Reasoning and Proof

DIAGNOSING READINESS *page 66*

1. $9x - 13 = 9(7) - 13 = 63 - 13 = 50$ **2.** $90 - 3x = 90 - 3(31) = 90 - 93 = -3$ **3.** $\frac{1}{2}x + 14 = \frac{1}{2}(23) + 14 = 11\frac{1}{2} + 14 = 25\frac{1}{2}$ **4.** $2x - 17 = 4; 2x = 21; x = 10.5$ **5.** $3x + 8 = 53; 3x = 45; x = 15$ **6.** $(10x + 5) + (6x - 1) = 180; 16x + 4 = 180; 16x = 176; x = 11$ **7.** $(x + 21) + (2x + 9) = 90; 3x + 30 = 90$. Divide by 3: $x + 10 = 30; x = 20$. **8.** $3x + 4 = 2x - 1; x + 4 = -1; x = -5$ **9.** $3(x + 8) = 12$. Divide by 3: $x + 8 = 4; x = -4$. **10.** $2(x + 4) = x + 13; 2x + 8 = x + 13; x + 8 = 13; x = 5$ **11.** $7x + 5 = 5x + 17; 2x + 5 = 17; 2x = 12; x = 6$ **12.** $14x = 2(5x + 14)$. Divide by 2: $7x = 5x + 14; 2x = 14; x = 7$. **13.** $2(3x - 4) + 10 = 5(x + 4); 6x - 8 + 10 = 5x + 20; 6x + 2 = 5x + 20; x + 2 = 20; x = 18$ **14.** Since the vertex C is the vertex of more than one angle, three letters must be used and the middle letter must be the vertex C: $\angle ACD$, $\angle DCA$. **15.** The vertex of $\angle 2$ is C. **16.** Since D is the midpt., $x + 8 = 2x + 5; -x = -3; x = 3$. **17.** Since D is the vertex of more than one angle, use 3 letters with the vertex D as the middle letter: $\angle ADB$ or $\angle BDA$. **18.** An $\angle$ bis. is a segment, ray, or line that divides the $\angle$ into 2 $\cong$ angles, so $\overline{CD}$ is the $\angle$ bisector. **19.** A rt. $\angle$ measures 90. $m\angle 1 + m\angle 2 = m\angle ACB$; $m\angle 1 + 45 = 90; m\angle 1 = 45$ **20.** By the Angle Addition Post., $m\angle 1 + m\angle 2 = m\angle ACB; 4x + (2x + 18) = 90; 6x + 18 = 90; 6x = 72; x = 12$. $m\angle 1 = 4x = 4(12) = 48$; $m\angle 2 = 2x + 18 = 2(12) + 18 = 24 + 18 = 42$

2-1 Conditional Statements *pages 68–74*

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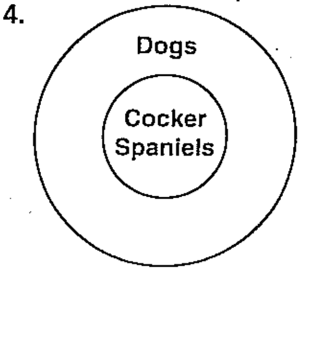
1. −1 **2.** −2, 2 **3.** 2 **4.** −4 **5.** 0 **6.** −1

Check Understanding **1.** The hypothesis follows *If*, and the conclusion follows *then*, so the hypothesis is $y - 3 = 5$, and the conclusion is $y = 8$. **2a.** Write it as an if-then statement: If an integer ends with 0, then it is divisible by 5. **2b.** Write it as an if-then statement: If a figure is a square, then it has 4 congruent sides. **3.** States that contain the word *New* are New Hampshire, New Jersey, New Mexico, and New York. Of those, New Mexico does not border the ocean. Thus, the conditional is false.

4.

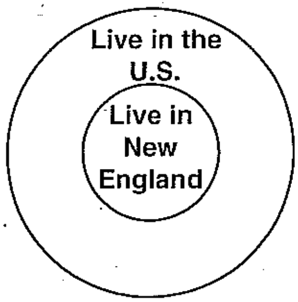


The hypothesis is a region inside the conclusion.

5. Switch the hypothesis and conclusion: If two lines are skew, then they are not parallel and do not intersect. **6a.** Original conditional: If two lines do not intersect, then they are parallel. Since skew lines are not parallel and do not intersect, the original conditional is false. The converse switches the hypothesis and conclusion: If two lines are parallel, then they do not intersect. By def. of $\parallel$ lines, the converse is true. **6b.** Original conditional: If $x = 2$, then $|x| = 2$. Since the absolute value measures positive distance, the conditional is true. The converse switches the hypothesis and conclusion: If $|x| = 2$, then $x = 2$. Since $|x| = 2$ for $x = 2$ and for $x = -2$, the converse is false. **7.** Answers may vary. Sample: The statement "I breathe when I sleep" can be rewritten as "If I sleep, then I breathe." This is always a true statement. The statement "I sleep when I breathe" can be rewritten as "If I breathe, then I sleep." This is sometimes a true statement. The two statements are converses and do not have the same meaning.

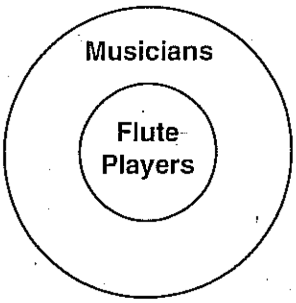
Exercises **1.** The hypothesis follows *If* and the conclusion follows *then*. Hypothesis: You send in the proof-of-purchase. Conclusion: They send you a get-well card. **2.** The hypothesis follows *If* and the conclusion follows *then*. Hypothesis: You want to be fit. Conclusion: Get plenty of exercise. **3.** The hypothesis follows *If* and the conclusion follows *then*. Hypothesis: $x + 20 = 32$. Conclusion: $x = 12$. **4.** The hypothesis follows *If* and the conclusion follows *then*. Hypothesis: You can see the magic in a fairy tale. Conclusion: You can face the future. **5.** The hypothesis follows *If* and the conclusion follows *then*. Hypothesis: Somebody throws a brick at me. Conclusion: I can catch it and throw it back. **6.** The hypothesis follows *If* and the conclusion follows *then*. Hypothesis: You can accept defeat and open your pay envelope without feeling guilty. Conclusion: You're stealing. **7.** The hypothesis follows *If* and the conclusion follows *then*. Hypothesis: My fans think that I can do everything I say I can do. Conclusion: They're crazier than I am. **8.** The hypothesis follows *If* and the conclusion follows *then*. Hypothesis: I could paint that flower in a huge scale. Conclusion: You could not ignore its beauty. **9.** Write the statement in if-then form: If an object is glass, then it is fragile. **10.** Write the statement in if-then form: If $3x - 7 = 14$, then $3x = 21$. **11.** Write the statement in if-then form: If a whole number has 2 as a factor, then it is even. **12.** Write the statement in if-then form: If something is an obtuse angle, then it has a measure greater than 90. **13.** Write the statement in if-then form: If the weather is good, then a picnic is enjoyable. **14.** Write the statement in if-then form: If two lines are skew, then they do not lie in the same plane. **15.** Both Saturday and Sunday are not weekdays, so Sunday is the counterexample. **16.** Odd numbers less than 10 are 1, 3, 5, 7, and 9, and 3, 5, and 7 are prime, but 1 and 9 are not. Answers may vary. Sample: 9 **17.** Both Canada and Mexico border the United States, so the counterexample is Mexico. **18.** Some European and Western Indian games involve a ball and bat. Answers may vary. Sample: softball

19.



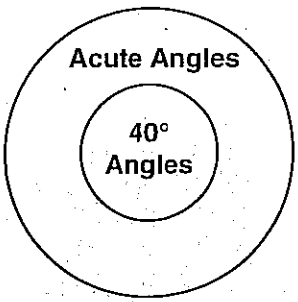
The hypothesis is inside the conclusion.

20.



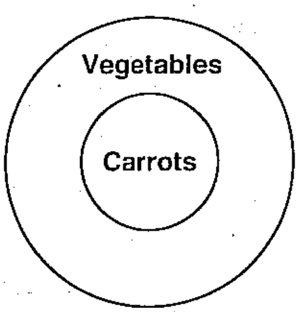
The hypothesis is inside the conclusion.

21.



The hypothesis is inside the conclusion.

22.



The hypothesis is inside the conclusion.

