

## DIAGNOSING READINESS

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1.  $\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{7}{6} = 1\frac{1}{6}$  2.  $\frac{3}{13} + \frac{6}{13} = \frac{9}{13}$   
 3.  $\frac{16}{25} + \frac{3}{10} = \frac{32}{50} + \frac{15}{50} = \frac{47}{50}$  4.  $\frac{5}{9} - \frac{5}{36} = \frac{20}{36} - \frac{5}{36} = \frac{15}{36} = \frac{5}{12}$  5.  $P(\text{two } \$1 \text{ bills}) = \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$   
 6.  $P(\text{two } \$5 \text{ bills}) = \frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20} = \frac{1}{10}$  7.  $P(\text{two bills of the same kind}) = P(\text{two } \$1 \text{ bills}) + P(\text{two } \$5 \text{ bills}) = \frac{3}{10} + \frac{1}{10} = \frac{4}{10} = \frac{2}{5}$  8.  $P(\text{two different kinds of bills}) = P(\$1, \text{then } \$5) + P(\$5, \text{then } \$1) = \frac{3}{5} \cdot \frac{2}{4} + \frac{2}{5} \cdot \frac{3}{4} = \frac{6}{20} + \frac{6}{20} = \frac{12}{20} = \frac{3}{5}$  9.  $\frac{6w^3x^2}{2wx} = 3w^2x$  10.  $\frac{81r^{10}s^6}{(3r^2s)^4} = \frac{81r^{10}s^6}{81r^8s^4} = r^2s^2$  11.  $\frac{(5k^5)(2k^3)}{(2k^2)^2} = \frac{10k^8}{4k^4} = \frac{5}{2}k^4$  12.  $\sqrt{x} - 4 = 6; \sqrt{x} = 10; (\sqrt{x})^2 = 10^2; x = 100$  13.  $\sqrt{3x} + 5 = 2; \sqrt{3x} = -3$ ; no solution 14.  $2x = \sqrt{3x + 1}; (2x)^2 = (\sqrt{3x + 1})^2; 4x^2 = 3x + 1; 4x^2 - 3x - 1 = 0; (4x + 1)(x - 1) = 0; 4x + 1 = 0 \text{ or } x - 1 = 0; 4x = -1 \text{ or } x = 1; x = -\frac{1}{4}$  (extraneous) or  $x = 1; x = 1$  15.  $f(x) = 5 - \sqrt{x}$ ; domain:  $x \geq 0$  16.  $y = -2 + \sqrt{3x}$ ; domain:  $3x \geq 0; x \geq 0$  17.  $y = \sqrt{10 - 3x}$ ; domain:  $10 - 3x \geq 0; 10 \geq 3x; x \leq \frac{10}{3}$

## 12.1 Inverse Variation pages 636–642

**Check Skills You'll Need** For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM*.

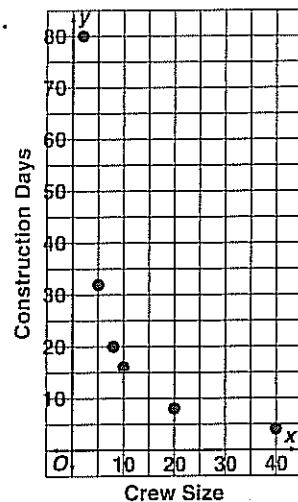
1. 5 2. -7 3.  $\frac{1}{3}$  4. 4 5.  $y = 2x$  6.  $y = 0.5x$

7.  $y = -0.25x$  8.  $y = 0.4x$

**Investigation** 1. If 20 people can complete a house in 8 days, then 40 people can complete a house in 4 days.

| Crew Size (x) | Construction Days (y) | Total Workdays |
|---------------|-----------------------|----------------|
| 2             | 80                    | 160            |
| 5             | 32                    | 160            |
| 8             | 20                    | 160            |
| 10            | 16                    | 160            |
| 20            | 8                     | 160            |
| 40            | 4                     | 160            |

3.



4. Construction time decreases.

- Check Understanding** 1.  $k = xy = 2(9) = 18; xy = 18$   
 2a.  $x_1 \cdot y_1 = x_2 \cdot y_2; 3y = 5(9); 3y = 45; y = 15$   
 2b.  $x_1 \cdot y_1 = x_2 \cdot y_2; 75(0.2) = x(3); 15 = 3x; x = 5$   
 3a.  $100(4) = 75x; 400 = 75x; x = \frac{400}{75} = 5.\bar{3}; 5.\bar{3} \text{ ft}$   
 3b.  $80(9) = w(6); 720 = 6w; w = 120; 120 \text{ lb}$   
 4a. check products  $xy$ : 3(12) = 36, 6(6) = 36, 9(4) = 36; inverse variation;  $xy = 36$  4b. check ratios  $\frac{y}{x} \cdot \frac{12}{3} = 4$ ,  $\frac{20}{5} = 4, \frac{32}{8} = 4$ ; direct variation;  $\frac{y}{x} = 4$ , or  $y = 4x$   
 5a. Direct variation, since the ratio  $\frac{\text{cost}}{\text{sweater}}$  is constant at \$15 each. 5b. Inverse variation, since the total number of miles walked each day is a constant product of 5.

- Exercises** 1.  $k = xy = 3(6) = 18; xy = 18$  2.  $k = xy = 2(1) = 2; xy = 2$  3.  $k = xy = 8(7) = 56; xy = 56$  4.  $k = xy = 0.5(3) = 1.5; xy = 1.5$  5.  $k = xy = 2.4(10) = 24; xy = 24$  6.  $k = xy = 2.2(3.5) = 7.7; xy = 7.7$  7.  $k = xy = \frac{1}{3}(6) = 2; xy = 2$  8.  $k = xy = 8\left(\frac{1}{16}\right) = \frac{1}{2} = 0.5; xy = 0.5$  9.  $k = xy = \frac{3}{5}\left(\frac{1}{10}\right) = \frac{3}{50} = 0.06; xy = 0.06$  10.  $6(12) = 9y; 72 = 9y; y = 8$  11.  $3(5) = 1n; 15 = n; n = 15$  12.  $x(11) = 1(66); 11x = 66; x = 6$  13.  $x(55) = 5(77); 55x = 385; x = 7$  14.  $9.4b = 6(4.7); 9.4b = 28.2; b = 3$  15.  $50(13) = t(5); 650 = 5t; t = 130$  16.  $4(3.6) = 1.2g; 14.4 = 1.2g; g = 12$  17.  $24(1.6) = c(0.4); 38.4 = 0.4c; c = 96$  18.  $500(25) = 4n; 12,500 = 4n; n = 3125$  19.  $\frac{1}{2}(24) = 6y; 12 = 6y; y = 2$  20.  $x\left(\frac{1}{2}\right) = \frac{1}{3}\left(\frac{1}{4}\right); \frac{1}{2}x = \frac{1}{12}$ ,  $x = \frac{2}{12}; x = \frac{1}{6}$  21.  $\frac{1}{2}(5) = b\left(\frac{1}{8}\right); \frac{5}{2} = \frac{b}{8}; 2b = 40; b = 20$  22.  $2.5(48) = x(40); 120 = 40x; x = 3$  3 h 23.  $2(10) = 1.5x; 20 = 1.5x; x = 13.\bar{3}$  13.  $\bar{3}$  mi/h 24. check ratios  $\frac{y}{x} \cdot \frac{1}{2} \cdot \frac{2.5}{5} = \frac{1}{2} \cdot \frac{4}{8} = \frac{1}{2}$ ; direct variation;  $\frac{y}{x} = \frac{1}{2}$ , or  $y = \frac{1}{2}x$ , or  $y = 0.5x$  25. check products  $xy$ : 4(15) = 60, 6(10) = 60, 10(6) = 60; inverse variation;  $xy = 60$  26. check products  $xy$ : 3(24) = 72, 9(8) = 72, 12(6) = 72; inverse variation;  $xy = 72$  27. Direct variation; the ratio  $\frac{\text{cost}}{\text{pound}}$  is

constant at \$1.79. **28.** Inverse variation; the total number of slices is constant at 8. **29.** Inverse variation; the product of the length and width remains constant with an area of 24 square units. **30.**  $k = xy = 4(8) = 32$ ;  $xy = 32$  **31.**  $k = rt = 3.3\left(\frac{1}{3}\right) = 1.1$ ;  $rt = 1.1$  **32.**  $k = xy = \frac{1}{2}(5) = 2.5$ ;  $xy = 2.5$  **33.**  $k = ab = 25(0.04) = 1$ ;  $ab = 1$  **34.**  $k = pq = 10.4(1.5) = 15.6$ ;  $pq = 15.6$  **35.**  $k = xy = 5(75) = 375$ ;  $xy = 375$  **36.** Direct variation; the ratio of the perimeter to the side length is constant at 3. **37.** Inverse variation; the product of the rate and the time is always 150. **38.** Direct variation; the ratio of the circumference to the radius is constant at  $2\pi$ . **39.**  $99(110) = 90x$ ;  $10,890 = 90x$ ;  $x = 121$ ; 121 ft **40.**  $4(3) = 5x$ ;  $12 = 5x$ ;  $x = \frac{12}{5} = 2.4$ ; 2.4 days **41.** check ratios  $\frac{y}{x} : \frac{4}{10} = 0.4$ ,  $\frac{32}{8} = 0.4$ ; direct variation; equation:  $\frac{y}{x} = 0.4$ , or  $y = 0.4x$ ; missing value:  $\frac{y}{20} = 0.4$ ,  $y = 20(0.4) = 8$  **42.** check ratios  $\frac{y}{x} : \frac{28}{0.4} = 70$ ,  $\frac{84}{1.2} = 70$ ; direct variation; equation:  $\frac{y}{x} = 70$ , or  $y = 70x$ ; missing value:  $\frac{63}{x} = 70$ ,  $63 = 70x$ ,  $x = \frac{63}{70} = 0.9$  **43.** check products  $xy$ :  $1.6(30) = 48$ ,  $4.8(10) = 48$ ; inverse variation; equation:  $xy = 48$ ; missing value:  $x(96) = 48$ ,  $96x = 48$ ,  $x = \frac{48}{96} = 0.5$  **44a.** greater **44b.** greater **44c.** less **45a.**  $\$5/h \cdot x h = \$80$ ,  $5x = 80$ ,  $x = \frac{80}{5} = 16$ , 16 h;  $\$8/h \cdot x h = \$80$ ,  $8x = 80$ ,  $x = \frac{80}{8} = 10$ , 10 h;  $\$10/h \cdot x h = \$80$ ,  $10x = 80$ ,  $x = \frac{80}{10} = 8$ , 8 h;  $\$20/h \cdot x h = \$80$ ,  $20x = 80$ ,  $x = \frac{80}{20} = 4$ , 4 h **45b.** hours worked, rate of pay **45c.**  $rt = 80$  **46.** Check students' work. **47.**  $15.3(40) = x(60)$ ;  $612 = 60x$ ;  $x = 10.2$ ;  $10.2$  L **48.**  $p$ , direct variation:  $\frac{y}{x} = \frac{2}{4} = 0.5$ ,  $y = 0.5x$ ;  $q$ , inverse variation:  $xy = 4(2) = 8$ ,  $xy = 8$  **49a.**  $y$  is doubled. **49b.**  $y$  is halved. **50.** by a factor of 4;  $s\left(\frac{1}{2}d\right)^2 = \frac{1}{4}sd^2 = k$ , so  $s = 4\frac{k}{d^2}$ . **51a.**  $x^4y = k$  **51b.**  $\frac{x^4y}{z} = k$  **52.**  $xy = 12(3) = 36$ ;  $xy = 36$ , or  $y = \frac{36}{x}$ ; C **53.**  $VP = 75(30) = 2250$ ; F **54.** [2] Direct variation:  $y = kx$ ,  $10 = 5k$ ,  $k = 2$ . So when  $x = 8$ ,  $y = 2 \cdot 8 = 16$ . Inverse variation:  $xy = k$ ,  $5 \cdot 10 = 50$ . So when  $x = 8$ ,  $y = \frac{50}{8}$ , or 6.25. [1] no work shown OR one computational error **55. [4]** **55a.**

| Distance ( $d$ ) | 100            | 100            | 100 | 100            |
|------------------|----------------|----------------|-----|----------------|
| Speed ( $r$ )    | 30             | 40             | 50  | 60             |
| Time ( $t$ )     | $3\frac{1}{3}$ | $2\frac{1}{2}$ | 2   | $1\frac{2}{3}$ |

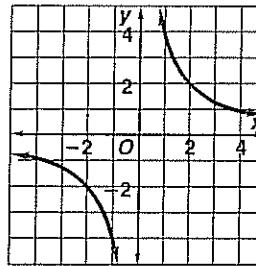
**55b.** The variables speed and time are inversely related. **55c.**  $2\frac{1}{2}$  h [3] one computational error [2] one part missing [1] two parts missing **56.**  $\sin F = \frac{16}{34} = \frac{8}{17}$  **57.**  $\sin G = \frac{30}{34} = \frac{15}{17}$  **58.**  $\cos F = \frac{30}{34} = \frac{15}{17}$  **59.**  $\cos G = \frac{16}{34} = \frac{8}{17}$  **60.**  $\tan F = \frac{16}{30} = \frac{8}{15}$  **61.**  $\tan G = \frac{30}{16} = \frac{15}{8}$  **62.**  $d = \sqrt{(5 - 4)^2 + (2 - 7)^2} = \sqrt{1^2 + (-5)^2} = \sqrt{1 + 25} = \sqrt{26} \approx 5.1$  **63.**  $d = \sqrt{(-2 - 6)^2 + (9 - 0)^2} = \sqrt{(-8)^2 + 9^2} = \sqrt{64 + 81} = \sqrt{145} \approx 12.0$

