Tools of Geometry

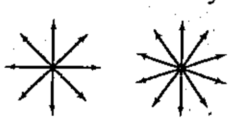
DIAGNOSING READINESS page 2

1. $3^2 = (3)(3) = 9$ **2.** $4^2 = (4)(4) = 16$ **3.** $11^2 = (11)(11) = 121$ **4.** $2 \cdot 7.5 + 2 \cdot 11 = 15 + 22 = 37$
5. $\pi(5)^2 \approx 3.14(5)(5)\ 3.14(25) = 78.5$
6. $\sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = \sqrt{13^2} = 13$
7. $\frac{a+b}{2} = \frac{4-2}{2} = \frac{2}{2} = 1$ **8.** $\frac{a-7}{3-b} = \frac{4-7}{3-(-2)} = \frac{-3}{5} = -\frac{3}{5}$
9. $\sqrt{(7-a)^2 + (2-b)^2} = \sqrt{(7-4)^2 + (2-(-2))^2} = \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5$ **10.** $|-8| = 8$ **11.** $|2-6| = |-4| = 4$ **12.** $|-5-(-8)| = |-5+8| = |3| = 3$
13. $2x + 7 = 13 \rightarrow 2x = 6 \rightarrow x = 3$ **14.** $5x - 12 = 2x + 6 \rightarrow 3x - 12 = 6 \rightarrow 3x = 18 \rightarrow x = 6$
15. $2(x+3) - 1 = 7x \rightarrow 2x + 6 - 1 = 7x \rightarrow 2x + 5 = 7x \rightarrow 5 = 5x \rightarrow x = 1$

1-1 Patterns and Inductive Reasoning pages 4–9

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*
1. 2, 4, 6, 8, 10, . . . **2.** 1, 3, 5, 7, 9, . . . **3.** $1^2 = 1, 2^2 = 4, 3^2 = 9, 4^2 = 16, 5^2 = 25, 6^2 = 36, 7^2 = 49, 8^2 = 64, 9^2 = 81, 10^2 = 100$ **4.** It is odd.

Check Understanding **1a.** The difference between the first term and the second is 2 − 1, or 1; between the second and the third is 4 − 2, or 2; between the third and the fourth is 7 − 4, or 3; between the fourth and the fifth is 11 − 7, or 4; and so on. The difference increases by 1 for each successive pair. So, if n is the kth term, then $n_{k+1} = n_k + k$. Thus, the eighth and ninth terms are 22 + 7, or 29, and 29 + 8, or 37, respectively.
1b. The days of the week are named in order, so the next two are Thursday and Friday.

1c.  Answers may vary. Sample: Each figure is the same as the previous figure with the addition of one more line passing through a point on the original line.

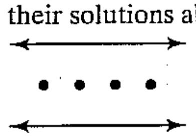
2. From Example 2, the sum of the first n odd numbers is n^2, so the sum of the first 35 odd numbers is 35^2, or 1225. The calculator verifies the conjecture. **3.** Answers may vary. Samples: A true conjecture is that the product of 5 and any odd number is odd. A false conjecture is that the product of 5 and any number is odd. **4a.** Since sales are decreasing at about 3 sales each month, about 45 − 3, or 42 skateboards will sell in June, and 42 − 3, or 39 skateboards will sell in July. **4b.** Not confident; December may be a month of high sales. I wouldn't use a graph to predict skateboard sales in December because it is too far into the future.

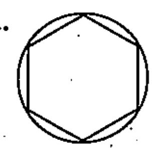
Exercises **1.** Each term is twice the previous term. The next two terms are 40 × 2 = 80 and 80 × 2 = 160.
2. To get the next term, multiply its previous term by 10 and add 3. The next two terms are 3333 × 10 + 3 = 33,333 and 33,333 × 10 + 3 = 333,333. **3.** The terms come in pairs, a positive number and its opposite, and each positive number is one more than the previous positive number. The next two terms are −3 and 4.
4. Each term is half the previous term. The next two terms are $\frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$ and $\frac{1}{16} \times \frac{1}{2} = \frac{1}{32}$. **5.** Each term is 3 less than the previous term. The next two terms are 6 − 3 = 3 and 3 − 3 = 0. **6.** Each term is a third of the previous term. The next two terms are $3 \times \frac{1}{3} = 1$ and $1 \times \frac{1}{3} = \frac{1}{3}$. **7.** Each term is the first letter of the numbers *one, two, three, . . .* The ninth and tenth numbers are *nine* and *ten*, so the ninth and tenth terms are N and T.
8. Each term is the first letter of the months *January, February, March, . . .* The sixth and seventh months are *June* and *July*, so the sixth and seventh terms are J and J.
9. The pattern is multiply by 2, then multiply by 3, then multiply by 4, then multiply by 5, . . . The next two terms are 120 × 6 = 720 and 720 × 7 = 5040. **10.** Each term is twice the previous term. So, the next two terms are 32 × 2 = 64 and 64 × 2 = 128. **11.** The pattern for the nth term is $\frac{1}{n^2}$, so the sixth and seventh terms are $\frac{1}{6^2} = \frac{1}{36}$ and $\frac{1}{7^2} = \frac{1}{49}$, respectively. **12.** The pattern for the nth term is $\frac{1}{n}$, so the fifth and sixth terms are $\frac{1}{5}$ and $\frac{1}{6}$, respectively. **13.** These are the first names of the first four U.S. presidents: George Washington, John Adams, Thomas Jefferson, and James Madison. The next two presidents were James Monroe and John Quincy Adams, so the next two names in the sequence are James and John. **14.** These are the names of the first four U.S. presidents' wives: Martha Washington, Abigail Adams, Martha Jefferson, and Dolley Madison. The next two presidents' wives are Elizabeth Monroe and Louisa Adams, so the next two names in the sequence are Elizabeth and Louisa. **15.** The first names of the people pictured on U.S. currency are George Washington ($1), Thomas Jefferson ($2), Abe Lincoln ($5), Alexander Hamilton ($10), Andrew Jackson ($20), and Ulysses Grant ($50). So the next two names after Alexander are Andrew and Ulysses. **16.** The first four names of the Zodiac in order are Aquarius, Pisces, Aries, and Taurus. The next two are Gemini and Cancer.

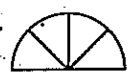
17.  The shaded region begins on the left and moves around the outermost regions of the figure in a clockwise direction.

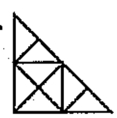
18.  Each term adds a square whose vertices connect the midpoints of the sides of the previous new square.

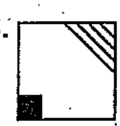
19. From the table, the sum of the first n even numbers is $n(n + 1)$. So, the sum of the first 6 even numbers is $6(6 + 1) = 6(7) = 42$. **20.** From the table, the sum of the first n even numbers is $n(n + 1)$. So, the sum of the first 30 even numbers is $30(30 + 1) = 30(31) = 930$. **21.** From the table, the sum of the first n even numbers is $n(n + 1)$. So, the sum of the first 100 even numbers is $100(100 + 1) = 100(101) = 10{,}100$. **22.** From Example 2, the sum of the first n odd numbers is n^2, so the sum of the first 100 odd numbers is 100^2, or 10,000. **23.** Each product is 9 identical digits, the repeated digit increasing by 1 for each subsequent product. So, a conjecture is that the next term is 555,555,555. The product checks with a calculator having a 10-digit display for the product of 12345679 and 45. **24.** Reading each product from left to right, the nth term ascends to n and then descends to 1. So, the fifth product should be 123,454,321. **25.** Answers may vary. Sample: Add positive and negative integers such as $8 + (-5) = 3$. Since $3 < 8$, the statement is false. **26.** Answers may vary. Sample: Multiply any number by a fraction between 0 and 1, such as $\frac{1}{3} \cdot 6 = 2$. Since $\frac{1}{3} < 2$, the statement is false. **27.** Answers may vary. Sample: Subtract two negative integers such as $-6 - (-4) = -2$. Since $-2 > -6$ and $-2 > -4$, the statement is false.
28. Answers may vary. Sample: Divide one fraction by a fraction that is less than it, such as $\frac{1}{2} \div \frac{1}{3} = \frac{1}{2} \cdot \frac{3}{1} = \frac{3}{2}$. Since $\frac{3}{2}$ is improper, the statement is false. **29.** The table shows that the temperature increases 10° for each increase of 5 chirps in a 14-second period. Since 20 chirps is 5 more than 15, the temperature should be $65 + 10$, or 75°F. **30.** The sequence is 14, 19, 25, ... He increased by 5 pushups in the second month and by 6 pushups in the third month. So, in the fourth month he should be able to do $25 + 7$, or 32 pushups; and in the fifth month he should do $32 + 8$, or 40 pushups. Answers may vary. Sample: Not very confident; assuming Dino does not want to increase his workout time substantially, he will reach his limit to the number of pushups he can do in his allotted time for exercises. **31.** The second term is the previous term increased by 2, the third term is the previous term increased by 4, the fourth is the previous term increased by 6, and the fifth is the previous term increased by 8. So, the sixth term is $21 + 10 = 31$, and the seventh term is $31 + 12 = 43$.
32. Odd terms are increased by 1 to get the subsequent even term, and even terms are increased by 3 to get the subsequent odd term. Since 9 is the fifth term and 5 is odd, the next term is $9 + 1 = 10$. Since 10 is the sixth term and 6 is even, the next term is $10 + 3 = 13$.
33. Each subsequent term is multiplied by 0.1, so the next term is $0.001(0.1) = 0.0001$, and the following term is $0.0001(0.1) = 0.00001$. **34.** Answers may vary. Sample: The odd terms are 2, 7, 22, 67, and they increase by 5, 15, and 45. Each subsequent increase of odd terms is multiplied by 3, so the next odd term, which is the ninth term, is $67 + 45(3) = 67 + 135 = 202$. The even terms are each one less than the subsequent odd term, so the eighth term is $202 - 1 = 201$. The eighth and ninth terms are 201 and 202. **35.** The pattern shows increases by 2, 4, 6, 8, and 16. Each increase is twice the previous increase, so the next term is $31 + 16(2) = 31 + 32 = 63$, and the one following that is $63 + 32(2) = 63 + 64 = 127$.
36. The pattern shows increases by $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$. Each increase is half the previous increase, so the next term is $\frac{15}{16} + \frac{1}{16} \cdot \frac{1}{2} = \frac{15}{16} + \frac{1}{32} = \frac{30}{32} + \frac{1}{32} = \frac{31}{32}$, and the one following that is $\frac{31}{32} + \frac{1}{32} \cdot \frac{1}{2} = \frac{31}{32} + \frac{1}{64} = \frac{62}{64} + \frac{1}{64} = \frac{63}{64}$.
37. The first letters of the planets in order from the sun are Mercury, Venus, Earth, Mars, Jupiter, and Saturn. So, the next letters following M, V, E, and M are J and S.
38. Listing the states in alphabetical order, the pairs of letters are the postal abbreviations for them. The states are Alabama (AL), Alaska (AK), Arizona (AZ), and Arkansas (AR). The next two states are California whose postal abbreviation is CA and Colorado whose postal abbreviation is CO. **39.** The first 6 chemical elements in order of atomic number (with the symbol) are Hydrogen (H), Helium (He), Lithium (Li), Beryllium (Be), Boron (B), and Carbon (C). So, the next two symbols after H, He, Li, and Be are B and C.
40. Answers may vary. Sample: In Exercise 31, each number increases by increasing multiples of 2. In Exercise 33, to get the next term, divide by 10. For others see their solutions above.

41.  You would get a third line halfway between and parallel to the first two lines.

42.  The number of sides of the polygon inside the figure increases by 1. So, the fourth should have 6 sides.

43.  Each subsequent semicircle has one more radius dividing it into equal portions. So, the fourth should have 3 radii.

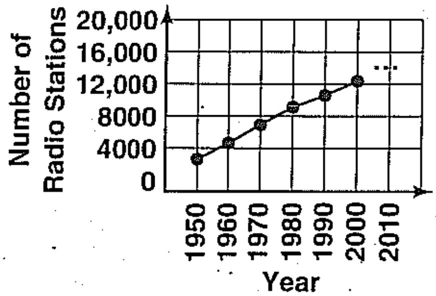
44.  In each right triangle, a segment is added from the right angle to the middle of the opposite side.

45.  The small green square moves clockwise about the large square, and the lines in the opposite corner also move clockwise and increase in number by 1.

46. For 1 triangle, the perimeter is 3 cm. For 2 triangles, the perimeter is 4 cm. For 3 triangles, the perimeter is 5. For 4 triangles, the perimeter is 6. The perimeter is always 2 cm more than the number of triangles in the figure, so 100 triangles arranged in a row have a perimeter of 102 cm. **47a.** Answers may vary. Sample: Women may soon outrun men in running competitions.

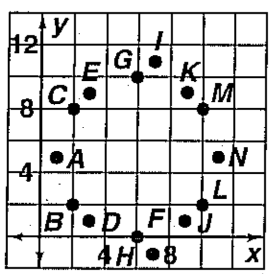
47b. Answers may vary. Sample: The conclusion was based on continuing the trend shown in past records. **47c.** Answers may vary. Sample: The conclusions are based on fairly recent records for women and those rates of improvement may not continue. The conclusion about the marathon is most suspect because records date only from 1955.

48a.



48b. The graph shows a reasonably steady increase every 10 years. If the line on the graph were extended, it would go to about 14,000 radio stations in 2010. **48c.** Answers may vary. Sample: Confident; the pattern has held for several decades. **49.** Answers may vary. Sample: Start with 1 and 3 for each. For one pattern, multiply each term by 3 to get the next term: 1, 3, 9, 27, 81, . . . For the other pattern, add 2 to each term to get the next term: 1, 3, 5, 7, 9, . . . **50.** His conjecture is probably false because most people's growth slows by 18 until they stop growing somewhere between 18 and 22 years.

51a.



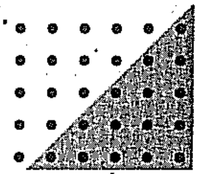
51b. The points that do not fit the pattern of the circle formed by the other 12 points are I and H. **51c.** If you continued graphing points that fit the pattern, you would get a circle whose center is at (6, 5) and whose radius is 5 units. **52.** Each term is the sum of the previous two terms. The eighth term is $8 + 13 = 21$, the ninth is $13 + 21 = 34$, and the tenth is $21 + 34 = 55$. **53a.** Of the years 1984, 1988, 1992, 1996, and 2000, each term is divisible by 4. A conjecture is that a leap year is divisible by 4. **53b.** 2020, 2100, and 2400 are each divisible by 4, so they are likely to be leap years. **53c.** Leap years are divisible by 4, so the conjecture for part (a) is correct. However, not all years divisible by 4 are leap years. Years ending in 00 are leap years only if they are divisible by 400. So, 2100 will not be a leap year, but 2400 will be. **54.** Answers may vary. Sample: Let x represent the sum of the first 100 numbers.

$$\begin{array}{rcl} 100 + 99 + 98 + \dots + 3 + 2 + 1 & = & x \\ \underline{1 + 2 + 3 + \dots + 98 + 99 + 100} & = & \underline{x} \\ 101 + 101 + 101 + \dots + 101 + 101 + 101 & = & 2x \end{array}$$

The grand sum is 100 sums of 101, which is also $2x$, so $100(101) = 2x$. Solving for x results in $\frac{100(101)}{2}$, or 5050. Similarly, the sum of the first n numbers is $\frac{n(n+1)}{2}$.

55a. Each term adds the number of its term to the term preceding it: $1 + 0 = 1$, $2 + 1 = 3$, $3 + 3 = 6$, $4 + 6 = 10$, $5 + 10 = 15$, $6 + 15 = 21$. **55b.** For $n = 1$, $\frac{n^2+n}{2} = \frac{1^2+1}{2} = \frac{2}{2} = 1$; for $n = 2$, $\frac{n^2+n}{2} = \frac{2^2+2}{2} = \frac{4+2}{2} = \frac{6}{2} = 3$; for $n = 3$, $\frac{n^2+n}{2} = \frac{3^2+3}{2} = \frac{9+3}{2} = \frac{12}{2} = 6$; for $n = 4$, $\frac{n^2+n}{2} = \frac{4^2+4}{2} = \frac{16+4}{2} = \frac{20}{2} = 10$; for $n = 5$, $\frac{n^2+n}{2} = \frac{5^2+5}{2} = \frac{25+5}{2} = \frac{30}{2} = 15$; for $n = 6$, $\frac{n^2+n}{2} = \frac{6^2+6}{2} = \frac{36+6}{2} = \frac{42}{2} = 21$. **55c.** The diagram shows the product of n and $(n + 1)$ divided by 2 for $n = 3$. The result is 6.

55d.

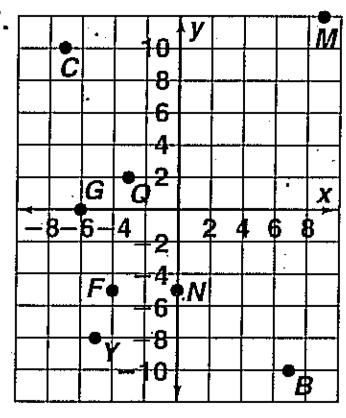


The result is 15.

56. The tens digit of the least of the addends matches the hundreds digit of their sum. Since 9 is the tens digit of the least addend from 91 to 100, their sum is 955. The answer is choice B. **57.** A counterexample for choice I is $3 + 9 = 12$, showing that the sum of two odd numbers can be even. The answer is choice I.

58. [2] **a.** The dots are in the pattern 1, 4, 8, 16, . . . These numbers can be represented by the squares $1^2, 2^2, 3^2, 4^2$. So, the next three numbers are 5^2, or 25; 6^2, or 36; and 7^2, or 49. The number of dots in figures E, F, and G, are 25, 36, and 49, respectively. **b.** Since the first term is 1^2, the second term is 2^2, the third term is 3^2, and so on, the nth term is n^2. [1] one part correct **59.** [4] **a.** The pattern is of equations showing 11 multiplied by a 3-digit factor on the left side and the product on the right side. The 3-digit factor starts at 101 and increases by 10 with each subsequent equation. The product starts at 1111 and increases by 110 with each subsequent equation. The product is a 4-digit number whose middle two terms are each one greater than the middle term of the 3-digit factor. The next two equations are $(151)(11) = 1661$ and $(161)(11) = 1771$. **b.** Based on the pattern described in part (a), $(181)(11) = 1991$. A calculator or longhand multiplication verifies the conjecture. **c.** A calculator verifies that the pattern does not continue after (181)(11). The next pair of factors in the pattern should be (191)(11) and their product is not a 4-digit number, since 1 greater than 9 is the two-digit number 10.

60–67.

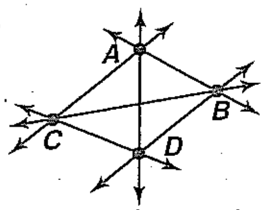


The first number in a coordinate pair tells how many units to move left or right from the origin, and the second tells how many units to move up or down.

68. The fourth quadrant is the lower right quadrant. Only point B is in Quadrant IV. **69.** The y-axis is the vertical axis. Only point N is on the y-axis. **70.** The x-axis is the horizontal axis. Only point G is on the x-axis.

1-2 Points, Lines, and Planes *pages 10–16*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*
1. (1, 6) **2.** (3, 2) **3.** (5, 10) **4.**



Investigation 1. Each line drawn contains exactly 2 of the 3 points, forming a triangle. At most, 3 lines can be drawn. **2.** The figure drawn using the maximum number of lines forms a 4-sided figure with its two diagonals. At most, 6 lines can be drawn. **3.** For 5 points, the figure formed is a 5-sided figure with its 5 diagonals, so 10 lines can be drawn. For 6 points, the figure formed is a 6-sided figure with its 9 diagonals, so 15 lines can be drawn.
4. The sequence is 3, 6, 10, 15, ... and follows the pattern increase by 3, increase by 4, increase by 5, ... The table continues this pattern.

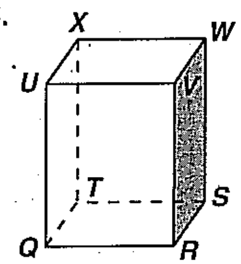
Number of points	3	4	5	6
Number of lines	3	6	10	15

If the number of lines is represented by ℓ and the number of points is represented by p, then $\ell = \frac{p(p-1)}{2}$.
When $p = 10$, then $\ell = \frac{p(p-1)}{2} = \frac{10(10-1)}{2} = \frac{10(9)}{2} = 5(9) = 45$. So, for 10 points in the constellation, there are 45 possible lines.

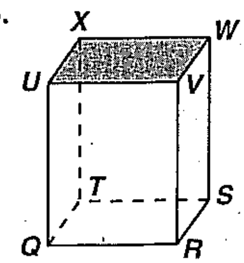
Check Understanding 1a. Since a single straight line cannot pass through all 3 points, they are not collinear. **1b.** A line can be named by any two points on it. Answers may vary. Samples: $\overleftrightarrow{AB}, \overleftrightarrow{AC}, \overleftrightarrow{CA}, \overleftrightarrow{BA}, \overleftrightarrow{BC}, \overleftrightarrow{CB}$ **1c.** Lines, rays, and segments need different symbols to distinguish them from one another. Since a line continues forever in both directions, two arrowheads are used to show that the figure is a line that extends in opposite directions without end. **2.** A plane can be named by 3 or more letters representing points contained in the plane. Answers may vary. Sample: *HEF*, *HEFG*, *FGH* **3.** Any two planes containing the points B and F also contain $\overleftrightarrow{BF}$. Planes *ABF* and *CBF* contain the line. **4a.** Plane *ABC* is the bottom plane of the box. The other labeled point in that plane is D. **4b.** Plane *EHC* is the green plane that appears inside the box. The other labeled point in that plane is B.

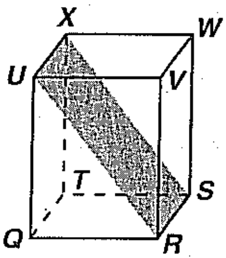
Exercises 1. no, because you cannot draw a single line that contains A, D, and E **2.** Yes; line n contains B, C, and D. **3.** Yes; line n contains B, C, and F. **4.** Yes; line m contains A, E, and C. **5.** Yes; line n contains F, B, and D. **6.** no, because you cannot draw a single line that contains F, A, and E **7.** no, because you cannot draw a single line that contains G, F, and C **8.** Yes; line m contains A, G, and C. **9.** A line can be named by any 2 points contained by the line. Answers may vary. Sample: $\overleftrightarrow{AE}, \overleftrightarrow{EC}, \overleftrightarrow{GA}$ **10.** A line can be named by any 2 points contained by the line. Answers may vary. Sample: $\overleftrightarrow{BF}, \overleftrightarrow{CD}, \overleftrightarrow{BF}$ **11.** Name the plane by 3 or more points contained in the plane. Answers may vary. Sample: *ABCD* **12.** Name the plane by 3 or more points contained in the plane. Answers may vary. Sample: *EFGH* **13.** Name the plane by 3 or more points contained in the plane. Answers may vary. Sample: *ABHF* **14.** Name the plane by 3 or more points contained in the plane. Answers may vary. Sample: *EDCG* **15.** Name the plane by 3 or more points contained in the plane. Answers may vary. Sample: *EFAD* **16.** Name the plane by 3 or more points contained in the plane. Answers may vary. Sample: *BCGH* **17.** The common points to both planes are on $\overleftrightarrow{RS}$. **18.** The 4 labeled points in *UXV* are U, X, V, and W. The 4 labeled points in *WVS* are W, V, S, and R. The common points to both planes are on $\overleftrightarrow{VW}$. **19.** The 4 labeled points in *XWV* are X, W, V, and U. The 4 labeled points in *UVR* are U, V, R, and Q. The common points to both planes are on $\overleftrightarrow{UV}$. **20.** The 4 labeled points in *TXW* are T, X, W, and S. The 4 labeled points in *TQU* are T, Q, U, and X. The common points to both planes are on $\overleftrightarrow{XT}$. **21.** The 6 planes are *UXWV*, *VWSR*, *SRQT*, *QTXU*, *XWST*, and *UVRQ*. If a plane contains 2 points on a line, it contains the line. The planes that contain both Q and U are *QTXU* and *UVRQ*. Other names for these planes are *QUX* and *QUV*, respectively. **22.** The 6 planes are *UXWV*, *VWSR*, *SRQT*, *QTXU*, *XWST*, and *UVRQ*. If a plane contains 2 points on a line, it contains the line. The planes that contain both T and S are *XWST* and *SRQT*. Other names for these planes are *XTS* and *QTS*, respectively. **23.** The 6 planes are *UXWV*, *VWSR*, *SRQT*, *QTXU*, *XWST*, and *UVRQ*. If a plane contains 2 points on a line, it contains the line. The planes that contain both X and T are *QTXU* and *XWST*. Other names for these planes are *UXT* and *WXT*, respectively. **24.** The 6 planes are *UXWV*, *VWSR*, *SRQT*, *QTXU*, *XWST*, and *UVRQ*. If a plane contains 2 points on a line, it contains the line. The planes that contain both V and W are *UXWV* and *VWSR*. Other names for these planes are *UVW* and *RVW*, respectively.