23. Switch the hypothesis and conclusion: If you grow, then you eat your vegetables. **24.** Switch the hypothesis and conclusion: If a triangle has a 90° angle, then it is a right triangle. **25.** Switch the hypothesis and conclusion: If two segments have the same length, then they are congruent. **26.** Switch the hypothesis and conclusion: If you do not get paid, then you do not work. **27.** Original conditional: If you travel from the United States to Kenya, then you have a passport. In order to travel outside your country, you must have a passport, so the conditional is true. To write the converse, switch the hypothesis and conclusion: If you have a passport, then you travel from the United States to Kenya. Not all people who have a passport travel to Kenya; some travel to the Far East, or to Europe, for example. The converse is false. **28.** Original conditional: If a point is in the first quadrant, then its coordinates are positive. Since both x- and y-values must be positive in Quadrant I, the conditional is true. To write the converse, switch the hypothesis and conclusion: If the coordinates of a point are positive, then it is in the first quadrant. This statement is also true. **29.** Original conditional: If a substance is water, then its chemical formula is H_2O. There are no counterexamples, so this conditional statement is true. To write the converse, switch the hypothesis and conclusion: If the chemical formula for a substance is H_2O, then the substance is water. The converse is also true. **30.** Original conditional: If the probability that an event will occur is 1, then the event is certain to occur. The meaning of 1 in probability is that an event is 100% certain to happen, so the conditional is true. To write the converse, switch the hypothesis and conclusion: If an event is certain to occur, then the probability of the event is 1. The converse is also true. **31.** Original conditional: If you are in Indiana, then you are in Indianapolis. A counterexample is that you could be in Gary, Indiana, and still be in Indiana, so the conditional is false. To write the converse, switch the hypothesis and conclusion: If you are in Indianapolis, then you are in Indiana. Since Indianapolis is the only city so named, it must be in Indiana, the converse is true. **32.** Original conditional: If two angles have measure 90, then the angles are congruent. Since the two angles have the same measure, then they are congruent, so the conditional is true. To write the converse, switch the hypothesis and conclusion: If two angles have the same measure, then they measure 90. The converse is false since a pair of angles can have any measure between 0 and 180. **33.** The hypothesis is inside the conclusion: If a person is an Olympian, then that person is an athlete. **34.** The hypothesis is inside the conclusion: If something is a robin, then it is a bird. **35.** The hypothesis is inside the conclusion: If something is a whole number, then it is an integer. **36a.** Switch the hypothesis and conclusion: If x^2 is an integer divisible by 3, then x is an integer divisible by 3. **36b.** The converse is false. 3 is an integer divisible by 3. If $x^2 = 3$, then $x = \sqrt{3}$ and $\sqrt{3}$ is not an integer divisible by 3. **37.** Answers may vary. Sample: The conditional, "If $x = 1$, then $2x = 2$," and its converse, "If $2x = 2$, then $x = 1$," are both true. **38.** Answers may vary. Sample: The conditional, "If $x = 2$, then $x^2 = 4$," is true. The converse, "If $x^2 = 4$, then $x = 2$," is false because the counterexample is $x = -2$. **39.** The conditional, "If $x = 3$, then $x^2 = 6$," is false because the counterexample is $x^2 = 9$. The converse, "If $x^2 = 6$, then $x = 3$," is also false because it has the counterexamples $x = \sqrt{6}$ and $x = -\sqrt{6}$. **40.** Write the statement in if-then form: If we're half the people, then we should be half the Congress. **41.** Write the statement in if-then form: If a work is great, then it is made out of a combination of obedience and liberty. **42.** Write the statement in if-then form: If a problem is well stated, then it is half solved. **43.** Switch the hypothesis and conclusion: If $x = 18$, then $x - 3 = 15$. There are no counterexamples, so the converse is true. **44.** Switch the hypothesis and conclusion: If $-y$ is positive, then y is negative. There are no counterexamples in the set of real numbers, so the converse is true. **45.** Switch the hypothesis and conclusion: If $|x| = 6$, then $x = -6$. The converse is false because x could also be 6. **46.** Switch the hypothesis and conclusion: If $x^2 > 0$, then $x < 0$. A counterexample is $x = 5$, since $5^2 > 0$ but 5 is not less than 0. The converse is false. **47.** Switch the hypothesis and conclusion: If $x^2 = 4$, then $x = 2$. The converse is false, since $x = -2$ is also a solution. **48.** Switch the hypothesis and conclusion: If $x^3 < 0$, then $x < 0$. There are no counterexamples, so the converse is true. **49a.** Write the statement in if-then form: If you want to look good at the beach this summer, then join GoodFit Health Club. **49b.** Write the statement in if-then form: If I join GoodFit Health Club, then I will look good at the beach this summer. **49c.** Answers may vary. Sample: Al's statement means that joining the club will make him look good. The ad statement does not guarantee that he will look good; the ad just invites people with a desire to look good. **50.** p = A figure is a square. q = A figure has four congruent angles. The first letter is the hypothesis and the second is the conclusion: If a figure is a square, then it has four congruent angles. All angles of a square measure 90, so they are congruent. Thus, the conditional is true. **51.** p = A figure is a square. q = A figure has four congruent angles. The first letter is the hypothesis and the second is the conclusion: If a figure has four congruent angles, then a figure is a square. A rectangle has four congruent angles but its sides are not necessarily =, so it's not necessarily a square. Also, all angles of a stop sign are =, but it is not a square. Thus, the conditional is false. **52.** Write the statement in if-then form. Answers may vary. Sample: If you had bought Treadmaster tires, you would not have had a flat tire. **53.** Check students' work. **54.** Write the statement in if-then form. Answers may vary. Sample: If two lines intersect, then they meet in exactly one point. **55.** Write the statement in if-then form. Answers may vary. Sample: If two planes intersect, then they meet in exactly one line. **56.** Write the statement in if-then form. Answers may vary. Sample: If two figures are congruent, then they have equal areas. **57.** Write the statement in if-then form. Answers may vary. Sample: If two points are given, then there is exactly one line through them. **58.** Write the statement in if-then form. Answers may vary. Sample: If three noncollinear points are given, then there is exactly one plane that contains them. **59.** Since the hypothesis is completely inside the conclusion, then *all* integers that are divisible by 8 are divisible by 2. **60.** Since the hypothesis and conclusion have nothing in common, *n* triangles are squares. (Or: *No* squares are triangles.) **61.** Since part of the hypothesis and conclusion partially overlap, *some* students are musicians. (Or: *Some* musicians are students.) **62.** Answers may vary. Sample: "All apples are fruits" as a conditional is "If something is an apple, then it is a fruit." "No segments are rays" as a conditional is "If something is a segment, then it is not a ray."

63. There are 21 possible statements:

Conclusion (columns); Hypothesis (rows)

$\Rightarrow$	r	s	t	u	v
r	T	F	T	F	T
s	F	T	T	F	T
t	F	F	T	F	T
u	F	F	T	T	T
v	F	F	T	F	T

64. The hypothesis follows *If*. The answer is choice A. **65.** The converse switches the hypothesis and conclusion of the conditional: If you can go with Sarah, then you can sing. The answer is choice I. **66.** The converse switches the hypothesis and conclusion. For A, "If a vehicle has four wheels, then it is a car" is false since a truck has four wheels, but it is not a car. For B, "If you cross an ocean from the United States, then you go to Asia" is false since you can cross an ocean to go to Africa, Europe, or Australia. For C, "If your pet is furry, then you own a dog" is false since you could own a cat instead. For D, "If you can walk, then you can stand up" is true because walking requires the ability to stand. The answer is choice D. **67.** [2] Switch the hypothesis and conclusion: If Marta is too young to vote, then she is five years old. The statement is false because a counterexample is Marta is 10 and too young to vote. [1] Only the converse or only the truth value is provided. **68.** $P = 2b + 2h = 2(6) + 2(12) = 12 + 24 = 36$ in. **69.** $P = 2b + 2h = 2(3.5) + 2(7) = 7 + 14 = 21$ cm **70.** Since 18 in. $= \frac{1}{2}$ yd, $P = 2b + 2h = 2\left(1\frac{3}{4}\right) + 2\left(\frac{1}{2}\right) = 2\left(\frac{7}{4}\right) + 2\left(\frac{1}{2}\right) = \frac{7}{2} + \frac{2}{2} = \frac{9}{2} = 4\frac{1}{2}$ yd. Since $1\frac{3}{4}$ yd $= 36 + \frac{3}{4}(36) = 36 + 27 = 63$ in., $P = 2b + 2h = 2(63) + 2(18) = 126 + 36 = 162$ in. **71.** Since 60 cm $=$ 0.6 m, $P = 2b + 2h = 2(11) + 2(0.6) = 22 + 1.2 = 23.2$ m. Since 11 m $=$ 1100 cm, $P = 2b + 2h = 2(1100) + 2(60) = 2200 + 120 = 2320$ cm. **72.** $r = \frac{d}{2} = \frac{10}{2} = 5$ in.; $A = \pi r^2 = \pi(5)^2 = 25\pi$ in.^{2} **73.** $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(1-4)^2 + (2-(-2))^2} = \sqrt{(1-4)^2 + (2+2)^2} = \sqrt{(-3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$ **74.** $MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-5-0)^2 + (1-5)^2} = \sqrt{(-5)^2 + (-4)^2} = \sqrt{25 + 16} = \sqrt{41} \approx 6.4$ **75.** $RT = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0-2)^2 + (-6-3)^2} = \sqrt{(-2)^2 + (-9)^2} = \sqrt{4 + 81} = \sqrt{85} \approx 9.2$ **76.** Each term is half the previous term, so the next two numbers after $\frac{1}{2}$ are $\frac{1}{2} \cdot \frac{1}{2}$, or $\frac{1}{4}$, and $\frac{1}{4} \cdot \frac{1}{2}$, or $\frac{1}{8}$. **77.** Each term is 3 less than the previous term, so the next two terms after -4 are $-4 - 3$, or -7, and $-7 - 3$, or -10. **78.** Each term is the letter just before the previous term in the alphabet, so the next two terms after K are J and I.

2-2 Biconditionals and Definitions

pages 75–81

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1. Hypothesis: $x > 10$. Conclusion: $x > 5$. **2.** Hypothesis: You live in Milwaukee. Conclusion: You live in Wisconsin. **3.** If a figure is a square, then it has four sides. **4.** If something is a butterfly, then it has wings. **5.** If we go on a picnic, then the sun shines. **6.** If two lines do not intersect, then they are skew. **7.** If $x^3 = -27$, then $x = -3$.

Check Understanding 1. To write the converse, switch the hypothesis and conclusion: If three points lie on the same line, then they are collinear. Both the conditional and the converse are the def. of collinear, so they are true and the statement can be combined as a biconditional by using the *if-and-only if* phrase: Three points are collinear if and only if they lie on the same line. Or: Three points lie on the same line if and only if they are collinear. **2.** Write an if-then statement using one part as the hypothesis and the other part as the conclusion: If a number is prime, then it has only two distinct factors, 1 and itself. If a number has only two distinct factors, 1 and itself, then it is prime.
3–4. (Answers follow the Investigation below.)

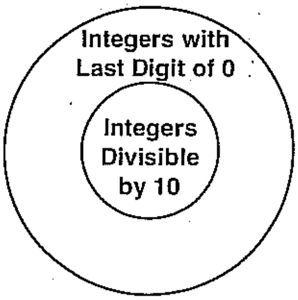
Investigation 1. Polyglobs can vary in color, but they have 3 solid black dots and 3 fingers. Figure B is a polyglob. **2.** Answers may vary. Sample: A polyglob has three fingers and three solid dots.

Check Understanding 3. To write the statement as a conditional, write it in if-then form: If an angle is a right angle, then its measure is 90. To write the converse, switch the hypothesis and conclusion: If an angle has measure 90, then it is a right angle. Both statements reflect the definition of a right angle, so they are both true. To write it as a biconditional, write *if and only if* between the hypothesis and conclusion: An angle is a right angle if and only if its measure is 90. Or: An angle measures 90 if and only if the angle is a right angle.
4. The conditional, if a figure is a square then it has four right angles, is true. The converse, if a figure has four right angles then it is a square, is false, since a rectangle has four right angles but is not necessarily a square.