- 64.**  $d = \sqrt{(-4 - 4)^2 + (-1 - 8)^2} = \sqrt{(-8)^2 + (-9)^2} = \sqrt{64 + 81} = \sqrt{145} \approx 12.0$  **65.**  $d = \sqrt{(8 - 1)^2 + (10 - 2)^2} = \sqrt{7^2 + 8^2} = \sqrt{49 + 64} = \sqrt{113} \approx 10.6$  **66.**  $d = \sqrt{(-3 - (-2))^2 + (-5 - (-7))^2} = \sqrt{(-1)^2 + 2^2} = \sqrt{1 + 4} = \sqrt{5} \approx 2.2$  **67.**  $d = \sqrt{(2 - 1.5)^2 + (3.5 - 1)^2} = \sqrt{(0.5)^2 + (2.5)^2} = \sqrt{0.25 + 6.25} = \sqrt{6.5} \approx 2.5$  **68.**  $3a^2 + 11a - 4 = (3a - 1)(a + 4)$  **69.**  $15x^2 + 41x + 14 = (5x + 2)(3x + 7)$  **70.**  $2y^2 + 13y - 24 = (2y - 3)(y + 8)$

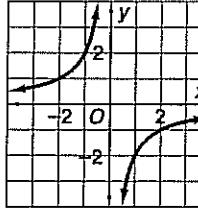
## TECHNOLOGY

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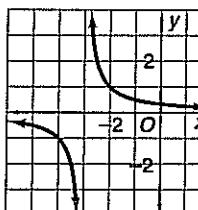
1.  $y = \frac{4}{x}$



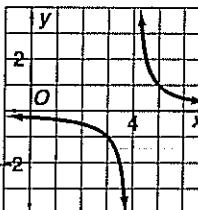
2.  $y = \frac{-2}{x}$



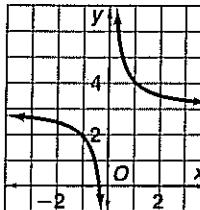
3.  $y = \frac{1}{x+3}$



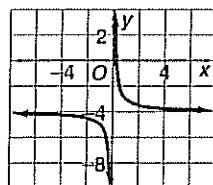
4.  $y = \frac{1}{x-4}$



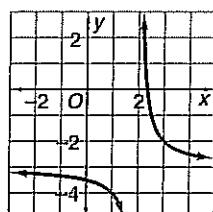
5.  $y = \frac{1}{x} + 3$



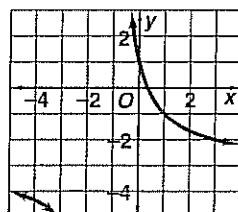
6.  $y = \frac{1}{x} - 4$



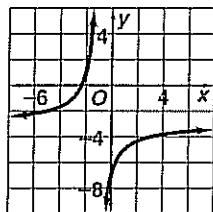
7.  $y = \frac{1}{x-2} - 3$



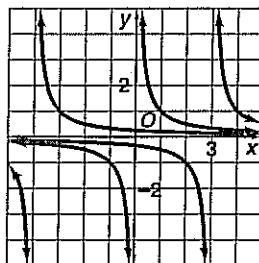
8.  $y = \frac{4}{x+1} - 3$



9.  $y = \frac{-4}{x+1} - 3$



10a.  $y = \frac{1}{x}, y = \frac{1}{x-3}, y = \frac{1}{x+4}$



10b. Adding translates the graph left, subtracting translates the graph right.

11a.  $y = \frac{1}{x}, y = \frac{1}{x} - 3, y = \frac{1}{x} + 4$

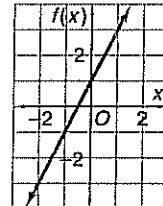
11b. Adding translates the graph up, subtracting translates the graph down.

## 12-2 Graphing Rational Functions

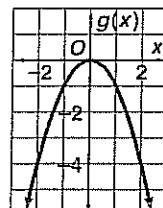
**Check Skills You'll Need** For complete solutions see Daily Skills Check and Lesson Quiz Transparencies or Presentation Pro CD-ROM.

1. -10; -8; -5 2. 8; 4; 13 3.  $\frac{1}{9}; 1; 27$

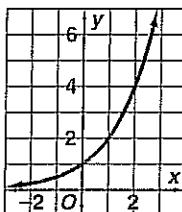
4.  $f(x) = 2x + 1$



5.  $g(x) = -x^2$



6.  $y = 2^x$



### Check Understanding

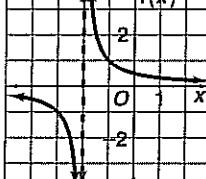
1a.  $t = \frac{40}{r}$ ; points  $(r, t)$ : (5, 8), (10, 4), (20, 2), (40, 1)



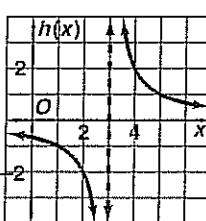
1b. Speed and time are both positive values, so graphing it in the first quadrant only makes sense.

2a.  $f(x) = \frac{1}{x+2}$ ; vertical asymptote:  $x + 2 = 0, x = -2$ ; points:

$$\left(-4, -\frac{1}{2}\right), \left(-3, -1\right), \left(-\frac{5}{2}, -2\right), \left(-\frac{3}{2}, 2\right), \left(-1, 1\right), \left(0, \frac{1}{2}\right)$$

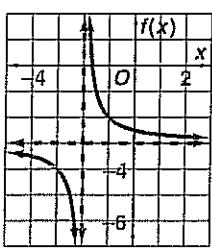


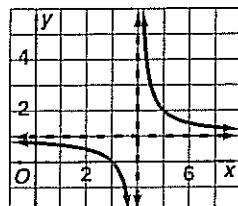
2b.  $h(x) = \frac{2}{x-3}$ ; vertical asymptote:  $x - 3 = 0, x = 3$ ; points:  $\left(0, -\frac{2}{3}\right), (1, -1), (2, -2), (4, 2), (5, 1), \left(6, \frac{2}{3}\right)$



3a.  $f(x) = \frac{1}{x+2} - 3$ ; vertical asymptote:  $x + 2 = 0, x = -2$ ; horizontal asymptote:  $y = -3$ ; points:

$$\left(-4, -3\frac{1}{2}\right), \left(-3, -4\right), \left(-2\frac{1}{2}, -5\right), \left(-1\frac{1}{2}, -1\right), \left(-1, -2\right), \left(0, -2\frac{1}{2}\right)$$

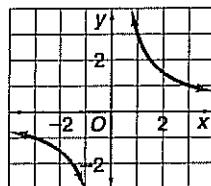




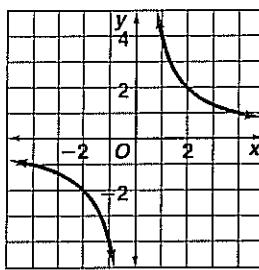
- 3b.**  $y = \frac{1}{x-4} + 1$ ; vertical asymptote:  $x - 4 = 0, x = 4$ ; horizontal asymptote:  $y = 1$ ; points:  $(0, \frac{3}{4}), (2, \frac{1}{2}), (3, 0), (3\frac{1}{2}, -1), (4\frac{1}{2}, 3), (5, 2), (6, 1\frac{1}{2})$

- 4a.**  $g(x) = |x + 4|$ , an absolute value function; the graph has vertex  $(-4, 0)$  **4b.**  $f(x) = 8 \cdot 2^x$ , an exponential function; the graph shows exponential growth **4c.**  $h(x) = \frac{2}{x+1}$ , a rational function; the graph has asymptotes  $x = -1$  and  $y = 0$

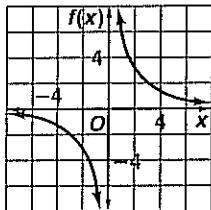
### Exercises



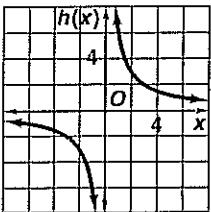
- 1.**  $y = \frac{3}{x}$ ; points  $(-3, -1), (-2, -1\frac{1}{2}), (-1, -3), (1, 3), (2, 1\frac{1}{2}), (3, 1)$



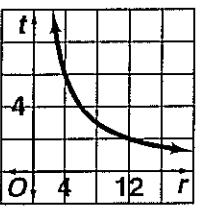
- 2.**  $y = \frac{4}{x}$ ; points:  $(-4, -1), (-2, -2), (-1, -4), (1, 4), (2, 2), (4, 1)$



- 3.**  $f(x) = \frac{5}{x}$ ; points:  $(-5, -1), (-2\frac{1}{2}, -2), (-1, -5), (1, 5), (2\frac{1}{2}, 2), (5, 1)$



- 4.**  $h(x) = \frac{6}{x}$ ; points:  $(-6, -1), (-3, -2), (-2, -3), (-1, -6), (1, 6), (2, 3), (3, 2), (6, 1)$



- 5.**  $t = \frac{24}{7}$ ; points:  $(4, 6), (8, 3), (12, 2), (20, 1.2)$

**6.**  $f(x) = \frac{3}{x}; x = 0$  **7.**  $y = \frac{1}{x-2}; x - 2 = 0, x = 2$

**8.**  $y = \frac{x}{x+2}; x + 2 = 0, x = -2$

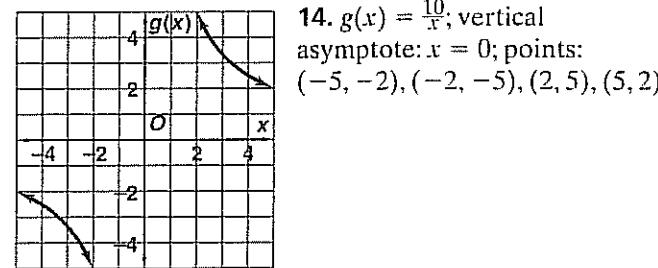
**9.**  $h(x) = \frac{3}{2x-4}; 2x - 4 = 0, 2x = 4, x = 2$

**10.** vertical asymptote:  $x = 2$   
horizontal asymptote:  $y = 0$

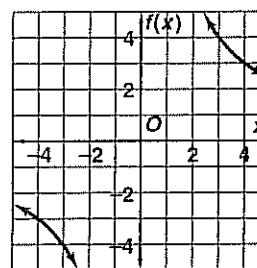
**11.** vertical asymptote:  $x = -1$   
horizontal asymptote:  $y = 0$

**12.** vertical asymptote:  $x = 1$   
horizontal asymptote:  $y = -1$

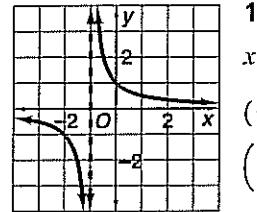
- 13.** vertical asymptote:  $x = 0$   
horizontal asymptote:  $y = 2$



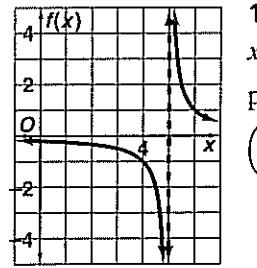
- 14.**  $g(x) = \frac{10}{x}$ ; vertical asymptote:  $x = 0$ ; points:  $(-5, -2), (-2, -5), (2, 5), (5, 2)$



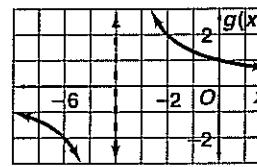
- 15.**  $f(x) = \frac{12}{x}$ ; vertical asymptote:  $x = 0$ ; points:  $(-4, -3), (-3, -4), (3, 4), (4, 3)$



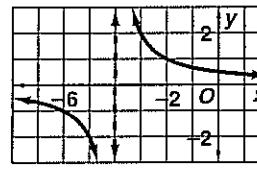
- 16.**  $y = \frac{1}{x+1}$ ; vertical asymptote:  $x + 1 = 0, x = -1$ ; points:  $(-3, -\frac{1}{2}), (-2, -1), (-1\frac{1}{2}, -2), (-\frac{1}{2}, 2), (0, 1), (1, \frac{1}{2})$



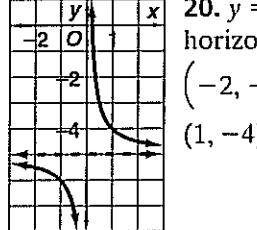
- 17.**  $f(x) = \frac{1}{x-5}$ ; vertical asymptote:  $x - 5 = 0, x = 5$ ; points:  $(0, -\frac{1}{5}), (4, -1), (4\frac{1}{2}, -2), (5\frac{1}{2}, 2), (6, 1)$



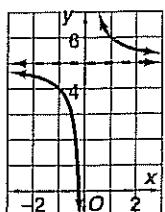
- 18.**  $g(x) = \frac{4}{x+4}$ ; vertical asymptote:  $x + 4 = 0, x = -4$ ; points:  $(-8, -1), (-6, -2), (-2, 2), (0, 1), (2, \frac{2}{3})$



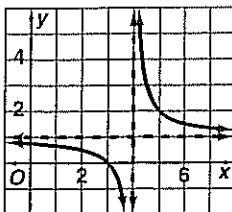
- 19.**  $y = \frac{2}{x+4}$ ; vertical asymptote:  $x + 4 = 0, x = -4$ ; points:  $(-6, -1), (-5, -2), (-3, 2), (-2, 1), (0, \frac{1}{2})$



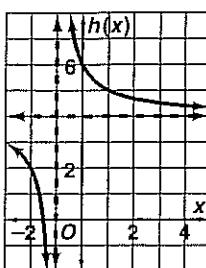
- 20.**  $y = \frac{1}{x} - 5$ ; vertical asymptote:  $x = 0$ ; horizontal asymptote:  $y = -5$ ; points:  $(-2, -5\frac{1}{2}), (-1, -6), (-\frac{1}{2}, -7), (\frac{1}{2}, -3), (1, -4), (2, -4\frac{1}{2})$



