6$$
;

18.
$$\nu$$
 < 1;

)

19. $150 + 35n \le 850$, where n = number of boxes; at most 20 boxes 20. $5(6) + 4n \le 62$, where n = number of tables seating four people; no more than 8 tables

21. $5 + 1.25n \le 15$, where n = number of rides; 8 rides **22.** $19.50 + 0.25n \le 44$, where n = number of miles; 98 mi **23.** $3(200) + 5n \ge 1000$, where n = number of adults; at least 80 adults

24. b < 7 25. n < -2 26. d < -0.5 27. t < -2 28. j < -12 29. $x \ge 5$ 30. z > 1 31. b < 6 32. $y \ge -8$ 33. f < 1 34. $k < \frac{3}{4}$ 35. $g \ge 5$ 36. g > -9 37. y < 0 38. t > -5 39. d > 3 40. n < 2 41. $d \le 4$

Practice 3-5

1. -10 < s < 0; -10-8-6-4-2=0

2. -1 < x < 2;

5. -3 < d < -2;

6. -6 < t < 2; -7 - 6 - 5 - 4 - 3 - 2 - 1 0 1 2 3

7. -6 < s < 4; $-8 - 6 - 4 - 2 \quad 0 \quad 2 \quad 4 \quad 6$

 $8. j \ge 2 \text{ or } j \le -2;$

9. -2 < x < 2; -3 - 2 - 1 0 1 2 3

10. g > -3 or g < -3; $\begin{array}{c}
+ & + & + & + & + \\
-5 & -4 & -3 & -2 & -1
\end{array}$ 11. -5 < y < -1; -6 - 5 - 4 - 3 - 2 - 1 = 0

13. d > 7 or d < -1;

14. -2 < h < -1; -4 - 3 - 2 - 1 0 1

15. a > 1 or a < -2; -3-2-1 0 1 2

16. z > 1.05 or z < -1.95; -1.95 1.05

18. h < -1 or h > 2; -2-1 0 1 2 3

19. $-750 \le n - 10,000 \le 750$, where n = number of people; from 9250 to 10,750 people;

9250 10000 10750

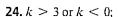
20. 25 < 5n < 30, where n = number of miles; between 5 and 6 mi;

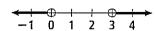
21. 629.4 < 5.5 + w < 630.6, where w = weight in grams; more than 623.9 but less than 625.1g;

623.9 624.5 625.1

22. 500 < 395 + d < 600, where d = dollar amount left to sell; between \$105 and \$205;

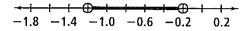
23. $n \ge 1$ or $n \le 0$; -2-1 0 1 2 3



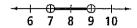


25.
$$1 < h < 3$$
;

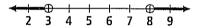
26.
$$-1.2 ;$$



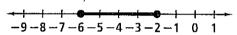
27.
$$7 < x < 9$$
;



28. m < 3 or m > 8;

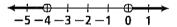


29.
$$-6 \le x \le -2$$
;



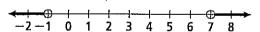
30. -2 < x < -1;

31. x > 0 or x < -4;



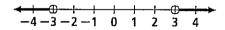
32. $2 \le s \le 6$;

33. w > 7 or w < -1;

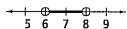


34. -1 < x < 2;

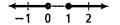
35. t < -3 or t > 3;



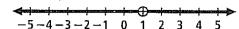
36. 6 < g < 8;



 $37. x \ge 1 \text{ or } x \le 0;$

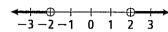


38. y < 1 or y > 1;

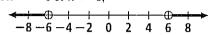


Practice 3-6

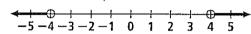
1. d > 2 or d < -2;



2. h < -6 or h > 6;

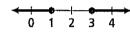


3. k > 4 or k < -4;

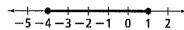


4. s < -6 or s > -2:

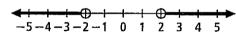
5. $c \ge 3 \text{ or } c \le 1$;



6. $-4 \le n \le 1$;



7. z > 2 or z < -2;



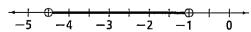
8. $-6 \le x \le 6$:

9. -5 < t < 1:

10. $j \ge 8$ or $j \le -8$;

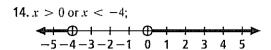
11. $-4 < \nu < 0$;

12. -4.5 < y < -1;



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13. $n \ge 1$ or $n \le 0$; -5-4-3-2-1 0 1 2 3 4 5