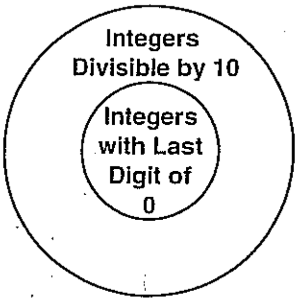
Exercises 1. For the converse, switch the hypothesis and conclusion: If two segments are congruent, then they have the same length. Since this is the definition of congruent segments, the converse is true. For the biconditional, separate the hypothesis and conclusion with *if and only if*: Two segments have the same length if and only if they are congruent. **2.** For the converse, switch the hypothesis and conclusion: If $2x - 5 = 19$, then $x = 12$. The converse is true since $2(12) - 5 = 19$. For the biconditional, separate the hypothesis and conclusion with *if and only if*: $x = 12$ if and only if $2x - 5 = 19$. **3.** For the converse, switch the hypothesis and conclusion: If a number is even, then it is divisible by 20. The statement is false, since 4 is even but not divisible by 20. **4.** For the converse, switch the hypothesis and conclusion: If $|x| = 3$, then $x = 3$. The converse is false since $|-3| = 3$, also. **5.** For the converse, switch the hypothesis and conclusion: If it is July 4th in the United States, then it is Independence Day. The statement is true since Independence Day is the name of the holiday on July 4th. For the biconditional, separate the hypothesis and conclusion with *if and only if*: In the United States, it is Independence Day if and only if it is July 4th. **6.** For the converse, switch the hypothesis and conclusion: If $x^2 = 100$, then $x = -10$. The statement is false since $x = 10$ also satisfies the equation. **7.** Write one part as the hypothesis and the other part as the conclusion in an if-then statement; then write the converse: If a line bisects a segment, then the line intersects the segment at its midpoint. If a line contains a segment's midpoint, then it bisects the segment.
8. Write one part as the hypothesis and the other part as the conclusion in an if-then statement, then write the converse: If an integer is divisible by 100, then its last two digits are zeros. If an integer's last two digits are zeros, then it is divisible by 100. **9.** Write one part as the hypothesis and the other part as the conclusion in an if-then statement, then write the converse: If you live in Washington, D.C., then you live in the capital of the United States. If you live in the capital of the United States, then you live in Washington, D.C. **10.** Write one part as the hypothesis and the other part as the conclusion in an if-then statement, then write the converse: If two lines are parallel, then they are coplanar and do not intersect. If two lines are coplanar and do not intersect, then they are parallel. **11.** Write one part as the hypothesis and the other part as the conclusion in an if-then statement, then write the converse: If two angles are congruent, then they have the same measure. If two angles have the same measure, then they are congruent.
12. Write one part as the hypothesis and the other part as the conclusion in an if-then statement, then write the converse: If $x^2 = 144$, then $x = 12$ or $x = -12$. If $x = 12$ or $x = -12$, then $x^2 = 144$. **13.** Conditional in if-then form: If a line, segment, or ray is perpendicular to a segment at its midpoint, then it is the perpendicular bisector of the segment. Converse: If a line, segment, or ray is the perpendicular bisector of a segment, then it is perpendicular to a segment at its midpoint. Both statements are true since they are the def. of $\perp$ bis. A line, segment, or ray is a perpendicular bisector of a segment if and only if it is perpendicular to the segment at its midpoint. **14.** Conditional: If two planes do not intersect, then they are parallel. Converse: If two planes are parallel, then they do not intersect. Both are true since they reflect the def. of parallel planes. Biconditional: Planes are parallel if and only if they do not intersect.
15. Conditional: If a person is a Tarheel, then the person was born in North Carolina. This statement is false. North Carolina is the Tar Heel State, and everyone living there, regardless of whether they were born there, is considered a Tar Heel. So, the statement is not reversible.

16. Conditional: If a figure is a rectangle, then it has at least one right angle. Converse: If a figure has at least one right angle, then it is a rectangle. The converse is false, since a pentagon in the shape of a house has at least one right angle, but it is not a rectangle. The statement is not reversible. **17.** Conditional: If a point divides a segment into two congruent segments, then it is a midpoint of a segment. Converse: If a point is a midpoint of a segment, then it divides it into two congruent segments. Biconditional: A point is a midpoint of a segment if and only if it divides the segment into two congruent segments. **18.** Conditional: If an animal is a cat, then it has whiskers. Cats have whiskers, so the conditional is true. Converse: If an animal has whiskers, then it is a cat. The converse is false, since a mouse has whiskers but is not a cat. The statement is not reversible, so it is not a good definition. **19.** This statement is false, since a dog is not considered to be a good pet for all people. The statement is not reversible and the term *good* is vague, so it is not a good definition. **20.** Conditional: If something is a segment, then it is part of a line. This is true. Converse: If something is part of a line, then it is a segment. This statement is false since a ray or a point is also part of a line. The definition is not precise and is not reversible, so it is not a good definition. **21.** Conditional: If two lines are parallel, then they do not intersect. This statement is true. Converse: If two lines do not intersect, then they are parallel. This statement is not true, since skew lines do not intersect. **22.** Conditional: If a figure is a square, then it is a figure with two pairs of parallel sides. The statement is true. Converse: If a figure has two pairs of parallel sides, then the figure is a square. A stop sign has two (or more) pairs of parallel sides, but a stop sign is not a square. The statement is not reversible, so it is not a good definition. **23.** Conditional: If an ray is an angle bisector, then it divides an angle into two congruent angles. This statement is true. Converse: If a ray divides an angle into two congruent angles, then it is an angle bisector. This statement is also true. Since both are true, the statement is a good definition. **24.** A good definition is reversible. This statement is not reversible since a straight angle has a measure that is greater than 90, but is not an obtuse angle. **25.** A good definition is precise, reversible, and clearly understood. Answers may vary. Sample: An acute angle is an angle whose measure is between 0 and 90. **26.** Conditional: If a line is parallel to a plane, then it does not intersect the plane. Biconditional: A line is parallel to a plane if and only if it does not intersect the plane. **27.** The linear pairs share a vertex and are adjacent to one another. Also, their sum is 180°. Pairs that are not linear share a vertex, but not a side, or share neither a vertex nor a side, and their sum is not 180°. Answers may vary. Sample: Two angles form a linear pair if and only if they share a side and a vertex and are supplementary. **28.** No; though the angles share a vertex and a side, their sum is not 180. **29.** Yes; the angles share a vertex and a side and their sum is 180. **30.** No; the angles share a side and their sum is 180, but they do not share a vertex.

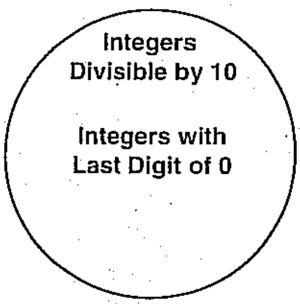
31. No; the angles share a vertex but no side, and their sum is not 180. **32.** Converse: If $2x - 3 = 35$, then $x = 19$. The converse is true. Biconditional: $2x - 3 = 35$ if and only if $x = 19$. **33.** Converse: If $x^2 = 9$, then $x = 3$. The converse is false. A counterexample is $x = -3$. **34.** Converse: If $|x| > 0$, then $x > 0$. $|x|$ is always > 0, even when $x < 0$, so the converse is false. **35.** Converse: If $x^3 - 125$, then $x = 5$. The converse is true. Biconditional: $x^3 = 125$ if and only if $x = 5$. **36.** This is a good definition because its terms are clearly understood and precise. **37.** This definition could be easily confused with the letter *V*, so it is not a good definition. **38.** This definition could be easily confused with the letter *L*, so it is not a good definition. **39.** This is a good definition because its terms are clearly understood and precise. **40.** This is a good definition because its terms are clearly understood and precise. **41.** Conditional: If angles are congruent, then they have equal measure. Biconditional: Angles are congruent if and only if they have equal measure. **42.** Conditional: If the sum of the digits of an integer is divisible by 9, then the integer is divisible by 9. Biconditional: The sum of the digits of an integer is divisible by 9 if and only if the integer is divisible by 9. **43.** Conditional: If a number is a whole number, then it is a nonnegative integer. Biconditional: A number is a whole number if and only if it is a nonnegative integer. **44.** $p = \angle A$ is an acute angle; $q = \angle A$ has measure between 0 and 90. So, $p \rightarrow q$ is: If $\angle A$ is an acute angle, then $\angle A$ has measure between 0 and 90. **45.** $p = \angle A$ is an acute angle; $q = \angle A$ has measure between 0 and 90. So $q \rightarrow p$ is; If $\angle A$ has measure between 0 and 90, then $\angle A$ is an acute angle. **46.** $p = \angle A$ is an acute angle; $q = \angle A$ has measure between 0 and 90. So $p \leftrightarrow q$ is $\angle A$ is an acute angle if and only if $\angle A$ has measure between 0 and 90. **47.** If Carla and the drummer wear different-colored shirts, then Carla must not be the drummer. Since the keyboard player is older than Bob, then Bob is not the keyboard player. Since Amy lives next door to the guitarist, she is not the guitarist. Since Amy is the youngest, she cannot be older than Bob, so she is not the keyboard player. Since Amy is neither the keyboard player nor the guitarist, she must be the drummer. Since neither Bob nor Carla is the keyboard player, the keyboard player must be Carla. The guitarist, who is not Amy or Carla, must be Bob. **48a.** Write the biconditional in if-then form, then write its converse: If an integer is divisible by 10, then its last digit is 0. Converse: If an integer's last digit is 0, then it is divisible by 10.

48b. 

Position the hypothesis completely inside the conclusion.

48c. 

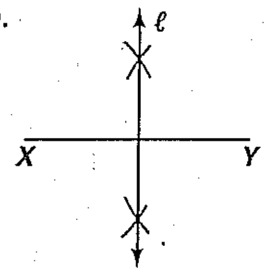
Position the hypothesis completely inside the conclusion.

48d. 

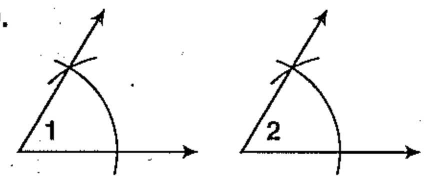
Both the hypothesis and conclusion are the same size, overlapping each other completely.

48e. Answers may vary. Sample: The two circles coincide. **48f.** Answers may vary. Sample: A good definition may be written as a biconditional because either of the coinciding circles of its Venn diagram can be the hypothesis of a conditional, and the other can be the conclusion. **49.** There are only 2 blue hats and all the rest are red, so, if the two hats in front of Alan were blue, he would know his own hat was red. Ben can tell from Alan's response that there are 1 or 2 red hats in front of Alan. Since Ben can't decide the color of his hat, Cal figures Ben must see a red hat. If Ben had seen Cal in a blue hat, Ben would have known his own hat was the red one seen by Alan. So, Cal's hat must be red. **50.** A counterexample for statement A is parallel lines. A counterexample for B is skew lines. A counterexample for D is intersecting lines that form 30° and 150° angles. The answer is choice C. **51.** A counterexample for statement G is intersecting lines, because they lie in the same plane, but they do intersect. The answer is choice G. **52.** Statement B is not true because a counterexample is $x = -1$. **53.** [2] Write a conditional by writing the statement in if-then form, then write its converse: If you can go to the movies, then you do your homework. If you do your homework, then you can go to the movies. **54.** [4] **54a.** To write the converse, switch the hypothesis and conclusion: If a person is old enough to vote, then that person is 18 years old. **54b.** A counterexample is that the person is any age older than 18 years. So, the converse is false.

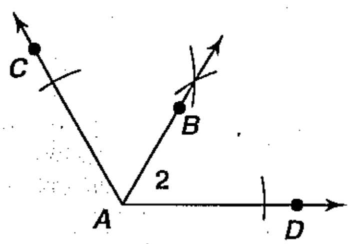
54c. Counterexamples may vary. Sample: A 20-year-old is old enough to vote, but is not 18 years old. (OR equivalent conditionals) [3] predominantly correct but with one error [2] at least one correct answer, and some appropriate information for one other part [1] some correct information **55.** Write the statement in if-then form: If a whole number ends in 0, then it is even. **56.** Write the statement in if-then form: If $-x = -5$, then $x^2 = 25$. **57.** Write the statement in if-then form: If a day is Sunday, then it is a weekend day. **58.** Write the statement in if-then form: If a prime number is greater than 2, then it is odd.

59. 

With the compass point on X, open the compass to more than half XY and swing an arc on each side of the segment. With the same setting, place the compass point on Y and swing arcs that intersect the first two arcs. Draw a line, ℓ, through the arc intersections. Line ℓ bisects $\overline{XY}$.

60. 