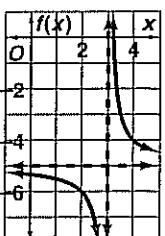
21.  $y = \frac{1}{x} + 5$ ; vertical asymptote:  $x = 0$ ; horizontal asymptote:  $y = 5$ ; points:  $(-2, 4\frac{1}{2}), (-1, 4), (-\frac{1}{2}, 3), (\frac{1}{2}, 7), (1, 6), (2, 5\frac{1}{2})$



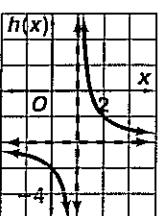
22.  $y = \frac{1}{x} - 6$ ; vertical asymptote:  $x = 0$ ; horizontal asymptote:  $y = -6$ ; points:  $(-2, -6\frac{1}{2}), (-1, -7), (-\frac{1}{2}, -8), (\frac{1}{2}, -4), (1, -5), (2, -5\frac{1}{2})$



23.  $h(x) = \frac{2}{x+1} + 4$ ; vertical asymptote:  $x + 1 = 0, x = -1$ ; horizontal asymptote:  $y = 4$ ; points:  $(-3, 3), (-2, 2), (-1\frac{1}{2}, 0), (-\frac{1}{2}, 8), (0, 6), (1, 5), (3, 4\frac{1}{2})$

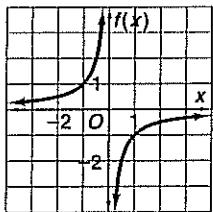


24.  $f(x) = \frac{1}{x-3} - 5$ ; vertical asymptote:  $x - 3 = 0, x = 3$ ; horizontal asymptote:  $y = -5$ ; points:  $(0, -5\frac{1}{3}), (2, -6), (1.5, -7), (3.5, -3), (4, -4)$

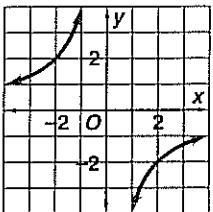


25.  $h(x) = \frac{1}{x-1} - 2$ ; vertical asymptote:  $x - 1 = 0, x = 1$ ; horizontal asymptote:  $y = -2$ ; points:  $(-1, -2\frac{1}{2}), (0, -3), (\frac{1}{2}, -4), (1\frac{1}{2}, 0), (2, -1), (3, -1\frac{1}{2})$

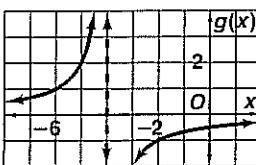
26.  $y = 4x + 1$ ; line with slope 4,  $y$ -intercept 1    27.  $h(x) = |x - 4|$ ; absolute value function with vertex (4, 0)  
 28.  $y = 0.4^x$ ; exponential decay    29.  $f(x) = \frac{x}{4}$ ; line with slope  $\frac{1}{4}$ ,  $y$ -intercept 0    30.  $y = \frac{4}{x} + 1$ ; rational function, with asymptotes  $x = 0, y = 1$     31.  $h(x) = \sqrt{x - 4} + 1$ ; radical function;  $y = \sqrt{x}$  shifted right 4, up 1    32.  $g(x) = x^2 - 4$ ; parabola with axis of symmetry  $x = 0$     33.  $f(x) = \frac{4}{x+4} - 1$ ; rational function with asymptotes  $x = -4, y = -1$     34.  $g(x) = 4x^2 + 2x + 1$ ; parabola with axis of symmetry  $x = -\frac{1}{4}$     35.  $g(x) = \frac{7}{x+1}$ ; moves graph 1 unit to the left    36.  $y = \frac{7}{x-3}$ ; moves graph 3 units to the right  
 37.  $y = \frac{7}{x} - 15$ ; lowers graph 15 units    38.  $f(x) = \frac{7}{x+12}$ ; moves graph 12 units left    39.  $g(x) = \frac{7}{x} + 12$ ; moves graph up 12 units    40.  $h(x) = \frac{7}{x+3}$ ; moves graph left 3 units    41.  $g(x) = \frac{7}{x} - 2$ ; moves graph down 2 units  
 42.  $y = \frac{7}{x+3} - 2$ ; moves graph 3 units left and 2 units down



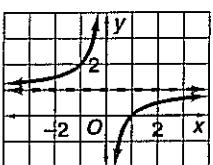
43.  $f(x) = \frac{-1}{x}$ ; vertical asymptote:  $x = 0$ ; horizontal asymptote:  $y = 0$ ; points:  $(-2, \frac{1}{2}), (-1, 1), (-\frac{1}{2}, 2), (\frac{1}{2}, -2), (1, -1), (2, -\frac{1}{2})$



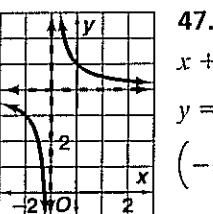
44.  $y = \frac{-4}{x}$ ; vertical asymptote:  $x = 0$ ; horizontal asymptote:  $y = 0$ ; points:  $(-4, 1), (-2, 2), (-1, 4), (1, -4), (2, -2), (4, -1)$



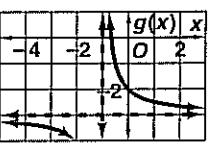
45.  $g(x) = \frac{-2}{x+4}$ ; vertical asymptote:  $x + 4 = 0, x = -4$ ; horizontal asymptote:  $y = 0$ ; points:  $(-8, \frac{1}{2}), (-6, 1), (-5, 2), (-3, -2), (-2, -1), (0, -\frac{1}{2})$



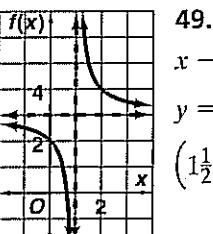
46.  $y = \frac{-1}{x} + 1$ ; vertical asymptote:  $x = 0$ ; horizontal asymptote:  $y = 1$ ; points:  $(-2, 1\frac{1}{2}), (-1, 2), (-\frac{1}{2}, 3), (\frac{1}{2}, -1), (1, 0), (2, \frac{1}{2})$



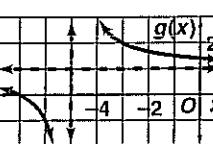
47.  $y = \frac{1}{x+1} + 4$ ; vertical asymptote:  $x + 1 = 0, x = -1$ ; horizontal asymptote:  $y = 4$ ; points:  $(-3, 3\frac{1}{2}), (-2, 3), (-1\frac{1}{2}, 2), (-\frac{1}{2}, 6), (0, 5), (1, 4\frac{1}{2})$



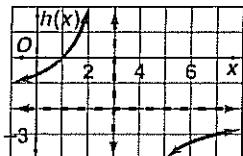
48.  $g(x) = \frac{1}{x+1} - 3$ ; vertical asymptote:  $x + 1 = 0, x = -1$ ; horizontal asymptote:  $y = -3$ ; points:  $(-3, -3\frac{1}{2}), (-2, -4), (-\frac{1}{2}, -1), (0, -2), (1, -2\frac{1}{2})$



49.  $f(x) = \frac{1}{x-1} + 3$ ; vertical asymptote:  $x - 1 = 0, x = 1$ ; horizontal asymptote:  $y = 3$ ; points:  $(-1, 2\frac{1}{2}), (0, 2), (\frac{1}{2}, 1), (1\frac{1}{2}, 5), (2, 4), (3, 3\frac{1}{2})$



50.  $g(x) = \frac{2}{x+5} + 1$ ; vertical asymptote:  $x + 5 = 0, x = -5$ ; horizontal asymptote:  $y = 1$ ; points:  $(-7, 0), (-6, -1), (-4, 3), (-3, 2), (-1, 1\frac{1}{2})$

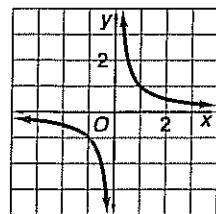


51.  $h(x) = \frac{-4}{x-3} - 2$ ; vertical asymptote:  $x = 3$ ,  $x = 3$ ; horizontal asymptote:  $y = -2$ ; points:  $(-1, -1), (1, 0), (2, 2), (5, -4), (7, -3)$

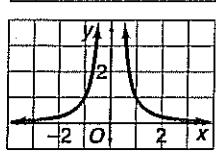
52. Answers may vary. Sample:  $f(x) = \frac{1}{x} + 3, g(x) = \frac{1}{x}$

53. 5 ft from the light bulb:  $I = \frac{445}{5^2} = 17.8$ ; 17.8 lumens;

15 ft from the light bulb:  $I = \frac{445}{15^2} = 1.9\bar{7}$ ; 1.9 $\bar{7}$  lumens



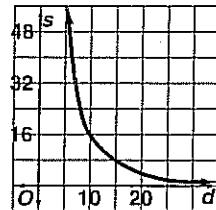
54a.  $y = \frac{1}{x}$ ; points:  $(-2, -\frac{1}{2}), (-1, -1), (-\frac{1}{2}, -2), (\frac{1}{2}, 2), (1, 1), (2, \frac{1}{2})$



$y = \frac{1}{x^2}$ ; points:  $(-2, \frac{1}{4}), (-1, 1), (-\frac{1}{2}, 4), (\frac{1}{2}, 4), (1, 1), (2, \frac{1}{4})$

54b. For both functions, the vertical asymptote is  $x = 0$  (the  $y$ -axis), and the horizontal asymptote is  $y = 0$  (the  $x$ -axis).

54c. The range of  $y = \frac{1}{x}$  is any real number except 0; the range of  $y = \frac{1}{x^2}$  is any real number greater than 0.



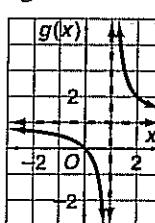
55a.  $s = \frac{1600}{d^2}$ ; points:  $(5, 64), (10, 16), (20, 4), (30, 1.8); d \geq 40$

55b.  $s = \frac{1600}{10^2} = 16; s = \frac{1600}{1^2} = 1600; s = \frac{1600}{0.1^2} = 160,000$

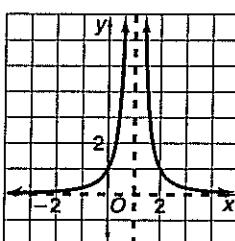
55c. The signal is extremely strong when you are in the immediate vicinity of a transmitter and it will interfere with the other station. 56. The graphs of  $y = \frac{3}{x}$  and  $y = -\frac{3}{x}$  are both composed of two curves with asymptotes  $x = 0$  and  $y = 0$ . The graph of  $y = -\frac{3}{x}$  is a reflection of the graph of  $y = \frac{3}{x}$  over the  $y$ -axis.

57–60. Use a graphing calculator, then sketch the graph.

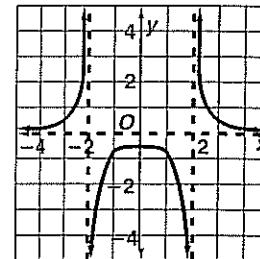
57.  $g(x) = \frac{x}{x-1}$



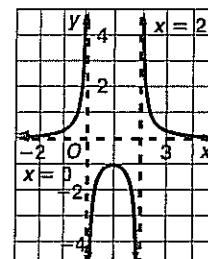
58.  $y = \frac{1}{(x-1)^2}$



59.  $y = \frac{2}{(x-2)(x+2)}$



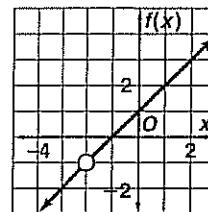
60.  $y = \frac{1}{x^2 - 2x}$



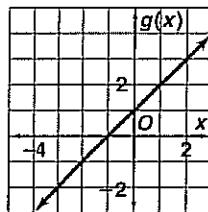
61a. vertical asymptote:  $x = -3$ ; horizontal asymptote:

61b.  $y = \frac{1}{x+3} - 2$

62.  $f(x) = \frac{(x+2)(x+1)}{x+2}$



$g(x) = x + 1$



No;  $f(x) = \frac{(x+2)(x+1)}{x+2}$  is equivalent to  $g(x) = x + 1$  for all values except  $x = -2$ .

63.  $y = \frac{3}{x} + 2$ , a rational function with asymptotes at  $x = 0$  and  $y = 2$ ; C 64.  $y = \frac{2}{x}$  translated down 3 units:  $y = \frac{2}{x} - 3$ ; I

65. A.  $x - 2 = 0, x = 2$  B.  $x + 2 = 0, x = -2; 2 > -2$ , so the answer is A. 66. A.  $y = 2$  B.  $y = 2; 2 = 2$ , so the answer is C. 67. [2] 67a. The graph of

$g(x) = \frac{4}{x-1} + 5$  is a translation of  $f(x) = \frac{4}{x}$ , 5 units up and 1 unit right. 67b.  $x = 1$  and  $y = 5$  [1] one part

answered correctly 68.  $k = xy = 3(7) = 21; xy = 21$

69.  $k = xy = 8(2) = 16; xy = 16$  70.  $k = xy = 4(5.5) = 22; xy = 22$  71.  $k = xy = 6.2(3.4) = 21.08; xy = 21.08$

72.  $x^2 + x + 1 = 0; b^2 - 4ac = 1^2 - 4(1)(1) = 1 - 4 = -3 < 0$ ; 0 real solutions

73.  $x^2 + 2x + 1 = 0; b^2 - 4ac = 2^2 - 4(1)(1) = 4 - 4 = 0$ ; 1 real solution

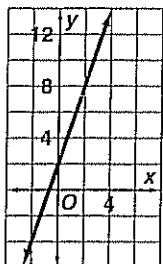
74.  $x^2 - 8x = 7; x^2 - 8x - 7 = 0; b^2 - 4ac = (-8)^2 - 4(1)(-7) = 64 + 28 = 92 > 0$ ; 2 real solutions

75.  $3d^2 - 108 = 3(d^2 - 36) = 3(d-6)(d+6)$  76.  $2m^2 - 14m - 120 = 2(m^2 - 7m - 60) = 2(m-12)(m+5)$

77.  $t^3 - t^2 + 3t - 3 = t^2(t-1) + 3(t-1) = (t^2 + 3)(t-1)$

1. In Quadrant I, as  $x$  increases,  $y$  gets closer and closer to  $-1$ . 2. In Quadrant III, as  $x$  decreases,  $y$  gets closer and closer to  $-1$ . 3. As  $x$  gets very close to 1 from the right,  $y$  gets very small. As  $x$  gets very close to 1 from the left,  $y$  gets very large. 4a. The limit of  $y$  as  $x$  increases is 3. 4b. The limit of  $x$  as  $y$  increases is 1. 4c. The limit of  $y$  as  $x$  decreases is 3. 4d. The limit of  $x$  as  $y$  decreases is 1. 5. The limit of  $y$  as  $|x|$  increases is 0. 6. The limit of  $y$  as  $|x|$  increases is  $-1$ . 7. The limit of  $y$  as  $|x|$  increases is 2.