25.

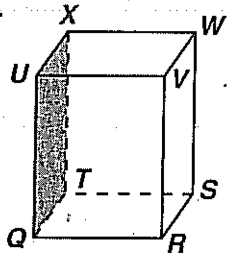


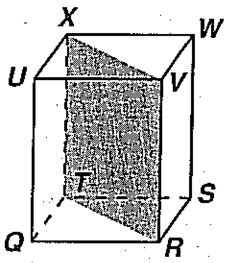
26.



27. 

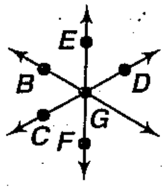
Since the 3 points are not on one face of the figure, the plane cuts across the middle of the figure.

28. 

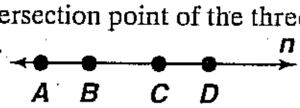
29. 

Since the 3 points are not on one face of the figure, the plane cuts across the middle of the figure.

30. R, V, and W are on the right face of the figure. The fourth point on that face is S. **31.** U, V, and W are on the top face of the figure. The fourth point on that face is X. **32.** U, X, and S are on a plane that cuts across the middle of the figure. The fourth point on that plane is R. **33.** T, U, and X are on the left face of the figure. The fourth point on that face is Q. **34.** T, V, and R are on a plane that cuts across the middle of the figure. The fourth point on that plane is X. **35.** No; V, W, and S are on the right side of the figure, while Q is on the left side. **36.** Yes; T, V, and S are on a plane that cuts across the middle of the figure. U is on that plane. **37.** No; X, V, and R are on a plane that cuts across the middle of the figure. W is not on that plane. **38.** Z, S, Y, and C are on the yellow plane, so they are coplanar. **39.** S, U, V, and Y are on the pink plane, so they are coplanar. **40.** Z is on the yellow plane and U is on the pink plane, so they are noncoplanar. **41.** X, S, V, and U are on the pink plane, so they are coplanar. **42.** Z is on the yellow plane and V is on the pink plane, so they are noncoplanar. **43.** C is on the yellow plane and V is on the pink plane, so they are noncoplanar. **44.** Answers may vary. Sample: The plane of the ceiling and the plane of a wall intersect in a line. Also, any two adjacent walls intersect in a line. **45.** Through any three noncollinear points there is exactly one plane. The ends of the legs of the tripod represent three noncollinear points, so they rest in one plane. Therefore, the tripod won't wobble. **46.** Post. 1-1: Through any two points there is exactly one line.

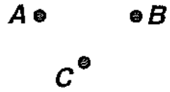
47. 

One of the named points must be the intersection point of the three lines.

48. 

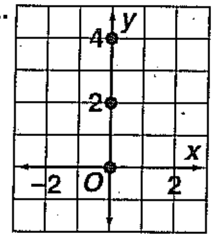
Any 4 points on the same line are collinear.

49. Since a line can pass through any two points (Post. 1-1), this is not possible.

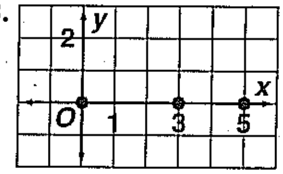
50. 

Any three points on a circle are not collinear. Or, the vertices (corners) of a triangle are not collinear.

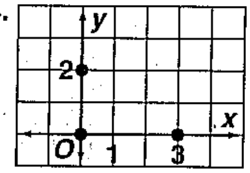
51. The three points are either collinear or noncollinear. If the three points are collinear, then a line contains them, and at least one plane contains the line. If the three points are noncollinear, then by Post. 1-4, there is one plane that contains them. Thus, three points that are noncoplanar is not possible.

52. 

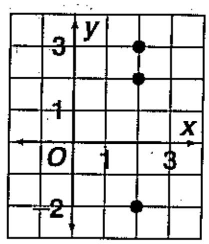
The graph of $x = 0$ passes through them, so they are collinear.

53. 

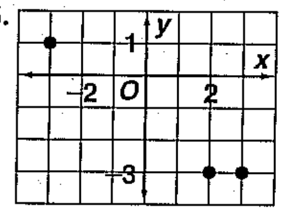
The graph of $y = 0$ passes through them, so they are collinear.

54. 

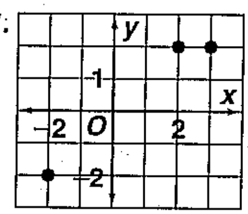
A different line passes through each possible pair of points, so the points are not collinear.

55. 

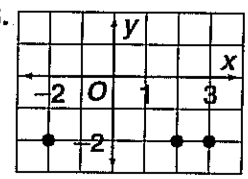
The graph of the line $x = 2$ passes through all three points, so they are collinear.

56. 

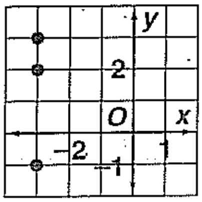
A different line passes through each possible pair of points, so the points are not collinear.

57. 

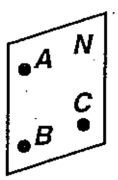
A different line passes through each possible pair of points, so the points are not collinear.

58. 

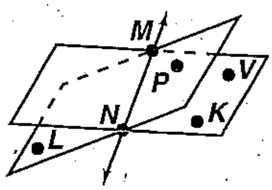
The graph of the line $y = -2$ passes through all three points, so they are collinear.

59. 

The graph of the line $x = -3$ passes through all three points, so they are collinear. **60.** Suppose that lines m and n are intersecting at P. Let A be another point on m and B be another point on n. Points A, B, and P are not collinear, so by Post. 1-4, the three points are coplanar. Thus, intersecting lines are *always* coplanar. **61.** By Post. 1-3, two planes intersect in exactly one line. Thus, two planes *never* intersect in exactly one point. **62.** Three points are either collinear or noncollinear. If they are collinear, then a line contains them, and at least one plane contains the line. If the three points are noncollinear, then by Post. 1-4, there is one plane that contains them. Thus, three points are *always* coplanar. **63.** Neither a line nor a plane curves. Thus, a plane containing two points of a line *always* contains the entire line. **64.** Any four points in a plane are coplanar. However, if three noncollinear points are in the plane containing a floor and the fourth point is on the ceiling, then the four points are not coplanar. Thus, four points are *sometimes* coplanar. **65.** For two lines to meet in more than one point, at least one of the lines would have to curve. Since lines do not curve, two lines *never* meet in more than one point. **66a.** Points E, F, and G are three noncollinear points, so by Post. 1-4, 1 plane contains the line and point. **66b.** Points P, H, and E are three noncollinear points, so by Post. 1-4, 1 plane contains the line and point. **66c.** Points F, G, and P are three noncollinear points, so by Post. 1-4, 1 plane contains the line and point. **66d.** Points E, P, and G are three noncollinear points, so by Post. 1-4, 1 plane contains the line and point. **66e.** A line and a point not on the line are always coplanar.

67. 

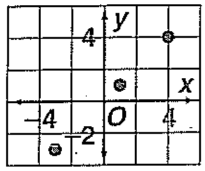
Post. 1-4: Through three noncollinear points there is exactly one plane.

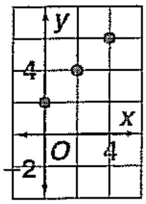
68. 

Answers may vary. Sample: Post. 1-3: If two planes intersect, then they intersect in exactly one line.

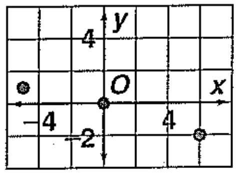
69. Place the edge of a paper on $\overleftrightarrow{AB}$. Point D lies on the edge of the paper, not point C. So, points A, B, and C are collinear. **70.** She stands on one point and sees a second point, so two points are represented. Post. 1-1 states that through any two points there is exactly one line. **71.** Each wall represents a plane and, by Post. 1-3, they intersect in exactly one line. **72.** The bottoms of the legs of the table represent points. If the bottom of the fourth leg of the table is not coplanar with the other three, then the table will rest on only three legs at a time. Post 1-4: Through any three noncollinear points, there is exactly one plane.

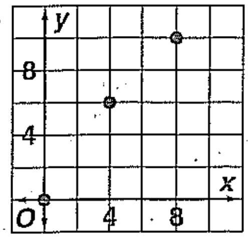
73–74. A line can be drawn through the three points, so they are collinear.

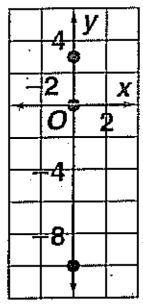
73. 

74. 

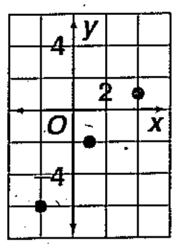
75–76. A single line cannot be drawn through the three points, so they are not collinear.

75. 

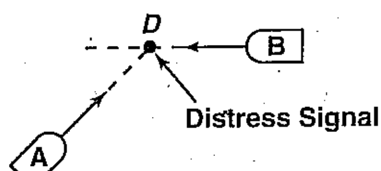
76. 

77. 

A line can be drawn through the three points, so they are collinear.

78. 

A single line cannot be drawn through the three points, so they are not collinear. **79.** Infinitely many; explanations may vary. Sample: Since two planes can intersect in one line, then so can three or four intersect in the same line. So, infinitely many planes can intersect in one line.

80. 

Points A, B, and C are noncollinear. By Post. 1-1, points D and B form a line and points A and D form a different line. The distress signal is on both lines, so it is at their intersection point. By Post. 1-2, there can be only one intersection point, so there can be only one distress signal. **81a.** Since the plane is flat, the line would have to curve so as to contain only the 2 points of the line. However, lines are straight, so, if two points of the line are in the plane, then all the points must be in the plane. **81b.** Points A, B, and C are noncollinear. By Post. 1-4, they are coplanar. By part ⓐ, all the points in each line lie in one plane, so $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ must be coplanar. **82.** By Post. 1-1, any two points are collinear, so the probability is 100%, or 1. **83.** The possible ways to pick 3 points are A-B-C, A-B-D, A-C-D, and B-C-D. The only group of points that are collinear is A-B-C. So, the probability is 1 out of 4, or $\frac{1}{4}$. **84.** Three points are either collinear or noncollinear. Since a line is contained by a plane, collinear points are coplanar. By Post. 1-4, any group of 3 noncollinear points is also coplanar, so the probability is 100%, or 1. **85.** H, F, B, and C are on the same line. The answer is choice A. **86.** The cheese represents space. Each slice represents a plane. Two intersecting planes divide a space into 4 spaces. The answer is choice I. **87.** The ends of the legs of the table represent points. By Post. 1-4, three points determine a plane, so the table can have at least 3 legs. The answer is choice B. **88.** If the points are collinear, then only one line can contain the three points. If they are noncollinear, then a different line passes through each pair of points. There are 3 ways to pair the points, so there are at most 3 possible lines. The answer is choice H. **89.** [2] **a.** A plane is named by at least three noncollinear points. The planes are ABD, ABC, ACD, and BCD. **b.** Each line will have point D in its name: $\overleftrightarrow{AD}, \overleftrightarrow{BD}, \overleftrightarrow{CD}$ [1] one part correct

90.

n	3^n	Last digit of 3^n
1	3	3
2	9	9
3	27	7
4	81	1
5	243	3
6	729	9
7	2187	7
8	6561	1
9	19,683	3
10	59,049	9

$$\begin{array}{r} 11\text{R}1 \\ 4\overline{)45} \end{array}$$

The pattern 3, 9, 7, 1 repeats 11 times for $n = 1$ to 44. For $n = 45$, the last digit is the first in the series, 3.

91. The pattern starts with A and skips every other letter of the alphabet. The next four letters after G are H, I, J, and K. Skip H and J to produce I and K as the next two letters of the sequence. **92.** The differences between the consecutive terms form the sequence 4, 6, 8, 10, ... The next two differences must be 12 and 14. So, the next two numbers are 30 + 12, or 42, and 42 + 14, or 56. **93.** $4^1 = 4$, $4^2 = 16, 4^3 = 64$, and $4^4 = 256$. The nth term is 4^n. The fifth term is 4^5, which is 1024. The sixth term is 4^6, which is 4096. **94.** The differences between the consecutive terms form the sequence 5, 10, 15, 20, ... The next two differences must be 25 and 30. So, the next two numbers are 50 − 25, or 25, and 25 − 30, or −5. **95.** For $a = 3$ and $b = -5, a^2 + b^2 = 3^2 + (-5)^2 = 9 + 25 = 34$. **96.** For $b = 8$ and $h = 11, \frac{1}{2}bh = \frac{1}{2}(8)(11) = 4(11) = 44$.

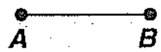
1-3 Segments, Rays, Parallel Lines and Planes *pages 17–23*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

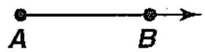
1. no **2.** yes **3.** no **4.** NMR **5.** PQL **6.** NKL **7.** PQR **8.** PKN **9.** LQR

Check Understanding **1.** The endpoint of $\overrightarrow{LP}$ is L, and the endpoint of $\overrightarrow{PL}$ is P. Since they don't have the same endpoint, they are not opposite rays. **2a.** From Plane $GJIH, \overline{GJ} \parallel \overline{HI}$. From Plane $ANJGD, \overline{GJ} \parallel \overline{DN}$. **2b.** Segments skew to $\overline{GJ}$ do not intersect $\overline{GJ}$, but are $\parallel$ to a line that does. All segments $\parallel$ to $\overline{JI}$ that do not intersect $\overline{GJ}$ are $\overline{AB}, \overline{CD}$, and $\overline{CH}$. **2c.** Answers may vary. Samples: $\overline{DN} \parallel \overline{HI}, \overline{JI} \parallel \overline{DC}, \overline{JN} \parallel \overline{CH}; \overline{DN}$ is skew to $\overline{HC}$ and also to $\overline{JI}$. **3a.** The parallel planes contain opposite faces of the diagram. $PSWT \parallel RQVU$, $PRUT \parallel SQVW, PSQR \parallel TWVU$ **3b.** P and Q are opposite corners of $PSQR$. The line $\parallel$ to $\overleftrightarrow{PQ}$ will pass through corresponding opposite corners in the plane $\parallel$ to $PSQR$. So, $\overleftrightarrow{PQ} \parallel \overleftrightarrow{TV}$. **3c.** Any line contained by a plane $\parallel$ to $QRUV$ is parallel to $QRUV$. Answers may vary. Samples: $\overleftrightarrow{SW}, \overleftrightarrow{TS}, \overleftrightarrow{PS}$

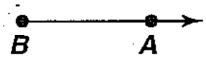
Exercises

1. 

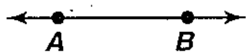
The symbol shows 2 endpoints and no arrows, so $\overline{AB}$ is a segment.

2. 

The symbol shows 1 endpoint at A and an arrow, so $\overrightarrow{AB}$ is a ray.

3. 

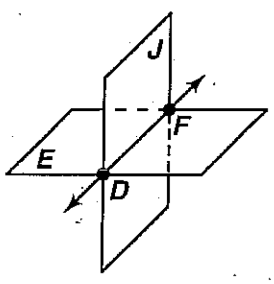
The symbol shows 1 endpoint at B and an arrow, so $\overrightarrow{BA}$ is a ray.

4. 

The symbol shows no endpoints and 2 arrows, so $\overleftrightarrow{AB}$ is a line.

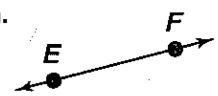
5. A systematic listing is to choose the left endpoints starting from the left and then list the right endpoints in order from that point: $\overline{RS}, \overline{RT}, \overline{RW}, \overline{ST}, \overline{SW}, \overline{TW}$. **6.** A systematic listing is to choose endpoints in order from left to right and select rays pointing right; then repeat selecting rays pointing left: $\overrightarrow{RS}, \overrightarrow{ST}, \overrightarrow{TW}, \overrightarrow{WT}, \overrightarrow{TS}, \overrightarrow{SR}$. **7a.** $\overrightarrow{TS}$ or $\overrightarrow{TR}, \overrightarrow{TW}$ **7b.** The endpoint must have a labeled point on either side of it. S is the only available endpoint besides T, so the rays are $\overrightarrow{SR}$ and $\overrightarrow{ST}$. **8.** Since Y is an endpoint, there are 4 choices for the other endpoint. The four segments are $\overline{RY}, \overline{SY}, \overline{TY}$, and $\overline{WY}$. **9.** Answers may vary. If Y is positioned to the left of R or to the right of W, then only one ray is possible; that ray is $\overrightarrow{YR}$ or $\overrightarrow{YW}$, respectively. If Y is positioned somewhere between R and W, then 2 rays are possible. Answers may vary. Sample: If Y is positioned between S and T, the two rays are $\overrightarrow{YS}$ (or $\overrightarrow{YR}$) and $\overrightarrow{YT}$ (or $\overrightarrow{YW}$). **10.** To name a ray, its position is defined by the endpoint and its direction is defined by a different point on the ray. Answers may vary. If Y is to the left of R or right of W, then a new direction can be named from R and Y, respectively. If either is the case, then one new ray can be named. If Y is between R and W, then no new rays can be named. Check students' diagrams for the position of Y. **11.** Find a segment in the same plane that does not intersect $\overline{AC}$: $\overline{DF}$. **12.** Find a segment in the same plane that does not intersect $\overline{EF}$: $\overline{BC}$. **13.** Find a segment in the same plane that does not intersect $\overline{AD}$: $\overline{BE}$ from Plane $ADEB$ and $\overline{CF}$ from Plane $ADFC$. **14.** Eliminate all segments that are $\parallel$ to $\overline{AC}$ and that intersect with $\overline{AC}$. Those remaining are skew to $\overline{AC}$: $\overline{DE}$, $\overline{EF}$, $\overline{BE}$. **15.** Eliminate all segments that are $\parallel$ to $\overline{EF}$ and that intersect with $\overline{EF}$. Those remaining are skew to $\overline{EF}$: $\overline{AD}$, $\overline{AB}$, $\overline{AC}$. **16.** Eliminate all segments that are $\parallel$ to $\overline{AD}$ and that intersect with $\overline{AD}$. Those remaining are skew to $\overline{AD}$: $\overline{BC}$ and $\overline{EF}$. **17.** Any planes that do not intersect are $\parallel$, so $ABC \parallel DEF$. **18.** Lines in the same plane that do not intersect are $\parallel$. Answers may vary. Sample: $\overleftrightarrow{BE} \parallel \overleftrightarrow{AD}$ **19.** Skew lines are not $\parallel$ and do not intersect. Answers may vary. Sample: $\overleftrightarrow{CF}$, $\overleftrightarrow{DE}$ **20.** A line is parallel to a plane if they do not intersect. Answers may vary. Sample: Plane $DEF \parallel \overleftrightarrow{BC}$ **21.** All lines $\parallel$ to $\overleftrightarrow{AB}$ are coplanar and do not intersect: $\overleftrightarrow{FG}$. **22.** Skew lines are not $\parallel$ and do not intersect with each other. Answers may vary. Samples: $\overleftrightarrow{HB}$, $\overleftrightarrow{DG}$, $\overleftrightarrow{CD}$, $\overleftrightarrow{AB}$ **23.** Lines $\parallel$ to a plane do not intersect the plane. $\overleftrightarrow{BG}$, $\overleftrightarrow{DH}$, $\overleftrightarrow{CL}$ **24.** Two planes intersect in a line, and two points determine a line. Name the intersection by the two known points shared by both planes, A and F: $\overleftrightarrow{AF}$ **25.** They are in Plane $BCGH$ and they will never intersect; true. **26.** Since the lines are not in the same plane, they can't be $\parallel$. They are skew lines, so the statement is false. **27.** The planes are on opposite sides of the "house" and will never intersect; true. **28.** If the "roof" and side of the "house" were extended, the two would intersect, so they can't be $\parallel$; false. **29.** The lines will not intersect and do not lie in the same plane; true. **30.** If the two lines were extended, they would intersect above pt. A; the statement is false. **31.** The two lines will not intersect and they lie in Plane $AEGC$, so the lines are $\parallel$, not skew; false. **32.** Points C, F, J, and A are coplanar and the lines will not intersect; thus they are $\parallel$, not skew; false. **33.** They are both segments that have the same endpoints, so they are the same. **34.** They are two rays that share some of the same points, but they do not have the same endpoint. They are not the same. **35.** By Post. 1-1, only one line passes through two given points, so both represent the same line through X and Y.

36.



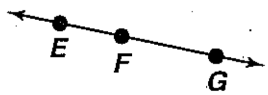
37. By def. of $\parallel$ lines, parallel lines are *always* coplanar. **38.** By def. of skew lines, two skew lines are *never* coplanar. **39.** By def. of opposite rays, two opposite rays *always* form a line. **40.** By Post. 1-1, only one line passes through 2 points, so $\overleftrightarrow{TQ}$ and $\overleftrightarrow{QT}$ are *always* the same line. **41.** To be the same ray, they would have the same endpoints and direction. Their endpoints are different. Thus, $\overrightarrow{GH}$ and $\overrightarrow{HG}$ are *never* the same ray. **42.** To be the same ray, they would have the same endpoints and direction. They have the same endpoints, but they have the same direction only if K and L are both on the same side of J. So, $\overrightarrow{JK}$ and $\overrightarrow{JL}$ are *sometimes* the same ray. **43.** They are both segments that have the same endpoint, so $\overline{AX}$ and $\overline{XA}$ are *always* the same segment. **44.** Two lines in the same plane either intersect or are $\parallel$. So two lines in the same plane are *sometimes* parallel. **45.** By def. of $\parallel$ planes, two planes that do not intersect are *always* parallel. **46.** Two lines that lie in parallel planes are either parallel or skew. So, two lines that lie in parallel planes are *sometimes* parallel. **47.** Two lines in intersecting planes are either intersecting or they are skew. So, two lines in intersecting planes are *sometimes* skew. **48.** Answers may vary. The point should be on the line $y = \frac{3}{2}x$ so that the x-coordinate is less than 2 and the y-coordinate is less than 3. Sample: (0, 0) **49a.** Answers may vary. Samples: east and west; northeast and southwest **49b.** Answers may vary. Sample: northwest and southeast **50.** Two lines can be parallel, skew, or intersecting in one point. Sample: Train tracks are parallel; vapor trail of a northbound jet and an eastbound jet at different altitudes are skew; streets that cross are intersecting. **51.** Answers may vary. Samples: Two planes flying at different altitudes escape crashing into one another. Or, skew lines cannot be contained in one plane. Therefore, they have "escaped" a plane. **52.** They are two lines in the same plane that do not intersect, so $\overleftrightarrow{ST} \parallel \overleftrightarrow{UV}$. **53.** They are two intersecting lines. Names of the lines may vary. Sample: $\overleftrightarrow{XY}$ and $\overleftrightarrow{ZW}$ intersect at R. **54.** Planes $ABCE$ and $DCBF$ intersect in $\overleftrightarrow{BC}$. **55a.** Two planes intersect in a line. The three planes intersect in 2 lines. Since the 2 lines are in the same plane (C) and do not intersect, they are $\parallel$. **55b.** Examples may vary. Sample: The floor and ceiling are parallel. A wall intersects both. The lines of intersection are parallel. **56.** Answers may vary. Sample: The diamond structure makes it tough, strong, hard, and durable. The graphite structure makes it soft and slippery.

57a.



From the two endpoints E and F, exactly one segment is possible: $\overline{EF}$.

57b.