Draw an angle whose measure is between 0 and 90. Label it $\angle 1$. Draw a ray. With the compass point on the vertex of $\angle 1$, swing an arc that intersects both sides of the angle. Keeping the setting, place the compass point on the endpoint of the ray and swing an arc of about the same length so that it intersects the ray. Place the compass point on the intersection of the arc and one side of $\angle 1$. Open the compass to where the arc intersects the other side of $\angle 1$. Place the compass point on the intersection of the arc with the ray. Swing an arc that intersects the arc. Draw a ray from the endpoint of the first ray through the intersection of the two arcs. Label the new angle $\angle 2$. $\angle 2 \cong \angle 1$

61. 

Draw an angle whose measure is between 90 and 180. Label it $\angle CAD$ with A at the vertex. With the compass point on A, swing an arc that intersects both sides of the angle. With the compass point on the intersection of the arc with one of the sides, swing an arc in the interior of the angle. Keeping the setting, place the compass point on the intersection of the arc with the other side of the angle and swing an arc that intersects the previous arc. Call that intersection B. Draw $\overrightarrow{AB}$. $\overrightarrow{AB}$ bisects $\angle CAD$. **62.** Intersecting lines are any two lines that share a point. Answers may vary. Sample: $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ **63.** Skew lines do not intersect and are not parallel. Answers may vary. Sample: $\overleftrightarrow{AB}$ and $\overleftrightarrow{CG}$

64. Parallel lines are coplanar lines that do not intersect. Answers may vary. Sample: $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ **65.** Parallel planes do not intersect. Answers may vary. Sample: ABC, EFG **66.** By Post. 1-1, any two points are collinear. Answers may vary. Samples: A and B, A and C, C and E **67.** By Post. 1-3, intersecting planes share the same line. Answers may vary. Sample: AEF, BFG **68.** Answers may vary. Samples: EFG, AEH, DHG **69.** By Post. 1-3, two planes intersect in a line. Answers may vary. Sample: Plane ABC and Plane FBC intersect in $\overleftrightarrow{BC}$.

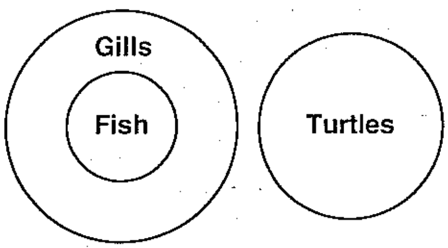
2-3 Deductive Reasoning pages 82–87

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM*.
1. If your grades suffer, then you don't sleep enough. **2.** If you must start early, then you want to arrive on time. **3.** If a year is a leap year, then it has 366 days. **4.** If students do not complete their homework, then they will have lower grades. **5.** If two lines are perpendicular, then they meet to form right angles. **6.** If a person is 16 years old, then that person is a teenager.

Check Understanding **1.** No; though a dead battery might be one of the things the mechanic will check, there could be other things wrong, such as a faulty starter. **2.** By the Law of Detachment, Vladimir Nuñez should not pitch two days in a row. Since he pitches a complete game on Monday, he should not pitch a complete game on Tuesday. **3.** You are given that a conditional and its conclusion are true. Since the second sentence refers to the conclusion and not the hypothesis of the conditional, you cannot apply the Law of Detachment. **4a.** Since the conclusion of one conditional is the hypothesis of the other, use the Law of Syllogism to conclude: If a number ends in 0, then it is divisible by 5. **4b.** Since the conclusion of one conditional is not the hypothesis of the other, you can not use the Law of Syllogism. **5.** From the first and third statements, use the Law of Detachment to conclude: The Volga River is less than 2300 miles long. From this statement and the second statement, use the Law of Detachment to conclude: The Volga River is not one of the world's ten longest rivers.

Exercises **1.** The second statement is the hypothesis of the first, so Felicia will pass the music theory course. **2.** The second statement is the hypothesis of the first, so Rashid must study hard. **3.** The second statement is the hypothesis of the first, so line ℓ and line m do not intersect. **4.** The second statement is the hypothesis of the first, so it is not safe for Marla to be out in the open. **5.** The second statement is the hypothesis of the true conditional, so figure $ABCD$ has two pairs of parallel sides. **6.** The second statement is not the hypothesis of the true conditional, so it is not possible to use the Law of Detachment to draw a conclusion. **7.** The second statement is the hypothesis of the true conditional, so points X, Y, and Z are collinear. **8.** The second statement is not the hypothesis of the true conditional, so it is not possible to use the Law of Detachment to draw a conclusion. **9.** The second statement is the hypothesis of the true conditional, so Nadine Muzerall attended the University of Minnesota. **10.** Since the conclusion of one conditional is the hypothesis of the other, use the Law of Syllogism to conclude: If an animal is a red wolf, then it is endangered. **11.** Since the conclusion of one conditional is the hypothesis of the other, use the Law of Syllogism to conclude: If two planes are not parallel, then they intersect in a line. **12.** Since the conclusion of one conditional is the hypothesis of the other, use the Law of Syllogism to conclude: If you read a good book, then your time is well spent. **13.** Since the conclusion of one conditional is the hypothesis of the other, use the Law of Syllogism to conclude: If you are studying botany, then you are studying a science. **14.** From the first and second statements, use the Law of Syllogism to conclude: If an Alaskan mountain is over 20,300 ft high, then it is the highest in the United States. Combine that statement with the third statement, and use the Law of Detachment to conclude that Alaska's Mount McKinley is the highest mountain in the United States. **15.** From the first and third statements, use the Law of Syllogism to conclude: If you live in Little Rock, then you live in the 25th state to enter the Union. Use that statement with the second statement to conclude: Levon lives in the 25th state to enter the Union. **16.** Must be true; by (E) and (A), it is breakfast time. Then by (D), Julio drinks juice. **17.** Must be true; by (E) and (A), it is breakfast time. Then, since breakfast is a mealtime, by (C), Curtis drinks water. **18.** May be true; by (E) and (A), it is breakfast time. We don't know what Kira drinks at breakfast time. **19.** Is not true; by (E) and (A), it is breakfast time. By (C), Curtis drinks water and nothing else. **20.** May be true; by (E), Maria drinks juice. We don't know if she also drinks water. **21.** Is not true; by (E) and (A), it is breakfast time. By (D), Julio drinks juice and nothing else, so he does not drink milk. **22.** Write the first statement in if-then form: If something is a national park, then it is interesting. The second sentence relates to the hypothesis, so: Mammoth Cave is interesting. **23.** Write the first statement in if-then form: If you are in Key West, Florida, then the temperature is always above 32°F. The second statement relates to the conclusion and not the hypothesis, so it is not possible to use the Law of Detachment. **24.** Write the first statement in if-then form: If you are a high-school student, then you like music. The second statement relates to the conclusion and not the hypothesis, so it is not possible to use the Law of Detachment. **25.** Write the first statement in if-then form: If a figure is a square, then it is a rectangle. The second statement relates to the hypothesis of the statement, so: $ABCD$ is a rectangle. **26.** Three statements must be a conditional, a fact related to the hypothesis of the conditional, and a third statement using the fact and the conclusion of the conditional. Answers may vary. Sample: If a student wears a hat to school, then the student must take it off indoors. Amy wears a hat to school. Then Amy must take off the hat indoors. **27.** No; red cars can never park. **28.** No; men with beards are not allowed to park on Monday. **29.** Yes; there are no restrictions on women with wigs, and parking is allowed at 10:00 A.M. on Saturday. **30.** No; there is no parking on Tuesday from 6:49 A.M. to 9:11 A.M. **31.** Yes; there are no restrictions on convertibles with leather seats, and parking is allowed at 6:00 P.M. on Sunday. **32.** Using the Law of Syllogism on the first two statements: If Anita goes to the concert, then Aisha will go, but also Beth will go. This is a total of 3 students. Since only 2 go, Anita must not go to the concert. Since the hypothesis in each subsequent statement is the conclusion of the previous statement, only the last two students are going to the concert. They are Aisha and Ramon.

33a.



Turtles will not overlap with gills, and the hypothesis is inside the conclusion of a conditional. **33b.** Turtles are not in the circle of animals with gills, so a turtle is not a fish. **34.** By the Law of Detachment, Evan will be tired. The answer is choice D. **35.** By the Law of Syllogism: If you have a job, then you must pay taxes. The answer is choice F. **36.** [2] **a.** Since Bert and Carl read the same books, Bert is also reading Hamlet. **b.** Andrea, Bert, and Carl; if Darla were reading *King Lear,* then all four people would be reading it. [1] one part correct **37.** [4] **a.** Each of the three categories of food was chosen by at least one of the three people. Since Mark won't eat bread, neither Clara nor Mark ate a sandwich. Thus, Harold had the sandwich. Since the person who had the sandwich also had the milk, Harold had the milk. **b.** Mark won't eat salad or a sandwich, so he must have had the soup, so he must have had the iced tea. From part a, Harold had the sandwich with milk. Clara must have had the salad with the mineral water. [3] correct answers with poor explanations [2] only one correct answer with poor explanations [1] one correct answer with no explanations **38.** The statement does not describe how the rays relate to one another, so it is not a good definition. Counterexamples may vary. Sample: Two rays that do not intersect do not form an angle. **39.** It is reversible, the terms are clear, and the statement is precise, so it is a good definition. **40.** A counterexample is North Carolina, since it does not border Canada. **41.** The square of any fraction between 0 and 1 will be less than the original fraction. Answers may vary. Sample: $\frac{1}{2}$, since $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$ and $\frac{1}{4}$ is not greater than $\frac{1}{2}$ **42.** Since two lines that do not intersect can be skew, two lines that do not intersect are *sometimes* parallel.

43. By definition of skew, two lines that intersect are *never* skew. **44.** By Post. 1-4, three noncollinear points are coplanar. By Post. 1-1, exactly one line can be drawn through any two points. So, two distinct lines can be drawn through two distinct pairs of the three points and the lines intersect at one of the three points. Since segments are parts of lines, two segments that intersect are *always* coplanar.

CHECKPOINT QUIZ 1 *page 88*

1. The hypothesis follows *If* and the conclusion follows *then.* Hypothesis: $x > 5$. Conclusion: $x^2 > 25$. **2.** Write the statement in if-then form: If something is a rose, then it is a beautiful flower. **3.** Switch the hypothesis and conclusion: If an integer is divisible by 2, then the integer ends with 0. **4.** Any integer ending in 0, 2, 4, 6, or 8 is divisible by 2, so choose a number ending in 2, 4, 6, or 8. Answers may vary. Sample: 42 is divisible by 2, but it does not end with 0. **5.** Write the statement in if-then form, then write its converse: If an angle is an acute angle, then its measure is between 0 and 90. If an angle's measure is between 0 and 90, then it is an acute angle. **6.** Write the statement in if-then form, then rewrite it by separating the hypothesis and conclusion by *if and only if.* If points are collinear, then they lie on the same line. Points are collinear if and only if they lie on the same line. **7.** Any computerized keyboard, such as a computerized piano, computer, or graphing calculator; answers may vary. Sample: A graphing calculator has a keyboard and a memory. **8.** The second statement relates to the hypothesis of the first statement, so use the Law of Detachment to conclude that Theresa has passing grades. **9.** The conclusion of one conditional is the hypothesis of the other, so use the Law of Syllogism to conclude: If a student studies geometry, then the student's mind is expanded. **10.** The second statement cannot be written as a conditional, so you cannot use the Law of Syllogism. Also, the second statement relates to the conclusion of the conditional, so you cannot use the Law of Detachment. It is *not possible* to draw a conclusion based on either law.