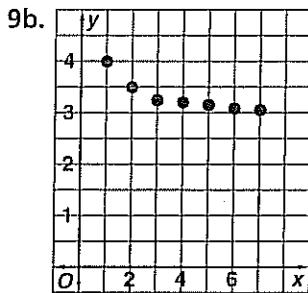
8a.  $y = 3x + 2$



8b. No; there is no asymptote.

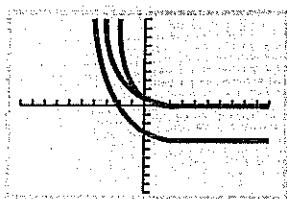
9a.

| Term Number | Term            |
|-------------|-----------------|
| 1           | 4               |
| 2           | $3\frac{1}{2}$  |
| 3           | $3\frac{1}{4}$  |
| 4           | $3\frac{1}{8}$  |
| 5           | $3\frac{1}{16}$ |
| 6           | $3\frac{1}{32}$ |
| 7           | $3\frac{1}{64}$ |



9c. 3

10a.  $y = 0.4^x$ ,  $y = 0.5^x$ ,  $y = 0.5^x - 4$

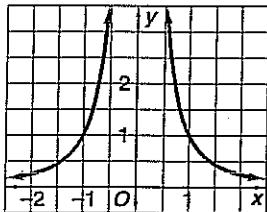


Xmin = -10, Ymin = -10, Xmax = 10, Ymax = 10.

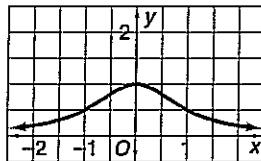
The limit of  $y = 0.4^x$  and  $y = 0.5^x$  as  $x$  increases is 0. The limit of  $y = 0.5^x - 4$  as  $x$  increases is -4.

- 10b. Answers may vary. Sample: Exponential decay functions have a limit as  $x$  gets larger, but not as  $x$  gets smaller.

11a.  $y = \frac{1}{x^2}$



$$y = \frac{1}{x^2 + 1}$$



- 11b. For  $y = \frac{1}{x^2}$ , the limit of  $y$  as  $|x|$  gets larger is 0, and the limit of  $x$  as  $y$  gets larger is 0. For  $y = \frac{1}{x^2 + 1}$ , the limit as  $x$  gets larger is 0.

**Check Skills You'll Need** For complete solutions see Daily Skills Check and Lesson Quiz Transparencies or Presentation Pro CD-ROM.

1. 4 2.  $-\frac{5}{8}$  3.  $\frac{5}{7}$  4.  $(x+4)(x-3)$  5.  $(x+4)(x+2)$   
6.  $(x-5)(x+3)$  7.  $(x+4)^2$  8.  $(x+3)(x-4)$   
9.  $(x-3)(x-4)$

**Check Understanding** 1a.  $\frac{15b}{25b^2} = \frac{3}{5b}$

1b.  $\frac{12c^2}{3c+6} = \frac{12c^2}{3(c+2)} = \frac{4c^2}{c+2}$  1c.  $\frac{4m-2}{2m-1} = \frac{2(2m-1)}{2m-1} = 2$  1d.  $\frac{20+4t}{t+5} = \frac{4(5+t)}{t+5} = 4$

2a.  $\frac{3x+12}{x^2-x-20} = \frac{3(x+4)}{(x-5)(x+4)} = \frac{3}{x-5}$

2b.  $\frac{2z-2}{z^2-4z+3} = \frac{2(z-1)}{(z-3)(z-1)} = \frac{2}{z-3}$

2c.  $\frac{8a+16}{2a^2+5a+2} = \frac{8(a+2)}{(2a+1)(a+2)} = \frac{8}{2a+1}$

2d.  $\frac{c^2-c-6}{c^2+5c+6} = \frac{(c-3)(c+2)}{(c+3)(c+2)} = \frac{c-3}{c+3}$

3a.  $\frac{x-4}{4-x} = \frac{x-4}{-1(x-4)} = \frac{1}{-1} = -1$

3b.  $\frac{8-m}{m^2-64} = \frac{-1(m-8)}{(m+8)(m-8)} = -\frac{1}{m+8}$

3c.  $\frac{8-4r}{r^2+2r-8} = \frac{4(2-r)}{(r+4)(r-2)} = \frac{-4(r-2)}{(r+4)(r-2)} = -\frac{4}{r+4}$  3d.  $\frac{2c^2-2}{3-3c^2} = \frac{2(c^2-1)}{3(1-c^2)} = \frac{2(c^2-1)}{-3(c^2-1)} = -\frac{2}{3}$

4a.  $\frac{30rh}{r+h} = \frac{30(3)(4)}{3+4} = \frac{360}{7} \approx 51.51 \text{ min}$

4b.  $\frac{60\pi r^2 h}{2\pi r^2 + 2\pi rh} = \frac{2\pi r(30rh)}{2\pi r(r+h)} = \frac{30rh}{r+h}$

**Exercises** 1.  $\frac{6a+9}{12} = \frac{3(2a+3)}{3(4)} = \frac{2a+3}{4}$  2.  $\frac{4x^3}{28x^4} = \frac{1}{7x}$

3.  $\frac{2m-5}{6m-15} = \frac{2m-5}{3(2m-5)} = \frac{1}{3}$  4.  $\frac{2p-24}{4p-48} =$

$\frac{2(p-12)}{4(p-12)} = \frac{2}{4} = \frac{1}{2}$  5.  $\frac{3x^2-9x}{x-3} = \frac{3x(x-3)}{x-3} = 3x$

6.  $\frac{3x+6}{3x^2} = \frac{3(x+2)}{3(x^2)} = \frac{x+2}{x^2}$  7.  $\frac{2x^2+2x}{3x^2+3x} = \frac{2x(x+1)}{3x(x+1)} = \frac{2}{3}$

8.  $\frac{2b-8}{b^2-16} = \frac{2(b-4)}{(b+4)(b-4)} = \frac{2}{b+4}$

9.  $\frac{m+6}{m^2-m-42} = \frac{m+6}{(m-7)(m+6)} = \frac{1}{m-7}$

10.  $\frac{w^2+7w}{w^2-49} = \frac{w(w+7)}{(w-7)(w+7)} = \frac{w}{w-7}$

11.  $\frac{a^2+2a+1}{5a+5} = \frac{(a+1)(a+1)}{5(a+1)} = \frac{a+1}{5}$

12.  $\frac{m^2+7m+12}{m^2+6m+8} = \frac{(m+3)(m+4)}{(m+2)(m+4)} = \frac{m+3}{m+2}$

13.  $\frac{c^2-6c+8}{c^2+c-6} = \frac{(c-4)(c-2)}{(c+3)(c-2)} = \frac{c-4}{c+3}$

14.  $\frac{b^2+8b+15}{b+5} = \frac{(b+5)(b+3)}{b+5} = b+3$

15.  $\frac{m+4}{m^2+2m-8} = \frac{m+4}{(m+4)(m-2)} = \frac{1}{m-2}$

16.  $\frac{5-4n}{4n-5} = \frac{-1(4n-5)}{4n-5} = -1$

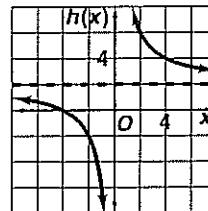
17.  $\frac{12-4t}{t^2-2t-3} = \frac{4(3-t)}{(t+1)(t-3)} = \frac{-4(t-3)}{(t+1)(t-3)} = \frac{-4}{t+1}$

18.  $\frac{4m-8}{4-2m} = \frac{4(m-2)}{2(2-m)} = \frac{4(m-2)}{-2(m-2)} = -2$

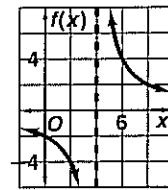
19.  $\frac{m-2}{4-2m} = \frac{m-2}{2(2-m)} = \frac{m-2}{-2(m-2)} = -\frac{1}{2}$

20.  $\frac{v-5}{25-v^2} = \frac{v-5}{(5-v)(5+v)} = \frac{v-5}{-1(v-5)(v+5)} = -\frac{1}{v+5}$   
 21.  $\frac{4-w}{w^2-8w+16} = \frac{-1(w-4)}{(w-4)(w-4)} = -\frac{1}{w-4}$   
 22.  $\frac{30rh}{r+h} = \frac{30(1.25)(26)}{1.25+26} = \frac{975}{27.25} \approx 36; 36 \text{ min}$   
 23.  $\frac{30rh}{r+h} = \frac{30(3.5)(0.5)}{3.5+0.5} = \frac{52.5}{4} \approx 13; 13 \text{ min}$   
 24.  $\frac{30rh}{r+h} = \frac{30(1)(0.75)}{1+0.75} = \frac{22.5}{1.75} \approx 13; 13 \text{ min}$   
 25.  $\frac{2r^2+9r-5}{r^2+10r+25} = \frac{(2r-1)(r+5)}{(r+5)(r+5)} = \frac{2r-1}{r+5}$   
 26.  $\frac{7z^2+23z+6}{z^2+2z-3} = \frac{(7z+2)(z+3)}{(z-1)(z+3)} = \frac{7z+2}{z-1}$   
 27.  $\frac{5t^2+6t-8}{3t^2+5t-2} = \frac{(5t-4)(t+2)}{(3t-1)(t+2)} = \frac{5t-4}{3t-1}$   
 28.  $\frac{32a^3}{16a^2-8a} = \frac{8a(4a^2)}{8a(2a-1)} = \frac{4a^2}{2a-1}$  29.  $\frac{3z^2+12z}{z^4} = \frac{3z(z+4)}{z^3} = \frac{3(z+4)}{z}$   
 30.  $\frac{2s^2+s}{s^3} = \frac{s(2s+1)}{s(s^2)} = \frac{2s+1}{s^2}$   
 31.  $\frac{4a^2-8a-5}{15-a-2a^2} = \frac{4a^2-8a-5}{-1(2a^2+a-15)} = \frac{(2a+1)(2a-5)}{-1(2a-5)(a+3)} = \frac{2a+1}{a+3}$   
 32.  $\frac{16+16m+3m^2}{m^2-3m-28} = \frac{3m^2+16m+16}{m^2-3m-28} = \frac{(3m+4)(m+4)}{(m-7)(m+4)} = \frac{3m+4}{m-7} = \frac{4+3m}{m-7}$   
 33.  $\frac{10c+c^2-3c^3}{5c^2-6c-8} = \frac{-c(3c^2-c-10)}{5c^2-6c-8} = \frac{-c(3c+5)(c-2)}{(5c+4)(c-2)}$  34. Answers may vary.  
 Sample:  $\frac{3}{(x-2)(x+3)}$  35ai.  $\frac{2(b^2+bh+bh)}{b \cdot b \cdot h} = \frac{2b(b+2h)}{b(bh)} = \frac{2(b+2h)}{bh} = \frac{2b+4h}{bh}$  35aii.  $\frac{2\pi r^2+2\pi rh}{\pi r^2 h} = \frac{\pi r(2r+2h)}{\pi r(rh)} = \frac{2r+2h}{rh} = \frac{2h+2r}{rh}$  35b. square prism:  
 $\frac{2b+4h}{bh} = \frac{2(12)+4(18)}{12(18)} = \frac{96}{216} = \frac{4}{9}$ ; cylinder:  $\frac{2h+2r}{rh} = \frac{2(18)+2(6)}{6(18)} = \frac{48}{108} = \frac{4}{9}$  36. The student canceled terms instead of factors. 37.  $-3$  is not in the domain of  $\frac{x^2-9}{x+3}$ .  
 38.  $\frac{\text{area of shaded part}}{\text{area of whole figure}} = \frac{(5w)^2}{\frac{1}{2}(5w)(5w+5w+12)} = \frac{(5w)^2}{\frac{1}{2}(5w)2(5w+6)} = \frac{(5w)(5w)}{(5w)(5w+6)} = \frac{5w}{5w+6}$   
 39.  $\frac{\text{area of shaded part}}{\text{area of whole figure}} = \frac{\pi(3x)^2}{\pi(6x)^2} = \frac{9\pi x^2}{36\pi x^2} = \frac{1}{4}$   
 40.  $\frac{\text{area of shaded part}}{\text{area of whole figure}} = \frac{\frac{1}{2}(8)(3y)}{(8)(8+2y)} = \frac{12y}{8(2)(4+y)} = \frac{12y}{16(4+y)} = \frac{3y}{4(y+4)}$  41.  $\frac{\text{area of shaded part}}{\text{area of whole figure}} = \frac{\frac{1}{2}(4)(2t+6)}{4(3t+6)} = \frac{\frac{1}{2}(2)(t+3)}{3(t+2)} = \frac{t+3}{3(t+2)}$   
 42.  $\frac{m^2-n^2}{m^2+11mn+10n^2} = \frac{(m-n)(m+n)}{(m+10n)(m+n)} = \frac{m-n}{m+10n}$   
 43.  $\frac{a^2-5ab+6b^2}{a^2+2ab-8b^2} = \frac{(a-3b)(a-2b)}{(a+4b)(a-2b)} = \frac{a-3b}{a+4b}$   
 44.  $\frac{36v^2-49w^2}{18v^2+9vw-14w^2} = \frac{(6v-7w)(6v+7w)}{(3v-2w)(6v+7w)} = \frac{6v-7w}{3v-2w}$   
 45.  $\frac{2b}{b} = 2$ ; not true if  $b = 0$ ; sometimes 46.  $\frac{ab^3}{b^4} = ab$ ; not true if  $b = 0$ ; sometimes 47.  $\frac{a^2+6a-5}{2a+2} \neq \frac{a+5}{2}$ ; never  
 48. A.  $\frac{x+1}{x-1} \neq -1$  B.  $\frac{r+3}{3-r} = \frac{r+3}{-1(r-3)} \neq -1$   
 C.  $\frac{n-2}{2-n} = \frac{n-2}{-1(n-2)} = \frac{1}{-1} = -1$  D.  $\frac{4-p}{4+p} \neq -1$ ;