From the 3 endpoints, exactly 3 segments are possible: $\overline{EF}$, $\overline{EG}$, $\overline{FG}$.

57c.

Number of points	Number of segments
2	1
3	3
4	6
5	10
6	15

Answers may vary. Sample: For each "new" point, the number of new segments equals the number of "old" points. So, for n points, there are $n - 1$ points added to the previous number of segments.

57d. Continue the chart started in part ⓒ. For 10 points, there are 45 possible segments.

Number of collinear points	Number of possible segments
2	1
3	3
4	6
5	10
6	15
7	$6 + 15 = 21$
8	$7 + 21 = 28$
9	$8 + 28 = 36$
10	$9 + 36 = 45$

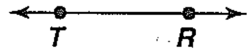
57e. 45 is half of 90, which is 10(9); 36 is half of 72, which is 9(8); 28 is half of 56, which is 8(7). The number of segments fits the pattern of $\frac{n(n-1)}{2}$. **58.** No; by Post. 1-3, two different planes cannot intersect in more than one line. **59.** Yes; Plane P intersects Plane A in one line $\parallel$ to $\overleftrightarrow{CD}$ and intersects Plane B in another line $\parallel$ to $\overleftrightarrow{CD}$.

60. Answers may vary. Samples: Lines that intersect at V are $\overleftrightarrow{SV}$, $\overleftrightarrow{RV}$, $\overleftrightarrow{QV}$, and $\overleftrightarrow{PV}$. Lines that intersect at S are $\overleftrightarrow{PS}$, $\overleftrightarrow{VS}$, and $\overleftrightarrow{RS}$. Lines that intersect at P are $\overleftrightarrow{VR}$, $\overleftrightarrow{QR}$, and $\overleftrightarrow{SR}$. **61.** A line in Plane $PSRQ$ that does not seem to intersect with $\overleftrightarrow{PS}$ is $\overleftrightarrow{QR}$, so $\overleftrightarrow{QR}$ may be $\parallel$ to $\overleftrightarrow{PS}$.

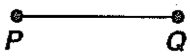
62. Explanations may vary. Samples are given: If 2 planes are $\parallel$, then a line in one plane is either parallel to or skew to a line in the other plane, so a line in the $\parallel$ plane through V can be $\parallel$ to $\overleftrightarrow{SR}$. Since the two planes are $\parallel$, they share no common points. Since a line in one plane cannot intersect a line in the other plane, a line in the $\parallel$ plane through V cannot intersect $\overleftrightarrow{SR}$. Being skew is one of the options mentioned in the first sentence, so a line in the plane $\parallel$ through V can be skew to $\overleftrightarrow{SR}$.

63. The segments are $\overline{AB}, \overline{AC}, \overline{AD}, \overline{AE}, \overline{BC}, \overline{BD}, \overline{BE}, \overline{CD}, \overline{CE}$, and $\overline{DE}$. There are 10 of them. The answer is choice D. **64.** The ray opposite $\overrightarrow{BC}$ has endpoint B and goes in the direction opposite C, so $\overrightarrow{BA}$ is the opposite ray. The answer is choice H. **65.** Any other name shows endpoint C and points in the direction of A. $\overrightarrow{CB}$ satisfies these conditions. The answer is choice B. **66.** By Post. 1-3, the answer is choice F. **67.** In column A the terms increase by 2 for each subsequent term, so the next number is $7 + 2$, or 9. In column B every other number is positive and negative and the numbers without the sign increase by 2; the next number is positive and is 2 more than 8, which is 10. Since $9 < 10$, the answer is choice B. **68.** For column A, Post. 1-1 indicates that the answer is 1. For column B, the number of points in each line is infinite. Since the amount in B is greater than 1, the answer is choice B. **69.** For column A, the possible segments are $\overline{AB}, \overline{AC}$, and $\overline{BC}$, so there are 3 possible segments. For column B, if the lines are collinear, then only 1 line is possible. If they are noncollinear, then 3 lines are possible. Since it's not clear if the lines are collinear or noncollinear, it is impossible to make a conclusion. The answer is choice D.

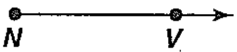
70. [2] **a.** Parallel and skew lines are alike in that they do not intersect. Parallel and skew lines are different because parallel lines lie in one plane and skew lines cannot be coplanar. **b.** No; of the 8 other lines shown, 4 ($\overleftrightarrow{PJ}, \overleftrightarrow{KJ}, \overleftrightarrow{LM}$, and $\overleftrightarrow{SM}$) intersect $\overleftrightarrow{JM}$ and 4 ($\overleftrightarrow{KQ}, \overleftrightarrow{LR}, \overleftrightarrow{SR}$, and $\overleftrightarrow{PQ}$) are skew to $\overleftrightarrow{JM}$. **71.** Answers may vary. Samples: $\overleftrightarrow{EF}$ or $\overleftrightarrow{FG}$ **72.** Answers may vary. Samples: A, B, C, D, E, F, G, or H **73.** By Post 1-2, the two lines intersect in one point, which is C. **74.** $\overleftrightarrow{EF}$ is the intersection of 2 planes, so it must lie in both: AEF and HEF. **75.** The plane can be named by any 3 or more of the 4 named points. Sample: ABH **76.** The plane can be named by any 3 or more of the 4 named points. Sample: EHG **77.** The points common to both planes are on the line $\overleftrightarrow{FG}$. **78.** Plane CGH is the right side of the figure and the fourth point on that side is B.

79. 

The figure has no endpoints, so it is a line.

80. 

The figure has exactly 2 endpoints, so it is a segment.

81. 

The figure has exactly one endpoint at N and one arrow, so it is a ray. **82.** Each term is 0.08 more than the previous term. The sixth term is $1.32 + 0.08$, or 1.4. The seventh term is $1.4 + 0.08$, or 1.48. **83.** The terms decrease by 1, 2, 3, 4, 5, so the seventh term decreases by 6 and is $-16 - 6$, or -22. The eighth term decreases by 7 and is $-22 - 7$, or -29. **84.** The first letter of each pair is the alphabet in order starting from A. The second letter of each pair is one more than the first letter. The next two pairs of letters after EF are FG and GH. **85.** Starting with A, each term is 3 letters more than the previous letter. The letters after M are N, O, P, Q, R, S, and every third letter is P and then S. **86.** If x is the original number, subtracting 5 from it results in $x - 5$ which is less than x. However, if you subtract -5 from it, the result is $x - (-5) = x + 5$, which is 5 more than x. Also, if you subtract 0, then the result is $x - 0$, or x. So, whenever you subtract a negative number, the answer is greater than the given number, and if you subtract 0, the answer stays the same.

CHECKPOINT QUIZ 1 *page 23*

1. Each term is 2.5 more than the previous term, so the fifth term is $2.5 + 26.5$, or 29. The sixth term is $2.5 + 29$, or 31.5. **2.** Starting with 3.4, the numerals to the right of the decimal are counting numbers in order. Following 3.4567 are 3.45678 and 3.456789. **3.** For 1: Add 2.5. For 2: Extend the decimal to one more place with a digit that is 1 more than the one to its left. **4.** All four points are in the top plane of the figure. One possible name for the plane is AEF. **5.** Yes; Plane $DCEF$ cuts across the middle of the space figure. **6.** No; H, G, and F are in the front plane $EFGH$. B, F, and G are in the right plane $BCGF$. **7.** No; A, E, and B are in the top plane, while C lies in the bottom plane. **8.** Parallel segments must be in the same plane without the lines that contain them intersecting. Segments $\parallel$ to $\overline{HG}$ are $\overline{CD}, \overline{AB}$, and $\overline{EF}$. **9.** Choose any line, and then choose a line that is not $\parallel$ to it and that does not intersect it. Answers may vary.

Sample: $\overleftrightarrow{AE}, \overleftrightarrow{BC}$ **10.** The only point the plane and line have in common is H, so H is the point of intersection.

ALGEBRA 1 REVIEW *page 24*

1. $10n + 12 = 14n - 12; 12 = 4n - 12; 24 = 4n; n = 6$ **2.** $(4w - 28) + (11w + 13) = 180; 15w - 15 = 180; 15w = 195; w = 13$ **3.** $(7a + 3) + (-a - 5) = -16; 6a - 2 = -16; 6a = -14; 3a = -7; a = -\frac{7}{3}$, or $-2\frac{1}{3}$
4. $7y + 44 = 12y + 11; -5y = -33; y = \frac{33}{5}$, or $6\frac{3}{5}$, or 6.6
5. $(7t - 21) + (t + 4) = 15; 8t - 17 = 15; 8t = 32; t = 4$
6. $8x - 4 - 2x = -10; 6x - 4 = -10; 6x = -6; x = -1$
7. $(8t + 30) + (-2t - 16) = -22; 6t + 14 = -22; 6t = -36; t = -6$ **8.** $6x + 17 = 9x + 2; -3x + 17 = 2; -3x = -15; x = 5$ **9.** $(3y - 5) + (5y + 20) = 135; 8y + 15 = 135; 8y = 120; y = 15$ **10.** $(11x - 37) + (5x + 59) = 54; 16x + 22 = 54; 16x = 32; x = 2$ **11.** $3x - 35 = 9x - 59; -6x - 35 = -59; -6x = -24; x = 4$ **12.** $9x - 3 = 8x - 7; x - 3 = -7; x = -4$ **13.** $(5w + 24) + (2w + 13) = 156; 7w + 37 = 156; 7w = 119; w = 17$ **14.** $(3x + 10) - 5x = 6x - 50; -2x + 10 = 6x - 50; -8x + 10 = -50; -8x = -60; x = 7.5$ **15.** $8y + 12 = 2y - 18; 6y + 12 = -18; 6y = -30; y = -5$ **16.** $7t - 8t + 4 = 5t - 2; -t + 4 = 5t - 2; -6t + 4 = -2; -6t = -6; t = 1$ **17.** $13c + 40 = 9c - 20 + c; 13c + 40 = 10c - 20; 3c + 40 = -20; 3c = -60; c = -20$ **18.** $(6a - 54) - (5a + 27) = 23; 6a - 54 - 5a - 27 = 23; a - 81 = 23; a = 104$
19. $(2 + 4y) - (y + 9) = 26; 2 + 4y - y - 9 = 26; 3y - 7 = 26; 3y = 33; y = 11$ **20.** $(12c + 35) - (5c - 11) = -2; 12c + 35 - 5c + 11 = -2; 7c + 46 = -2; 7c = -48; c = -\frac{48}{7}$, or $-6\frac{6}{7}$

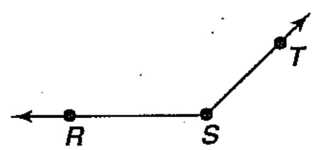
1-4 Measuring Segments and Angles *pages 25–33*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM*.
1. 6 **2.** 3.5 **3.** 3 **4.** 6 **5.** 2 **6.** 9 **7.** 4 **8.** 9 **9.** $\frac{1}{3}$

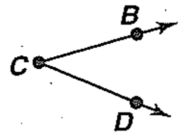
Check Understanding **1a.** $CD = |-2 - 0| = |-2| = 2; DE = |0 - 2| = |-2| = 2$; so $CD = DE$. **1b.** Yes; the distance between the two endpoints is the same: $|-5 - (-8)| = |-5 + 8| = |3| = 3$. **2.** Solve for x: $EF + FG = EG; (4x - 20) + (2x + 30) = 100; 6x + 10 = 100; 6x = 90; x = 15$. Solve for EF: $EF = 4x - 20 = 4(15) - 20 = 60 - 20 = 40$. Solve for FG: $FG = 2x + 30 = 2(15) + 30 = 30 + 30 = 60$. **3.** $XZ = ZY; XZ + ZY = 27; XZ + XZ = 27; 2XZ = 27; XZ = 13.5$ **4a.** Since E is the vertex of more than one angle, either 3 letters or a number must be used to name the angle: $\angle 2, \angle DEC$. **4b.** No; $\angle 1, \angle 2$, and $\angle AED$ all have E for a vertex. The name of the angle should distinguish it from any other angles having the same vertex, so more information is needed in the name. **5a.** The measure is 30. Since the measure is between 0 and 90, the angle is acute. **5b.** The measure is 90. Since the measure is exactly 90, the angle is right. **5c.** The measure is 140. Since the measure is between 90 and 180, the angle is obtuse.
6. $m\angle DEG + m\angle GEF = m\angle DEF$; $\angle DEF$ is a straight $\angle$, so its measure is 180. Substituting, $145 + m\angle GEF = 180; m\angle GEF = 180 - 145 = 35$.

Exercises **1.** $AC = |-8 - 1| = |-9| = 9; BD = |-6 - 3| = |-9| = 9$. The measures are =, so the segments are congruent. **2.** $BD = |-6 - 3| = |-9| = 9; CE = |1 - 7| = |-6| = 6$. The measures are not =, so the segments are not congruent. **3.** $AD = |-8 - 3| = |-11| = 11; BE = |-6 - 7| = |-13| = 13$. The measures are congruent. **4.** $BC = |-6 - 1| = |-7| = 7; CE = |1 - 7| = |-6| = 6$. The measures are not =, so the segments are not congruent. **5.** For $X = -7, Y = -3, Z = 1$, and $W = 5, XY = |-7 - (-3)| = |-4| = 4; ZW = |1 - 5| = |-4| = 4; \overline{XY} \cong \overline{ZW}$. **6.** For $X = -7, Y = -3, Z = 1$, and $W = 5, ZX = |1 - (-7)| = |8| = 8; WY = |5 - (-3)| = |8| = 8; \overline{ZX} \cong \overline{WY}$. **7.** For $X = -7, Y = -3, Z = 1$, and $W = 5, YZ = |-3 - 1| = |-4| = 4; XW = |-7 - 5| = |-12| = 12$. Since $4 < 12, YZ < XW$. **8.** $RS + ST = RT; 15 + 9 = RT; RT = 24$ **9.** $RS + ST = RT; RS + 15 = 40; RS = 25$ **10a.** $RS + ST = RT; (3x + 1) + (2x - 2) = 64; 5x - 1 = 64; 5x = 65; x = 13$ **10b.** $RS = 3x + 1 = 3(13) + 1 = 39 + 1 = 40; ST = 2x - 2 = 2(13) - 2 = 26 - 2 = 24$ **11a.** $RS + ST = RT; (8y + 4) + (4y + 8) = (15y - 9); 12y + 12 = 15y - 9; -3y + 12 = -9; -3y = 21; y = 7$ **11b.** $RS = 8y + 4 = 8(7) + 4 = 56 + 4 = 60; ST = 4y + 8 = 4(7) + 8 = 28 + 8 = 36; RT = 15y - 9 = 15(7) - 9 = 105 - 9 = 96$ **12a.** By def. of midpoint, $3x = 5x - 6$, so $-2x = -6$, then $x = 3$. $XA = 3x = 3(3) = 9$ **12b.** $AY = 5x - 6 = 5(3) - 6 = 15 - 6 = 9; XY = XA + AY = 9 + 9 = 18$ **13.** By the tick marks on the figure, $PT = TQ$. So, $5x + 3 = 7x - 9; -2x = -12; x = 6$. $PT = 5x + 3 = 5(6) + 3 = 30 + 3 = 33$ **14.** By the tick marks on the figure, $PT = TQ$. $4x - 6 = 3x + 4; x - 6 = 4; x = 10$. $PT = 4x - 6 = 4(10) - 6 = 40 - 6 = 34$
15. By the tick marks on the figure, $PT = TQ$. $7x - 24 = 6x - 2; x - 24 = -2; x = 22$. $PT = 7x - 24 = 7(22) - 24 = 154 - 24 = 130$ **16.** The vertex is always the middle letter, and, since the angle doesn't share a vertex with another angle, it can also be named by the vertex alone: $\angle XYZ, \angle ZYX, \angle Y$. **17.** The vertex is always the middle letter, and, since the angle doesn't share a vertex with another angle, it can also be named by the vertex alone, or by the number in its interior: $\angle MCP, \angle PCM, \angle C$, or $\angle 1$. **18.** The vertex is B and, since it's the vertex of more than one angle, 3 letters are needed to distinguish it from the others: $\angle ABC, \angle CBA$. **19.** The vertex is B and, since it's the vertex of more than one angle, 3 letters are needed in its name: $\angle CBD, \angle DBC$.

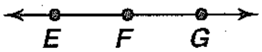
20. An obtuse angle measures between 90 and 180. Answers may vary. Sample:



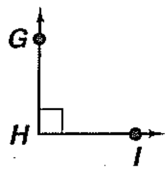
21. An acute angle measures between 0 and 90. Answers may vary. Sample:



22. A straight angle measures exactly 180.

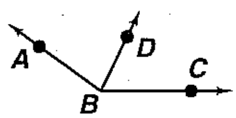


23. A right angle measures exactly 90.



24. The measure is 60. Since it is between 0 and 90, the angle is acute. **25.** The measure is 90. Since it is exactly 90, the angle is right. **26.** The measure is 135. Since it is between 90 and 180, the angle is obtuse. **27.** $m\angle CBD + m\angle ABC = m\angle ABD$; $m\angle CBD + 45 = 79$; $m\angle CBD = 34$ **28.** $m\angle GFJ + m\angle EFG = m\angle JFE$. Since $\angle JFE$ is a straight angle, its measure is 180. $m\angle GFJ + 110 = 180$; $m\angle GFJ = 70$ **29.** $AB = |0 - 8| = |-8| = 8$. The length of each segment formed by the midpoint is $\frac{8}{2}$, or 4, so the distance from A to the midpt. is 4: $|0 - 4| = 4$. The point at 4 is Q. **30.** By def. of midpt., it is half way between the two endpoints. Half way between 4 and 8 is 6. **31.** By def. of midpt., it is half way between the two endpoints. Half way between -8 and 0 is -4. **32.** The midpt. of $\overline{QB}$ is 6 and the midpt. of $\overline{WA}$ is -4. The distance between them is $|6 - (-4)| = |6 + 4| = |10| = 10$. So, the midpt. is $\frac{10}{2}$, or 5 units in a negative direction from 6, or 5 units in a positive direction from -4. $6 - 5 = 1 = -4 + 5$, so the coordinate of the midpt. is 1. **33.** $AR = 5$, and R could be either a positive or a negative direction from A. So, R is either 5 or -5. The midpt. forms a segment that is $\frac{5}{2}$, or 2.5 units long in a positive or negative direction from A. Its location is at -2.5 or 2.5. **34.** $AT = 7$, and T could be either a positive or negative direction from A. So T is either 7 or -7. The midpt. forms a segment that is $\frac{7}{2}$, or 3.5 units long in a positive or negative direction from A. It is at either -3.5 or 3.5. **35.** If R is at -2.5 and T is at -3.5, then $RT = |-2.5 - (-3.5)| = |-2.5 + 3.5| = |1| = 1$, and the midpt. is 0.5 units left of -2.5, or $-2.5 - 0.5 = -3$. If R is at -2.5 and T is at 3.5, then $RT = |-2.5 - 3.5| = |-6| = 6$, and the midpt. is 3 units right of -2.5, or $-2.5 + 3 = 0.5$. If R is at 2.5 and T is at -3.5, then $RT = |2.5 - (-3.5)| = |2.5 + 3.5| = |6| = 6$, and the midpt. is 3 units left of 2.5, or $2.5 - 3 = -0.5$. If R is at 2.5 and T is at 3.5, then $RT = |2.5 - 3.5| = |1| = 1$, and the midpt. is 0.5 unit right or 2.5, or $2.5 + 0.5 = 3$. So, the possible midpts. are -3, 0.5, -0.5, and 3. **36a.** $|237 - 159| = 78$. The distance is 78 mi. **36b.** Answers may vary. Samples: measuring with a ruler; thermometer **37–41.** Check students' work. **42.** $AB = |-8 - (-6)| = |-8 + 6| = |-2| = 2$; $CD = |1 - 3| = |-2| = 2$. Since the lengths are =, the segments are $\cong$. The statement is true.