2-4 Reasoning in Algebra *pages 89–94*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*
1. $\angle AOB$, $\angle BOA$ **2.** O **3.** $\overrightarrow{OB}$ **4.** 45 **5.** $\overrightarrow{OA}$ and $\overrightarrow{OC}$
Check Understanding **1.** Since $m\angle MLN = 4x$ and $m\angle KLM = 2x + 40$, and $m\angle MLN = m\angle KLM$, $4x = 2x + 40$ by the Substitution Property. Subtract $2x$ from both sides to get $2x = 40$ by the Subtraction Prop. of Equality. Divide both sides by 2 to get $x = 20$ by the Division Prop. of Equality. **2.** $AB = 2y = 2(6) = 12$; $BC = 3y - 9 = 3(6) - 9 = 18 - 9 = 9$; $AB + BC = 12 + 9 = 21$ **3a.** Since the same figures are on each side of the congruence symbol, it is the Reflexive Prop. of $\cong$.

3b. Since the conclusion of one statement is the hypothesis of the other statement, the conclusion can be made from either the Transitive Property of ≅ or the Substitution Property of Equality.

Exercises **1a.** The measures of angles are added: Angle Addition Post. **1b.** Values are being substituted: Substitution Prop. of =. **1c.** Values are combined: Simplify. **1d.** Subtract 20 from both sides: Subtraction Prop. of Equality. **1e.** Divide both sides by 4: Division Prop. of Equality. **2a.** Segments are added: Segment Addition Post. **2b.** Values are substituted: Substitution Prop. of Equality. **2c.** 3 is distributed over $(n + 4)$: Distributive Property. **2d.** $3n$ and $3n$ are combined: Simplify. **2e.** Subtract 12 from both sides: Subtraction Prop. of Equality. **2f.** Divide both sides by 6: Division Prop. of Equality. **3a.** Multiply both sides by 2: Multiplication Prop. of Equality. **3b.** Distribute 2 over $\left(\frac{1}{2}x - 5\right)$: Distributive Prop. **3c.** Add 10 to both sides: Addition Prop. of Equality. **4a.** 5 is distributed over $(x + 3)$: Distributive Property. **4b.** Subtract 15 from both sides: Subtraction Prop. of Equality. **4c.** Divide both sides by 5: Division Prop. of Equality. **5.** Each side of the congruence symbol is identical to the other: Reflexive Prop. of ≅. **6.** 2 is distributed over $(3x + 5)$: Distributive Prop. **7.** Both sides are ÷ by 12: Division Prop. of Equality. **8.** The information on each side of the ≅ symbol is switched: Symmetric Prop. of ≅. **9.** Both sides are multiplied by 3: Multiplication Prop. of Equality. **10.** Both sides of the = symbol are identical: Reflexive Property of Equality. **11.** 14 is subtracted from both sides: Subtraction Prop. of =. **12.** Information on each side of the = symbol is switched: Reflexive Prop. of =. **13.** x is substituted for y in the first equation: Substitution Prop. **14.** BC is added to both sides: Addition Prop. of =. **15.** The hypothesis of one statement is the conclusion of the other, so a conclusion is made by the Transitive Prop. of =. **16.** Adding 5 to both sides, $2x = 10 + 5 = 15$. **17.** Subtracting 6 from both sides results in $5x = 15$. **18.** Switch sides: $YU = AB$. **19.** Switch sides: $\angle K = \angle H$. **20.** For the Reflexive Prop., the same thing is on both sides: $\angle PQR \cong \angle PQR$. **21.** Multiply both terms of $(x - 1)$ by 3: $3x - 3$. **22.** Substitute 7 for LM in $EF + LM = NP$: $EF + 7 = NP$. **23.** Two figures are ≅ to the same figure, so use the Transitive Prop. of ≅. **24.** Multiplying both sides by 3 results in $TR = 3UW$. **25.** Answers may vary. Sample: $\overline{LR}$ and $\overline{RL}$ are different ways to name the same segment, and $\angle CBA$ and $\angle ABC$ are different ways to name the same ∠. **26.** Falling is transferred from A to B to C, so it's ultimately transferred from A to C. So, domino A causes domino C to fall. **27a.** Given **27b.** Def. of midpt. **27c.** Values are substituted for other values: Substitution Prop. **27d.** Subtract $2x$ from both sides: Subtraction Prop. of =. **27e.** Divide both sides by 2: Division Prop. of =. **28a.** ① $KL + LM = KM$ (Segment Addition Post.) ② $(2x - 5) + 2x = 35$ (Substitution Prop.) ③ $4x - 5 = 35$ (Simplify.) ④ $4x = 40$ (Add. Prop. of =) ⑤ $x = 10$ (Div. Prop. of =)

28b. $KL = 2x - 5 = 2(10) - 5 = 20 - 5 = 15$
29a. ① $m\angle GFE + m\angle EFI = m\angle GFI$ (∠ Add. Post.) ② $(9x - 2) + 4x = 128$ (Substitution Prop.) ③ $13x - 2 = 128$ (Simplify.) ④ $13x = 130$ (Add. Prop. of =) ⑤ $x = 10$ (Div. Prop. of =) **29b.** $m\angle EFI = 4x = 4(10) = 40$ **30a.** Given **30b.** Def. of ∠ Bisector **30c.** Values are substituted: Substitution Prop. **30d.** Subtract $4n + 1$ from both sides: Subtraction Prop. of =. **30e.** Divide each side by 2: Division Prop. of =. **31.** Since $a = b$, then $b - a = 0$. In the fifth step, both sides are divided by $b - a$, which is 0, and division by 0 is not defined. **32.** You can have the same birthday as yourself, so "has the same birthday as" is reflexive. If you have the same birthday as a friend, then your friend has the same birthday as you, so "has the same birthday as" is symmetric. If you have the same birthday as a sister, and your sister has the same birthday as her friend, then you have the same birthday as her friend, so "has the same birthday as" is transitive. **33.** You cannot be taller than yourself, so "is taller than" is not reflexive. If you are taller than your friend, then your friend cannot be taller than you, so "is taller than" is not symmetric. If you are taller than your friend, and your friend is taller than the fence, then you are taller than the fence, so "is taller than" is transitive. **34.** You can live in the same state as yourself, so "lives in the same state as" is reflexive. If you live in the same state as Lynda, then Lynda lives in the same state as you, so "lives in the same state as" is symmetric. If you live in the same state as Ken, and Ken lives in the same state as Roy, then you live in the same state as Roy, so "lives in the same state as" is transitive. **35.** You cannot live in a different state than yourself, so "lives in a different state than" is not reflexive. If Joy lives in a different state than Ron, then Ron lives in a different state than Joy, so "lives in a different state than" is symmetric. If you live in a different state than Cam, and Cam lives in a different state than Gene, you and Gene can live in the same state or different states, so "lives in a different state than" is not transitive. **36.** You are the same height as yourself, so "is the same height as" is reflexive. If you are the same height as a tree, then the tree is the same height as you, so "is the same height as" is symmetric. If John is the same height as Jan, and Jan is the same height as Jo, then John is the same height as Jo, so "is the same height as" is transitive. **37.** You cannot be a descendent of yourself, so "is a descendant of" is not reflexive. If you are a descendant of your mother, then your mother cannot be a descendant of you, so "is a descendant of" is not symmetric. If you are a descendant of your mother, and your mother is a descendant of your grandmother, then you are a descendant of your grandmother, so "is a descendant of" is transitive. **38.** Since information on each side of the = symbol is switched, it the Symmetric Prop. of = justifies the statement. The answer is choice D. **39.** The radius of the circle in Column A is 3 cm, so $A = \pi r^2 = \pi(3)^2 = 9\pi$. For the circle in Column B, $A = \pi r^2 = \pi(6)^2 = 36\pi$. Since $9\pi < 36\pi$, the answer is choice B. **40.** The perimeter of a square with 4-in. sides is $4s = 4(4) = 16$ in. For the

rectangle in Column B, the two unknown sides could be greater than 4 in., less than 4 in. or equal to 4 in., so it's impossible to tell what the perimeter of that figure is. The answer is choice D. **41.** For Column A, $A = bh = (912)(5) = 60$ cm^2. For Column B, $A = s^2 = (6)^2 = 36$ cm^2. Since 60 cm > 36 cm, the answer is choice A. **42.** [2] **a.** Substitute $2y + 15$ for x to get $2y + 15 + y = 120$ (Substitution Prop.). Combine $2y$ and y to get $3y + 15 = 120$ (Simplify.). **b.** Subtract 15 from both sides to get $3y = 105$ (Subtraction Prop. of =). Divide both sides by 3 to get $y = 35$ (Division Prop. of =); $x = 2y + 15 = 2(35) + 15 = 70 + 15 = 85$. **43.** By the Law of Detachment, Elena's teacher is concerned. **44.** By the Law of Syllogism, if a person has a job, then that person can save money each week. **45.** $m\angle AOC = m\angle AOB + m\angle BOC = 60 + 20 = 80$ **46.** $m\angle AOD = m\angle AOB + m\angle BOC + m\angle COD = 60 + 20 + 45 = 125$ **47.** $m\angle DOB = m\angle DOC + m\angle COB = 45 + 20 = 65$ **48.** $m\angle BOE = m\angle BOC + m\angle COD + m\angle DOE = 20 + 45 + 25 = 90$ **49.** An obtuse angle measures between 90 and 180; a right triangle measures exactly 90. Answers may vary. Samples: Obtuse angles are $\angle AOD$ and $\angle AOE$. The only right angle is $\angle BOE$. **50.** Each term is 2.5 more than the previous term, so the next two terms after 26.5 are 26.5 + 2.5, or 29, and 29 + 2.5, or 31.5. **51.** The digits to the right of the decimal point are the counting numbers in order starting from 4. The next two terms after 3.4567 are 3.45678 and 3.456789. **52.** Each term is –3 times the previous term, so the next two terms after 54 are $-3(54)$, or -162, and $-3(-162)$, or 486.
53. Each term is $-\frac{1}{2}$ times the previous term, so the next two terms after -1 are $-\frac{1}{2}(-1)$, or $\frac{1}{2}$, and $-\frac{1}{2}(\frac{1}{2})$, or $-\frac{1}{4}$.

READING MATH — page 95

a. ① Since $\angle 1$ and $\angle 2$ are supplementary, $m\angle 1 + m\angle 2 = 180$ (Def. of suppl.). ② Since $m\angle 1 = 4y + 15$ and $m\angle 2 = 7y - 11$ (Given), substitute these values into the equation in step 1: $(4y + 15) + (7y - 11) = 180$ (Substitution Prop.). ③ Combine like terms: $11y + 4 = 180$ (Simplify.). ④ Subtract 4 from both sides: $11y = 176$ (Subtraction Prop. of =). ⑤ Divide both sides by 11: $y = 16$ (Division Prop. of =). **b.** Since $y = 16$, $m\angle 2 = 7y - 11 = 7(16) - 11 = 112 = 11 = 101$.