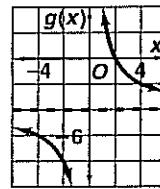
the answer is C. 49.  $\frac{y^2+8y-9}{y^2-81} = \frac{(y+9)(y-1)}{(y+9)(y-9)} = \frac{y-1}{y-9}$ ; I 50. A.  $\frac{t+1}{t^2-1} = \frac{t+1}{(t+1)(t-1)} = \frac{1}{t-1}$  B.  $\frac{2n-1}{n^2+4}$ , simplest form C.  $\frac{c-7}{7-c} = \frac{c-7}{-1(c-7)} = \frac{1}{-1} = -1$   
 D.  $\frac{2r-4}{8+6r} = \frac{2(r-2)}{2(3r+4)} = \frac{r-2}{3r+4}$ ; the answer is B.  
 51. A.  $\frac{-5(x+5)}{x+1}$  B.  $-10x \cdot \frac{2x}{4x^2} = \frac{-20x^2}{4x^2} = -5$ ; the answer is D. 52. A.  $\frac{4}{8-2b} = \frac{2(2)}{2(4-b)} = \frac{2}{4-b}$  B.  $\frac{2}{4-b}$ ; the answer is C. 53. [2] The student put the 4 in the numerator rather than in the denominator.  $\frac{x-5}{4x-20} = \frac{x-5}{4(x-5)} = \frac{1}{4}$  [1] no explanation OR no correctly simplified expression



54.  $h(x) = \frac{8}{x} + 2$ ; vertical asymptote:  $x = 0$ ; horizontal asymptote:  $y = 2$ ; points:  $(-8, 1), (-4, 0), (-2, -2), (-1, -6), (1, 10), (2, 6), (4, 4), (8, 3)$



55.  $f(x) = \frac{8}{x-4}$ ; vertical asymptote:  $x-4 = 0, x = 4$ ; horizontal asymptote:  $y = 0$ ; points:  $(0, -2), (2, -4), (6, 4), (8, 2)$



56.  $g(x) = \frac{8}{x} - 4$ ; vertical asymptote:  $x = 0$ ; horizontal asymptote:  $y = -4$ ; points:  $(-4, -6), (-2, -8), (2, 0), (4, -2)$

57.  $\sqrt{20} \cdot \sqrt{10} = \sqrt{200} = \sqrt{100 \cdot 2} = 10\sqrt{2}$

58.  $\sqrt{a^4b^7c^8} = \sqrt{(a^2b^3c^4)^2b} = a^2b^3c^4\sqrt{b}$

59.  $\frac{\sqrt{80}}{\sqrt{10}} = \sqrt{\frac{80}{10}} = \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2}$

60.  $\sqrt{\frac{2m}{25m^5}} = \sqrt{\frac{2}{25m^4}} = \frac{\sqrt{2}}{5m^2}$  61.  $y = x^2$ ,

$y = -2x^2$ ,  $y = 3x^2$  62.  $y = \frac{1}{4}x^2$ ,  $y = \frac{1}{3}x^2$ ,  $y = \frac{2}{5}x^2$

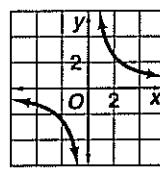
63.  $y = 0.5x^2$ ,  $y = 2x^2$ ,  $y = -4x^2$  64.  $y = -x^2$ ,  $y = 2.3x^2$ ,  $y = -3.8x^2$

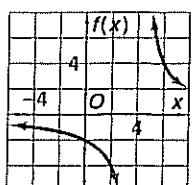
### CHECKPOINT QUIZ 1

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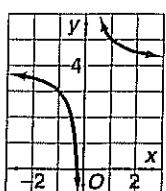
1.  $9(2) = x(6); 18 = 6x; x = 3$  2.  $8.2(-3) = 12.3y; -24.6 = 12.3y; y = -2$  3.  $0.5(7.2) = 0.9y; 3.6 = 0.9y; y = 4$

4.  $y = \frac{5}{x}$ ; vertical asymptote:  $x = 0$ ; horizontal asymptote:  $y = 0$ ; points:  $(-5, -1), (-2.5, -2), (-2, -2.5), (-1, -5), (1, 5), (2, 2.5), (2.5, 2), (5, 1)$





5.  $f(x) = \frac{8}{x-4}$ ; vertical asymptote:  $x-4=0, x=4$ ; horizontal asymptote:  $y=0$ ; points:  $(-4, -1), (0, -2), (2, -4), (6, 4), (8, 2)$



6.  $y = \frac{1}{x} + 4$ ; vertical asymptote:  $x=0$ ; horizontal asymptote:  $y=4$ ; points:  $(-2, 3\frac{1}{2}), (-1, 3), (-\frac{1}{2}, 2), (\frac{1}{2}, 6), (1, 5), (2, 4\frac{1}{2})$

7.  $\frac{6x^2 - 24}{x+2} = \frac{6(x^2 - 4)}{x+2} = \frac{6(x-2)(x+2)}{x+2} = 6(x-2)$

8.  $\frac{3c+9}{3c-9} = \frac{3(c+3)}{3(c-3)} = \frac{c+3}{c-3}$

9.  $\frac{k-2}{k^2+2k-8} = \frac{k-2}{(k-2)(k+4)} = \frac{1}{k+4}$

10. Answers may vary. Sample:  $\frac{15}{x-3}$

5.  $\frac{2x}{x+1} \cdot \frac{x-1}{3} = \frac{2x(x-1)}{3(x+1)} \quad 6. \frac{6x^2}{5} \cdot \frac{2}{x+1} = \frac{12x^2}{5(x+1)}$

7.  $\frac{4c}{2c+2} \cdot \frac{c+1}{c-1} = \frac{4c}{2(c+1)} \cdot \frac{c+1}{c-1} = \frac{2c}{c-1}$

8.  $\frac{5t^3}{x^2} \cdot \frac{3x^4}{6x} = \frac{15x^7}{6x^3} = \frac{5x^4}{2} \quad 9. \frac{3t}{t-2} = \frac{3t-6}{t^2} = \frac{3(t-2)}{t^2} = \frac{9}{t}$

10.  $\frac{m-2}{3m+9} \cdot \frac{2m+6}{2m-4} = \frac{m-2}{3(m+3)} \cdot \frac{2(m+3)}{2(m-2)} = \frac{1}{3}$

11.  $\frac{x-5}{4x+6} \cdot \frac{6x+9}{3x-15} = \frac{x-5}{2(2x+3)} \cdot \frac{3(2x+3)}{3(x-5)} = \frac{1}{2}$

12.  $\frac{4x+1}{5x+10} \cdot \frac{30x+60}{2x-2} = \frac{4x+1}{5(x+2)} \cdot \frac{30(x+2)}{2(x-1)} = \frac{3(4x+1)}{x-1} \quad 13. \frac{4t+4}{t-3} \cdot (t^2-t-6) = \frac{4(t+1)}{t-3} \cdot \frac{(t-3)(t+2)}{1} = 4(t+1)(t+2)$

14.  $\frac{2m+1}{3m-6} \cdot (9m^2-36) = \frac{2m+1}{3(m-2)} \cdot 9(m^2-4) =$

$\frac{2m+1}{3(m-2)} \cdot \frac{9(m-2)(m+2)}{1} = 3(2m+1)(m+2)$

15.  $(x^2-1) \cdot \frac{x-2}{3x+3} = \frac{(x-1)(x+1)}{1} \cdot \frac{x-2}{3(x+1)} =$

$\frac{(x-1)(x-2)}{3} \quad 16.$  reciprocal of  $\frac{2}{x+1} \cdot \frac{x+1}{2}$

17. reciprocal of  $\frac{-6d^2}{2d-5} : -\frac{2d-5}{6d^2} \quad 18.$  reciprocal of

$c^2-1 : \frac{1}{c^2-1} \quad 19.$  reciprocal of  $s+4 : \frac{1}{s+4}$

20.  $\frac{x-1}{x+4} \div \frac{x+3}{x+4} = \frac{x-1}{x+4} \cdot \frac{x+4}{x+3} = \frac{x-1}{x+3}$

21.  $\frac{3t+12}{5t} \div \frac{t+4}{10t} = \frac{3t+12}{5t} \cdot \frac{10t}{t+4} = \frac{3(t+4)}{5t} \cdot \frac{10t}{t+4} =$

$\frac{30t}{5t} = 6 \quad 22. \frac{y-4}{10} \div \frac{4-y}{5} = \frac{y-4}{10} \cdot \frac{5}{4-y} =$

$\frac{y-4}{10} \cdot \frac{5}{-1(y-4)} = -\frac{1}{2} \quad 23. \frac{x-3}{6} \div \frac{3-x}{2} =$

$\frac{x-3}{6} \cdot \frac{2}{3-x} = \frac{x-3}{6} \cdot \frac{2}{-1(x-3)} = -\frac{1}{3}$

24.  $\frac{x^2+6x+8}{x^2+x-2} \div \frac{x+4}{2x+4} = \frac{x^2+6x+8}{x^2+x-2} \cdot \frac{2x+4}{x+4} =$

$\frac{(x+4)(x+2)}{(x-1)(x+2)} \cdot \frac{2(x+2)}{x+4} = \frac{2(x+2)}{x-1}$

25.  $\frac{2n^2-5n-3}{4n^2-12n-7} \div \frac{4n+5}{2n-7} = \frac{2n^2-5n-3}{4n^2-12n-7} \cdot \frac{2n-7}{4n+5} =$

$\frac{(2n+1)(n-3)}{(2n+1)(2n-7)} \cdot \frac{2n-7}{4n+5} = \frac{n-3}{4n+5} \quad 26. \frac{3x+9}{x} \div (x+3) =$

$\frac{3x+9}{x} \cdot \frac{1}{x+3} = \frac{3(x+3)}{x} \cdot \frac{1}{x+3} = \frac{3}{x} \quad 27. \frac{11k+121}{7k-15} \div$

$(k+11) = \frac{11k+121}{7k-15} \cdot \frac{1}{k+11} = \frac{11(k+11)}{7k-15} \cdot \frac{1}{k+11} =$

$\frac{11}{7k-15} \quad 28. \frac{x^2+10x-11}{x^2+12x+11} \div (x-1) =$

$\frac{x^2+10x-11}{x^2+12x+11} \cdot \frac{1}{x-1} = \frac{(x-1)(x+11)}{(x+1)(x+11)} \cdot \frac{1}{x-1} = \frac{1}{x+1}$

29.  $\frac{t^2+5t+6}{t-3} \cdot \frac{t^2-2t-3}{t^2+3t+2} =$

$\frac{(t+3)(t+2)}{t-3} \cdot \frac{(t-3)(t+1)}{(t+2)(t+1)} = t+3$

30.  $\frac{c^2+3c+2}{c^2-4c+3} \div \frac{c+2}{c-3} = \frac{c^2+3c+2}{c^2-4c+3} \cdot \frac{c-3}{c+2} =$

$\frac{(c+2)(c+1)}{(c-3)(c-1)} \cdot \frac{c-3}{c+2} = \frac{c+1}{c-1}$

31.  $\frac{7t^2-28t}{2t^2-5t-12} \cdot \frac{6t^2-t-15}{49t^3} =$

$\frac{7t(t-4)}{(2t+3)(t-4)} \cdot \frac{(3t-5)(2t+3)}{49t^3} = \frac{3t-5}{7t^2}$