43. $BD = |-6 - 3| = |-9| = 9$; $CD = |1 - 3| = |-2| = 2$. Since 9 is not < 2, the statement is false. **44.** $AC = |-8 - 1| = |-9| = 9$; $BD = |-6 - 3| = |-9| = 9$; $AD = |-8 - 3| = |-11| = 11$. Since $9 + 9 \neq 11$, the statement is false. **45.** $AC = |-8 - 1| = |-9| = 9$; $CD = |1 - 3| = |-2| = 2$; $AD = |-8 - 3| = |-11| = 11$. Since $9 + 2 = 11$, the statement is true. **46.** G could be 5 units to the right or left of E, so there are 2 possible locations: $7 - 5$, or 2, or $7 + 5$, or 12. **47.** $m\angle ACB + m\angle BCD = m\angle ACD$. Since $\angle ACD$ is a straight angle, its measure is 180; $65 + m\angle BCD = 180$; $m\angle BCD = 115$. **48.** From Exercise 47, $m\angle BCD = 115$. Since $\angle ECB$ is a straight angle, its measure is 180. Substituting in $m\angle ECD + m\angle BCD = m\angle ECB$ results in $m\angle ECD + 115 = 180$. $m\angle ECD = 65$ **49.** Graph (3, 0) and use that as the center of a circle with a 12-unit radius. Answers may vary. Sample: The coordinates 12 units directly right, left, up, and down from (3, 0) are (15, 0), (−9, 0), (3, 12), and (3, −12). **50–54.** Check students' work. **55.** Trace the figure and extend the outer edges of the skis until they intersect. Place the center of the protractor on the intersection to measure the angle. It is about 42°. **56.** The clock is divided into 12 congruent angles, each measuring $\frac{360}{12}$, or 30. Each "minute" measures $\frac{30}{5}$, or 6. The minute hand and the hour hand should be in exactly the same location between 2 numbers and exactly 15-minute angles from one another. Answers may vary. Samples: 1:21:49$\frac{9}{11}$, 1:54:32$\frac{8}{11}$, 2:27:16$\frac{4}{11}$, 3:00, 9:00 **57.** An obtuse angle measures between 90 and 180. Answers may vary. Samples: 1:30, 5:00, 7:00 **58.** An hour hand travels 0.5° per minute. A minute hand travels 6° per minute. For $d = rt$, $180 = |0.5 - 6|t$, so $t = \frac{360}{11}$, or $32\frac{8}{11}$ minutes. This means that after 12:00, the first straight angle is at 12:32$\frac{8}{11}$, or 12:32:43$\frac{9}{11}$, and then every 65 minutes $27\frac{3}{11}$ seconds thereafter. Answers may vary. Samples: 1:38$\frac{2}{11}$, 2:43$\frac{7}{11}$, 3:49$\frac{1}{11}$, 6:00, 12:32$\frac{8}{11}$ **59.** Every 5-minute interval is $\frac{360}{12}$, or 30°, and each 1-minute interval is 6°. The hour hand and minute hand are 30 minutes apart, or $30(6) = 180°$. **60.** Every 1-minute interval is 6°. The hour hand and minute hand are 25 minutes apart, or $25(6) = 150°$. **61.** Every 1-minute interval is 6°. The hour hand and minute hand are 5 minutes apart, or $5(6) = 30°$. **62.** Every 5-minute interval is 30°. Since the minute hand is $\frac{2}{3}$ the distance around the clock, the hour hand is $\frac{2}{3}$ the distance between the 4 and 5, which is $\frac{2}{3}(30) = 20°$ past the 4, or 10° before the 5. At 40 minutes after the hour, the minute hand is on the 8, which is 3(30), or 90° from the 5. The total degrees between the two hands are $90 + 10 = 100$. **63.** Every 5-minute interval is 30°. Since the minute hand is $\frac{1}{3}$ the distance around the clock, the hour hand is $\frac{1}{3}$ the distance between the 5 and 6 on the clock, which is $\frac{1}{3}(30) = 10°$ past the 5. At 20 minutes after the hour, the minute hand is on the 4. The degrees between 4 and 5 are 30. The total degrees between the two hands are $30 + 10 = 40$. **64.** Every 5-minute interval is 30°. Since the minute hand is $\frac{2}{3}$ the distance around the clock, the hour hand is $\frac{2}{3}$ the distance between the 10 and 11 on the clock, or 20° past the 10. At 40 minutes after the hour, the minute hand is on the 8. The degrees between the 8 and 10 are 2(30) or 60°. The total degrees between the two hands are $60 + 20 = 80$. **65.** $m\angle MQP = m\angle MQV + m\angle VQP = 90 + 35 = 125$ **66.** $m\angle QVP + m\angle MVQ = m\angle MVP$. Since $\angle MVP$ is a straight angle, its measure is 180. $m\angle QVP + 55 = 180$; $m\angle QVP = 125$ **67.** An acute angle measures between 0 and 90. Answers may vary. Samples: $\angle QVM$ and $\angle VPN$ **68.** An obtuse angle measures between 90 and 180. Answers may vary. Samples: $\angle MNP$ and $\angle PNQ$ **69.** A right angle measures exactly 90. The only angles that are not obviously acute or obtuse are $\angle MQV$ and $\angle PNQ$. **70a.** $m\angle RQS + m\angle TQS = m\angle RQT$. Since $\angle RQT$ is a straight angle, its measure is 180. $(2x + 4) + (6x + 20) = 180$; $8x + 24 = 180$; $8x = 156$; $x = 19.5$ **70b.** $m\angle RQS = 2x + 4 = 2(19.5) + 4 = 39 + 4 = 43$; $m\angle TQS = 6x + 20 = 6(19.5) + 20 = 117 + 20 = 137$ **70c.** Answers may vary. Since $\angle RQS$ is a straight angle, the sum of the angle measures should be 180. **71.** By segment addition, $AD + DC = AC$, and the tick marks indicate $AD = DC$ so $DC = 12$. Now $2AD = AC$; $2(12) = 4y - 36$; $24 = 4y - 36$; $60 = 4y$; $y = 15$. Also, $AC = 2AD = 2(12) = 24$. **72.** By the tick marks, $ED = DB$, so $x + 4 = 3x - 8$; $-2x = -12$; $x = 6$. $ED = x + 4 = 6 + 4 = 10$; $ED = DB = 10$; $EB = ED + DB = 10 + 10 = 20$ **73a.** Answers may vary. Sample: The two rays that make up the sides of the angle come together at a sharp point. **73b.** Answers may vary. Sample: Molly had an *acute* pain in her knee. **74.** The Japanese 0 is actually 90 on a protractor, and the Japanese 90 is either 0 or 180 on a protractor. So, the Japanese 45 is, on a protractor, half way between 0 and 90 or half way between 90 and 180, so it is either 45 or 135. The Japanese 15 is $90 - 15$, which is 75 on a protractor, or $90 + 15$, which is 105 on a protractor. The Japanese 75 is $90 - 75$, which is 15 on a protractor, or $180 - 15$, which is 165 on a protractor. **75.** By angle addition, $m\angle AOB + m\angle BOC = m\angle AOC$. Then $(2x + 8) + (3x + 14) = 7x - 2$; $5x + 22 = 7x - 2$; $-2x + 22 = -2$; $-2x = -24$; $x = 12$. The $m\angle AOC$ is $7x - 2 = 7(12) - 2 = 84 - 2 = 82$. The $m\angle AOB$ is $2x + 8 = 2(12) + 8 = 24 + 8 = 32$. The $m\angle BOC$ is $3x + 14 = 3(12) + 14 = 36 + 14 = 50$. $82 = 32 + 52$, so the answer checks. **76.** The tick marks show $m\angle AOB = m\angle COD$, so $4x - 2 = 2x + 14$; $2x - 2 = 14$; $2x = 16$; $x = 8$. For $m\angle AOB$: $4x - 2 = 4(8) - 2 = 32 - 2 = 30$. For $m\angle BOC$: $5x + 10 = 5(8) + 10 = 40 + 10 = 50$. For $m\angle COD$: $2x + 14 = 2(8) + 14 = 16 + 14 = 30$. $30 = 30$, so the answer checks. **77.** $m\angle AOB + m\angle BOC + m\angle COD = m\angle AOD$; $m\angle COD = m\angle AOB = 28$; $28 + (3x - 2) + 28 = 6x$; $3x + 54 = 6x$; $54 = 3x$; $x = 18$. For $m\angle BOC$: $3x - 2 = 3(18) - 2 = 54 - 2 = 52$. For $m\angle AOD$: $6x = 108$. $28 + 52 + 28 = 108$, so the answer checks. **78.** $m\angle COD = m\angle AOB = 4x + 3$; $m\angle AOB + m\angle BOC + m\angle COD = m\angle AOD$; $(4x + 3) + (7x) + (4x + 3) = 16x - 1$; $15x + 6 = 16x - 1$; $-x = -7$; $x = 7$. For $m\angle AOB$: $4x + 3 = 4(7) + 3 = 28 + 3 = 31$. For $m\angle BOC$: $7x = 7(7) = 49$. For $m\angle COD$: $m\angle AOB = 31$. For $m\angle AOD$: $16x - 1 = 16(7) - 1 = 112 - 1 = 111$. $31 + 49 + 31 = 111$, so the answer checks. **79.** Since D is the midpt. of $\overline{AC}$ and since $DC = 16$, then $AD = 16$. Assign positions to each. Let $A = 0$, then $D = 16$, and $C = 32$, and since C is the midpt. of $\overline{AB}$, then $B = 64$. Since E is the midpt. of $\overline{AD}$, E is midway between 0 and 16, so $E = 8$. Since F is the midpt. of $\overline{ED}$, F is midway between 8 and 16, so $F = 12$. Since G is the midpt. of $\overline{EF}$, it is midway between 8 and 12, so $G = 10$. Since H is the midpt. of $\overline{DB}$, it is midway between 16 and 64, so $H = 40$. $GH = |G - H| = |10 - 40| = |-30| = 30$ **80a.** Answers may vary. Sample: 16 cm; 70° **80b.** Check students' work. **80c.** Check students' work. **81.** $81 + 24 = 105$ and $63 + 42 = 105$. They support the Angle Addition Post. **82.** $KC = 2x + 10 = 31$, so $2x = 21$ and $x = 10.5$. $KN = (2x + 10) + (4x + 1) = 6x + 11 = 6(10.5) + 11 = 63 + 11 = 74$. The answer is choice C. **83.** $KN = (2x + 10) + (4x + 1) = 29$, so $6x + 11 = 29$, then $6x = 18$, and $x = 3$. $CN = 4x + 1 = 4(3) + 1 = 12 + 1 = 13$. The answer is choice F. **84.** $2x + 10 = 4x + 1$; $-2x + 10 = 1$; $-2x = -9$; $x = 4.5$. $KC = 2x + 10 = 2(4.5) + 10 = 9 + 10 = 19$. The answer is choice D. **85.** Let n = the measure of the original angle. The measure of a right angle is 90. $n - 15 = 90$, so $n = 105$. The answer is choice H.

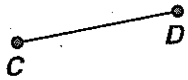
86. 

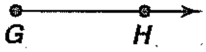
[2] **a.** The drawing should show $\overrightarrow{BD}$ in the interior of $\angle ABC$. **b.** An obtuse angle measures between 90 and 180 degrees; the least and greatest whole number values are 91 and 179 degrees. Part of $\angle ABC$ is 12°. So, the least measure for $\angle DBC$ is $91 - 12$, or 79. The greatest measure for $\angle DBC$ is $179 - 12$, or 167. **87.** By def. of skew lines, skew lines are *never* coplanar. **88.** By def. of skew lines, skew lines *never* intersect. **89.** By def. of opposite rays, opposite rays *always* form a line.
90. By def. of parallel lines, parallel lines *never* intersect.
91. Three points are either collinear or they are noncollinear. If they are collinear, then they can be on the intersection of two or more planes and lie in each of those planes. If they are noncollinear, then they lie in exactly one plane. Thus, three points are *always* coplanar.
92. By Post. 1-1, two points are *always* collinear.
93. By Post. 1-3, the intersection of two planes is *always* a line. **94.** By def. of parallel, intersecting lines are *never* parallel. **95.** Each term is 5 more than the previous term, so the next two terms are 25 and 30. **96.** Each term is 5 times the previous term, so the next two terms are 5(625) and 5(5)(625), or 3125 and 15,625. **97.** Each term is 4 more than the previous term, so the next two terms after 26 are 30 and 34.

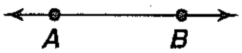
1-5 Basic Constructions

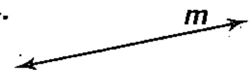
pages 34–40

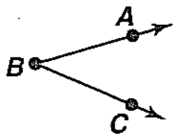
Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

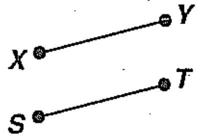
1. 

2. 

3. 

4. 

5. 

6. 

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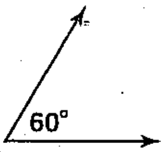
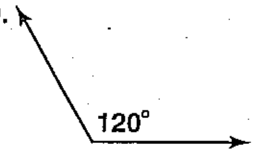
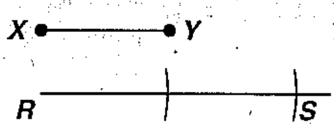
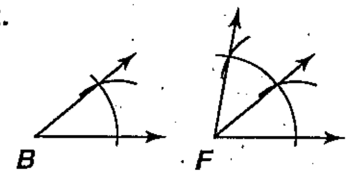
8. 

9. 

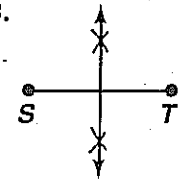
Check Understanding

1. 

Draw $\overrightarrow{RS}$. Place the point of the compass on X and open it to Y. Keeping the same setting, place the point of the compass on R and swing an arc that intersects $\overrightarrow{RS}$. Keeping the same setting, place the point of the compass on the intersection of the arc and the ray and swing a second arc that intersects with $\overrightarrow{RS}$. Label that intersection S.

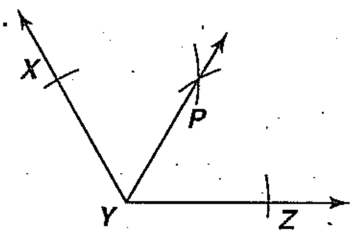
2. 

Draw a ray. Place the point of the compass on B and swing an arc. With the same setting, place the point of the compass on the endpt. of the ray and swing an arc that intersects the ray. Place the point of the compass on the arc of $\angle B$ where it intersects one of the sides and open it as far as the intersection of the arc with the other side. Keeping that setting, place the point of the compass on the intersection of the ray and arc and swing an arc that intersects the arc. Keeping the same setting, place the point of the compass on the intersection of the 2 rays and swing another arc that intersects the first arc on that figure. Draw a ray from the ray's endpt. through the second intersection of the two arcs.

3. 

With the compass open to more than half ST, place the point of the compass on S and swing an arc above and below the segment. Keep the same setting and repeat with the compass point on T. Draw a line through the two arc intersections.

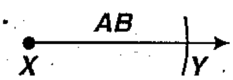
4. From Example 4, $m\angle JKN = 50$ and $\overrightarrow{KN}$ bisects $\angle JKL$. By def. of $\angle$ bisector, $m\angle NKL = m\angle JKN = 50$. Also by def. of $\angle$ bisector, $m\angle JKL = m\angle NKL + m\angle JKN = 50 + 50 = 100$.

5a. 

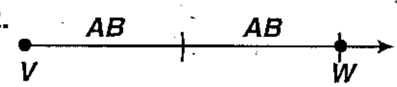
Place the point of the compass on Y and swing an arc that intersects both sides of the angle. With the point of the compass on one intersection, swing an arc in the interior of the angle. Keeping the same setting, place the point of the compass on the other arc intersection and swing an arc that intersects the arc in the interior of the angle. Label the intersection P. Draw $\overrightarrow{XP}$.

5b. Measure $\angle XYP$ and $\angle PYZ$ to see that they are $\cong$.

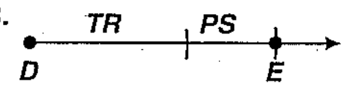
Exercises

1. 

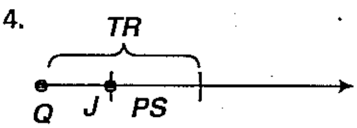
Draw $\overrightarrow{XY}$. Label the endpt. X. Place the point of the compass on A and open it to B. Keeping the same setting, place the point of the compass on X and swing an arc that intersects $\overrightarrow{XY}$. Label the intersection Y.

2. 

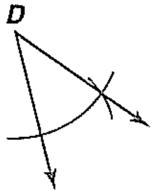
Draw $\overrightarrow{VW}$. Label the endpt. V. Place the point of the compass on A and open it to B. Keeping the same setting, place the point of the compass on V and swing an arc that intersects $\overrightarrow{VW}$. Keeping the same setting, place the point of the compass on the intersection and swing a second arc that intersects $\overrightarrow{VW}$. Label that intersection W.

3. 

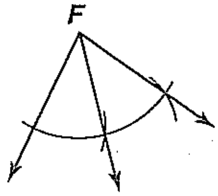
Draw $\overrightarrow{DE}$. Label the endpt. D. Place the point of the compass on T and open it to R. Keeping the setting, place the point of the compass on D and swing an arc that intersects $\overrightarrow{DE}$. Place the point of the compass on P and open it to S. Keeping the setting, place the point of the compass on the intersection of the arc and $\overrightarrow{DE}$. Swing an arc that intersects $\overrightarrow{DE}$ and label that intersection E.

4. 

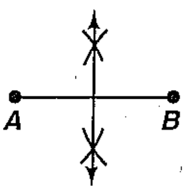
Draw $\overrightarrow{QJ}$. Label the endpt. Q. Place the point of the compass on T and open it to R. Keeping the setting, place the point of the compass on Q and swing an arc that intersects $\overrightarrow{QJ}$. Place the point of the compass on P and open it to S. Keeping the setting, place the point of the compass on the intersection of the arc and $\overrightarrow{QJ}$. Swing an arc towards Q that intersects $\overrightarrow{QJ}$ and label that intersection J.

5. 

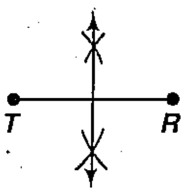
Draw a ray. Label the endpt. D. Place the point of the compass on C and swing an arc. With the same setting, place the point of the compass on D and swing an arc that intersects the ray. Place the point of the compass on the arc of $\angle C$ where it intersects one of the sides and open it as far as the intersection of the arc with the other side. Keeping that setting, place the point of the compass on the intersection of the ray and arc and swing an arc that intersects the arc. Draw a ray from D through the intersection of the two arcs.

6. 

Draw a ray. Label its endpt. F. Place the point of the compass on C and swing an arc. With the same setting, place the point of the compass on F and swing an arc that intersects the ray. Place the point of the compass on the arc of $\angle C$ where it intersects one of the sides and open it as far as the intersection of the arc with the other side. Keeping that setting, place the point of the compass on the intersection of the ray and arc and swing an arc that intersects the arc. Keeping the setting, place the point of the compass on the intersection of the 2 rays and swing another arc that intersects the first arc on that figure. Draw a ray from F through the second intersection of the two arcs.

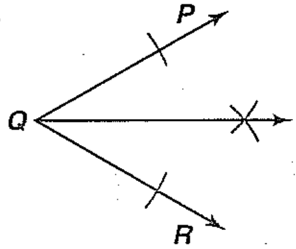
7. 

With the compass open to more than half AB, place the point of the compass on A and swing an arc above and below the segment. Keep the same setting and repeat with the compass point on B. Draw a line through the two arc intersections.

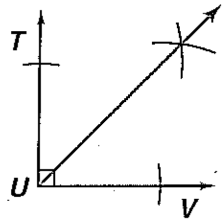
8. 

With the compass open to more than half TR, place the point of the compass on T and swing an arc above and below the segment. Keep the same setting and repeat with the compass point on S. Draw a line through the two arc intersections.

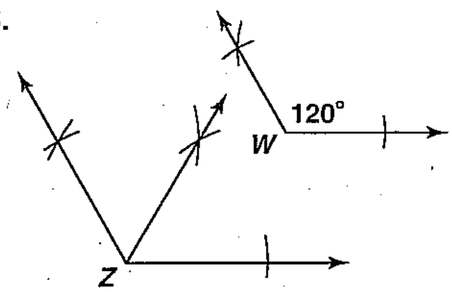
9a. By def. of $\angle$ bis., $m\angle FGH = m\angle HGI$, so $3x - 3 = 4x - 14$. Then $-x = -11$, so $x = 11$. Substituting, $3x - 3 = 3(11) - 3 = 33 - 3 = 30$, so $m\angle FGH = 30$. **9b.** Since, by def. of $\angle$ bis., $m\angle FGH = m\angle HGI$, $m\angle HGI = 30$. **9c.** By def. of $\angle$ bis., $m\angle FGI = m\angle FGI + m\angle HGI = 30 + 30 = 60$. **10.** By def. of $\angle$ bis., $m\angle XBC = m\angle ABX$, so $3x + 10 = 5x$; $10 = 2x$; $x = 5$. By def. of $\angle$ bis., $m\angle ABC = 2m\angle ABX = 2(5x) = 10x = 10(5) = 50$. **11.** By def. of $\angle$ bis., $m\angle ABC = 2m\angle ABX$, so $4x - 12 = 2(24)$; $4x - 12 = 48$; $4x = 60$; $x = 15$. $m\angle ABC = 2(24) = 48$ **12.** By def. of $\angle$ bis., $m\angle ABX = m\angle CBX$, so $4x - 16 = 2x + 6$; $2x - 16 = 6$; $2x = 22$; $x = 11$. $m\angle ABC = 2m\angle CBX = 2(2x + 6) = 4x + 12 = 4(11) + 12 = 44 + 12 = 56$

13. 

Place the point of the compass on Q and swing an arc that intersects both sides of the angle. With the point of the compass on one intersection, swing an arc in the interior of the angle. Keeping the same setting, place the point of the compass on the other arc intersection and swing an arc that intersects the arc in the interior of the angle. Draw a ray whose endpoint is Q through the intersection of the two arcs.

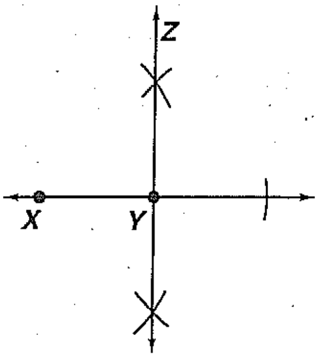
14. 

Place the point of the compass on U and swing an arc that intersects both sides of the angle. With the point of the compass on one intersection, swing an arc in the interior of the angle. Keeping the same setting, place the point of the compass on the other arc intersection and swing an arc that intersects the arc in the interior of the angle. Draw a ray whose endpoint is U through the intersection of the two arcs.

15. 

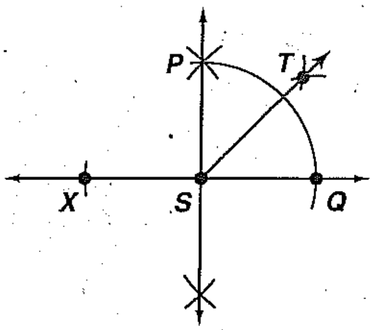
Draw a ray with endpt. Z. With the compass point on W, swing an arc that intersects both sides of the angle. Keeping the setting, place the compass point on Z and swing an arc similar to the first arc that intersects the ray. With the compass point on the intersection of the angle side and the arc, open the compass to the intersection of the arc with the other side of the angle. Keeping the setting, place the compass point on the intersection of the ray and arc and swing an arc that intersects the first arc of that figure. With endpt. Z, draw a ray through the intersection of the two arcs. To construct the bisector, place the compass point on the arc and its intersection with one of the sides of $\angle Z$. Swing an arc in the interior of the angle. Keeping the setting, place the compass point on the intersection of the first arc and the other side of the angle. Swing an arc that intersects with the previous arc. With endpt. Z, draw a ray through the intersection of the two arcs.

16.



Find a segment on $\overleftrightarrow{XY}$ and construct $\overleftrightarrow{YZ}$ as its $\perp$ bis. Place the compass point on X and open it to Y. Keep the setting and place the compass point on Y and swing an arc to intersect $\overleftrightarrow{XY}$ away from X. Place the compass point on X and open it to more than half XY. Swing an arc on both sides of the segment. Keep the setting and place the compass point on the intersection of the first arc with $\overleftrightarrow{XY}$. Swing arcs that intersect with the arcs above and below the segment. Draw $\overleftrightarrow{YZ}$ through the two arc intersections.

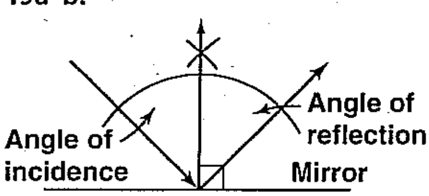
17.



Find a segment on $\overleftrightarrow{SQ}$ to construct $\overleftrightarrow{SP}$ as the $\perp$ bis. Construct a segment measuring $2SQ$. Call it $\overline{XQ}$. With the compass point on X, swing congruent arcs on both sides of $\overleftrightarrow{SP}$. Keep the same setting and with the compass point on Q, swing arcs that intersect those arcs. Draw $\overleftrightarrow{SP}$ through the arc intersections. To bisect $\angle PSQ$, place the compass point on S and swing an arc that intersects both sides of the angle. Place the compass point on one of the intersections of the arc and side and swing an arc in the interior of the angle. Keep the same setting and place the compass point on the other intersection of the arc and side. Swing an arc to intersect the previous arc. Label that intersection T. Draw $\overrightarrow{ST}$.

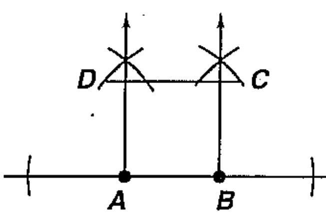
18a. According to the diagram, the angle of reflection is $\angle CBD$. $m\angle CBD = m\angle CBD = 41$ **18b.** $m\angle ABD = m\angle ABC + m\angle CBD = 41 + 41 = 82$ **18c.** $m\angle CBE = 90$. From part ⓑ, $m\angle ABC = 41$. $m\angle ABE + m\angle ABC = m\angle CBE$; $m\angle ABE + 41 = 90$; $m\angle ABE = 49$. Similarly, $m\angle DBF = 49$.

19a–b.



The angle of incidence and angle of reflection should be $\cong$.

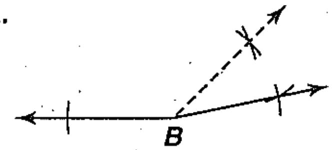
20.



Locate points A and B on a line. Then construct a $\perp$ ray at A and B as in Exercise 16. Construct $\overline{AD}$ and $\overline{BC}$ so that $AB = AD = BC$. Draw $\overline{DC}$.

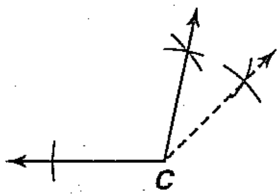
21a. One midpoint; explanations may vary. Sample: A midpt. divides a segment into two $\cong$ segments. If there were more than one midpt., the segments wouldn't be $\cong$. **21b.** A segment has infinitely many bisectors, since infinitely many lines can pass through the midpt. A segment has exactly one $\perp$ bisecting line, since it's impossible to construct more than one $\perp$ bisector of a segment, there can be only one line $\perp$ to a segment at its midpt. **21c.** There are an infinite number of lines in space that are $\perp$ to a segment at its midpt. Since the $\perp$ bisectors intersect, the lines are coplanar.

22.



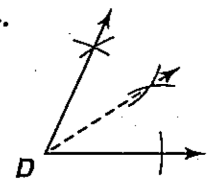
Construct an angle $\cong$ to $\angle 1$. Then, using one of the sides of the constructed angle, construct an angle $\cong$ to $\angle 2$. Make sure the second angle is not in the interior of the first.

23.



Construct an angle $\cong$ to $\angle 1$. Then, using one of the sides of the constructed angle, construct an angle $\cong$ to $\angle 2$. Make sure the second angle is in the interior of the first.

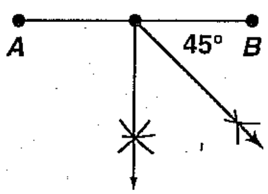
24.



Construct an angle $\cong$ to $\angle 2$. Then, using one of the sides of the constructed angle, construct a second angle $\cong$ to $\angle 2$.