15.
$$1 < h < 3$$
;
 $-5 - 4 - 3 - 2 - 1$ 0 1 2 3 4 5

16.
$$-2 \le x \le 2$$
;
 $-5-4-3-2-1$ 0 1 2 3 4 5

17.
$$2 \le s \le 6$$
;
 $-2-1$ 0 1 2 3 4 5 6 7 8

18.
$$w > 7$$
 or $w < -1$;
 $-2-1$ 0 1 2 3 4 5 6 7 8

19.
$$x \ge 1$$
 or $x \le 0$;
 $-5-4-3-2-1$ 0 1 2 3 4 5

21. ±9.5 **22.** no solution **23.** ±12 **24.** ±3 **25.** ±9

26. ± 14 **27.** no solution **28.** 4, -18

29. $|s - 25| \le 3$; between 22 and 28 seeds, inclusive

30. $|d-93| \le 1.6$; between 91.4 and 94.6 million mi, inclusive

31. $|s - 72| \le 2$; between 70 and 74, inclusive

32. $|a - 75| \le 6.50$; between \$68.50 and \$81.50, inclusive

33. $|t - 58.2| \le 6.4$; between 51.8 and 64.6 s, inclusive

34. $|l - 25.5| \le 0.025$; between 25.475 and 25.525 cm, inclusive

35. $|w - 125.2| \le 0.4$; between 124.8 and 125.6 lb, inclusive

36. $|r - 63| \le 3.8$; between 59.2% and 66.8%, inclusive

Reteaching 3-1

1. yes 2. yes 3. yes 4. yes 5. no 6. yes 7. no 8. no

Reteaching 3-2

1. y < 6 2. x < 5 3. w > 5 4. x > 16 5. $y \ge 2$ 6. a < 4 7. $h \le 3$ 8. s > 5 9. b < 5 10. x < 15 11. a > 12 12. b < 14 13. c > 2 14. d < 5 15. f > 19 16. $x + 35 \ge 75$; at least \$40

Reteaching 3-3

1. <; > 2. <; > 3. >; < 4. >; < 5. <; > 6. >; < 7. a < 6 8. b < -7 9. c < 4

Reteaching 3-4

1.
$$x < 2$$
;

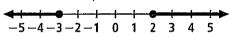
2.
$$x < 3$$
;

3.
$$x < 2$$
;

6.
$$x > -3$$
; $\xrightarrow{-3}$ $\xrightarrow{-2}$ $\xrightarrow{-1}$ 0 1 2 3

Reteaching 3-5

$$1. x \le -3 \text{ or } x \ge 2;$$



2.
$$x \le -1$$
 or $x > 3$;

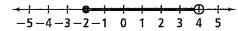
3.
$$-4 \le x < 2$$
;

4.
$$x \le -4 \text{ or } x > 2$$
;

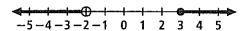
5.
$$-4 \le x < 2$$
;

6.
$$-4 < x \le 6$$
;

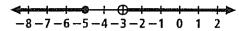
7. $-2 \le x < 4$;



8. x < -2 or $x \ge 3$;

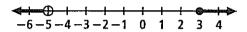


9. $x \le -5$ or x > -3;

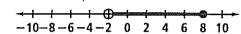


10. $-4 \le x < -2$:

11. x < -5 or $x \ge 3$:

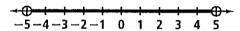


12. $-2 < x \le 8$;



Reteaching 3-6

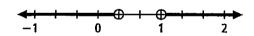
1. -5 < c < 5;



2. $u \le -1$ or $u \ge 1$;

3. $-3 \le a \le 1$;

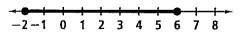
4. $m < \frac{1}{3}$ or m > 1;



5. $y \le 5$ or $y \ge 7$;

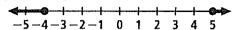
6. -4 < n < 3:

7. $-2 \le u \le 6$;



8. g < -4 or g > -1:

9. $y \le -4$ or $y \ge 5$;



Enrichment 3-1

1.
$$10 < g < 20$$
 2. $A < K < J$ **3.** $J = \frac{T + A}{2}$

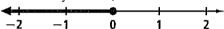
4. $J \neq 12, 14, 16, 18$ **5.** $S \neq \text{oldest}, S \neq \text{youngest}$

6. K - A = T - S; Karen: 14; Angela: 12; Janet: 15;

Tamara: 18; Susan: 16

Enrichment 3-2

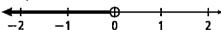
1. $x \le 0$; If $w \ge y + z$ and w + x = y + z, then by substitution $w \ge w + x$. Applying the Subtr. Prop. of Inequ., subtract w from both sides to obtain $0 \ge x$. The original statement is true only if $x \le 0$;



If w + x = y + z and $x \le 0$, then $w \ge y + z$.

2. none; Applying the Subtr. Prop. of Inequ. to x + z > y + z, subtract z from both sides to obtain x > y. Since this restriction is given, no additional restrictions are necessary.

3. y < 0; Applying the Add. Prop. of Inequ. to x > 0, add yto both sides to obtain x + y > y. Since x + y = 0, use substitution to obtain 0 > y. The original statement is true only if y < 0;



If x + y = 0 and y < 0, then x > 0.

4. none; Applying the Add. Prop. of Inequ. to x > y, add x to both sides to obtain 2x > x + y. Then, applying the Subtr. Prop. of Inequ. to 2x > x + y, subtract y from both sides to obtain 2x - y > x. Since this restriction is given, no additional restrictions are necessary.

Enrichment 3-3

1. Statements

$$1.x + a > b$$

$$2.x + a + (-a) > b + (-a)$$

$$3.x + 0 > b + (-a)$$

$$4.x > b + (-a)$$

$$5. x > b - a$$

1.
$$ax > b$$

 $2, \frac{ax}{a} > \frac{b}{a}$

 $3.\,\frac{1}{a}(ax)>\frac{b}{a}$

$$4.\left(\frac{1}{a}\cdot a\right)x > \frac{b}{a}$$

Reasons

1. Given

2. Add. Prop. of Eq.

3. Inv. Prop. of Add.

4. Identity Prop. of Add.

5. Def. of Subtr.

Reasons

1. Given

2. Div. Prop. of Ineq.

3. Def. of Div.

4. Assoc. Prop. of Mult.