32.  $\frac{5x^2+10x-15}{5-6x+x^2} \div \frac{2x^2+7x+3}{4x^2-8x-5} =$

$\frac{5(x^2+2x-3)}{x^2-6x+5} \cdot \frac{4x^2-8x-5}{2x^2+7x+3} =$

$$\frac{5(x+3)(x-1)}{(x-5)(x-1)} \cdot \frac{(2x+1)(2x-5)}{(2x+1)(x+3)} = \frac{5(2x-5)}{x-5}$$

$$33. \frac{x^2+x-6}{x^2-x-6} \div \frac{x^2+5x+6}{x^2+4x+4} =$$

$$\frac{x^2+x-6}{x^2-x-6} \cdot \frac{x^2+4x+4}{x^2+5x+6} =$$

$$\frac{(x+3)(x-2)}{(x-3)(x+2)} \cdot \frac{(x+2)(x+2)}{(x+2)(x+3)} = \frac{x-2}{x-3}$$

$$34. \left( \frac{x^2-25}{x^2-4x} \right) \left( \frac{x^2+x-20}{x^2+10x+25} \right) =$$

$$\frac{(x-5)(x+5)(x+5)(x-4)}{x(x-4)(x+5)(x+5)} = \frac{x-5}{x}$$

35. The student forgot to rewrite the expression using the reciprocal before canceling. 36. Answers may vary.

Sample:  $\frac{9(m+1)}{3m^2} \cdot \frac{m+2}{3(m+1)} \cdot \frac{m+2}{m^2}$

$$37. \frac{2x^2-5x-12}{6x} \div \frac{-3x-12}{x^2-16} =$$

$$\frac{2x^2-5x-12}{6x} \cdot \frac{x^2-16}{-3x-12} =$$

$$\frac{(2x+3)(x-4)}{6x} \cdot \frac{(x-4)(x+4)}{-3(x+4)}; \text{ undefined for } x = 0, 4, -4$$

$$38. m = \frac{A\left(\frac{r}{12}\right)\left(1 + \frac{r}{12}\right)^n}{\left(1 + \frac{r}{12}\right)^n - 1} = \frac{\$1500\left(\frac{0.08}{12}\right)\left(1 + \frac{0.08}{12}\right)^{18}}{\left(1 + \frac{0.08}{12}\right)^{18} - 1} \approx$$

$$\$88.71 \quad 39. m = \frac{A\left(\frac{r}{12}\right)\left(1 + \frac{r}{12}\right)^n}{\left(1 + \frac{r}{12}\right)^n - 1} =$$

$$\frac{\$3000\left(\frac{0.06}{12}\right)\left(1 + \frac{0.06}{12}\right)^{24}}{\left(1 + \frac{0.06}{12}\right)^{24} - 1} \approx \$132.96$$

$$40a. \$115,000 - \$15,000 = \$100,000 \quad 40b. 30(12) = 360;$$

$$360 \text{ payments} \quad 40c. m = \frac{A\left(\frac{r}{12}\right)\left(1 + \frac{r}{12}\right)^n}{\left(1 + \frac{r}{12}\right)^n - 1} =$$

$$\frac{\$100,000\left(\frac{0.06}{12}\right)\left(1 + \frac{0.06}{12}\right)^{360}}{\left(1 + \frac{0.06}{12}\right)^{360} - 1} \approx \$599.55$$

$$40d. 360(\$599.55) = \$215,838$$

$$41. \frac{x-5}{3x+2} \cdot \frac{x-2}{x^2+2x-35} \cdot \frac{3x+2}{4} =$$

$$\frac{(x-5)(x-2)(3x+2)}{(3x+2)(x+7)(x-5)(4)} = \frac{x-2}{4(x+7)}$$

$$42. \frac{m^2-m-6}{m^2+m-2} \cdot \frac{m^3}{m^2+m-12} \cdot \frac{2m+4}{m} =$$

$$\frac{(m-3)(m+2)(m^3)(2)(m+2)}{(m+2)(m-1)(m+4)(m-3)(m)} = \frac{2m^2(m+2)}{(m-1)(m+4)}$$

$$43. \frac{3a^2+8a-3}{2a^2+5a-3} \cdot \frac{4a-6}{3a-1} \cdot \frac{2a-1}{2a^2+7a-15} =$$

$$\frac{(3a-1)(a+3)(2)(2a-3)(2a-1)}{(2a-1)(a+3)(3a-1)(2a-3)(a+5)} = \frac{2}{a+5}$$

$$44. \frac{r+2}{r^2-1} \cdot \frac{r-3}{r^2+3r+2} \cdot \frac{r^2-9}{r^2-6r+9} =$$

$$\frac{(r+2)(r-3)(r+3)(r-3)}{(r+1)(r-1)(r+2)(r+1)(r-3)(r-3)} = \frac{r+3}{(r-1)(r+1)^2}$$

$$45. \text{Robin wrote } w^5 \text{ as a fraction to easily see what could cancel.} \quad 46a. P(\text{shaded}) = \frac{2x(x+1)}{(2x+1)(4x+4)} =$$

$$\frac{2x(x+1)}{(2x+1)(4)(x+1)} = \frac{x}{2(2x+1)}; P(\text{both points shaded}) =$$

$$\frac{x}{2(2x+1)} \cdot \frac{x}{2(2x+1)} = \frac{x^2}{4(2x+1)^2}$$

$$46b. P(\text{not shaded}) = \frac{(2x+1)(4x+4) - 2x(x+1)}{(2x+1)(4x+4)} =$$

$$\frac{8x^2+12x+4-2x^2-2x}{4(2x+1)(x+1)} = \frac{6x^2+10x+4}{4(2x+1)(x+1)} =$$

$$\frac{(3x+2)(2x+2)}{4(2x+1)(x+1)} = \frac{2(3x+2)(x+1)}{4(2x+1)(x+1)} = \frac{3x+2}{2(2x+1)},$$

$$P(\text{shaded and not shaded}) = \frac{x}{2(2x+1)} \cdot \frac{3x+2}{2(2x+1)} =$$

$$\frac{x(3x+2)}{4(2x+1)^2} \quad 47. \frac{3m^3-3m}{4m^2+4m-8} \cdot (6m^2+12m) =$$

$$\frac{3m(m^2-1)}{4(m^2+m-2)} \cdot \frac{6m(m+2)}{1} =$$

$$\frac{3m(m+1)(m-1)(6m)(m+2)}{4(m+2)(m-1)} = \frac{18m^2(m+1)}{4} =$$

$$\frac{9m^2(m+1)}{2} \quad 48. \frac{t^2-r^2}{t^2+tr-2r^2} \cdot \frac{t^2+3tr+2r^2}{t^2+2tr+r^2} =$$

$$\frac{(t-r)(t+r)}{(t+2r)(t-r)} \cdot \frac{(t+2r)(t+r)}{(t+r)(t+r)} = 1$$

$$49. \frac{5x^2}{y^2-25} \div \frac{5xy-25x}{y^2-10y+25} = \frac{5x^2}{y^2-25} \cdot \frac{y^2-10y+25}{5xy-25x} =$$

$$\frac{5x^2}{(y-5)(y+5)} \cdot \frac{(y-5)^2}{5x(y-5)} = \frac{x}{y+5}$$

$$50. \frac{2a^2-ab-6b^2}{2b^2+9ab-5a^2} \div \frac{2a^2-7ab+6b^2}{a^2-4b^2} =$$

$$\frac{2a^2-ab-6b^2}{2b^2+9ab-5a^2} \cdot \frac{a^2-4b^2}{2a^2-7ab+6b^2} =$$

$$\frac{(2a+3b)(a-2b)}{(2b-a)(b+5a)} \cdot \frac{(a-2b)(a+2b)}{(2a-3b)(a-2b)} =$$

$$\frac{(2a+3b)(a-2b)(a-2b)(a+2b)}{(-1)(a-2b)(5a+b)(2a-3b)(a-2b)} =$$

$$\frac{-(2a+3b)(a+2b)}{(5a+b)(2a-3b)} \quad 51. \frac{3m}{m-1} \div \frac{6m^2}{m-2} =$$

$$\frac{3m}{m-1} \cdot \frac{m-2}{6m^2} = \frac{m-2}{2m(m-1)} \quad 52. \frac{3x}{x^2-1} \div \frac{6}{x^2-x-2} =$$

$$\frac{3x}{x^2-1} \cdot \frac{x^2-x-2}{6} = \frac{3x}{(x-1)(x+1)} \cdot \frac{(x-2)(x+1)}{6} =$$

$$\frac{x(x-2)}{2(x-1)} \quad 53. \frac{w-3}{w^2-4} \div \frac{w^2-9}{w-2} = \frac{w-3}{w^2-4} \cdot \frac{w-2}{w^2-9} =$$

$$\frac{w-3}{(w-2)(w+2)} \cdot \frac{w-2}{(w-3)(w+3)} = \frac{1}{(w+2)(w+3)}$$

$$54. (2x-5) \cdot \frac{2x}{2x^2-9x+10} = \frac{2x(2x-5)}{(2x-5)(x-2)} = \frac{2x}{x-2}; \text{B}$$

$$55. \frac{a^2-9}{a^2} \div \frac{a+3}{a} = \frac{a^2-9}{a^2} \cdot \frac{a}{a+3} = \frac{(a-3)(a+3)(a)}{a^2(a+3)} =$$

$$\frac{a-3}{a}; \text{G} \quad 56. \frac{r^2-1}{r} \div (2r^2-2) = \frac{r^2-1}{r} \cdot \frac{1}{2r^2-2}; \text{D}$$

$$57. \text{G} \quad 58. [2] \frac{x^2-1}{x} \cdot \frac{3x}{x-1} = \frac{(x+1)(x-1)(3x)}{x(x-1)} =$$

$3(x+1)$  [1] one computational error OR answer with no work shown

$$59. \frac{5b-25}{10} = \frac{5(b-5)}{10} = \frac{b-5}{2}$$

$$60. \frac{36k^3}{48k^4} = \frac{3}{4k} \quad 61. \frac{7m-14}{3m-6} = \frac{7(m-2)}{3(m-2)} = \frac{7}{3} \quad 62. \frac{7q^5}{28q} = \frac{q^4}{4}$$

$$63. \frac{15t^2-27}{24} = \frac{3(5t^2-9)}{3(8)} = \frac{5t^2-9}{8} \quad 64. \frac{6m^3}{12m-18m^2} =$$

$$\frac{6m^3}{6m(2-3m)} = \frac{m^2}{2-3m} \quad 65. \frac{5a^2}{10a^4-15a^2} = \frac{5a^2}{5a^2(2a^2-3)} =$$

$$\frac{1}{2a^2-3} \quad 66. \frac{2z^2-11z-21}{z^2-6z-7} = \frac{(2z+3)(z-7)}{(z+1)(z-7)} = \frac{2z+3}{z+1}$$

$$67. \frac{4c^2-36c+81}{4c^2-2c-72} = \frac{(2c-9)^2}{2(2c^2-c-36)} = \frac{(2c-9)^2}{2(2c-9)(c+4)} =$$

$$\frac{2c-9}{2(c+4)}, \text{ or } \frac{2c-9}{2c+8} \quad 68. a^2+b^2=c^2; 2^2+8^2=c^2;$$

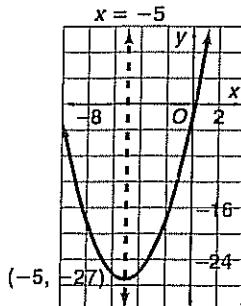
$$4+64=c^2; 68=c^2; c=\sqrt{68} \approx 8.2 \quad 69. a^2+b^2=c^2;$$

$$3.12+4.3^2=c^2; 9.61+18.49=c^2; 28.1=c^2;$$

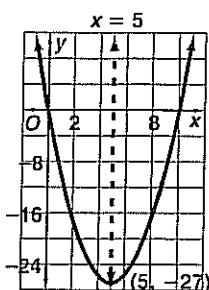
$$c=\sqrt{28.1} \approx 5.3 \quad 70. a^2+b^2=c^2; (\sqrt{7})^2+b^2=$$

$$(\sqrt{32})^2; 7+b^2=32; b^2=25; b=\sqrt{25}=5$$

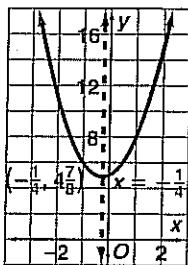
71.  $a^2 + b^2 = c^2$ ;  $(\sqrt{10})^2 + (\sqrt{111})^2 = c^2$ ;  $10 + 111 = c^2$ ;  $121 = c^2$ ;  $c = \sqrt{121} = 11$  72.  $a^2 + b^2 = c^2$ ;  $(\frac{1}{5})^2 + (\frac{1}{12})^2 = c^2$ ;  $\frac{1}{25} + \frac{1}{144} = c^2$ ;  $\frac{169}{3600} = c^2$ ;  $c = \sqrt{\frac{169}{3600}} = \frac{13}{60} \approx 0.2$  73.  $a^2 + b^2 = c^2$ ;  $(2\frac{1}{3})^2 + (6\frac{2}{3})^2 = c^2$ ;  $\frac{49}{9} + \frac{400}{9} = c^2$ ;  $\frac{449}{9} = c^2$ ;  $c = \sqrt{\frac{449}{9}} \approx 7.1$



74.  $y = x^2 + 10x - 2$ ; axis of symmetry:  $x = \frac{-b}{2a} = \frac{-10}{2(1)} = -5$ ,  $x = -5$ ;  $y$ -coordinate of vertex:  $y = (-5)^2 + 10(-5) - 2 = -27$ ; vertex  $(-5, -27)$



75.  $y = x^2 - 10x - 2$ ; axis of symmetry:  $x = \frac{-b}{2a} = \frac{-(-10)}{2(2)} = 5$ ,  $x = 5$ ;  $y$ -coordinate of vertex:  $y = 5^2 - 10(5) - 2 = -27$ ; vertex  $(5, -27)$



76.  $y = 2x^2 + x + 5$ ; axis of symmetry:  $x = \frac{-b}{2a} = \frac{-1}{2(2)} = -\frac{1}{4}$ ,  $x = -\frac{1}{4}$ ;  $y$ -coordinate of vertex:  $y = 2(-\frac{1}{4})^2 + (-\frac{1}{4}) + 5 = \frac{47}{8}$ ; vertex  $(-\frac{1}{4}, \frac{47}{8})$

## 12-5 Dividing Polynomials

pages 662–666

**Check Skills You'll Need** For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies or Presentation Pro CD-ROM*.

1.  $-4a^2 + 9a + 1$  2.  $-x^3 + 3x^2 + 5x - 6$  3.  $8t - 2$

4.  $2x^2 + 10x + 12$  5.  $-3n^2 + 11n + 20$

6.  $6a^3 - 21a^2 + 2a - 7$

**Check Understanding** 1a.  $(3m^3 - 6m^2 + m) \div 3m^2 =$

$$(3m^3 - 6m^2 + m) \cdot \frac{1}{3m^2} = \frac{3m^3}{3m^2} - \frac{6m^2}{3m^2} + \frac{m}{3m^2} =$$

$m - 2 + \frac{1}{3m}$  1b.  $(8t^5 + 16t^3 - 4t^2 + 2t) \div 4t^2 =$

$$(8t^5 + 16t^3 - 4t^2 + 2t) \cdot \frac{1}{4t^2} = \frac{8t^5}{4t^2} + \frac{16t^3}{4t^2} - \frac{4t^2}{4t^2} + \frac{2t}{4t^2} =$$