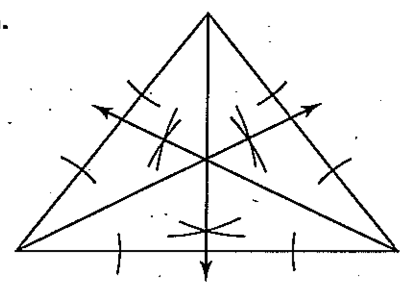
25. They are both correct. Multiplying both sides of Lani's equation by 2 results in Denyse's equation. Or, multiplying both sides of Denyse's equation by $\frac{1}{2}$ results in Lani's equation. **26.** Construct the $\perp$ bisector of the segment: Open the compass to more than half the measure of the segment. Swing large congruent arcs from each endpt. to intersect on each side of the segment. Draw a line through the two arc intersections. The intersection of the line and segment is the midpt. of the segment.

27.



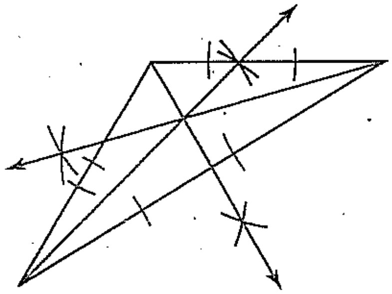
Construct a $\perp$ bisector. Then bisect one of the angles by construction.

28a.



The angle bisectors appear to meet at one point.

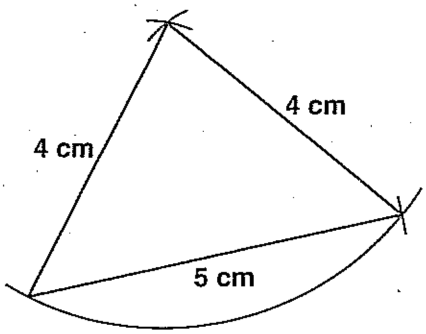
28b.



The angle bisectors appear to meet at one point.

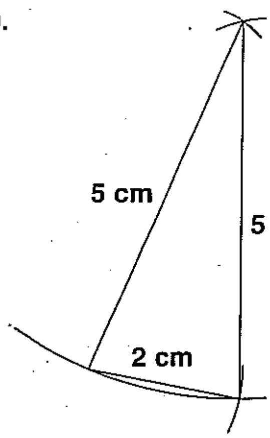
28c. The angle bisectors of a triangle intersect in one point.

29.



Draw a ray. Open the compass to 5 cm and construct a 5-cm segment on the ray. Open the compass to 4 cm. Swing a 4-cm arc from each endpt. of the 5-cm segment so that the two arcs intersect. Draw segments from each endpt. of the 5-cm segment to the intersection of the two arcs.

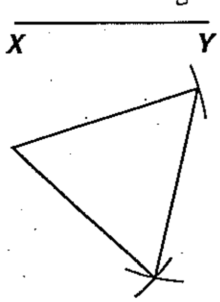
30.



Draw a ray. Open the compass to 2 cm and construct a 2-cm segment on the ray. Open the compass to 5 cm. Swing a 5-cm arc from each endpt. of the 2-cm segment so that the two arcs intersect. Draw segments from each endpt. of the 2-cm segment to the intersection of the two arcs.

31. Draw a ray. Construct a 5-cm segment. 2-cm arcs from the endpts. of the 5-cm segment will not intersect. The triangle is impossible because $2 + 2 < 5$. The short segments are not long enough to form a triangle.
32. Draw a ray. Construct a 4-cm segment. 2-cm arcs from the endpts. of the 4-cm segment intersect on the segment. The triangle is impossible because $2 + 2 = 4$. The short segments are not long enough to form a triangle.

33a.

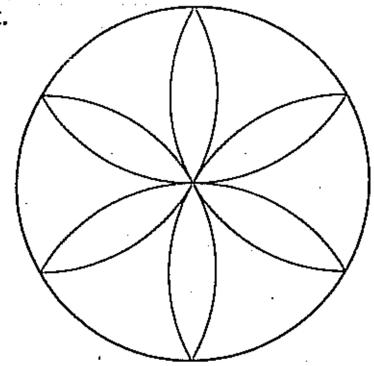


Draw $\overline{XY}$, then draw a ray. Construct a segment of length XY. From the endpts. of the constructed segment, swing arcs of length XY so they intersect. Draw segments from the arc intersection to each endpt.

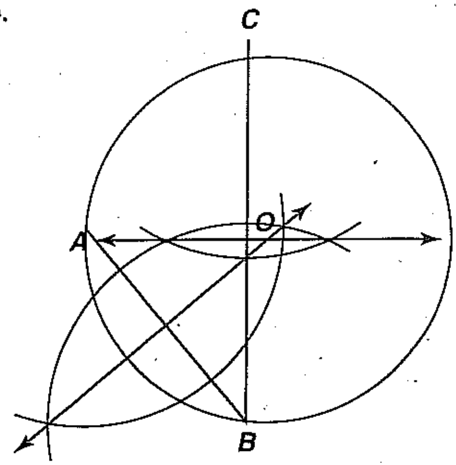
33b. Each angle measures 60.

33c. Answers may vary. Sample: to construct a 60° angle, mark a pt. A. Swing a long arc from A. From a pt. P on the arc, swing another arc the same size that intersects the first arc and label that pt. Q. Draw $\angle PAQ$. To construct a 30° angle, bisect the 60° angle.

34a–c.

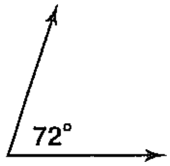


35a–b.



35c. Point O is the center of the circle. **36.** r is $\perp$ to two lines in plane M and is $\perp$ to the plane. t is $\perp$ to only one line in the plane and is not $\perp$ to the plane. A line is perpendicular to a plane if it is *perpendicular* to every line in the plane that *the line intersects*. **37.** Construction does not use numbered measuring tools, so eliminate choices A and B. The perpendicular bisector bisects the segment, thereby locating its midpt. The answer is choice D. **38.** Construction of a congruent segment can begin with a ray or a line. Points are then labeled on the ray or line. The answer is choice F. **39.** [2] **a.** Draw $\overleftrightarrow{XY}$. With the compass pt. on B, swing an arc that intersects $\overrightarrow{BA}$ and $\overrightarrow{BC}$. Label the intersections P and Q, respectively. With the compass point on X, swing a congruent arc intersecting $\overrightarrow{XY}$. Label the intersection K. Open the compass to PQ. With compass pt. on K, swing the PQ arc to intersect the first arc. Label the intersection R. Draw $\overrightarrow{XR}$. **b.** With compass open to XK, put the compass point on X and swing the XK arc intersecting $\overrightarrow{XR}$. With compass on R, swing an arc open to KR to intersect the first arc. Label the intersection T. Draw $\overrightarrow{XT}$. [1] one part correct **40.** [4] **a.** Construct the $\perp$ bisector of the segment. **b.** Construct the $\perp$ bisector. Then construct the $\perp$ bisector of each of the two new segments.
c. Draw $\overrightarrow{AB}$. Construct a congruent segment. Construct the $\perp$ bisector of the segment. Construct the $\perp$ bisector of one of the 2 new segments. Open the compass to the

length of the shortest segment. With the point of the compass on B, swing an arc in the opposite direction from A intersecting $\overrightarrow{AB}$ at C. $AC = 1.25AB$ [3] Explanations are not thorough. [2] two correct explanations [1] part (a) correct **41.** $AC = |-7-(-1)| = |-7+1| = |-6| = 6$ **42.** $AD = |-7-3| = |-10| = 10$ **43.** $CD = |-1-3| = |-4| = 4$ **44.** $BC = |-4-(-1)| = |-4+1| = |-3| = 3$

45. 

Draw a ray. Place the center of the protractor on the endpoint of the ray so the ray intersects the 0 on the protractor. Count up from 0 to 72 and mark it with a point. Draw a ray from the endpt. of the first ray through the point.

46. By def. of straight angle, $m\angle DEF = 180$; $m\angle DEG + m\angle GEF = m\angle DEF$; $80 + m\angle GEF = 180$; $m\angle GEF = 100$. **47.** If W is in the interior of $\angle TUV$, then $m\angle VUW + m\angle TUW = m\angle TUV$; $80 + m\angle TUW = 100$; $m\angle TUW = 20$. If W is not in the interior of $\angle TUV$, then $m\angle TUV + m\angle VUW = m\angle TUW$; $100 + 80 = m\angle TUW$; $m\angle TUW = 180$.

48. 

The name of the figure shows two arrows and no endpoints, so the figure is a line.

49. By def. of opposite rays, they are not opposite rays because they do not have the same endpoint. **50.** The endpoints of $\overline{RS}$ are R and S, the same endpoints as $\overline{SR}$. So, the two segments are the same.

TECHNOLOGY — *page 41*

1a. Answers may vary. Sample: With "Draw" you can change measures by moving points. The tools allow you to *draw* objects with very few constraints. A *construction* attaches some type of geometric property to the object being created. It will retain that property no matter how the figure is manipulated. **1b.** Construction is exact, and drawing is not. **2a–b.** Check students' work. **2c.** The angle measures are always =, so $\overrightarrow{KM}$ is always the angle bisector of $\angle JKL$. **3a.** Check students' work. **3b.** Since $\overrightarrow{OQ}$ was drawn and not constructed, it is not always the angle bisector of $\angle NOP$.

INVESTIGATION — *page 42*

1. On the map, it is about $\frac{27}{32}$ in. from 44th and 7th east to 5th, and $2\frac{11}{16}$ in. from 44th and 5th to Madison Square Park, for a total of $3\frac{17}{32}$ in. The key shows 1 in. is about 0.4 mi, so the distance is $3\frac{17}{32}(0.4)$, or about 1.4 mi. In metric units, the distance is $3\frac{17}{32}(0.64)$, or about 2.3 km. **2.** On the map, it is $2\frac{11}{16}$ in. south on 7th and about $\frac{27}{32}$ in. east to the park for a total of $3\frac{17}{32}$ in. The key shows 1 in. is about 0.4 mi, so the distance is $3\frac{17}{32}(0.4)$, or about 1.4 mi. In metric units, the distance is $3\frac{17}{32}(0.64)$, or about 2.3 km. **3.** On the map, it is $1\frac{11}{32}$ in. from 44th to 34th, and $1\frac{1}{2}$ in. from 34th to the park for a total of $2\frac{27}{32}$ in. The key shows 1 in. is about 0.4 mi, so the distance is $2\frac{27}{32}(0.4)$, or about $1\frac{1}{8}$ mi. In metric units, the distance is $2\frac{27}{32}(0.64)$, or about 1.8 km. **4.** Yvonne's route is the shortest because it is the most direct. **5.** Yvonne's father's route and her mother's route tie for the longest route. They walked the sides of a rectangle.

1-6 The Coordinate Plane — *pages 43–49*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*
1. 5.0 **2.** 4.1 **3.** 11.1 **4.** 100 **5.** 100 **6.** 58 **7.** 196 **8.** 10 **9.** −1

Check Understanding **1a.** $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(-4-1)^2+(4-(-3))^2} = \sqrt{(-4-1)^2+(4+3)^2} = \sqrt{(-5)^2+(7)^2} = \sqrt{25+49} = \sqrt{74} \approx 8.6$ **1b.** $d = \sqrt{(5-(-4))^2+(2-(-1))^2} = \sqrt{(5+4)^2+(2+1)^2} = \sqrt{(9)^2+(3)^2} = \sqrt{81+9} = \sqrt{90}$. The results are the same because the differences are opposites, and the square of a number and the square of its opposite are the same. **2a.** Elm Station is $(-3,-6)$ and Symphony Station is $(1,2)$. $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(-3-1)^2+(-6-2)^2} = \sqrt{(-4)^2+(-8)^2} = \sqrt{16+64} = \sqrt{80}$, or about 8.9 mi. **2b.** Maple Station is $(-6,2)$ and Cedar Station is $(-3,1)$. $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(-6-(-3))^2+(2-1)^2} = \sqrt{(-6+3)^2+(2-1)^2} = \sqrt{(-3)^2+(1)^2} = \sqrt{9+1} = \sqrt{10}$, or about 3.2 mi. **3.** The midpt. is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{2+6}{2}, \frac{-5+13}{2}\right) = \left(\frac{8}{2}, \frac{8}{2}\right) = (4,4)$. **4.** The midpt. is $(4,-6)$ and the unknown endpt. is (x_2, y_2). For the x-coordinate, $\frac{2+x_2}{2} = 4$; $2 + x_2 = 8$; $x_2 = 6$. For the y-coordinate, $\frac{-3+y_2}{2} = -6$; $-3 + y_2 = -12$; $y_2 = -9$. The coordinates of Y are $(6,-9)$.

Exercises **1.** $JI = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(2-2)^2+(-1-5)^2} = \sqrt{(0)^2+(-6)^2} = \sqrt{0+36} = \sqrt{36} = 6$
2. $LM = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(10+8)^2+(0)^2} = \sqrt{(18)^2+0} = \sqrt{(18)^2} = 18$
3. $NP = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(-1-(-1))^2+(-11-(-3))^2} = \sqrt{(-1+1)^2+(-11+3)^2} = \sqrt{(0)^2+(-8)^2} = \sqrt{64} = 8$ **4.** $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} =$

$\sqrt{(0-0)^2+(3-12)^2}=\sqrt{(0)^2+(-9)^2}=$ $\sqrt{0+81}=\sqrt{81}=9$ **5.** $CD=$ $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=$ $\sqrt{(12-(-8))^2+(6-18)^2}=$ $\sqrt{(12+8)^2+(6-18)^2}=\sqrt{(20)^2+(-12)^2}=$ $\sqrt{400+144}=\sqrt{544}\approx 23.3$

6. $EF=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=$ $\sqrt{(6-(-2))^2+(-2-4)^2}=$ $\sqrt{(6+2)^2+(-2-4)^2}=\sqrt{(8)^2+(-6)^2}=$ $\sqrt{64+36}=\sqrt{100}=10$ **7.** $QT=$ $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=$ $\sqrt{(12-5)^2+(-12-12)^2}=\sqrt{(7)^2+(-24)^2}=$ $\sqrt{49+576}=\sqrt{625}=25$

8. $RS=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=$ $\sqrt{(0-12)^2+(5-3)^2}=\sqrt{(-12)^2+(2)^2}=$ $\sqrt{144+4}=\sqrt{148}\approx 12.2$

9. $XY=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=$ $\sqrt{(-3-5)^2+(-4-5)^2}=$ $\sqrt{(-8)^2+(-9)^2}=\sqrt{64+81}=\sqrt{145}\approx 12.0$

10. North Station is $(0, 5)$, and South Station is $(0, -4)$. $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=$ $\sqrt{(0-0)^2+(5-(-4))^2}=\sqrt{(0)^2+(5+4)^2}=$ $\sqrt{(9)^2}=9$, or 9 mi. **11.** Oak Station is $(-1, -2)$, and Symphony Station is $(1, 2)$. $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=$ $\sqrt{(-1-1)^2+(-2-2)^2}=$ $\sqrt{(-2)^2+(-4)^2}=\sqrt{4+16}=\sqrt{20}\approx 4.5$. The distance is about 4.5 mi. **12.** City Plaza is $(0, 0)$, and Cedar Station is $(-3, 1)$. $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=$ $\sqrt{(0-(-3))^2+(0-1)^2}=\sqrt{(3)^2+(-1)^2}=$ $\sqrt{9+1}=\sqrt{10}\approx 3.2$. The distance is about 3.2 mi.

13. Station A is $(0, 0)$ and Station B is $(-4, -5)$. $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=$ $\sqrt{(0-(-4))^2+(0-(-5))^2}=$ $\sqrt{(0+4)^2+(0+5)^2}=\sqrt{(4)^2+(5)^2}=\sqrt{16+25}=$ $\sqrt{41}\approx 6.4$ **14.** Station B is $(-4, -5)$ and Station C is $(5, 8)$. $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=$ $\sqrt{(-4-5)^2+(-5-8)^2}=$ $\sqrt{(-9)^2+(-13)^2}=\sqrt{81+169}=\sqrt{250}\approx 15.8$

15. Station B is $(-4, -5)$ and Station D is $(1, 10)$. $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=$ $\sqrt{(-4-1)^2+(-5-10)^2}=$ $\sqrt{(-5)^2+(-15)^2}=\sqrt{25+225}=\sqrt{250}\approx 15.8$

16. Station E is $(2, 12)$ and Station F is $(5, 16)$. $d=$ $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=$ $\sqrt{(2-5)^2+(12-16)^2}=\sqrt{(-3)^2+(-4)^2}=$ $\sqrt{9+16}=\sqrt{25}=5.0$ **17.** From Exercise 13, Station A is $\sqrt{41}$ from Station A. Station C is at $(5, 8)$, so $AC=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=$ $\sqrt{(5-0)^2+(8-0)^2}=\sqrt{(5)^2+(8)^2}=$ $\sqrt{25+64}=\sqrt{89}$. Station D is $(1, 10)$, so $AD=$ $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=$ $\sqrt{(1-0)^2+(10-0)^2}=\sqrt{(1)^2+(10)^2}=$ $\sqrt{1+100}=\sqrt{101}$. Station E is $(2, 12)$, so $AE=$ $\sqrt{(2)^2+(12)^2}=\sqrt{4+144}=\sqrt{148}$. Station F is $(5, 16)$, so $AF=\sqrt{(5)^2+(16)^2}=\sqrt{25+256}=\sqrt{281}$. Since $\sqrt{41}<\sqrt{89}<\sqrt{101}<\sqrt{148}<\sqrt{281}$, the stations in order of least to greatest distance from Station A are B, C, D, E, and F. **18.** The midpt. is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)=\left(\frac{0+8}{2}, \frac{0+4}{2}\right)=\left(\frac{8}{2}, \frac{4}{2}\right)=(4, 2)$.

19. The midpt. is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)=\left(\frac{-1+7}{2}, \frac{3+(-1)}{2}\right)=$ $\left(\frac{6}{2}, \frac{2}{2}\right)=(3, 1)$. **20.** The midpt. is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)=$ $\left(\frac{13+(-6)}{2}, \frac{8+(-6)}{2}\right)=\left(\frac{7}{2}, \frac{2}{2}\right)=(3.5, 1)$.

21. The midpt. is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)=\left(\frac{7+5}{2}, \frac{10+(-8)}{2}\right)=$ $\left(\frac{12}{2}, \frac{2}{2}\right)=(6, 1)$. **22.** The midpt. is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)=$ $\left(\frac{-4.5}{2}, \frac{4.2}{2}\right)=(-2.25, 2.1)$. **23.** The midpt. is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)=\left(\frac{5\frac{1}{2}+2\frac{1}{4}}{2}, \frac{-4\frac{3}{4}+(-1\frac{1}{4})}{2}\right)=$ $\left(\frac{5\frac{2}{4}+2\frac{1}{4}}{2}, \frac{-4\frac{3}{4}-1\frac{1}{4}}{2}\right)=\left(\frac{7\frac{3}{4}}{2}, \frac{-5\frac{4}{4}}{2}\right)=\left(\frac{7\frac{3}{4}}{2}, \frac{-6}{2}\right)=$ $\left(\frac{\frac{31}{4}}{2}, \frac{-6}{2}\right)=\left(\frac{31}{4}\div 2, \frac{-6}{2}\right)=\left(\frac{31}{4}\cdot\frac{1}{2}, \frac{-6}{2}\right)=\left(\frac{31}{8}, \frac{-6}{2}\right)=$ $\left(3\frac{7}{8}, -3\right)$. **24.** The midpt. is $(5, -8)$, T is $(0, 4)$, and the unknown endpt. is (x_2, y_2). For the x-coordinate, $\frac{0+x_2}{2}=5; x_2=10$. For the y-coordinate, $\frac{4+y_2}{2}=-8;$ $4+y_2=-16; y_2=-20$. The coordinates of Y are $(10, -20)$. **25.** The midpt. is $(5, -8)$, T is $(5, -15)$, and the unknown endpt. is (x_2, y_2). For the x-coordinate, $\frac{5+x_2}{2}=$ $5; 5+x_2=10; x_2=5$. For the y-coordinate, $\frac{-15+y_2}{2}=$ $-8; -15+y_2=-16; y_2=-1$. The coordinates of Y are $(5, -1)$. **26.** The midpt. is $(5, -8)$, T is $(10, 18)$, and the unknown endpt. is (x_2, y_2). For the x-coordinate, $\frac{10+x_2}{2}=5; 10+x_2=10; x_2=0$. For the y-coordinate, $\frac{18+y_2}{2}=-8; 18+y_2=-16; y_2=-34$. The coordinates of Y are $(0, -34)$. **27.** The midpt. is $(5, -8)$, T is $(-2, 8)$, and the unknown endpt. is (x_2, y_2). For the x-coordinate, $\frac{-2+x_2}{2}=5; -2+x_2=10; x_2=12$. For the y-coordinate, $\frac{8+y_2}{2}=-8; 8+y_2=-16; y_2=-24$. The coordinates of Y are $(12, -24)$. **28.** The midpt. is $(5, -8)$, T is $(1, 12)$, and the unknown endpt. is (x_2, y_2). For the x-coordinate, $\frac{1+x_2}{2}=5; 1+x_2=10; x_2=9$. For the y-coordinate, $\frac{12+y_2}{2}=-8; 12+y_2=-16; y_2=-28$.

The coordinates of Y are $(9, -28)$. **29.** The midpt. is $(5, -8)$, T is $(4.5, -2.5)$, and the unknown endpt. is (x_2, y_2). For the x-coordinate, $\frac{4.5 + x_2}{2} = 5; 4.5 + x_2 = 10;$ $x_2 = 5.5$. For the y-coordinate, $\frac{-2.5 + y_2}{2} = -8;$ $-2.5 + y_2 = -16; y_2 = -13.5$. The coordinates of Y are $(5.5, -13.5)$. **30.** The endpt. is $(2, 6)$, the midpt. is $(5, 12)$, and the unknown endpt. is (x_2, y_2). For the x-coordinate, $\frac{2 + x_2}{2} = 5; 2 + x_2 = 10; x_2 = 8$. For the y-coordinate, $\frac{6 + y_2}{2} = 12; 6 + y_2 = 24; y_2 = 18$. $(x_2, y_2) = (8, 18)$ **31.** The endpt. is $(2, 3)$, the midpt. is $(3, -4)$, and the unknown endpt. is (x_2, y_2). For the x-coordinate, $\frac{2 + x_2}{2} = 3; 2 + x_2 = 6; x_2 = 4$. For the y-coordinate, $\frac{3 + y_2}{2} = -4; 3 + y_2 = -8; y_2 = -11$. $(x_2, y_2) = (4, -11)$ **32a.** $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3 - 6)^2 + (2 - 6)^2} = \sqrt{(-3)^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5.0$ **32b.** The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{3 + 6}{2}, \frac{2 + 6}{2}\right) = \left(\frac{9}{2}, \frac{8}{2}\right) = (4.5, 4)$.

33a. $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0 - 3)^2 + (-2 - 3)^2} = \sqrt{(-3)^2 + (-5)^2} = \sqrt{9 + 25} = \sqrt{34} \approx 5.8$ **33b.** The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0 + 3}{2}, \frac{-2 + 3}{2}\right) = \left(\frac{3}{2}, \frac{1}{2}\right) = (1.5, 0.5)$.

34a. $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-4 - 1)^2 + (-2 - 3)^2} = \sqrt{(-5)^2 + (-5)^2} = \sqrt{25 + 25} = \sqrt{50} \approx 7.1$ **34b.** The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-4 + 1}{2}, \frac{-2 + 3}{2}\right) = \left(\frac{-3}{2}, \frac{1}{2}\right) = (-1.5, 0.5)$. **35a.** $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-5 - 0)^2 + (2 - 4)^2} = \sqrt{(-5)^2 + (-2)^2} = \sqrt{25 + 4} = \sqrt{29} \approx 5.4$ **35b.** The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-5 + 0}{2}, \frac{2 + 4}{2}\right) = \left(\frac{-5}{2}, \frac{6}{2}\right) = (-2.5, 3)$.