2-5 Proving Angles Congruent — pages 96–103

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*
1. 50 **2.** 90 **3.** 35 **4.** right angles **5.** vertex

Check Understanding **1a.** Adjacent angles share the same vertex and one side and have no common interior points. Answers may vary. Sample: $\angle AFB$ and $\angle BFC$; $\angle BFD$ and $\angle DFE$ **1b.** Since $\angle AFE$ is a straight angle, then $m\angle EFD + m\angle AFD = 180$; $27 + m\angle AFD = 180$; $m\angle AFD = 180 - 27 = 153$. **2a.** According to the tick marks in the diagram, $\overline{TW} \cong \overline{WV}$. **2b.** There are no markings that indicate this congruence is true, so it cannot be assumed. **2c.** There are no markings that indicate this congruence is true, so it cannot be assumed. **2d.** There are no markings that indicate this congruence is true, so it cannot be assumed. **2e.** By def. of midpoint, W is the midpt. of $\overline{TV}$.

Investigation Vertical angles are congruent.

Check Understanding **3.** The postulates, definitions, and theorems used in the proof are true for all values, so the size of the angles does not affect the proof or the truth value of the theorem. **4a.** $3x + 35 = 4x = 4(35) = 140$ **4b.** For both of the other angles, $180 - 140 = 40$. **4c.** $140 + 40 = 180$; it checks.

Exercises **1.** Look for angles that form a straight angle with $\angle AOD$: $\angle AOB$ or $\angle DOC$. **2.** $m\angle AOE = 90$, so look for a right angle. Also, adjacent angles share a vertex and a side and have no interior points the same: $\angle EOC$. **3.** The sum of the measures of two suppl. angles is 180: $\angle EOC$. **4.** The sum of the measures of two complementary angles is 90: $\angle DOC$ and, since vertical angles are $\cong$, $\angle AOB$. **5.** Answers may vary. Possible answers: $\angle AOB$ and $\angle DOC$; $\angle BOC$ and $\angle AOD$ **6.** Since $\angle AOE$ and $\angle EOC$ are suppl., and since $m\angle AOE = 90$, $m\angle EOC = 180 - 90 = 90$. **7.** From Exercise 6, $m\angle EOC = 90$, $m\angle EOD = 60$, and $m\angle EOD + m\angle DOC = m\angle EOC$. By substitution, $60 + m\angle DOC = 90$. By the Subtraction Prop. of =, $m\angle DOC = 90 - 60 = 30$. **8.** Since vertical angles are $\cong$, $m\angle BOC = m\angle AOD = m\angle AOE + m\angle EOD = 90 + 60 = 150$. **9.** From Exercise 7, $m\angle DOC = 30$. Since vertical angles are $\cong$, $m\angle AOB = m\angle DOC = 30$. **10.** Yes; the $\angle$ tick marks on the diagram show that the congruence statement is true. **11.** The diagram shows no congruence markings on the angles, so the congruence statement may not be true. **12.** Yes; the diagram shows that the angles share a vertex and side, and that they have no interior points in common, so you can conclude that the angles are adjacent and supplementary from the diagram. **13.** No; there are no markings to tell if the angle measures are exactly the same. Measures on the diagram may be incorrect unless stated. **14.** Yes; since the angles are adjacent and together form a straight angle, you can conclude from the diagram that they are supplementary. **15.** Yes; the tick marks on the segments indicate that the congruence statement is true. **16.** No; there are no markings to tell if the C is in the middle of $\overline{JD}$. Measures on the diagram may be inaccurate unless stated. **17.** Since the angles are formed by two intersecting lines, you can conclude that the angles are vertical angles from the diagram. **18.** No; there are no angle markings to indicate $\angle$ congruence. **19a.** Since they are complementary, the sum of the measures of the angles is 90. **19b.** Since they are complementary, the sum of the measures of the angles is 90. **19c.** Since the sum of the measures of the angles are = to an equal amount, you can substitute the sum of one pair of angles for 90: substitution. **19d.** Subtracting $m\angle 2$ from the right side leaves $m\angle 3$. **20.** Since vertical angles are $\cong$, $3x = 80 - x$. By the Addition Post., $4x = 80$. By the Division Post., $x = 20$. **21.** Since vertical angles are $\cong$, $3x = 75$. By the Division Post., $x = 25$. By def. of suppl. angles, $y + 75 = 180$. By the Subtraction Post., $y = 105$. **22.** Since vertical angles are $\cong$, $x + 90 = 4x$. By the Subtraction Post., $90 = 3x$. By the Division Post., $30 = x$. By the Symmetric Prop., $x = 30$. **23.** From Exercise 20, $x = 20$, so $3x = 3(20) = 60$. Since vertical angles are $\cong$, both angles measure 60. **24.** The diagram shows that each angle of one pair of vertical angles measures 75. Exercise 21 shows that $y = 105$, so each angle in the other pair of vertical angles measures 105. **25.** From Exercise 22, $x = 30$, so $x + 90 = 30 + 90 = 120$. So, each angle of the labeled pair of vertical angles measures 120. **26.** Answers may vary. Sample: A theorem is a proven statement, and a postulate is assumed to be true. **27.** Answers may vary. Sample: scissors, legs of an ironing board **28.** $m\angle 1 + m\angle 2 = 180, m\angle 3 + m\angle 4 = 180$ (Given). Substitute $m\angle 3 + m\angle 4$ for 180 in the first equation: $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$ (Substitution). Subtract $m\angle 2$ from both sides: $m\angle 1 = m\angle 3$ (Subtraction Post. of =). **29.** Since vertical angles are $\cong$, $x + 10 = 4x - 35$; $-3x + 10 = -35$; $-3x = -45$; $x = 15$. So, $x + 10 = 15 + 10 = 25$ and $4x - 35 = 4(15) - 35 = 60 - 35 = 25$. **30.** Since vertical angles are $\cong$, $3x + 8 = 5x - 20$; $-2x + 8 = -20$; $-2x = -28$; $x = 14$. So, $3x + 8 = 3(14) + 8 = 42 + 8 = 50$ and $5x - 20 = 5(14) - 20 = 70 - 20 = 50$. Solve for the other $\angle$: $(5x + 4y) + 50 = 180$; $5x + 4y = 130$; $5(14) + 4y = 130$; $70 + 4y = 130$; $4y = 60$; $y = 15$. $5x + 4y = 5(14) + 4(15) = 70 + 60 = 130$. So, $x = 14$, $y = 15$, and the angle measures are 50, 50, and 130. **31a.** By the def. of a *right angle*, $m\angle X = 90$ and $m\angle Y = 90$. **31b.** By substituting $m\angle Y$ for 90 in $m\angle X = 90$, $m\angle X = m\angle Y$. **32.** Pairs of vertical angles are $\cong$, and all straight angles are $\cong$ to one another. Answers may vary. Sample: $\angle DOB \cong \angle AOC$ and $\angle DOA \cong \angle BOC$, since they are vertical angles. **33.** $\angle EIG \cong \angle FIH$, since all right angles are congruent; $\angle EIF \cong \angle HIG$, since they are complements of the same angle. **34.** $\angle KPJ \cong \angle MPJ$, since they are marked congruent; $\angle KPL \cong \angle MPL$, since they are supplements of congruent angles. **35a.** Congruent angles have = measures: V. **35b.** by def. of suppl. angles: 180 **35c.** Both sides are divided by 2: Division. **35d.** since angles measuring 90 are right angles: right **36.** The other pair of vertical angles are supplements of 72, so their measure is $180 - 72$, or 108. **37a.** B can be any point on the positive y-axis or any point on $y = -\frac{1}{3}x$, such that $x > 0$. Answers may vary. Sample: $B = (0, 5)$ **37b.** If B is a point on the y-axis in part (a), then B must be on $y = -\frac{1}{3}x$, such that $x > 0$, here. Otherwise B is on the y-axis here. Answers may vary. Sample: $B = (3, -1)$ **38.** $\overrightarrow{OF}$ must form a straight angle with one of the existing segments. F must lie on $y = \frac{1}{5}x$, such that $x < 0$, or F must lie on $y = \frac{3}{2}x$, such that $x < 0$. Answers may vary. Sample: $F = (-5, -1)$ **39.** Solve for x: $54 + 4x = 90$; $4x = 36$; $x = 9$. Solve for the $\angle$ measure: $4x = 4(9) = 36$. **40.** Solve for x: $12x - 15 + 3x + 45 = 180$; $15x + 30 = 180$; $15x = 150$; $x = 10$. Solve for the $\angle$ measures: $12x - 15 = 12(10) - 15 = 120 - 15 = 105$. $3x + 45 = 3(10) + 45 = 30 + 45 = 75$ **41.** Solve for x: $3x + 2x = 90$; $5x = 90$; $x = 18$. Solve for the $\angle$ measures: $2x = 2(18) = 36$ and $3x = 3(18) = 54$. **42.** Solve for x: $4x + 5 + 3x + 8 = 90$; $7x + 13 = 90$; $7x = 77$; $x = 11$. Solve for the $\angle$ measures: $4x + 5 = 4(11) + 5 = 44 + 5 = 49$ and $3x + 8 = 3(11) + 8 = 33 + 8 = 41$. **43.** $\angle 1$ and $\angle 3$ are supplements, as are $\angle 2$ and $\angle 4$. Supplements of $\cong$ angles are $\cong$. **44.** Let x be the measure of one of the angles. Since they are $\cong$, x is the measure of the other angle. Since they are supplementary, $x + x = 180$. Solving results in $x = 90$. So, both angles measure 90. **45.** The angles are $\cong$, so both angles measure x. Since they are complementary, their sum is 90. $2x = 90$, so $x = 45$. Both angles measure 45. **46.** Since vertical angles are always $\cong$ for all measures between 0 and 180, it is not possible to establish a set measure. **47.** Solve for x: $m\angle A + m\angle B = 90$; $3x + 12 + 2x - 22 = 90$; $5x - 10 = 90$; $5x = 100$; $x = 20$. Solve for $m\angle A$: $3x + 12 = 3(20) + 12 = 60 + 12 = 72$. Solve for $m\angle B$: $2x - 22 = 2(20) - 22 = 40 - 22 = 18$. **48.** Solve for x: $m\angle A + m\angle B = 180$; $3x + 12 + 2x - 22 = 180$; $5x - 10 = 180$; $5x = 190$; $x = 38$. Solve for $m\angle A$: $3x + 12 = 3(38) + 12 = 114 + 12 = 126$. Solve for $m\angle B$: $2x - 22 = 2(38) - 22 = 76 - 22 = 54$. **49.** Solve for $m\angle A$: $2m\angle B$; $m\angle B = 90 - m\angle A$; $m\angle A = 2(90 - m\angle A)$; $m\angle A = 180 - 2m\angle A$; $3m\angle A = 180$; $m\angle A = 60$. Solve for $m\angle B$: $90 - m\angle A = 90 - 60 = 30$. **50.** Solve for $m\angle A$: $\frac{1}{2}m\angle B$; $m\angle B = 90 - m\angle A$; $m\angle A = \frac{1}{2}(90 - m\angle A)$; $2m\angle A = 90 - m\angle A$; $3m\angle A = 90$; $m\angle A = 30$. Solve for $m\angle B$: $90 - m\angle A = 90 - 30 = 60$. **51.** Solve for $m\angle A$: $2m\angle B$; $m\angle B = 180 - m\angle A$; $m\angle A = 2(180 - m\angle A)$; $m\angle A = 360 - 2m\angle A$; $3m\angle A = 360$; $m\angle A = 120$. Solve for $m\angle B$: $180 - m\angle A = 180 - 120 = 60$. **52.** $m\angle A = \frac{1}{2}(2)(m\angle B)$; $m\angle A = m\angle B$; $m\angle B = 180 - m\angle A$; $m\angle A = 180 - m\angle A$; $2m\angle A = 180$; $m\angle A = 90 = m\angle B$ **53.** Solve for $m\angle C$: $m\angle B = 4m\angle C$; $m\angle B = 180 - m\angle A$, so $4m\angle C = 180 - m\angle A$, or $m\angle A = 180 - 4m\angle C$; $m\angle C = 90 - m\angle A$, so $m\angle A = 90 - m\angle C$; $180 - 4m\angle C = 90 - m\angle C$; $90 = 3m\angle C$; $m\angle C = 30$. Solve for $m\angle B$: $m\angle B = 4m\angle C = 4(30) = 120$. Solve for $m\angle A$: $m\angle C = 90 - m\angle A$; $30 = 90 - m\angle A$; $m\angle A = 60$. **54.** Solve for $m\angle A$: $m\angle B = 90 - m\angle A$; $m\angle C = 180 - m\angle A$; $m\angle B = \frac{1}{6}m\angle C$; $90 - m\angle A = \frac{1}{6}(180 - m\angle A)$; $540 - 6m\angle A = 180 - m\angle A$; $360 = 5m\angle A$; $m\angle A = 72$. Solve for $m\angle B$: $90 - m\angle A = 90 - 72 = 18$. Solve for $m\angle C$: $180 - m\angle A = 180 - 72 = 108$. **55.** By the def. of suppl. angles, $m\angle 1 + m\angle 2 = 180$ and $m\angle 3 + m\angle 4 = 180$. Substitute $m\angle 3 + m\angle 4$ for 180 in the first equation. Then $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$ by the Substitution Prop. It is given that $\angle 2 \cong \angle 4$, so, by def. of $\cong$, $m\angle 2 = m\angle 4$. Then, by the Subtr. Prop. of =, $m\angle 1 = m\angle 3$, or $\angle 1 \cong \angle 3$. **56.** By the def. of compl. angles, $m\angle 1 + m\angle 2 = 90$ and $m\angle 3 + m\angle 4 = 90$. Substitute $m\angle 3 + m\angle 4$ for 90 in the first equation. Then $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$ by the Substitution Prop. It is given that $\angle 2 \cong \angle 4$, so, by def. of $\cong$, $m\angle 2 = m\angle 4$. Then, by the Subtr. Prop.