$2t^3 + 4t - 1 + \frac{1}{2t}$

2a. 
$$\begin{array}{r} 2b - 3 \\ b + 1 ) 2b^2 - b - 3 \\ \underline{2b^2 + 2b} \\ -3b - 3 \\ \underline{-3b - 3} \\ 0 \end{array}$$

quotient:  $2b - 3$

2b. 
$$\begin{array}{r} 3m - 4 \\ 2m + 1 ) 6m^2 - 5m - 7 \\ \underline{6m^2 + 3m} \\ -8m - 7 \\ \underline{-8m - 4} \\ -3 \end{array}$$

quotient:  $3m - 4 - \frac{3}{2m + 1}$

3a. 
$$\begin{array}{r} t^3 + t^2 + 2t + 3 \\ t - 1 ) t^4 + 0t^3 + t^2 + t - 3 \\ \underline{t^4 - t^3} \\ t^3 + t^2 \\ \underline{t^3 - t^2} \\ 2t^2 + t \\ \underline{2t^2 - 2t} \\ 3t - 3 \\ \underline{3t - 3} \\ 0 \end{array}$$

quotient:  $t^3 + t^2 + 2t + 3$

3b. 
$$\begin{array}{r} c^2 - 3c + 5 \\ c + 3 ) c^3 + 0c^2 - 4c + 12 \\ \underline{c^3 + 3c^2} \\ -3c^2 - 4c \\ \underline{-3c^2 - 9c} \\ 5c + 12 \\ \underline{5c + 15} \\ -3 \end{array}$$

quotient:  $c^2 - 3c + 5 - \frac{3}{c + 3}$

4a. 
$$\begin{array}{r} 4x + 3 \\ 2x + 1 ) 8x^2 + 10x - 1 \\ \underline{8x^2 + 4x} \\ 6x - 1 \\ \underline{6x + 3} \\ -4 \end{array}$$

quotient:  $4x + 3 - \frac{4}{2x + 1}$

4b. 
$$\begin{array}{r} -2a - 5 \\ 3a - 2 ) -6a^2 - 11a + 9 \\ \underline{-6a^2 + 4a} \\ -15a + 9 \\ \underline{-15a + 10} \\ -1 \end{array}$$

quotient:  $-2a - 5 - \frac{1}{3a - 2}$

**Exercises** 1.  $(x^6 - x^5 + x^4) \div x^2 =$

$$(x^6 - x^5 + x^4) \cdot \frac{1}{x^2} = \frac{x^6}{x^2} - \frac{x^5}{x^2} + \frac{x^4}{x^2} = x^4 - x^3 + x^2$$

2.  $(12x^8 - 8x^3) \div 4x^4 = (12x^8 - 8x^3) \cdot \frac{1}{4x^4} = \frac{12x^8}{4x^4} - \frac{8x^3}{4x^4} = 3x^4 - \frac{2}{x}$  3.  $(9c^4 + 6c^3 - c^2) \div 3c^2 =$

$$(9c^4 + 6c^3 - c^2) \cdot \frac{1}{3c^2} = \frac{9c^4}{3c^2} + \frac{6c^3}{3c^2} - \frac{c^2}{3c^2} =$$

$$3c^2 + 2c - \frac{1}{3}$$

$$4. (n^5 - 18n^4 + 3n^3) \div n^3 =$$

$$(n^5 - 18n^4 + 3n^3) \cdot \frac{1}{n^3} = \frac{n^5}{n^3} - \frac{18n^4}{n^3} + \frac{3n^3}{n^3} =$$

$$n^2 - 18n + 3$$

$$5. (8q^2 - 32q) \div 2q^2 =$$

$$(8q^2 - 32q) \cdot \frac{1}{2q^2} = \frac{8q^2}{2q^2} - \frac{32q}{2q^2} = 4 - \frac{16}{q}$$

$$6. (-7t^5 + 14t^4 - 28t^3 + 35t^2) \div 7t^2 =$$

$$(-7t^5 + 14t^4 - 28t^3 + 35t^2) \cdot \frac{1}{7t^2} =$$

$$\frac{-7t^5}{7t^2} + \frac{14t^4}{7t^2} - \frac{28t^3}{7t^2} + \frac{35t^2}{7t^2} = -t^3 + 2t^2 - 4t + 5$$

$$7.$$

$$\begin{array}{r} x - 3 \\ x - 2 ) x^2 - 5x + 6 \\ \underline{x^2 - 2x} \\ -3x + 6 \\ \underline{-3x + 6} \\ 0 \end{array}$$

quotient:  $x - 3$

$$8.$$

$$\begin{array}{r} 2t + 9 \\ t - 3 ) 2t^2 + 3t - 11 \\ \underline{2t^2 - 6t} \\ 9t - 11 \\ \underline{9t - 27} \\ 16 \end{array}$$

quotient:  $2t + 9 + \frac{16}{t - 3}$

$$9.$$

$$\begin{array}{r} n - 1 \\ n - 4 ) n^2 - 5n + 4 \\ \underline{n^2 - 4n} \\ -n + 4 \\ \underline{-n + 4} \\ 0 \end{array}$$

quotient:  $n - 1$

$$10.$$

$$\begin{array}{r} y - 3 \\ y + 2 ) y^2 - y + 2 \\ \underline{y^2 + 2y} \\ -3y + 2 \\ \underline{-3y - 6} \\ 8 \end{array}$$

quotient:  $y - 3 + \frac{8}{y + 2}$

$$11.$$

$$\begin{array}{r} 3x - 1 \\ x - 3 ) 3x^2 - 10x + 3 \\ \underline{3x^2 - 9x} \\ -x + 3 \\ \underline{-x + 3} \\ 0 \end{array}$$

quotient:  $3x - 1$

$$12.$$

$$\begin{array}{r} -2q - 10 \\ 2q + 1 ) -4q^2 - 22q + 12 \\ \underline{-4q^2 - 2q} \\ -20q + 12 \\ \underline{-20q - 10} \\ 22 \end{array}$$

quotient:  $-2q - 10 + \frac{22}{2q + 1}$

$$13.$$

$$\begin{array}{r} 5t - 50 \\ t + 10 ) 5t^2 + 0t - 500 \\ \underline{5t^2 + 50t} \\ -50t - 500 \\ \underline{-50t - 500} \\ 0 \end{array}$$

quotient:  $5t - 50$

$$14.$$

$$\begin{array}{r} 2w^2 + 2w + 5 \\ w - 1 ) 2w^3 + 0w^2 + 3w - 15 \\ \underline{2w^3 - 2w^2} \\ 2w^2 + 3w \\ \underline{2w^2 - 2w} \\ 5w - 15 \end{array}$$

$$\begin{array}{r} 5w - 5 \\ -10 \end{array}$$

quotient:  $2w^2 + 2w + 5 - \frac{10}{w - 1}$

$$15.$$

$$\begin{array}{r} b^2 - 3b - 1 \\ 3b - 1 ) 3b^3 - 10b^2 + 0b + 4 \\ \underline{3b^3 - b^2} \\ -9b^2 + 0b \\ \underline{-9b^2 + 3b} \\ -3b + 4 \\ \underline{-3b + 1} \\ 3 \end{array}$$

quotient:  $b^2 - 3b - 1 + \frac{3}{3b - 1}$

$$16.$$

$$\begin{array}{r} c^2 \\ c - 1 ) c^3 - c^2 + 0c - 1 \\ \underline{c^3 - c^2} \\ 0 + 0c - 1 \end{array}$$

quotient:  $c^2 - \frac{1}{c - 1}$

$$17.$$

$$\begin{array}{r} t^2 - 2t - 2 \\ t + 2 ) t^3 + 0t^2 - 6t - 4 \\ \underline{t^3 + 2t^2} \\ -2t^2 - 6t \\ \underline{-2t^2 - 4t} \\ -2t - 4 \\ \underline{-2t - 4} \\ 0 \end{array}$$

quotient:  $t^2 - 2t - 2$

$$18.$$

$$\begin{array}{r} n^2 - 2n - 21 \\ n + 2 ) n^3 + 0n^2 - 25n - 50 \\ \underline{n^3 + 2n^2} \\ -2n^2 - 25n \\ \underline{-2n^2 - 4n} \\ -21n - 50 \\ \underline{-21n - 42} \\ -8 \end{array}$$

quotient:  $n^2 - 2n - 21 - \frac{8}{n + 2}$

$$19. \begin{array}{r} r^2 + 5r + 1 \\ r - 5 ) r^3 + 0r^2 - 24r - 5 \\ \underline{r^3 - 5r^2} \\ 5r^2 - 24r \\ \underline{5r^2 - 25r} \\ r - 5 \\ \underline{r - 5} \\ 0 \end{array}$$

length:  $(r^2 + 5r + 1)$  cm

$$20. A = \frac{1}{2}bh, 2A = bh, \frac{2A}{b} = h; h = \frac{2(2c^3 + 16)}{c+2} = \frac{4c^3 + 32}{c+2}$$

$$\begin{array}{r} 4c^2 - 8c + 16 \\ c+2 ) 4c^3 + 0c^2 + 0c + 32 \\ \underline{4c^3 + 8c^2} \\ -8c^2 + 0c \\ \underline{-8c^2 - 16c} \\ 16c + 32 \\ \underline{16c + 32} \\ 0 \end{array}$$

height:  $(4c^2 - 8c + 16)$  ft

$$21. \begin{array}{r} b + 12 \\ b + 4 ) b^2 + 16b + 49 \\ \underline{b^2 + 4b} \\ 12b + 49 \\ \underline{12b + 48} \\ 1 \end{array}$$

quotient:  $b + 12 + \frac{1}{b+4}$

$$22. \begin{array}{r} a - 1 \\ a + 4 ) a^2 + 3a - 6 \\ \underline{a^2 + 4a} \\ -a - 6 \\ \underline{-a - 4} \\ -2 \end{array}$$

quotient:  $a - 1 - \frac{2}{a+4}$

$$23. \begin{array}{r} 10w - 681 \\ w + 72 ) 10w^2 + 39w + 14 \\ \underline{10w^2 + 720w} \\ -681w + 14 \\ \underline{-681w - 49,032} \\ 49,046 \end{array}$$

quotient:  $10w - 681 + \frac{49,046}{w+72}$

$$24. \begin{array}{r} t \\ t + 4 ) t^2 + 4t - 9 \\ \underline{t^2 + 4t} \\ 0 - 9 \end{array}$$

quotient:  $t - \frac{9}{t+4}$

$$25. \begin{array}{r} 2x^2 + 5x + 2 \\ x - 3 ) 2x^3 - x^2 - 13x - 6 \\ \underline{2x^3 - 6x^2} \\ 5x^2 - 13x \\ \underline{5x^2 - 15x} \\ 2x - 6 \\ \underline{2x - 6} \\ 0 \end{array}$$

quotient:  $2x^2 + 5x + 2$

$$26. \begin{array}{r} 3q^2 + 2q + 3 \\ q - 2 ) 3q^3 - 4q^2 - q + 6 \\ \underline{3q^3 - 6q^2} \\ 2q^2 - q \\ \underline{2q^2 - 4q} \\ 3q + 6 \\ \underline{3q - 6} \\ 12 \end{array}$$

quotient:  $3q^2 + 2q + 3 + \frac{12}{q-2}$

$$27. (6x^4 + 4x^3 - x^2) \div 2x^3 = (6x^4 + 4x^3 - x^2) \cdot \frac{1}{2x^3} =$$

$$\frac{6x^4}{2x^3} + \frac{4x^3}{2x^3} - \frac{x^2}{2x^3} = 3x + 2 - \frac{1}{2x}$$

$$28. (c^3 + 11c^2 - 15c + 8) \div c =$$

$$(c^3 + 11c^2 - 15c + 8) \cdot \frac{1}{c} = \frac{c^3}{c} + \frac{11c^2}{c} - \frac{15c}{c} + \frac{8}{c} =$$

$$c^2 + 11c - 15 + \frac{8}{c}$$

$$29. \begin{array}{r} 2b^2 + 2b + 10 \\ b - 1 ) 2b^3 + 0b^2 + 8b + 0 \\ \underline{2b^3 - 2b^2} \\ 2b^2 + 8b \\ \underline{2b^2 - 2b} \\ 10b + 0 \\ \underline{10b - 10} \\ 10 \end{array}$$

quotient:  $2b^2 + 2b + 10 + \frac{10}{b-1}$

$$30. \begin{array}{r} y^2 + 5y + 29 \\ y - 5 ) y^3 + 0y^2 + 4y - 7 \\ \underline{y^3 - 5y^2} \\ 5y^2 + 4y \\ \underline{5y^2 - 25y} \\ 29y - 7 \\ \underline{29y - 145} \\ 138 \end{array}$$

quotient:  $y^2 + 5y + 29 + \frac{138}{y-5}$

$$31. \begin{array}{r} 28a - 12 \\ 2a + 1 ) 56a^2 + 4a - 12 \\ \underline{56a^2 + 28a} \\ -24a - 12 \\ \underline{-24a - 12} \\ 0 \end{array}$$

quotient:  $28a - 12$

$$32. \begin{array}{r} 5t^3 - 25t^2 + 115t - 575 \\ t + 5 ) 5t^4 + 0t^3 - 10t^2 + 0t + 6 \\ \underline{5t^4 + 25t^3} \\ -25t^3 - 10t^2 \\ \underline{-25t^3 - 125t^2} \\ 115t^2 + 0t \\ \underline{115t^2 + 575t} \\ -575t + 6 \\ \underline{-575t - 2875} \\ 2881 \end{array}$$

quotient:  $5t^3 - 25t^2 + 115t - 575 + \frac{2881}{t+5}$

33.  $(3k^3 - 0.9k^2 - 1.2k) \div 3k =$   
 $(3k^3 - 0.9k^2 - 1.2k) \cdot \frac{1}{3k} = \frac{3k^3}{3k} - \frac{0.9k^2}{3k} - \frac{1.2k}{3k} =$   
 $k^2 - 0.3k - 0.4$