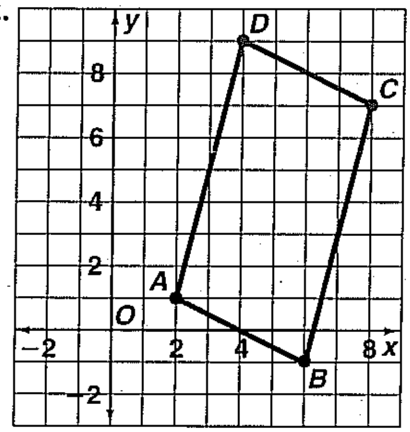
36a. $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-3 - 5)^2 + (-1 - (-7))^2} = \sqrt{(-3 - 5)^2 + (-1 + 7)^2} = \sqrt{(-8)^2 + (6)^2} = \sqrt{64 + 36} = \sqrt{100} = 10$ **36b.** The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3 + 5}{2}, \frac{-1 + (-7)}{2}\right) = \left(\frac{2}{2}, \frac{-8}{2}\right) = (1, -4)$. **37a.** $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-5 - (-3))^2 + (-3 - (-5))^2} = \sqrt{(-5 + 3)^2 + (-3 + 5)^2} = \sqrt{(-2)^2 + (2)^2} = \sqrt{4 + 4} = \sqrt{8} \approx 2.8$ **37b.** The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-5 + (-3)}{2}, \frac{-3 + (-5)}{2}\right) = \left(\frac{-8}{2}, \frac{-8}{2}\right) = (-4, -4)$. **38a.** $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-4 - (-1))^2 + (-5 - 1)^2} = \sqrt{(-4 + 1)^2 + (-5 - 1)^2} = \sqrt{(-3)^2 + (-6)^2} = \sqrt{9 + 36} = \sqrt{45} \approx 6.7$ **38b.** The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-4 + (-1)}{2}, \frac{-5 + 1}{2}\right) = \left(\frac{-5}{2}, \frac{-4}{2}\right) = (-2.25, -2)$.

39a. $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2 - 4)^2 + (3 - (-2))^2} = \sqrt{(2 - 4)^2 + (3 + 2)^2} = \sqrt{(-2)^2 + (5)^2} = \sqrt{4 + 25} = \sqrt{29} \approx 5.4$

39b. The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2 + 4}{2}, \frac{3 + (-2)}{2}\right) = \left(\frac{6}{2}, \frac{1}{2}\right) = (3, 0.5)$.

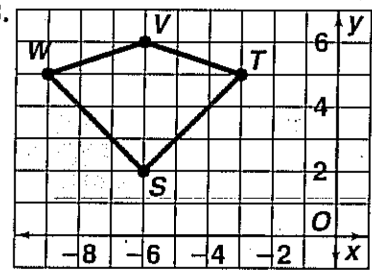
40a. $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4 - 3)^2 + (2 - 0)^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{1 + 4} = \sqrt{5} \approx 2.2$ **40b.** The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{4 + 3}{2}, \frac{2 + 0}{2}\right) = \left(\frac{7}{2}, \frac{2}{2}\right) = (3.5, 1)$. **41.** By symmetry, Quadrant IV contains S.

42.



The midpt. of $\overline{AC}$ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{2 + 8}{2}, \frac{1 + 7}{2}\right) = \left(\frac{10}{2}, \frac{8}{2}\right) = (5, 4)$. The midpt. of $\overline{BD}$ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{6 + 4}{2}, \frac{-1 + 9}{2}\right) = \left(\frac{10}{2}, \frac{8}{2}\right) = (5, 4)$. The midpts are both at $(5, 4)$. The diagonals bisect each other.

43.



$ST = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-6 - (-3))^2 + (2 - 5)^2} = \sqrt{(-6 + 3)^2 + (2 - 5)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9 + 9} = \sqrt{18}$; $TV = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-3 - (-6))^2 + (5 - 6)^2} = \sqrt{(-3 + 6)^2 + (5 - 6)^2} = \sqrt{(3)^2 + (-1)^2} = \sqrt{9 + 1} = \sqrt{10}$; $VW = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-6 - (-9))^2 + (6 - 5)^2} = \sqrt{(-6 + 9)^2 + (6 - 5)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9 + 1} = \sqrt{10}$; $SW = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} =$

$\sqrt{(-6-(-9))^2+(2-5)^2} = \sqrt{(-6+9)^2+(2-5)^2} = \sqrt{(3)^2+(-3)^2} = \sqrt{9+9} = \sqrt{18}$. Since $\sqrt{10} \neq \sqrt{18}$, the sides are not $\cong$. However, $ST = SW$ and $TV = VW$.

44a. $A = (-9, -6)$ and $B = (6, 6)$, so $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(-9-6)^2+(-6-6)^2} = \sqrt{(-15)^2+(-12)^2} = \sqrt{225+144} = \sqrt{369}$, or about 19.2 units. **44b.** The midpt. is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-9+6}{2}, \frac{-6+6}{2}\right) = \left(\frac{-3}{2}, \frac{0}{2}\right) = (-1.5, 0)$.

45a. $A = (-2, -2)$ and $B = (8, -6)$, so $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(-2-8)^2+(-2-(-6))^2} = \sqrt{(-2-8)^2+(-2+6)^2} = \sqrt{(-10)^2+(4)^2} = \sqrt{100+16} = \sqrt{116}$, or about 10.8 units. **45b.** The midpt. is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{-2+8}{2}, \frac{-2+(-6)}{2}\right) = \left(\frac{6}{2}, \frac{-8}{2}\right) = (3, -4)$. **46a.** $A = (0, 3)$ and $B = (-2, -2)$, so $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(0-(-2))^2+(3-(-2))^2} = \sqrt{(0+2)^2+(3+2)^2} = \sqrt{(2)^2+(5)^2} = \sqrt{4+25} = \sqrt{29}$, or about 5.4 units. **46b.** The midpt. is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{0+(-2)}{2}, \frac{3+(-2)}{2}\right) = \left(\frac{-2}{2}, \frac{1}{2}\right) = (-1, 0.5)$. **47.** $XY = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(5-(-6))^2+(-2-9)^2} = \sqrt{(5+6)^2+(-2-9)^2} = \sqrt{(11)^2+(-11)^2} = \sqrt{121+121} = \sqrt{242}$, or about 15.6 units. $XZ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(5-17)^2+(-2-(-3))^2} = \sqrt{(5-17)^2+(-2+3)^2} = \sqrt{(-12)^2+(1)^2} = \sqrt{144+1} = \sqrt{145} \approx 12.0$, or about 12 units. Since $15.6 > 12.0$, Z is closer. Z is about 12 units from X.

48. The possible routes are $TU + UV$ or $TV + UV$, so a comparison of TU and TV will expose the shortest route. $TU = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(80-20)^2+(20-60)^2} = \sqrt{(60)^2+(-40)^2} = \sqrt{3600+1600} = \sqrt{5200}$; $TV = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(80-110)^2+(20-85)^2} = \sqrt{(30)^2+(-65)^2} = \sqrt{900+4225} = \sqrt{5125}$. Since $\sqrt{5125} < \sqrt{5200}$, T to V to U is the shortest route. **49.** Houston = (8936, 3542) and Chicago = (5985, 3439), so $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(8936-5985)^2+(3542-3439)^2} = \sqrt{(2951)^2+(103)^2} = \sqrt{8{,}708{,}401+10{,}609} = \sqrt{8{,}719{,}010} \approx 2952.8$; $2952.8(\sqrt{0.1}) \approx 934$, or about 934 mi.

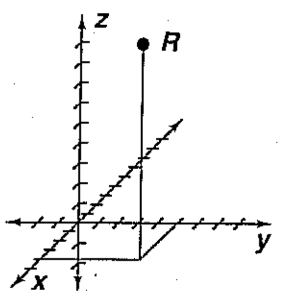
50. Denver = (7490, 5881) and New Orleans = (8448, 2625), so $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(7490-8448)^2+(5881-2625)^2} = \sqrt{(958)^2+(3256)^2} = \sqrt{917{,}764+10{,}601{,}536} = \sqrt{11{,}519{,}300} \approx 3394.0$; $3394.0(\sqrt{0.1}) \approx 1073$, or about 1073 mi. **51.** Boston = (4422, 1241) and San Francisco = (8495, 8720), so $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(4422-8495)^2+(1241-8720)^2} = \sqrt{(-4073)^2+(-7479)^2} = \sqrt{16{,}589{,}329+55{,}935{,}441} = \sqrt{72{,}524{,}770} \approx 8516.1$; $8516.1(\sqrt{0.1}) \approx 2693$, or about 2693 mi. **52.** New Orleans = (8448, 2625) and Houston = (8936, 3542), so $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(8448-8936)^2+(2625-3542)^2} = \sqrt{(-488)^2+(-917)^2} = \sqrt{238{,}144+840{,}889} = \sqrt{1{,}079{,}033} \approx 1038.8$; $1038.8(\sqrt{0.1}) \approx 328$, or about 328 mi. **53.** From algebra, $\overleftrightarrow{AD} \parallel \overleftrightarrow{XY}$ if their slopes are =. The slope of $\overleftrightarrow{XY} = \frac{y_1-y_2}{x_1-x_2} = \frac{1-3}{-2-2} = \frac{-2}{-4} = \frac{1}{2}$. So, the slope of $\overleftrightarrow{AD} = \frac{1}{2}$. For every 2 units x changes, y changes 1 unit in the same direction. Since $A = (-1, 4)$, the coordinates of D satisfy $(-1+2k, 4+k)$. Answers may vary. Sample: If $k = 2$, then $D = (-1+4, 4+2) = (3, 6)$. If $k = 0.5$, then $D = (-1+2(0.5), 4+(0.5)) = (0, 4.5)$. **54.** From algebra, $\overleftrightarrow{EC} \parallel \overleftrightarrow{XY}$ if their slopes are =. From Exercise 54, the slope of $\overleftrightarrow{XY} = \frac{1}{2}$. For every 2 units x changes, y changes 1 unit in the same direction. Since $C = (4, 2)$, the coordinates of E satisfy $(4+2k, 2+k)$. Answers may vary. Sample: If $k = -2$, then $C = (4+2(-2), 2+(-2)) = (0, 0)$. If $k = 2$, then $D = (4+2(2), 2+(2)) = (8, 4)$. **55.** From algebra, $\overleftrightarrow{FB} \perp \overleftrightarrow{XY}$ if the product of their slopes is -1. From Exercise 54, the slope of $\overleftrightarrow{XY} = \frac{1}{2}$, so the slope of $\overleftrightarrow{FB}$ must be -2, or $\frac{-2}{1}$. For every unit x changes, y changes 2 units in the opposite direction. Since $B = (0, 2)$, the coordinates of F satisfy $(0+k, 2-2k)$. Answers may vary. Sample: If $k = 1$, then $F = (0+1, 2-2(1)) = (1, 0)$. If $k = -1$, then $F = (0-1, 2-2(-1)) = (-1, 4)$.

56. From algebra, $\overleftrightarrow{GC} \perp \overleftrightarrow{XY}$ if the product of their slopes is -1. From Exercise 54, the slope of $\overleftrightarrow{XY} = \frac{1}{2}$, so the slope of $\overleftrightarrow{GC}$ must be -2, or $\frac{-2}{1}$. For every unit x changes, y changes 2 units in the opposite direction. Since $C = (4, 2)$, the coordinates of G satisfy $(4+k, 2-2k)$. Answers may vary. Sample: If $k = -4$, then $F = (4+(-4), 2-2(-4)) = (0, 10)$. If $k = 1$, then $F = (4+1, 2-2(1)) = (5, 0)$. **57.** H is the intersection of two lines, so there is exactly one pt. for H. Since A is 1 unit up and 3 units left of Y; H is 1 unit up and 3 units left of X, so $H = (-2-3, 1+1) = (-5, 2)$. **58.** J is the intersection of two lines, so there is exactly one pt. for J. From algebra, the product of the slopes of $\perp$ lines $= -1$. From Exercise 54, the slope of $\overleftrightarrow{XY}$ is $\frac{1}{2}$, so the slope of $\overleftrightarrow{JX} = -2$. Write an equation in $y = mx + b$ form where m is the slope and b is the y-intercept. Since $(-2, 1)$ lies on $\overleftrightarrow{JX}$, $1 = -2(-2) + b$, so $b = -3$. So, J satisfies the equation for $\overleftrightarrow{JX}$ which is $y = -2x - 3$.

The slope of $\overleftrightarrow{CY}$ is $-\frac{1}{2}$, so the slope of $\overleftrightarrow{JC}$ is 2. Since C lies on $\overleftrightarrow{JC}$, if $y = 2x + b$, then $2 = 2(4) + b$, so $b = -6$. So, J satisfies the equation for $\overleftrightarrow{JC}$, which is $y = 2x - 6$. Solving the system $y = -2x - 3$ and $y = 2x - 6$, $-2x - 3 = 2x - 6$, so $4x = 3$, thus $x = \frac{3}{4}$. Solving for y in either equation results in $y = -\frac{9}{2}$. So, $J = \left(\frac{3}{4}, -\frac{9}{2}\right)$. **59a.** If $A = (0, 1)$ and $B = (0, 5)$, then $AB = 4$ units and is vertical. If $D = (4, 0)$ and $C = (4, 4)$, then CD is 4 units and is vertical. Then $BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0 - 4)^2 + (5 - 4)^2} = \sqrt{(-4)^2 + (1)^2} = \sqrt{16 + 1} = \sqrt{17}$; $AD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0 - 4)^2 + (1 - 0)^2} = \sqrt{(-4)^2 + (1)^2} = \sqrt{16 + 1} = \sqrt{17}$. $BC = AD$ **59b.** If one pair of opposite sides of a quadrilateral are both $\parallel$ and $\cong$, then the other pair of opposite sides are $\cong$. **59c.** The midpt. of $\overline{AC}$ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0 + 4}{2}, \frac{1 + 4}{2}\right) = \left(\frac{4}{2}, \frac{5}{2}\right) = (2, 2.5)$; the midpt. of $\overline{BD}$ is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0 + 4}{2}, \frac{5 + 0}{2}\right) = \left(\frac{4}{2}, \frac{5}{2}\right) = (2, 2.5)$. The midpts. are the same. **59d.** Answers may vary. Sample: If one pair of opposite sides of a quadrilateral are both $\parallel$ and $\cong$, then its diagonals bisect each other. **59e.** The midpt. of $\overline{AD} = E = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0 + 4}{2}, \frac{1 + 0}{2}\right) = \left(\frac{4}{2}, \frac{1}{2}\right) = (2, 0.5)$; the midpt of $\overline{BC} = F = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0 + 4}{2}, \frac{5 + 4}{2}\right) = \left(\frac{4}{2}, \frac{9}{2}\right) = (2, 4.5)$. From Part ⓐ, $AB = 4$ units. $EF = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2 - 2)^2 + (0.5 - 4.5)^2} = \sqrt{(0)^2 + (-4)^2} = \sqrt{0 + 16} = \sqrt{16} = 4$ units. Answers may vary. Sample: $EF = AB$ **59f.** Answers may vary. Sample: If a pair of opposite sides of a quadrilateral are both $\parallel$ and $\cong$, then the segment joining the midpts. of the other two sides has the same length as each of the first pair of sides.

60. A = (front-back, right-left, up-down) = (0, 0, 0);
B = (front-back, right-left, up-down) = (6, 0, 0);
C = (front-back, right-left, up-down) = (6, −3.5, 0);
D = (front-back, right-left, up-down) = (0, −3.5, 0);
E = (front-back, right-left, up-down) = (0, 0, 9);
F = (front-back, right-left, up-down) = (6, 0, 9);
G = (front-back, right-left, up-down) = (0, −3.5, 9)

61



(4, 5, 9) is 4 units front, 5 units right, and 9 units up.

62. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = \sqrt{(2 - (-2))^2 + (3 - 4)^2 + (4 - 9)^2} = \sqrt{(2 + 2)^2 + (3 - 4)^2 + (4 - 9)^2} = \sqrt{(4)^2 + (-1)^2 + (-5)^2} = \sqrt{16 + 1 + 25} = \sqrt{42} \approx 6.5$, or about 6.5 units.

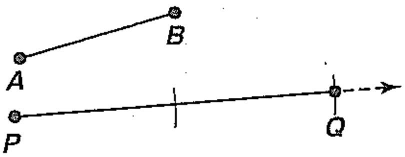
63. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = \sqrt{(0 - (-8))^2 + (12 - 20)^2 + (15 - 12)^2} = \sqrt{(0 + 8)^2 + (12 - 20)^2 + (15 - 12)^2} = \sqrt{(8)^2 + (-8)^2 + (3)^2} = \sqrt{64 + 64 + 9} = \sqrt{137} \approx 11.7$, or about 11.7 units. **64.** The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{4 + (-22)}{2}, \frac{1 + 8}{2}\right) = \left(\frac{-18}{2}, \frac{9}{2}\right) = (-9, 4.5)$. The answer is choice B. **65.** $(0, 0)$ to $(0, -7) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0 - 0)^2 + (-7 - 0)^2} = \sqrt{(-7)^2} = \sqrt{49} = 7$; $(0, 0)$ to $(5, 1) = \sqrt{(5 - 0)^2 + (1 - 0)^2} = \sqrt{(5)^2 + (1)^2} = \sqrt{25 + 1} = \sqrt{26} \approx 5.1$; $(0, 0)$ to $(-4, -3) = \sqrt{(-4)^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$; $(0, 0)$ to $(-3, 8) = \sqrt{(-3)^2 + (8)^2} = \sqrt{9 + 64} = \sqrt{73} \approx 8.5$. Since 8.5 is the greatest distance, the answer is choice D. **66.** The distance from $(2, -3)$ to $(0, 19) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2 - 0)^2 + (-3 - 19)^2} = \sqrt{(2)^2 + (-22)^2} = \sqrt{4 + 484} = \sqrt{488}$; the distance from $(-12, 6)$ to $(-4, -10) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-12 - (-4))^2 + (6 - (-10))^2} = \sqrt{(-12 + 4)^2 + (6 + 10)^2} = \sqrt{(-8)^2 + (16)^2} = \sqrt{64 + 256} = \sqrt{320}$. Since $\sqrt{488} > \sqrt{320}$, the answer is choice A. **67.** The distance from $(-31, -17)$ to $(-23, -16) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-31 - (-23))^2 + (-17 - (-16))^2} = \sqrt{(-31 + 23)^2 + (-17 + 16)^2} = \sqrt{(-8)^2 + (-1)^2} = \sqrt{64 + 1} = \sqrt{65}$; the distance from $(8, 0)$ to $(0, -1) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(8 - 0)^2 + (0 - (-1))^2} = \sqrt{(8 - 0)^2 + (0 + 1)^2} = \sqrt{(8)^2 + (1)^2} = \sqrt{64 + 1} = \sqrt{65}$. Since the distances are =, the answer is choice C.

68. The midpt. of the segment whose endpts. are $(14, -5)$ and $(6, 14) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{14 + 6}{2}, \frac{-5 + 14}{2}\right) = \left(\frac{20}{2}, \frac{9}{2}\right) = (10, 4.5)$. The x-coordinate, 10, > the y-coordinate, 4.5, so the answer is choice A.

69. [2] **a.** If R is the midpt. then $R = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-4 + 2}{2}, \frac{6 + 4}{2}\right) = \left(\frac{-2}{2}, \frac{10}{2}\right) = (-1, 5)$. If P is the midpt., then $(-4, 6) = \left(\frac{2 + x_2}{2}, \frac{4 + y_2}{2}\right)$, so $x_2 = -4(2) - 2 = -10$ and $y_2 = 6(2) - 4 = 8$, so $R = (-10, 8)$. If Q is the midpt., then $(2, 4) = \left(\frac{-4 + x_2}{2}, \frac{6 + y_2}{2}\right)$, so $x_2 = 2(2) + 4 = 8$ and $y_2 = 4(2) - 6 = 2$, so $R = (8, 2)$. **b.** For $R = (-10, 8)$, $RQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-10 - 2)^2 + (8 - 4)^2} = \sqrt{(-12)^2 + (4)^2} =$

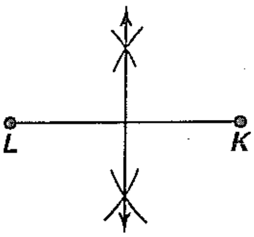
$\sqrt{144 + 16} = \sqrt{160}$. For $R = (-1, 5)$, $RQ = \sqrt{(-1 - 2)^2 + (5 - 4)^2} = \sqrt{(-3)^2 + (1)^2} = \sqrt{9 + 1} = \sqrt{10}$. For $R = (8, 2)$, $RQ = \sqrt{(8 - 2)^2 + (2 - 4)^2} = \sqrt{(6)^2 + (-2)^2} = \sqrt{36 + 4} = \sqrt{40}$. Only when $R = (-10, 8)$ is $RQ = \sqrt{160}$, so the new information does affect the answer in part ⓐ. [1] part ⓐ correct or plausible explanation for part ⓑ

70.



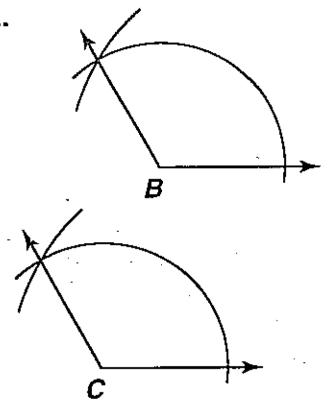
Draw a ray and construct a segment ≅ to $\overline{AB}$. Then repeat starting from the newly constructed endpoint.

71.



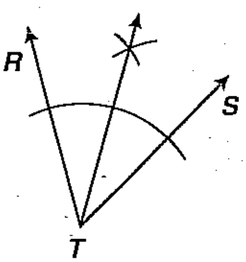
With the compass open to more than half the length of the segment, swing arcs from each endpoint on each side of the segment so the arcs from the second endpoint intersect with the arcs from the first endpoint.

72.



Draw $\angle B$ so its measure is between 90 and 180. Draw a ray. Label the endpt. of the ray C. With the compass point on B, swing an arc that intersects both sides of $\angle B$. Keeping the same setting, swing a similar arc with the compass pt. on C. Place the compass pt. on an intersection of the arc and the side of $\angle B$. Open the compass to the intersection of the arc with the other side. Keeping the same setting, place the compass pt. on the intersection of the arc and the ray. Swing an arc to intersect the arc. Draw a ray from C through the intersection of the two arcs.

73.



Draw $\angle RTS$ to measure between 0 and 90. With the compass pt. on T, swing an arc that intersects both sides of the angle. With the compass pt. on an intersection of the arc and one side of the angle, swing an arc in the interior of the angle. Keeping the same setting, swing an arc from the intersection of the first arc with the other side. Draw a ray from T through the intersection of the second and third arcs.

74. By segment addition, $AB + BC = AC$, so $(x + 8) + (3x - 3) = 45$; $4x + 5 = 45$; $4x = 40$; $x = 10$. **75.** $AB = |5 - (-5)| = |5 + 5| = |10|$ **76.** By def. of midpt., $2x + 10 = 5x - 11$; $-3x = -21$; $x = 7$. $EF = 2x + 10 + 5x - 11 = 2(7) + 10 + 5(7) - 11 = 14 + 10 + 35 - 11 = 48$ **77.** When using 3 letters to name an $\angle$, the vertex letter is always in the middle: $\angle TAP$, $\angle PAT$. **78.** $m\angle RQS = m\angle PQR + m\angle PQS = 60 + 90 = 150$

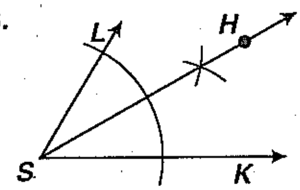
CHECKPOINT QUIZ 2 *page 49*

1. By the tick marks, $4x + 5 = 3x + 8$; $x + 5 = 8$; $x = 3$. $AC = 4x + 5 = 4(3) + 5 = 12 + 5 = 17$ **2.** $m\angle BCD = m\angle BCE + m\angle ECD = 45 + 65 = 110$ **3.** Since $\angle ACD$ is a straight angle, it measures 180. $m\angle FCD = m\angle ACD - m\angle FCA = 180 - 40 = 140$ **4a.** By def. of $\perp$, $\angle AMS$ is a rt. $\angle$, so $m\angle AMS = 90$. **4b.** By def. of bis., $QM = MS$, so $QS = 2QM = 2(30) = 60$. **5.** By def. of $\angle$ bis., $\angle APT \cong \angle RPT$. **6.** Since an angle bisector divides an angle into 2 ≅ angles, and a rt. $\angle$ measures 90, the measure of $m\angle TOR = \frac{90}{2} = 45$.