of $=$, $m\angle 1 = m\angle 3$; or $\angle 1 \cong \angle 3$. **57.** Solve for x: $(y + x) + (y - x) = 180; 2y = 180; y = 90$. Since vertical angles are $\cong$, $2x = y - x; 2x = 90 - x; 3x = 90; x = 30$. The angles measure $2x = 2(30) = 60$, $y + x = 90 + 30 = 120$, and $y - x = 90 - 30 = 60$. **58.** Since vertical angles are congruent, $y = 2x; 2x + x + y + 5 = 180;$ $2x + x + 2x + 5 = 180; 5x + 5 = 180; 5x = 175; x = 35$. The angles measure $y = 2x = 2(35) = 70$, $x + y + 5 = 35 + 70 + 5 = 110$, and $2x = 2(35) = 70$. **59.** Solve for y: $2x + 4y = 180; x + 2y = 90; x = 90 - 2y$. Since vertical angles are congruent, $4y = x + y + 10$. By substitution, $4y = (90 - 2y) + y + 10; 4y = 100 - y; 5y = 100; y = 20$. Solve for x: $2x + 4y = 180$, so $2x + 4(20) = 180;$ $x + 2(20) = 90; x + 40 = 90; x = 50$. The angles measure $4y = 4(20) = 80$, $2x = 2(50) = 100$, and $x + y + 10 = 50 + 20 + 10 = 80$. **60.** Let n represent the measure of the angle. Then $90 - n$ is the measure of its complement. $n = (90 - n) - 8; n = 82 - n; 2n = 82; n = 41$
61. $x + y = 90$ and $x = y$, so $2x = 90; x = 45$.
62. Let $w =$ the measure of the angle. Then $180 - w$ is the measure of its supplement. $w = 3(180 - w); w = 540 - 3w; 4w = 540; w = 135$ **63.** $m\angle 1 = 90 - 70 = 20$ **64.** $m\angle 2 + 90 = 180$, so $m\angle 2 = 90$. **65.** Since vertical angles are congruent, $m\angle 3 = 70$. **66.** $m\angle 4 + 70 = 180$, so $m\angle 4 = 180 - 70 = 110$. **67.** Subtract 7 from both sides: 12. **68.** Reflexive Property has the same information on both sides: $\overline{AB}$. **69.** Substitute 3 for MN in $MN + NP = 15$: $3 + NP = 15$. **70.** The second statement reflects the conclusion and not the hypothesis of the first statement, so a conclusion is not possible.
71. The second statement refers to the hypothesis of the first statement, so $\angle 1$ and $\angle 2$ are congruent. **72.** To write the converse, switch the hypothesis and conclusion: If $y = 25$, then $y + 7 = 32$. To write a biconditional, write if and only if between the hypothesis and conclusion: $y + 7 = 32$ if and only if $y = 25$. **73.** To write the converse, switch the hypothesis and conclusion: If you live south of the equator, then you live in Australia. The converse is false. A counterexample is you live in Chile.
74. To write the converse, switch the hypothesis and conclusion: If $n^2 > 0$, then $n > 0$. The converse is false. A counterexample is any number less than -1, since its square is also greater than 0.

TEST-TAKING STRATEGIES *page 104*

1. Answers may vary. Sample: 1 point because the explanation was not thorough enough; it does not say they have a common vertex or a common side.
2. 0 points because the angles are not adjacent and the explanation is incorrect **3.** Answers may vary. Sample: 2 points because the description includes everything except that they are coplanar; since most problems in this course relate to only one plane, this is acceptable.
4. $\angle ABD$ and $\angle DBC$ are suppl. because they form a linear pair. **5.** By def. of compl., the sum of the two angles must be 90. $m\angle ABD + m\angle BDC = 90;$ $6x + (2x + 2) = 90; 8x + 2 = 90; 8x = 88; x = 11$

CHAPTER REVIEW *pages 105–107*

1. Since the same information is on both sides, it is an example of the Reflexive Property of Congruence.
2. The part that follows *if* is the hypothesis. **3.** By def. of adjacent angles, two coplanar angles with a common side, a common vertex, and no common interior points are adjacent. **4.** The congruence transfers from A to B and from B to C, so it is an example of the Transitive Property of Congruence. **5.** By def. of compl., if the sum of the measures of two angles is 90, then the angles are complementary. **6.** By def. of biconditional, a true conditional and its true converse can be combined as a biconditional. **7.** By def. of vert. angles, two angles whose sides are opposite rays are vertical angles.
8. The *converse* of a conditional switches the hypothesis and the conclusion. **9.** When a statement is reversed, it is an example of the Symmetric Property of Congruence.
10. By def. of suppl., if the sum of the measures of two angles is 180, then the angles are supplementary.
11a. Switch the hypothesis and conclusion: If you are younger than 20, then you are a teenager. **11b.** The conditional is true, but the converse is false. A counterexample for the converse is that you could be younger than 20 at age 10 and not be a teenager.
12a. Switch the hypothesis and conclusion: If an angle has measure greater than 90 and less than 180, then it is obtuse. **12b.** Both the conditional and converse are true by def. of obtuse. **12c.** Write *if and only if* between the hypothesis and conclusion of either the conditional or its converse: An angle is obtuse if and only if its measure is greater than 90 and less than 180. **13a.** Switch the hypothesis and conclusion: If a figure has four sides, then it is a square. **13b.** The conditional is true, but its converse is false. A counterexample for the converse is a trapezoid, which has four sides but is not a square.
14. Write the sentence as an if-then statement: If something is a flower, then it is beautiful. **15.** Rico's definition is not reversible, since a magazine is a counterexample. You can read a magazine, but it is not a book. **16.** First write the sentence as a conditional by making an if-then statement: If a phrase is an oxymoron, then it contains contradictory terms. Next, write a biconditional by separating the hypothesis and conclusion with *if and only if*: A phrase is an oxymoron if and only if it contains contradictory terms. **17.** To write a conditional, write an if-then sentence with one of the two phrases as the hypothesis and the other as the conclusion: If two angles are complementary, then the sum of their measures is 90. To write the converse, switch the hypothesis and conclusion: If the sum of the measures of two angles is 90, then the angles are complementary. **18.** The second sentence refers to the hypothesis of the conditional, so Lucy will become a better player. **19.** The first sentence refers to the hypothesis of the conditional, so lines ℓ and m intersect to form right angles. **20.** The second sentence refers to the hypothesis of the conditional, so the sum of the

measures of $\angle 1$ and $\angle 2$ is 180. **21.** Remove the hypothesis and conclusion that relate and combine what is left in an if-then statement: If Kate studies, then she will graduate. **22.** Remove the hypothesis and conclusion that relate and combine what is left in an if-then statement: If a, then c. **23.** Remove the hypothesis and conclusion that relate and combine what is left in an if-then statement: If the weather is wet, then Nathan can stop at the ice cream shop. **24a.** Two segments are parts of a longer segment: Segment Addition Postulate. **24b.** $x + 3$ is substituted for QR, $2x$ is substituted for RS, and 42 is substituted for QS: Substitution Property. **24c.** Combine like terms: Simplify. **24d.** Subtract 3 from both sides: Subtraction Property of Equality. **24e.** Divide both sides by 3: Division Property of Equality. **25.** Add 3 to both sides, so $5 + 3 = 8$. **26.** Divide both sides by 2, so $AX = BY$. **27.** For the Reflexive Prop., the information is the same on both sides: $m\angle Y$. **28.** The Symmetric Property switches the left and right sides: $RS = XY$. **29.** Since x relates to 5 and 5 relates to y, then x relates to y: y. **30.** By the Distributive Property, $2(4x + 5) = 2(4x) + 2(5) = 8x + 10$: 10. **31.** Factor out a 3 to use the Distributive Property backwards: $p - 2q$. **32.** For the Reflexive Prop., the information is the same on both sides: $\overline{NM}$. **33.** Since vertical angles are congruent, $3y + 20 = 5y - 16$; $-2y + 20 = -16$; $-2y = -36$; $y = 18$. **34.** The angles form a linear pair, so $3x + 31 + 2x - 6 = 180$; $5x + 25 = 180$; $x + 5 = 36$; $x = 31$. **35.** The sum of all three angles is 180, so $90 + (k + 10) + (3k) = 180$; $4k + 100 = 180$; $4k = 80$; $k = 20$. **36.** $m\angle KJD + m\angle DJH = m\angle KJH$ by the Angle Addition Post.; $m\angle KJD = m\angle DJH$ by the markings on the diagram; $\overrightarrow{JD}$ bisects $\angle KJH$ by the definition of angle bisector. **37.** $AB = CD$ by the markings on the diagram; $AC = BD$ by the Addition Property of $=$ and by the Segment Addition Post. **38.** $\angle 1 \cong \angle 4$ by the markings on the diagram; $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$ because vertical angles are $\cong$; $\angle 2 \cong \angle 3$ by the Trans. Prop. of $\cong$.

CHAPTER TEST *page 108*

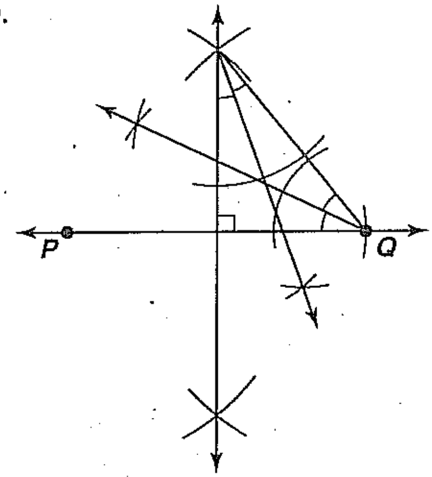
1. The hypothesis follows *If* and the conclusion follows *then*. Hypothesis: $x + 9 = 11$. Conclusion: $x = 2$. **2.** Write the sentence in if-then form: If something is a baby, then it is cute. **3.** Counterexample: Two 45° angles are complementary and congruent. **4a.** Switch the hypothesis and conclusion: If a figure has two right angles, then it is a rectangle. **4b.** The converse is false. A counterexample is a trapezoid with two right angles, or a pentagon in the shape of a "house." **5a.** Switch the hypothesis and conclusion: If two lines lie in the same plane, then they intersect. **5b.** The converse is false. A counterexample is parallel lines. They lie in the same plane but do not intersect. **6a.** Switch the hypothesis and conclusion: If it is not summer, then it is snowing in South Carolina. **6b.** The converse is false. It could be spring or fall or even winter and not be snowing anywhere in South Carolina. **7.** A good definition is precise, reversible, and uses terms that have been previously defined or are commonly accepted: The definition is not reversible. A pen is a writing instrument, but it is not a pencil. **8.** A good definition is precise, reversible, and uses terms that have been previously defined or are commonly accepted: The definition is not precise enough to be true. Two nonadjacent angles may be complementary. Three angles may form a right angle, but they are not complementary. **9.** A good definition is precise, reversible, and uses terms that have been previously defined or are commonly accepted: The definition is not precise. Any two angles can be congruent. **10.** Transitive Prop. of $=$ or Substitution Prop. **11.** Subtract $m\angle 2$ from both sides: Subtraction Prop. of $=$. **12.** Both sides are the same: Reflexive Prop. of $\cong$. **13.** Both sides are mult. by 2: Mult. Prop. of $=$. **14.** The statements are reversed: Symmetric Prop. of $\cong$. **15a.** The sum of the 2 congruent angles is 90, so they are both 45. $m\angle CDM = 90 + 45 = 135$ **15b.** Since vertical angles are congruent, $m\angle KDB = 90$. The sum of the 2 congruent angles is 90, so they are both 45. $m\angle KDM = 90 + 45 = 135$ **15c.** $m\angle JDK = 180$, because it is a straight angle. **15d.** The sum of the 2 congruent angles is 90, so they are both 45. $m\angle JDM = 45$ **15e.** $m\angle CDB = 180$, because it is a straight angle. **15f.** $m\angle CDK = 180 - 90 = 90$ **16.** Let n be the measure of its supplement. Then $n + 2z = 180$, by def. of suppl. So, $n = 180 - 2z$. **17.** If x is the measure of an angle, then $90 - x$ is the measure of its complement. $x = 52 + 90 - x$; $2x = 142$; $x = 71$ **18.** $m\angle VNM = 62$ because vertical angles are congruent; $m\angle LNV = m\angle PNM = 118$ because both angles are suppl. to $\angle LNP$. **19.** $\angle BCE \cong \angle DCF$ by the markings on the diagram; $\angle BCF \cong \angle ECD$ by the $\angle$ Add. Post. **20.** Write one of the two phrases as the hypothesis and the other as the conclusion of an if-then statement, then write its converse: If a fish is a bluegill, then it is a bluish, freshwater sunfish. If a fish is a bluish freshwater sunfish, then it is a bluegill. **21.** By the Congruent Complements Thm., the angles are congruent. **22.** The second statement refers to the conclusion of the conditional, so it is not possible to make any conclusion. **23.** Since the first sentence refers to the hypothesis of the conditional, James must graduate from college. **24.** By the Law of Syllogism, $p \rightarrow r$ is true. **25.** By the Law of Detachment, q is true. **26.** Since q refers to the conclusion of the conditional, it is not possible to make a conclusion from the information. **27a.** Complementary angles total 90: 90. **27b.** Angle Addition Post. **27c.** Substitution **27d.** Def. of right angle

STANDARDIZED TEST PREP *page 109*

1. Switch the hypothesis and conclusion: If a strawberry is ripe, then it is red. The answer is choice B. **2.** By Post. 1-3, the intersection of two planes is a line. The answer is choice G. **3.** Both sides are divided by 4, so the Division Prop. of $=$ is used. The answer is choice A. **4.** For choice

F, $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} =$ $\sqrt{(0 - 0)^2 + (-7 - 0)^2} = \sqrt{(0)^2 + (-7)^2} =$ $\sqrt{0 + 49} = \sqrt{49} = 7$. For choice G, $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} =$ $\sqrt{(-3 - 0)^2 + (8 - 0)^2} = \sqrt{(-3)^2 + (8)^2} =$ $\sqrt{9 + 64} = \sqrt{73}$. For choice H, $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} =$ $\sqrt{(-4 - 0)^2 + (-3 - 0)^2} = \sqrt{(-4)^2 + (-3)^2} =$ $\sqrt{16 + 9} = \sqrt{25}$. For I, $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} =$ $\sqrt{(5 - 0)^2 + (1 - 0)^2} = \sqrt{(5)^2 + (1)^2} = \sqrt{25 + 1} =$ $\sqrt{26}$. Since $\sqrt{73}$ is the greatest, the answer is choice G. **5.** $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} =$ $\sqrt{(1 - (-3))^2 + (7 - (-1))^2} =$ $\sqrt{(1 + 3)^2 + (7 + 1)^2} = \sqrt{(4)^2 + (8)^2} = \sqrt{16 + 64} =$ $\sqrt{80}$. The answer is choice C. **6.** Every even term is negative. The number is the square of the number of the term. The next term is positive and is 5^2, or 25. The answer is choice H. **7.** $x = (90 - x) - 78; x = 12 - x;$ $2x = 12; x = 6$. The answer is choice A. **8.** $m\angle A =$ $m\angle B$ and $m\angle A + m\angle B = 180; 2m\angle B = 180; m\angle B =$ 90. The answer is choice G.
9. $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0 + (-4)}{2}, \frac{-4 + 7}{2}\right) =$ $\left(\frac{-4}{2}, \frac{3}{2}\right) = \left(-2, \frac{3}{2}\right)$. The answer is choice C.
10. $n = 2(180 - n) - 12; n = 360 - 2n - 12; 3n = 348;$ $n = 116$. The answer is choice I. **11.** $VT = \frac{1}{4}(12x) = 3x$. The quantities are =. The answer is choice C. **12.** $m\angle A$ could be greater than, equal to, or less than $m\angle B$. The answer is choice D. **13.** By def. of complement, $m\angle A + m\angle B = 90$, which is less than 180. The answer is choice B. **14.** Since $m\angle A + m\angle B = 90, m\angle B < 90$. The answer is choice B. **15.** $m\angle A + m\angle B = 90;$ $m\angle B = 90 - m\angle A$; suppl. of $\angle B =$ $180 - (90 - m\angle A) = 90 + m\angle A$. Since $m\angle A < 90 + m\angle A$, the answer is choice B.
16. $A = \pi r^2; 10\pi = \pi r^2; 10 = r^2; r = \sqrt{10}$, so $d = 2r = 2\sqrt{10} \approx 6.32$. **17.** $n = \frac{1}{3}(180 - n); 3n = 180 - n;$ $4n = 180; n = 45$ **18** [2] **a.** $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) =$ $\left(\frac{3 + 9}{2}, \frac{6 + (-2)}{2}\right) = \left(\frac{12}{2}, \frac{4}{2}\right) = (6, 2)$
b. $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} =$ $\sqrt{(3 - 9)^2 + (6 - (-2))^2} = \sqrt{(-6)^2 + (6 + 2)^2} =$ $\sqrt{(-6)^2 + (8)^2} = \sqrt{36 + 64} = \sqrt{100} = 10$ [1] one answer correct

19.



[4] Correct construction: Draw $\overleftrightarrow{PQ}$. With the compass point on P and open to more than half PQ, swing an arc on each side of the line. Keep the same setting. Swing arcs from Q to intersect the first two arcs. Draw a line through the intersections of the arcs. Draw a segment from Q to the line just drawn. This forms a right triangle. To bisect each angle, place the compass point on the vertex of the angle and swing an arc to intersect both sides of the angle. With the compass point on the intersection of the previous arc and the side of the angle, swing an arc in the interior of the angle. Keep the same setting and swing an arc to intersect that previous arc with the compass point on the intersection of the arc with the other side of the angle. Draw a ray through the angle vertex and the intersection of the last two arcs. [3] correct right triangle, but only one correct angle bisector [2] correct right triangle [1] correctly started triangle

REAL-WORLD SNAPSHOTS *pages 110–111*

Activity 1 If you want to divide the cake into two = regions, you can divide the perimeter by 2. To divide it into 4 = regions, you can divide the perimeter by 4. To divide the cake into 8 = regions, you can divide the perimeter by 8. This works if the cake is square or rectangular. Thus, find the center of the cake, the perimeter of the cake, divide the cake into n regions for n = pieces, and cut from the center to the edge of each section.

Activity 2 **a.** The pizza slice is about $\frac{1}{3}$ of half the pizza, which is $\frac{1}{3} \cdot \frac{1}{2}$, or $\frac{1}{6}$ of the pizza. The entire pizza is $\pi\left(\frac{12}{2}\right)^2 = 36\pi$, or about 3(36), which is 108 in.^{2}. $\frac{1}{6}$ of $108 = 18$. So, the area of the piece is about 18 in.^{2}. **b.** The area of the entire pizza is 36π in.^{2}. Answers may vary. Sample: Cut circular pieces from the center of the pizza. For 2 = pieces, $\pi r^2 = \frac{36\pi}{2} = 18\pi$, so cut out 1 circle whose diameter is $2\sqrt{18}$, or about 8.5 in. For 4 = pieces, $\pi r^2 = \frac{36\pi}{4} = 9\pi$, so cut out 3 circles whose diameter is $2\sqrt{9}$, or 6 in. For 6 = pieces, $\pi r^2 = \frac{36\pi}{6} = 6\pi$, so cut out 5 circles whose diameter is $2\sqrt{6}$, or about 4.9 in. For 8 = pieces, $\pi r^2 = \frac{36\pi}{8} = 4.5\pi$, so cut out 7 circles whose diameter is $2\sqrt{4.5}$, or about 4.25 in.