7.

Draw a ray with endpt. F. Open the compass to CD and swing that arc from F to intersect the ray. Open the compass to RT and swing that arc from the intersection of the arc and the ray away from the endpoint to intersect the ray.

8.



Construct the angle bisector of $\angle LSK$.

9. $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-4 - 6)^2 + (5 - (-2))^2} = \sqrt{(-4 - 6)^2 + (5 + 2)^2} = \sqrt{(-10)^2 + (7)^2} = \sqrt{100 + 49} = \sqrt{149} \approx 12.2$, or about 12.2 units.

10. The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-4 + 6}{2}, \frac{5 + (-2)}{2}\right) = \left(\frac{2}{2}, \frac{3}{2}\right) = (1, 1.5)$.

READING MATH *page 50*

a. $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-4 - 1)^2 + (4 - (-3))^2} = \sqrt{(-4 - 1)^2 + (4 + 3)^2} = \sqrt{(-5)^2 + (7)^2} = \sqrt{25 + 49} = \sqrt{74} \approx 8.6$

b. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(5 - (-4))^2 + (2 - (-1))^2} = \sqrt{(5 + 4)^2 + (2 + 1)^2} = \sqrt{(9)^2 + (3)^2} = \sqrt{81 + 9} = \sqrt{90} \approx 9.5$ You get the same result because the differences are opposites, and the square of a number and the square of its opp. are the same.

1-7 Perimeter, Circumference, and Area *pages 51–58*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

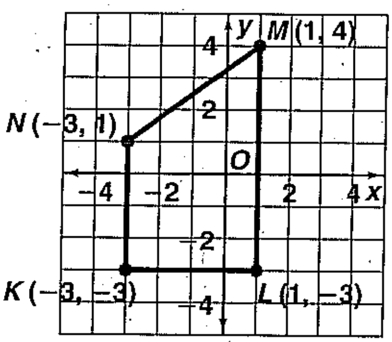
1. 4 **2.** 15 **3.** 8 **4.** 6.7 **5.** 3.2 **6.** 7.8 **7.** 4.5 **8.** 13.0 **9.** 7.8

Investigation **1.** For the 5-by-3 rectangle, $P = 5 + 3 + 5 + 3 = 16$ cm. For the 8-by-2 rectangle, $8 + 2 + 8 + 2 = 20$ cm. For the 4-by-4 rectangle, $4 + 4 + 4 + 4 = 16$ cm. **2.** The 5-by-3 rectangle has 5 rows of 3, so its area is 15 cm^2. The 8-by-2 rectangle has 8 rows of 2, so its area is 16 cm^2. The 4-by-4 rectangle has 4 rows of 4, so its area is 16 cm^2. **3.** The 5-by-3 rectangle and the 4-by-4 rectangle have = perimeters but their areas are 15 cm^2 and 16 cm^2, respectively. Rectangles with equal perimeters do not necessarily have the same area. **4.** The 8-by-2 rectangle and the 4-by-4 rectangle have = areas but their perimeters are 20 cm and 16 cm, respectively. Rectangles with equal areas do not necessarily have the same perimeter.

Check Understanding **1a.** $P = 2(6) + 2(7) = 12 + 14 = 26$, or 26 in. **1b.** The width of the outside edge of the frame $= 6 + \frac{1}{2} + \frac{1}{2} = 7$, or 7 in. The length of the outside frame $= 7 + \frac{1}{2} + \frac{1}{2} = 8$, or 8 in. $P = 2(7) + 2(8) = 14 + 16 = 30$, or 30 in.

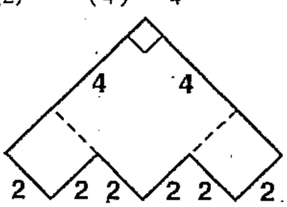
2a. A diameter is twice the radius, so $d = 2(18) = 36$, or 36 m. $C = \pi d = \pi(36) = 36\pi$ **2b.** $C = \pi d = \pi(18) \approx 56.5$, or about 56.5 m.

3.



Counting units from the graph, $NK = 4$, $LK = 4$, and $ML = 7$. $NM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-3 - 1)^2 + (1 - 4)^2} = \sqrt{(-4)^2 + (-3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$; $P = 4 + 4 + 7 + 5 = 20$, or 20 units.
4. $4 \text{ ft} = 1\frac{1}{3} = \frac{4}{3}$ yd; $A = 7\left(\frac{4}{3}\right) = \left(\frac{7}{1}\right)\left(\frac{4}{3}\right) = \frac{28}{3} = 9\frac{1}{3}$ yd^2; $\frac{28}{3} = 84x$; $x = \frac{28}{3} \div 84 = \frac{28}{3} \cdot \frac{1}{84} = \frac{28}{3} \cdot \frac{1}{28 \cdot 3} = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$, so $9\frac{1}{3}$ is one-ninth of 84. **5a.** $r = \frac{d}{2} = \frac{5}{2}$, so $A = \pi r^2 = \pi\left(\frac{5}{2}\right)^2 = \pi\left(\frac{25}{4}\right) = \frac{25}{4}\pi$ ft^2. **5b.** $\frac{25}{4}\pi \approx 19.6$, or about 19.6 ft^2.

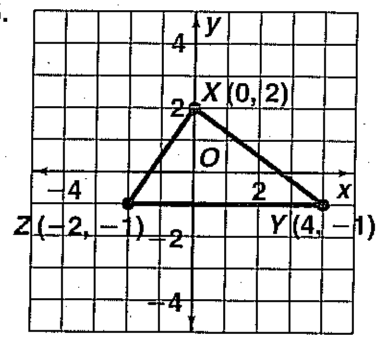
6.



Divide the figure into rectangles whose sides are ∥ to the sides of the figure. Answers may vary. Sample: $A_1 = (2)(2) = 4$; $A_2 = (4)(4) = 16$; $A_3 = (2)(2) = 4$; $4 + 16 + 4 = 24$, or 24 units^2.

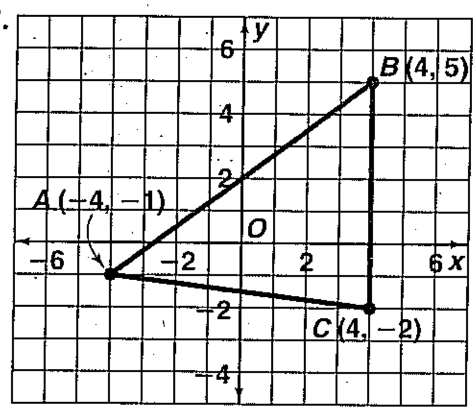
Exercises **1.** $P = 2b + 2h = 2(4) + 2(7) = 8 + 14 = 22$, or 22 in. **2.** $P = 2b + 2h = 2(9) + 2(9) = 18 + 18 = 36$, or 36 cm. **3.** $P = 2b + 2h = 2(21) + 2(7) = 42 + 14 = 56$, or 56 in. **4.** $P = 2b + 2h = 2(16) + 2(23) = 32 + 46 = 78$, or 78 cm. **5.** $P = 2b + 2h = 2(24) + 2(36) = 48 + 72 = 120$, or 120 m. **6.** $b = 8 + 1\frac{1}{2} + 1\frac{1}{2} = 8 + 3 = 11$ in.; $h = 10 + 1\frac{1}{2} + 1\frac{1}{2} = 10 + 3 = 13$, or 13 in.; $P = 2b + 2h = 2(11) + 2(13) = 22 + 26 = 48$, or 48 in. **7.** $b = 5 + 2 + 2 = 9$, or 9 ft; $h = 6 + 2 + 2 = 10$, or 10 ft; $P = 2b + 2h = 2(9) + 2(10) = 18 + 20 = 38$, or 38 ft. **8.** $C = \pi d = 15\pi$ cm **9.** $d = 2r = 2(5) = 10$, or 10 ft; $C = \pi d = 10\pi$ ft. **10.** $C = \pi d = 3.7\pi$, or 3.7π in. **11.** $d = 2r = 2\left(\frac{1}{4}\right) = \frac{1}{2}$, or $\frac{1}{2}$ m; $C = \pi d = \frac{1}{2}\pi$, or $\frac{1}{2}\pi$ m. **12.** $d = 2r = 2(9) = 18$, or 18 in.; $C = \pi d = \pi(18) \approx 56.5$, or about 56.5 in. **13.** $C = \pi d = \pi(7.3) \approx 22.9$, or about 22.9 m. **14.** $C = \pi d = \pi\left(\frac{1}{2}\right) = \pi(0.5) \approx 1.6$, or about 1.6 yd. **15.** $d = 2r = 2(56) = 112$, or 112 cm; $C = \pi d = \pi(112) \approx 351.9$, or about 351.9 cm.

16.



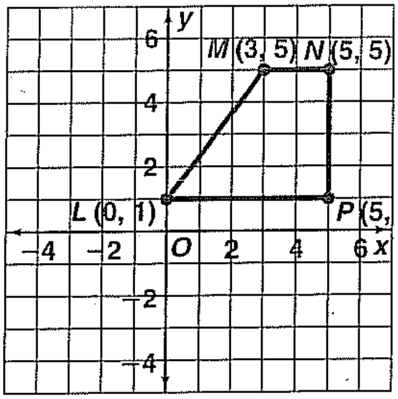
$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0 - 4)^2 + (2 - (-1))^2} = \sqrt{(0 - 4)^2 + (2 + 1)^2} = \sqrt{(-4)^2 + (3)^2} = \sqrt{16 + 9} = \sqrt{25} = 5$; $XZ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0 - (-2))^2 + (2 - (-1))^2} = \sqrt{(0 + 2)^2 + (2 + 1)^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{4 + 9} = \sqrt{13} \approx 3.6$. By counting, $YZ = 6$. $P \approx 5 + 3.6 + 6 = 14.6$, or 14.6 units.

17.

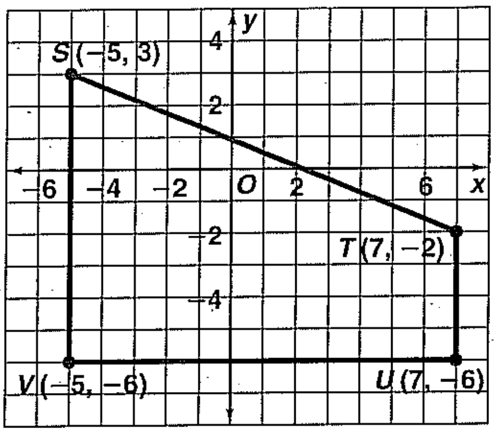


$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-4 - 4)^2 + (-1 - 5)^2} = \sqrt{(-8)^2 + (-6)^2} = \sqrt{64 + 36} = \sqrt{100} = 10$; $AC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} =$

$\sqrt{(-4-4)^2+(-1-(-2))^2} =$
$\sqrt{(-4-4)^2+(-1+2)^2} = \sqrt{(-8)^2+(1)^2} =$
$\sqrt{64+1} = \sqrt{65} \approx 8.1$. By counting, $BC = 7$;
$P \approx 10 + 8.1 + 7 = 25.1$, or 25.1 units.

18.


$LM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} =$
$\sqrt{(0-3)^2 + (1-5)^2} = \sqrt{(-3)^2 + (-4)^2} =$
$\sqrt{9+16} = \sqrt{25} = 5$. By counting, $MN = 2$, $NP = 4$, and $PL = 5$. $P = 5 + 2 + 4 + 5 = 16$, or 16 units.

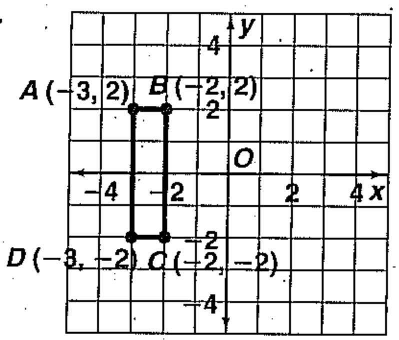
19.


$ST = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} =$
$\sqrt{(-5-7)^2 + (3-(-2))^2} =$
$\sqrt{(-5-7)^2 + (3+2)^2} = \sqrt{(-12)^2 + (5)^2} =$
$\sqrt{144+25} = \sqrt{169} = 13$. By counting, $TU = 4$, $UV = 12$, and $VS = 9$. $P = 13 + 4 + 12 + 9 = 38$, or 38 units.
20. In ft^2, 4 in. $= \frac{4}{12}$, or $\frac{1}{3}$ ft, so $A = bh = (4)\left(\frac{1}{3}\right) = \frac{4}{3}$, or $1\frac{1}{3}$ ft^2. In in.^{2}, 4 ft $= 4(12)$, or 48 in., so $A = bh = 48(4) = 192$, or 192 in.^{2}. **21.** In in.^{2}, 4 yd $= 4(36$ in.$) = 144$ in, so $A = bh = 30(144) = 4320$, or 4320 in.^{2}. In yd^2, 30 in. $= \frac{30}{36}$, or $\frac{5}{6}$ yd, so $A = bh = \left(\frac{5}{6}\right)(4) = \frac{20}{6} = \frac{10}{3} = 3\frac{1}{3}$, or $3\frac{1}{3}$ yd^2.
22. In ft^2, 3 in. $= \frac{1}{4}$ ft, and 6 in. $= \frac{1}{2}$ ft, so $b = 2\frac{1}{4}$, or $\frac{9}{4}$ ft, and $h = \frac{1}{2}$ ft, thus $A = bh = \frac{9}{4}\left(\frac{1}{2}\right) = \frac{9}{8} = 1\frac{1}{8}$, or $1\frac{1}{8}$ ft^2. In in.^{2}, 2 ft $= 2(12$ in.$) = 24$ in., so $b = 24 + 3$, or 27 in. and $h = 6$ in., thus $A = bh = 27(6) = 162$, or 162 in.^{2}. **24.** In m^2, 190 cm $= 1.9$ m, so $A = bh = 3(1.9) = 5.7$, or 5.7 m^2. In cm^2, 3 m $= 300$ cm, so $A = bh = 300(190) = 57{,}000$, or 57,000 cm^2. **25.** In cm^2, 5 m $= 500$ cm, so $A = bh = 240(500) = 120{,}000$, or 120,000 cm^2. In m^2, 240 cm $= 2.4$ m, so $A = bh = 2.4(5) = 12$, or 12 m^2. **26.** In ft^2, 100 yd $= 100(3$ ft$) = 300$ ft, so $A = bh = 20(300) = 6000$, or 6,000 ft^2. In yd^2, 20 ft $= 20\left(\frac{1}{3}\text{yd}\right) = \frac{20}{3}$ yd, so $A = \frac{20}{3}(100) = \frac{2000}{3} = 666\frac{2}{3}$, or $666\frac{2}{3}$ yd^2. **27.** $A = \pi r^2 = \pi(20)^2 = 400\pi$, or 400π m^2. **28.** $r = \frac{d}{2} = \frac{16}{2} = 8$, so $A = \pi r^2 = \pi(8)^2 = 64\pi$, or 64π ft^2. **29.** $r = \frac{d}{2} = \frac{\frac{3}{4}}{2} = \frac{3}{4} \div 2 = \frac{3}{4}\left(\frac{1}{2}\right) = \frac{3}{8}$, so $A = \pi r^2 = \pi\left(\frac{3}{8}\right)^2 = \pi\left(\frac{3^2}{8^2}\right) = \frac{9}{64}\pi$, or $\frac{9}{64}\pi$ in.^{2}. **30.** $A = \pi r^2 = \pi(0.5)^2 = 0.25\pi$, or 0.25π m^2. **31.** $r = \frac{d}{2} = \frac{6.3}{2} = 3.15$, so $A = \pi r^2 = \pi(3.15)^2 = 9.9225\pi$, or 9.9225π ft^2. **32.** $A = \pi r^2 = \pi(0.1)^2 = 0.01\pi$, or 0.01π m^2. **33.** $A = \pi r^2 = \pi(7)^2 = 49\pi \approx 153.9$, or about 153.9 ft^2. **34.** $r = \frac{d}{2} = \frac{8.3}{2} = 4.15$, so $A = \pi r^2 = \pi(4.15)^2 = 17.2225\pi \approx 54.1$, or about 54.1 m^2. **35.** $r = \frac{d}{2} = \frac{24}{2} = 12$ cm, so $A = \pi r^2 = \pi(12)^2 = 144\pi \approx 452.4$, or about 452.4 cm^2. **36.** $A = \pi r^2 = \pi(12)^2 = 144\pi \approx 452.4$, or about 452.4 in.^{2}. **37.** The area of the shaded region is the area of the 20-by-18 rectangle minus the area of the 10-by-5 rectangle; $(20)(18) - (10)(5) = 360 - 50 = 310$, or 310 m^2. **38.** The area of the shaded region is the area of the 6-by-5 rectangle minus the 2-by-1 rectangle minus the 3-by-3 rectangle; $(6)(5) - (2)(1) - (3)(3) = 30 - 2 - 9 = 28 - 9 = 19$, or 19 yd^2. **39.** The left vertical portion consists of three 2-by-2 rectangles, the middle vertical portion consists of two 2-by-2 rectangles, and the right portion consists of one 2-by-2 rectangle. The total area is $3[(2)(2)] + 2[(2)(2)] + 1[(2)(2)] = 6[(2)(2)] = 6[4] = 24$, or 24 cm^2. **40.** The area of the shaded portion is the area of the 12-by-8 rectangle minus the 4-by-4 rectangle; $(12)(8) - (4)(4) = 96 - 16 = 80$, or 80 in.^{2}. **41a.** $b = 12$ in. and $h = 12$ in., so $A = bh = (12)(12) = 144$, or 144 in.^{2}. **41b.** $b = 1$ ft and $h = 1$ ft, so $A = bh = (1)(1) = 1$, or 1 ft^2. **41c.** There are 144, or 144 in.^{2} per 1, or 1 ft^2. A square whose sides are 12 in. long and a square whose sides are 1 ft long are the same size. **42a.** Counting, there are 30 squares. **42b.** The area of the 4-by-4 square is $(4)(4) = 16$, or 16 in.^{2}. The area of the 3-by-3 square is $(3)(3) = 9$, or 9 in.^{2}. The area of the 2-by-2 square is $(2)(2) = 4$, or 4 in.^{2}. The area of the 1-by-1 square is $(1)(1) = 1$, or 1 in.^{2}. **42c.** $16 + 9 + 4 + 2 + 1 = 30$, so the results for the total area are =. This supports Post. 10. **43.** The center region is 14 m by 14 m and its area is $(14)(14) = 196$, or 196 m^2. There are 4 "corner" regions that are 23 m by 23 m, so the total area of these 4 regions is $4(23)(23) = 2116$, or 2116 m^2. There are 4 "spoke" regions that are 14 m by $(23 + 6)$ m, or 14 m by 29 m, so the total area of these 4 spokes is $4(14)(29) = 1624$, or 1624 m^2. Thus, the total area of the "footprint" is $196 + 2116 + 1624$, or 3936 m^2. **44.** Estimates may vary. Sample: The base is 9 in. and the height is 10 in., so $P = 2(9) + 2(10) = 18 + 20 = 38$, or 38 in.; $A = bh = (9)(10) = 90$, or 90 in.^{2}. **45.** Estimates may vary. Sample: For an 8.5-by-11 notebook, $P = 2(8.5) + 2(11) = 17 + 22 = 39$, or 39 in.; $A = (8.5)(11) = 93.5$, or 93.5 in.^{2}. **46.** Estimates may vary. Sample: For a 2 ft-by-4 ft bulletin board, $P = 2(2) + 2(4) = 4 + 8 = 12$, or 12 ft; $A = (2)(4) = 8$, or 8 ft^2. **47.** Estimates may vary. Sample: For a 1.5-by-2.5 ft desk top, $P = 2(1.5) + 2(2.5) = 3 + 5 = 8$, or 8 ft. $A = (1.5)(2.5) = 3.75$, or 3.75 ft^2.

48. Answers may vary. Sample: For Exercise 43, you use feet because the bulletin board is too big for inches. You do not use yards because your estimated lengths in feet were not divisible by 3. **49.** $A = bh; 176 = 11h; h = 16$ cm **50.** $P = 2b + 2h; 40 = 2(12) + 2h; 20 = 12 + h; h = 8;$ $A = bh = (12)(8) = 96$, or 96 cm^2. **51.** The area of the rectangle is $bh = (64)(81) = 5184$, or 5184 cm^2 = area of the square. If s represents the side of the square, the $s \cdot s$, or s^2 = the area, so $s^2 = 5184; s = \sqrt{5184} = 72 =$ $b = h; P = 2b + 2h = 2(72) + 2(72) = 144 + 144 = 288$, or 288 cm. **52a.** Both rectangles and squares have a base and a height. In a square, the base and height are =. So, yes, the same formula can be used for the perimeter of a square and perimeter of a rectangle. **52b.** Answers may vary. Sample: No; if the base and height differ, then you can not use $P = 4s$. **52c.** $P = 4x$, so $s = \frac{P}{4}; A = s^2 = \left(\frac{P}{4}\right)^2$, or $A = \frac{P^2}{4^2} = \frac{P^2}{16}$.

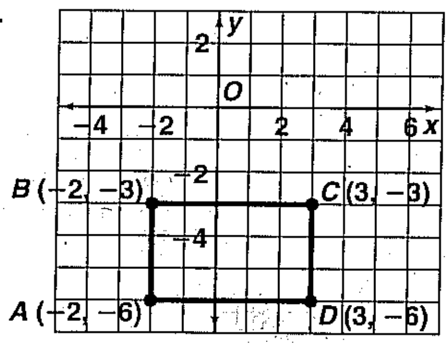
53. Since 6 in. $= \frac{1}{2}$ ft, each square foot can be covered by exactly 4 tiles. $A = bh = (8)(16) = 128$, or 128 ft^2; $128(4) = 512$, or 512 tiles.

54.



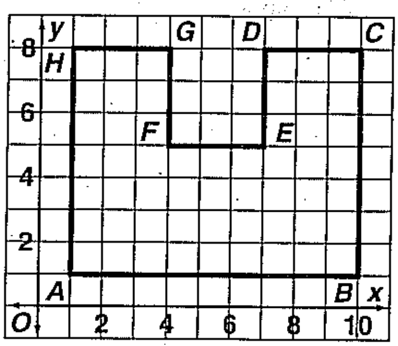
By counting, $AB = CD = 1$ and $BC = DA = 4$. $P = 2b + 2h = 2(1) + 2(4) = 2 + 8 = 10$, or 10 units; $A = bh = (1)(4) = 4$, or 4 units^2.

55.



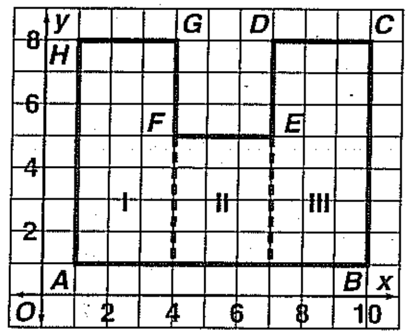
By counting, $AB = CD = 3$ and $BC = DA = 5$. $P = 2b + 2h = 2(5) + 2(3) = 10 + 6 = 16$, or 16 units; $A = bh = (5)(3) = 15$, or 15 units^2.

56.



$AB = 9, BC = 7, CD = 3, DE = 3, EF = 3, FG = 3, GH = 3, HA = 7; P = AB + BC + CD + DE + EF + FG + GH + HA = 9 + 7 + 3 + 3 + 3 + 3 + 3 + 7 = 38$, or 38 units.

57.

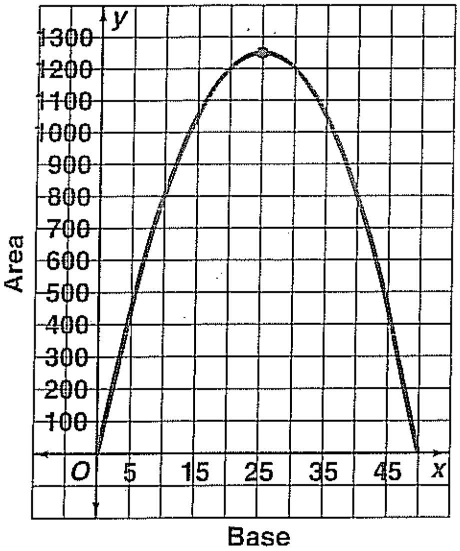


Divisions of the polygon may vary. Sample: Rectangle I is 3 by 7, so its area is $(3)(7) = 21$, or 21 units^2. Rectangle II is 3 by 4, so its area is $(3)(4) = 12$, or 12 units^2. Rectangle III is 3 by 7, the same as Rectangle I, so its area is 21 units^2. By Post. 10, the total area is $21 + 12 + 21 = 54$, or 54 units^2. **58.** $A = \pi r^2 = \pi(718)^2 = 515{,}524\pi \approx 1{,}620{,}000$, or about 1,620,000 m^2. **59.** $A = \pi r^2; 225\pi = \pi r^2; r^2 = 225; r = \sqrt{225} = 15$. Since $d = 2r$, $d = 2(15) = 30$, or 30 m. **60.** $A = bh; 4x^2 - 2x = xh;$ $x(4x - 2) = xh$. Because x is a length, $x > 0$, so $h = 4x - 2$. **61.** Use area because the wall is a surface. **62.** Use perimeter because you must measure the length of the strip to fit the edges of the door. **63.** Use perimeter because the fence sits on the perimeter of the garden. **64.** Use area because you are painting the surface of the floor. **65.** The diameter is AB; $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2 - 5)^2 + (1 - 5)^2} = \sqrt{(-3)^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5; r = \frac{d}{2} = \frac{5}{2} = 2.5;$ $A = \pi r^2 = \pi(2.5)^2 = 6.25\pi$, or 6.25π units^2.

66a.

base	height	area
1	98	98
2	96	192
3	94	282
	⋮	
24	52	1248
25	50	1250
26	48	1248
	⋮	
47	6	282
48	4	192
49	2	98

66b.



66c. The greatest area is 1250 for a corral that is 25 ft by 50 ft. **67a.** $A_{1\text{-in.}} = \pi r^2 = \pi(1)^2 = \pi$; $A_{3\text{-in.}} = \pi r^2 = \pi(3)^2 = 9\pi$. Since $\frac{9\pi}{\pi} = 9$, nine 1-in. circles are needed. **67b.** $A_{2\text{-in.}} = \pi r^2 = \pi(2)^2 = 4\pi$; $A_{6\text{-in.}} = \pi r^2 = \pi(6)^2 = 36\pi$. Since $\frac{36\pi}{4\pi} = 9$, nine 2-in. circles are needed. **67c.** $A_{3\text{-in.}} = \pi r^2 = \pi(3)^2 = 9\pi$; $A_{9\text{-in.}} = \pi r^2 = \pi(9)^2 = 81\pi$. Since $\frac{81\pi}{9\pi} = 9$, nine 3-in. circles are needed. **67d.** Nine circles with a radius of n in. are needed for the sum of their areas to equal the area of a circle with a radius of $3n$ in. **68.** $A = bh = \left(\frac{2a}{5b}\right)\left(\frac{3b}{8}\right) = \frac{2a \cdot 3b}{5b \cdot 8} = \frac{a \cdot 3}{5 \cdot 4} = \frac{3a}{20}$, or $\frac{3a}{20}$ units^2. **69.** $P = 4s = 10n$, so $s = \frac{10n}{4}$, or $\frac{5n}{2}$. $A = s^2 = \left(\frac{5n}{2}\right)^2 = \frac{25n^2}{4}$, or $\frac{25n^2}{4}$ units^2. **70.** $A = s^2 = (3m - 4n)^2 = (3m - 4n)(3m - 4n) = 3m \cdot 3m - 3m \cdot 4n - 4n \cdot 3m + (-4n)(-4n) = 9m^2 - 12mn - 12mn + 16n^2$, or $(9m^2 - 24mn + 16n^2)$ units^2. **71.** Answers may vary. Samples: one 8 in.-by-8 in. square + one 5 in.-by-5 in. square + two 4 in.-by-4 in. squares; one 2 in.-by-2 in. square + one $\sqrt{117}$ in.-by-$\sqrt{117}$ in. square **72.** The outer perimeter is equivalent to the circumference of a circle whose diameter is 40 + 10 + 10, or 60 yd, + two 100-yd widths of the rectangular portion of the field $= \pi d + 2b = \pi(60) + 2(100) = 60\pi + 200 \approx 188.5 + 200 = 388.5$, or 388.5 yd. **73.** If the width of the garden is b and the length is h, then the perimeter of the garden is $2b + 2h$. The width of the outer walkway is $(b + 8 + 8)$, or $(b + 16)$ and the length of the outer walkway is $(h + 8 + 8)$, or $(h + 16)$, so the perimeter of the outer walkway is $2(b + 16) + 2(h + 16)$. Subtracting the two perimeters, $[2(b + 16) + 2(h + 16)] - [2b + 2h] = 2b + 32 + 2h + 32 - 2b - 2h = 64$, or 64 ft greater. To answer the question, grid 64. **74.** Since $P = 2b + 2h$, $260 = 2b + 2h$, so $b + h = 130$. The walkway can be divided into two 8-by-b rectangles whose total area is $2(8b)$, or $16b$ ft^2, two 8-by-h rectangles whose total area is $2(8h)$, or $16h$, and four 8-by-8 corners whose total area is $4(8)(8)$, or 256. The total area is $16b + 16h + 256 = 16(b + h) + 256$. Substitute 130 for $(b + h)$: $16(130) + 256 = 2080 + 256 = 2336$, or 2336 ft^2. To answer the question, grid 2336. **75.** Since 12 in. = 1 ft, 1 tile covers 1 ft^2 of flooring. The area of the floor is $(12)(15) = 180$, or 180 ft^2, so 180 tiles are needed. The cost is \$3 per ft^2, so the total cost is 3(180), or \$540. To answer the question, grid 540. **76.** Since 1 ft = 12 in., the size of the floor is 12(12 in.) by 12(15 in.) = 144 ft by 180 ft. The floor area is (144)(180), or 25,920 in.^{2}. Each tile has an area of (10)(12), or 120 in.^{2}. $25{,}920 \div 120 = 216$, or 216 tiles are needed. To answer the question, grid 216. **77.** From Exercise 75, the area of the floor is 180 ft^2. It costs \$4.50 per ft^2, so \$4.50(180) = \$810. To answer the question, grid 810. **78.** If D is (x, y), then $\frac{x - 5}{2} = 5$ and $\frac{y - 1}{2} = 6$. Solving for x, $x - 5 = 10$, so $x = 15$. Solving for y, $y - 1 = 12$, so $y = 13$. $(x, y) = (15, 13)$

79a. $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4 - 7)^2 + (1 - 9)^2} = \sqrt{(-3)^2 + (-8)^2} = \sqrt{9 + 64} = \sqrt{73} \approx 8.5$, or about 8.5 units. **79b.** The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{4 + 7}{2}, \frac{1 + 9}{2}\right) = \left(\frac{11}{2}, \frac{10}{2}\right) = (5.5, 5)$.
80a. $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0 - 3)^2 + (3 - 8)^2} = \sqrt{(-3)^2 + (-5)^2} = \sqrt{9 + 25} = \sqrt{34} \approx 5.8$, or about 5.8 units. **80b.** The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0 + 3}{2}, \frac{3 + 8}{2}\right) = \left(\frac{3}{2}, \frac{11}{2}\right) = (1.5, 5.5)$.
81a. $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(9 - (-3))^2 + (2 - 9)^2} = \sqrt{(9 + 3)^2 + (2 - 9)^2} = \sqrt{(12)^2 + (-7)^2} = \sqrt{144 + 49} = \sqrt{193} \approx 13.9$, or about 13.9 units.
81b. The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{9 + (-3)}{2}, \frac{2 + 9}{2}\right) = \left(\frac{6}{2}, \frac{11}{2}\right) = (3, 5.5)$.
82a. $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0 - (-4))^2 + (1 - 6)^2} = \sqrt{(0 + 4)^2 + (1 - 6)^2} = \sqrt{(4)^2 + (-5)^2} = \sqrt{16 + 25} = \sqrt{41} \approx 6.4$ **82b.** The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{0 + (-4)}{2}, \frac{1 + 6}{2}\right) = \left(\frac{-4}{2}, \frac{7}{2}\right) = (-2, 3.5)$. **83a.** $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4 - (-2))^2 + (10 - 3)^2} = \sqrt{(4 + 2)^2 + (10 - 3)^2} = \sqrt{(6)^2 + (7)^2} = \sqrt{36 + 49} = \sqrt{85} \approx 9.2$, or about 9.2 units. **83b.** The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{4 + (-2)}{2}, \frac{10 + 3}{2}\right) = \left(\frac{2}{2}, \frac{13}{2}\right) = (1, 6.5)$.
84a. $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-1 - (-4))^2 + (1 - (-5))^2} = \sqrt{(-1 + 4)^2 + (1 + 5)^2} = \sqrt{(3)^2 + (6)^2} = \sqrt{9 + 36} = \sqrt{45} \approx 6.7$, or about 6.7 units. **84b.** The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-1 + (-4)}{2}, \frac{1 + (-5)}{2}\right) = \left(\frac{-5}{2}, \frac{-4}{2}\right) = (-2.5, -2)$. **85.** By def. of $\perp$, $\angle BIR$ is a rt $\angle$, so it measures 90. **86.** By def. of segment bisector, $\overline{WI} \cong \overline{RI}$. **87.** By def. of segment bisector, $IR = \frac{1}{2}WR = \frac{1}{2}(124) = 62$, or 62 units. **88.** $PQ = |12 - (-6)| = |12 + 6| = |18| = 18$, or 18 units. **89.** $PQ = |3 - 9| = |-6| = 6$, or 6 units. **90.** $PQ = |-23 - 10| = |-33| = 33$

TECHNOLOGY *page 59*

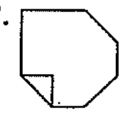
1. From the table and the graph, the maximum area occurs when the base and height are =, so the figure with the greatest area is a square. **2.** $C = \pi d; 32 = \pi d;$ $d = \frac{32}{\pi}$, so $r = \frac{1}{2} \cdot \frac{32}{\pi}$, or $\frac{16}{\pi}$; $A = \pi r^2 = \pi\left(\frac{16}{\pi}\right)^2 = \pi\left(\frac{256}{\pi^2}\right) = \frac{256}{\pi} \approx 81.5$, or about 81.5 yd^2; $81.5 > 64$, so the circular pen has greater area. **3a.** If one side measures n, then the other side measures $\frac{900}{n}$. Answers may vary. Sample: A 25 ft-by-36 ft rectangle has a perimeter of $2(25) + 2(36) = 50 + 72 = 122$, or 122 ft. A 30 ft-by-30 ft rectangle has a perimeter of $2(30) + 2(30) = 60 + 60 = 120$, or 120 ft. A 10 ft-by-90 ft rectangle has a perimeter of $2(10) + 2(90) = 20 + 180 = 200$, or 200 ft. **3b.** Let b be the integers from 1 through 900 and h be $\frac{900}{b}$. The minimum perimeter is a 30 ft-by-30 ft square.

TEST-TAKING STRATEGIES *page 60*

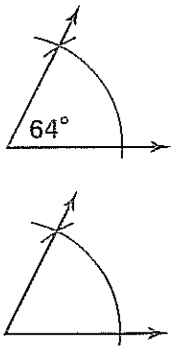
1. The y-coordinate is the average of the two y-values: $\frac{-1 + 12}{2} = \frac{11}{2}$, or 5.5. **2.** $C = \pi d; 0.5\pi = \pi d; d = 0.5$
3. $LM = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0 - 0.3)^2 + (0 - 0.4)^2} = \sqrt{(-0.3)^2 + (-0.4)^2} = \sqrt{0.09 + 0.16} = \sqrt{0.25} = 0.5$
4. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3 - 12)^2 + (5 - 5)^2} = \sqrt{(-9)^2 + (0)^2} = \sqrt{81} = 9;$ $r = \frac{d}{2} = \frac{9}{2} = 4.5; A = \pi r^2 = \pi(4.5)^2 = 20.25\pi \approx 63.62$
5. $A = \pi r^2; 10\pi = \pi r^2; r^2 = 10; r = \sqrt{10} \approx 3.16$

CHAPTER REVIEW *pages 61–63*

1. By def. of coplanar, figures that are in the same plane are *coplanar.* **2.** By def. of segment, a *segment* is the part of a line consisting of two endpoints and all points between them. **3.** By def. of congruent, two segments with the same length are *congruent.* **4.** By def. of midpoint, a *midpoint* of a segment is the point that divides the segment into two congruent segments.
5. By def. of angle bisector, an *angle bisector* is a ray that divides an angle into two congruent angles. **6.** By def. of conjecture, a conclusion based upon inductive reasoning is sometimes called a *conjecture.* **7.** Answers may vary. By def. of postulate, a *postulate* is an accepted statement of fact. By def. of axiom, an *axiom* is an accepted statement of fact. **8.** By def. of parallel lines, *parallel lines* are coplanar lines that do not intersect. **9.** By def. of obtuse angle, an *obtuse angle* is an angle whose measure is between 90 and 180. **10.** By def. of perpendicular bisector, a *perpendicular bisector* is a line, segment, or ray that is perpendicular to a segment at its midpoint. **11.** Each term is 5 less than the preceding term, so the next two terms after 25 are $25 - 5$, or 20, and $20 - 5$, or 15. **12.** Answers may vary. Samples: Each term is -1 times the preceding term so the next two terms after -5 are $-5(-1)$, or 5, and $5(-1)$, or -5. Or, every odd term is 5 and every even term is -5, so the next two terms after -5 are 5 and -5. **13.** Each term is 7 less than the preceding term, so the next two terms after 6 are $6 - 7$, or -1, and $-1 - 7$, or -8. **14.** Each term is 4 times the preceding term, so the next two terms after 384 are 4(384), or 1536, and 4(1536), or 6144.
15. Each term is twice the preceding term, so the next two terms after 32 are 2(32), or 64, and 2(64), or 128.
16. Even terms are 1 more than the preceding term and odd terms are 3 more than the preceding term, so the next two terms after the fifth term, 9, are $9 + 1$, or 10, and $10 + 3$, or 13.

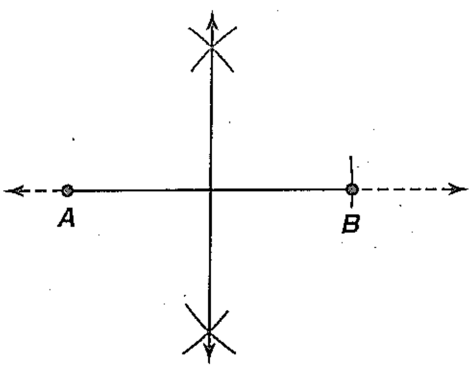
17.  Starting from the upper right corner, the corner is folded down. The next corner in clockwise rotation is folded down while the previous fold is removed. The next corner to fold in clockwise direction is the lower left corner and the two previous folds will have been removed.

18. Answers may vary. Sample: $\overleftrightarrow{AQ}$ and $\overleftrightarrow{QR}$ intersect at Q. **19.** Skew lines are noncoplanar lines, so they are lines that do not intersect and that are not $\parallel$. Answers may vary. Sample: $\overleftrightarrow{AQ}$ and $\overleftrightarrow{BC}$ **20.** Three noncollinear points form a plane. Answers may vary. Sample: A, Q, R
21. Noncoplanar points are points that lie in different planes. Answers may vary. Sample: A, Q, R, S
22. Parallel planes do not intersect. Answers may vary. Samples: $AQTD$ and $BRSC$, $QTSR$ and $ADCB$, $ABRQ$ and $DCST$ **23.** The lines are those that can have D in their name: $\overleftrightarrow{AD}, \overleftrightarrow{TD}, \overleftrightarrow{CD}$. **24.** Choose any two points on the line. By Post. 1-4, those two points and the point not on the line determine a plane, so a line and a point not on it are *always* coplanar. **25.** Two segments can be parts of lines that are skew, parallel, or intersecting. If the lines are parallel or intersecting, then the segments are coplanar. If the lines are skew, then the segments are not coplanar. Two segments are *sometimes* coplanar.
26. By def. of skew lines, skew lines are *never* coplanar.
27. By def. of skew and parallel, parallel lines are *never* skew. **28.** By Post. 1-1, two points are *always* collinear.
29. By def. of parallel lines, parallel lines are *always* coplanar. **30.** Q can be 5 units to the left of P, so it would be $-2 - 5$, or -7. Q can be 5 units to the right of P, so it would be $-2 + 5$, or 3. **31.** The midpt. is the mean of the two coordinates: $\frac{-2 + 3}{2} = \frac{1}{2} = 0.5$.
32. By the tick marks, $3m + 5 = 4m - 10; -m + 5 = -10; -m = -15; m = 15$. **33.** The two angles are supplementary, so $(3x + 31) + (2x - 6) = 180; 5x + 25 = 180; 5x = 155; x = 31$. **34.** $AB = |-5 - (-3)| = |-2| = 2; AC = |-5 - 1| = |-6| = 6; AD = |-5 - 3| = |-8| = 8; AE = |-5 - 7| = |-12| = 12; BC = |-3 - 1| = |-4| = 4; BD = |-3 - 3| = |-6| = 6; BE = |-3 - 7| = |-10| = 10; CD = |1 - 3| = |-2| = 2; CE = |1 - 7| = |-6| = 6; DE = |3 - 7| = |-4| = 4; 2 = AB = CD$, so $\overline{AB} \cong \overline{CD}$; $4 = BC = DE$, so $\overline{BC} \cong \overline{DE}$; $6 = AC = BD = CE$, so $\overline{AC} \cong \overline{BD} \cong \overline{CE}$. **35.** Since X is the vertex of more than one angle, the angles must be named by 3 letters with X being the middle letter: $\angle 1 = \angle WXY = \angle YXW; \angle 2 = \angle YXZ = \angle ZXY$.

36. 

Draw a ray. With the compass point on the vertex of the 64-degree angle, swing an arc that intersects both sides of the angle. Keep the same setting and swing a similar arc with the compass point on the endpoint of the ray. Place the compass point on the intersection of the arc with the side of the angle and open it to the intersection of the arc with the other side of the angle. Keep the setting and place the compass point on the intersection of the arc and the ray. Swing an arc that intersects the arc. Draw a ray from the endpoint of the ray through the intersection of the two arcs.

37a. Draw a line. Label one pt. A. Open the compass to PQ. Keep the setting and place the compass point on A. Swing an arc that intersects the line. Label the intersection B. See 37b for construction.

37b. 

Place the compass point on A and open it to more than half AB. Swing an arc on each side of the segment. Keep the same setting and place the compass point on B. Swing arcs that intersect with the previous two arcs. Draw a line through the intersections of the arcs.

38. $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-1 - 0)^2 + (5 - 4)^2} = \sqrt{(-1)^2 + (1)^2} = \sqrt{1 + 1} = \sqrt{2} \approx 1.4$, or about 1.4 units. **39.** $CD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-1 - 6)^2 + (-1 - 2)^2} = \sqrt{(-7)^2 + (-3)^2} = \sqrt{49 + 9} = \sqrt{58} \approx 7.6$, or about 7.6 units.

40. $EF = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-7 - 5)^2 + (0 - 8)^2} = \sqrt{(-12)^2 + (-8)^2} = \sqrt{144 + 64} = \sqrt{208} \approx 14.4$, or about 14.4 units.

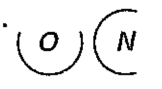
41. The midpt. is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-3 + 3}{2}, \frac{2 + (-2)}{2}\right) = \left(\frac{0}{2}, \frac{0}{2}\right) = (0, 0)$. **42.** $GH = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-3 - 3)^2 + (2 - (-2))^2} = \sqrt{(-3 - 3)^2 + (2 + 2)^2} = \sqrt{(-6)^2 + (4)^2} = \sqrt{36 + 16} = \sqrt{52} \approx 7.2$, or about 7.2 units.

43. The figure is a square, so $P = 4s = 4(8) = 32$, or 32 cm. $A = s^2 = (8)^2 = 64$, or 64 cm^2.

44. The figure is a rectangle, so $P = 2b + 2h = 2(13) + 2(6) = 26 + 12 = 38$, or 38 ft. $A = bh = (13)(6) = 78$, or 78 ft^2. **45.** The figure is comprised of a 3-by-5 rectangle at the upper right and a 5-by-5 rectangle (or square) in the lower left. There are four 5-in. sides and three 3-in. sides. The unmarked side is also 3 in. since the "neck" must be 2 in. By def. of perimeter, $P = 4(5) + 3(3) + 3 = 20 + 9 + 3 = 32$, or 32 in. By Post. 10, the total area is the sum of their areas: $(3)(5) + (5)(5) = 15 + 25 = 40$, or 40 in.^{2}. **46.** $d = 2r = 2(3) = 6$, or 6 in., so $C = \pi d = 6\pi \approx 18.8$, or about 18.8 in.; $A = \pi r^2 = \pi(3)^2 = 9\pi \approx 28.3$, or about 28.3 in.^{2}. **47.** $C = \pi d = \pi(15) \approx 47.1$, or about 47.1 m; $r = \frac{d}{2} = \frac{15}{2} = 7.5$ m, so $A = \pi r^2 = \pi(7.5)^2 = 56.25\pi \approx 176.7$, or about 176.7 m^2. **48.** $d = 2r = 2(26) = 52$, or 52 m, so $C = \pi d = 52\pi \approx 163.4$, or about 163.4 m; $A = \pi r^2 = \pi(26)^2 = 676\pi \approx 2123.7$, or about 2123.7 m^2.

CHAPTER TEST *page 64*

1. Answers may vary. Sample: Each term is the preceding term divided by -2. After -1, the next two terms are $-1 \div -2 = \frac{-1}{-2} = \frac{1}{2}$ and $\frac{1}{2} \div -2 = \frac{1}{2} \cdot -\frac{1}{2} = -\frac{1}{4}$. **2.** Each term is 2 more than the previous term. After 8, the next two terms are $8 + 2 = 10$ and $10 + 2 = 12$.

3. 

The letters are in reverse alphabetical order, so the letter after P is O and the letter after O is N. The "horseshoe" opens up and turns 90° in a clockwise direction.

4. Possible answers include using addition or multiplication to reach subsequent terms. Other answers may include repeating the patterns as parts of other patterns. Answers may vary. Sample: Each subsequent term is twice the preceding term: 1, 2, 4, 8, 16, 32, . . . ; the kth term is $(k - 1)$ more than the preceding term: 1, 2, 4, 7, 11, 16, . . . **5.** Three pts. on a single line are A, B, and C. **6.** Since intersecting lines are coplanar, all pts. on them are coplanar: A, B, C, and D. **7.** Choose 4 pts. such that 1 is not in the same plane as the other 3. Answers may vary. Samples: A, B, D, E; A, C, B, E; B, C, D, E

8. The only pt. that $\overleftrightarrow{AC}$ and plane Q have in common is pt. B. **9a.** B, D, and A are noncollinear pts., so by Post. 1-4, exactly 1 plane contains them. **9b.** By Post. 1-3, 2 planes intersect in a line. Several planes could intersect in the same line, so the answer is "infinitely many" planes can share that intersection. **9c.** B, E, and C are noncollinear pts, so by Post. 1-4, exactly 1 plane contains them. **9d.** B, D, and E are noncollinear pts, so by Post. 1-4, exactly 1 plane contains them. **10.** The figure consists of 2 half circles with a 50-ft radius, which is the same as 1 full circle with a 50-ft radius, and one 100-by-212 rectangle. By Post. 10, the area of the figure is the sum of the areas of its parts. $A = \pi r^2 + bh = \pi(50)^2 + (100)(212) = 2500\pi + 21{,}200 \approx 7854.0 + 21{,}200 = 29{,}054.0$, or 29,054.0 ft^2.

11. Opposite rays share an endpt., so $\overrightarrow{LT}$ and $\overrightarrow{TJ}$ are *never* opposite rays. **12.** Three points are always coplanar; the fourth point may or may not be coplanar with the first three pts., so four points are *sometimes* coplanar. **13.** By def. of skew lines, skew lines are *never* coplanar. **14.** By def. of $\parallel$ segments, two segments that lie in parallel lines are *always* parallel. **15.** By Post. 1-3, two planes intersect in exactly 1 line, so the intersection