34.  $\begin{array}{r} 3s - 8 \\ 2s + 3 \overline{)6s^2 - 7s + 5} \\ \underline{6s^2 + 9s} \\ -16s + 5 \\ \underline{-16s - 24} \\ 29 \end{array}$

quotient:  $3s - 8 + \frac{29}{2s + 3}$

35.  $\begin{array}{r} -2z^2 + 3z - 4 \\ z + 1 \overline{-2z^3 + z^2 - z + 1} \\ \underline{-2z^3 - 2z^2} \\ 3z^2 - z \\ \underline{3z^2 + 3z} \\ -4z + 1 \\ \underline{-4z - 4} \\ 5 \end{array}$

quotient:  $-2z^2 + 3z - 4 + \frac{5}{z + 1}$

36.  $\begin{array}{r} 6m^2 - 24m + 99 \\ m + 4 \overline{6m^3 + 0m^2 + 3m + 70} \\ \underline{6m^3 + 24m^2} \\ -24m^2 + 3m \\ \underline{-24m^2 - 96m} \\ 99m + 70 \\ \underline{99m + 396} \\ -326 \end{array}$

quotient:  $6m^2 - 24m + 99 - \frac{326}{m + 4}$

37.  $\begin{array}{r} -16c^2 - 20c - 25 \\ -4c + 5 \overline{64c^3 + 0c^2 + 0c - 125} \\ \underline{64c^3 - 80c^2} \\ 80c^2 + 0c \\ \underline{80c^2 - 100c} \\ 100c - 125 \\ \underline{100c - 125} \\ 0 \end{array}$

quotient:  $-16c^2 - 20c - 25$

38.  $\begin{array}{r} 2r^4 + r^2 - 7 \\ r^2 - 3 \overline{2r^6 - 5r^4 - 10r^2 + 21} \\ \underline{2r^6 - 6r^4} \\ r^4 - 10r^2 \\ \underline{r^4 - 3r^2} \\ -7r^2 + 21 \\ \underline{-7r^2 + 21} \\ 0 \end{array}$

quotient:  $2r^4 + r^2 - 7$

39.  $\begin{array}{r} t - 1 \\ 2t^3 + 1 \overline{2t^4 - 2t^3 + 0t^2 + 3t - 1} \\ \underline{2t^4} \\ -2t^3 + 0t^2 + 2t - 1 \\ \underline{-2t^3} \\ 2t \end{array}$

quotient:  $t - 1 + \frac{2t}{2t^3 + 1}$

40.  $\begin{array}{r} z^3 - 3z^2 + 10z - 30 \\ z + 3 \overline{z^4 + 0z^3 + z^2 + 0z - 2} \\ \underline{z^4 + 3z^3} \\ -3z^3 + z^2 \\ \underline{-3z^3 - 9z^2} \\ 10z^2 + 0z \\ \underline{10z^2 + 30z} \\ -30z - 2 \\ \underline{-30z - 90} \\ 88 \end{array}$

quotient:  $z^3 - 3z^2 + 10z - 30 + \frac{88}{z + 3}$

41. Answers may vary. Sample:

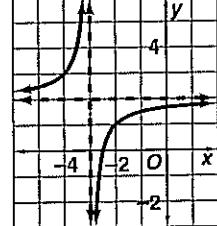
41a.  $(c^3 + 3c^2 - 2c - 4); (c + 1)$

41b.  $\begin{array}{r} c^2 + 2c - 4 \\ c + 1 \overline{c^3 + 3c^2 - 2c - 4} \\ \underline{c^3 + c^2} \\ 2c^2 - 2c \\ \underline{2c^2 + 2c} \\ -4c - 4 \\ \underline{-4c - 4} \\ 0 \end{array}$

$(c^3 + 3c^2 - 2c - 4) \div (c + 1) = c^2 + 2c - 4$

42a.  $\begin{array}{r} 2 \\ x + 3 \overline{2x + 5} \\ \underline{2x + 6} \\ -1 \end{array}$

$y = 2 - \frac{1}{x + 3}$



42b. Answers may vary. Sample:

|   |    |               |               |               |               |
|---|----|---------------|---------------|---------------|---------------|
| x | -2 | -1            | 0             | 1             | 2             |
| y | 1  | $\frac{3}{2}$ | $\frac{5}{3}$ | $\frac{7}{4}$ | $\frac{9}{5}$ |

42c. vertical asymptote:  $x = -3$ ; horizontal asymptote:  $y = 2$  43. The binomial is a factor of the polynomial if there is no remainder from the division.

44.  $\begin{array}{r} m^2 + 5m + 4 \\ m + 3 \overline{m^3 + 8m^2 + 19m + 12} \\ \underline{m^3 + 3m^2} \\ 5m^2 + 19m \\ \underline{5m^2 + 15m} \\ 4m + 12 \\ \underline{4m + 12} \\ 0 \end{array}$

area of base:  $m^2 + 5m + 4$

45a.  $\begin{array}{r} d - 2 \\ d + 1 \overline{d^2 - d + 1} \\ \underline{d^2 + d} \\ -2d + 1 \\ \underline{-2d - 2} \\ 3 \end{array}$

quotient:  $d - 2 + \frac{3}{d + 1}$

45b.

$$d+1 \overline{)d^3 - d^2 + d - 1}$$

$$\begin{array}{r} d^3 + d^2 \\ \hline -2d^2 + d \\ -2d^2 - 2d \\ \hline 3d - 1 \\ 3d + 3 \\ \hline -4 \end{array}$$

quotient:  $d^2 - 2d + 3 - \frac{4}{d+1}$

45c.

$$d+1 \overline{)d^4 - d^3 + d^2 - d + 1}$$

$$\begin{array}{r} d^4 + d^3 \\ \hline -2d^3 + d^2 \\ -2d^3 - 2d^2 \\ \hline 3d^2 - d \\ 3d^2 + 3d \\ \hline -4d + 1 \\ -4d - 4 \\ \hline 5 \end{array}$$

quotient:  $d^3 - 2d^2 + 3d - 4 + \frac{5}{d+1}$

45d. Answers may vary.

Sample:  $d^4 - 2d^3 + 3d^2 - 4d + 5 - \frac{6}{d+1}$

45e.

$$d+1 \overline{)d^5 - d^4 + d^3 - d^2 + d - 1}$$

$$\begin{array}{r} d^5 + d^4 \\ \hline -2d^4 + d^3 \\ -2d^4 - 2d^3 \\ \hline 3d^3 - d^2 \\ 3d^3 + 3d^2 \\ \hline -4d^2 + d \\ -4d^2 - 4d \\ \hline 5d - 1 \\ 5d + 5 \\ \hline -6 \end{array}$$

quotient:  $d^4 - 2d^3 + 3d^2 - 4d + 5 - \frac{6}{d+1}$

46.

$$x+3 \overline{x^2 - x - k}$$

$$\begin{array}{r} x^2 + 3x \\ \hline -4x - k \\ -4x - 12 \\ \hline 0 \text{ if } k = 12 \end{array}$$

47a.  $d = rt; \frac{d}{r} = t; t = \frac{d}{r}$

47b.

$$t+1 \overline{t^3 - 6t^2 + 5t + 12}$$

$$\begin{array}{r} t^3 + t^2 \\ \hline -7t^2 + 5t \\ -7t^2 - 7t \\ \hline 12t + 12 \\ 12t + 12 \\ \hline 0 \end{array}$$

time:  $(t^2 - 7t + 12)$  hours

48.  $(4a^3b^4 - 6a^2b^5 + 10a^2b^4) \div 2ab^2 =$   
 $(4a^3b^4 - 6a^2b^5 + 10a^2b^4) \cdot \frac{1}{2ab^2} = \frac{4a^3b^4}{2ab^2} - \frac{6a^2b^5}{2ab^2} + \frac{10a^2b^4}{2ab^2} =$   
 $2a^2b^2 - 3ab^3 + 5ab^2$

49.

$$5x - y \overline{15x^2 + 7xy - 2y^2}$$

$$\begin{array}{r} 15x^2 - 3xy \\ \hline 10xy - 2y^2 \\ 10xy - 2y^2 \\ \hline 0 \end{array}$$

quotient:  $3x + 2y$

50.

$$9r + 1 \overline{90r^6 + 28r^5 + 2r^4 + 45r^3 + 5r^2}$$

$$\begin{array}{r} 90r^6 + 10r^5 \\ \hline 18r^5 + 2r^4 \\ 18r^5 + 2r^4 \\ \hline 0 + 45r^3 + 5r^2 \\ 45r^3 + 5r^2 \\ \hline 0 \end{array}$$

quotient:  $10r^5 + 2r^4 + 5r^2$

51.

$$b^3 + 2b^2 - 1 \overline{2b^6 + 2b^5 - 4b^4 + b^3 + 8b^2 + 0b - 3}$$

$$\begin{array}{r} 2b^6 + 4b^5 \\ \hline -2b^5 - 4b^4 \\ -2b^5 - 4b^4 \\ \hline 3b^3 + 6b^2 - 3 \\ 3b^3 + 6b^2 - 3 \\ \hline 0 \end{array}$$

quotient:  $2b^3 - 2b^2 + 3$

52a.

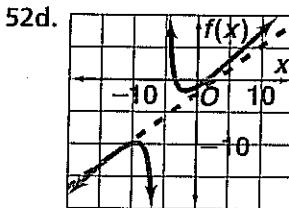
$$x+5 \overline{x^2 + 2x - 5}$$

$$\begin{array}{r} x^2 + 5x \\ \hline -3x - 5 \\ -3x - 15 \\ \hline 10 \end{array}$$

52b.  $f(x) = x - 3 + \frac{10}{x+5}$

52c.  $y = x - 3$

quotient:  $x - 3 + \frac{10}{x+5}$



53.

$$x+1 \overline{3x^3 + 0x^2 - 4x - 1}$$

$$\begin{array}{r} 3x^3 + 3x^2 \\ \hline -3x^2 - 4x \\ -3x^2 - 3x \\ \hline -x - 1 \\ -x - 1 \\ \hline 0 \end{array}$$

quotient:  $3x^2 - 3x - 1; C$

$$54. \begin{array}{r} x+3 \\ x-3 ) \underline{x^2 + 0x - 4} \\ \underline{x^2 - 3x} \\ 3x - 4 \\ \underline{3x - 9} \\ 5 \end{array}$$

The remainder is 5, so the answer is G.

$$55. \begin{array}{r} x-1 \\ x+3 ) \underline{x^2 + 2x + 1} \\ \underline{x^2 + 3x} \\ -x + 1 \\ \underline{-x - 3} \\ 4 \end{array}$$

I is not true, II is true, III is not true; B

$$56. [2] \begin{array}{r} x^2 + 3x + 2 \\ 2x - 1 ) \underline{2x^3 + 5x^2 + x - 2} \\ \underline{2x^3 - x^2} \\ 6x^2 + x \\ \underline{6x^2 - 3x} \\ 4x - 2 \\ \underline{4x - 2} \\ 0 \end{array}$$
  

$$\begin{array}{r} x+1 \\ x+2 ) \underline{x^2 + 3x + 2} \\ \underline{x^2 + 2x} \\ x+2 \\ \underline{x+2} \\ 0 \end{array}$$

The width is  $x + 1$ . [1] one computational error OR correct answer with no work shown

57. [4]

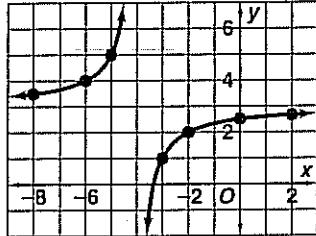
$$57a. \begin{array}{r} 3 \\ x+4 ) \underline{3x+10} \\ \underline{3x+12} \\ -2 \end{array}$$

$$y = 3 - \frac{2}{x+4}$$

57b. Tables may vary.

| x  | y              |
|----|----------------|
| 0  | $2\frac{1}{2}$ |
| 2  | $2\frac{2}{3}$ |
| -2 | 2              |
| -6 | 4              |
| -5 | 5              |
| -3 | 1              |
| -8 | $3\frac{1}{2}$ |

vertical asymptote:  $x = -4$ ;  
horizontal asymptote:  $y = 3$



[3] appropriate methods, but with one computational error [2] no asymptotes OR no graph [1] one part only

$$58. \begin{array}{l} \frac{n^2 + 7n - 8}{n-1} \cdot \frac{n^2 - 4}{n^2 + 6n - 16} = \\ \frac{(n+8)(n-1)}{n-1} \cdot \frac{(n-2)(n+2)}{(n+8)(n-2)} = n+2 \end{array}$$

$$59. \begin{array}{l} \frac{6t^2 - 30t}{2t^2 - 55t - 55} \cdot \frac{6t^2 + 35t + 11}{18t^2} = \\ \frac{6t(t-5)}{(2t-55)(t+1)} \cdot \frac{(3t+1)(2t+11)}{6t(3t)} = \\ \frac{(t-5)(3t+1)(2t+11)}{(2t-55)(t+1)(3t)} \end{array}$$

$$60. \frac{\frac{3c^2 - 4c - 32}{2c^2 + 17c + 35}}{\frac{3c^2 - 4c - 32}{2c^2 + 17c + 35}} \div \frac{c-4}{c+5} =$$

$$\frac{3c^2 - 4c - 32}{2c^2 + 17c + 35} \cdot \frac{c+5}{c-4} = \frac{(3c+8)(c-4)}{(2c+7)(c+5)} \cdot \frac{c+5}{c-4} = \frac{3c+8}{2c+7}$$

$$61. \frac{\frac{x^2 + 9x + 20}{x^2 + 5x - 24}}{\frac{x^2 + 9x + 20}{x^2 + 15x + 56}} =$$

$$\frac{x^2 + 9x + 20}{x^2 + 5x - 24} \cdot \frac{x^2 + 15x + 56}{x^2 + 15x + 56} =$$

$$\frac{(x+4)(x+5)}{(x+8)(x-3)} \cdot \frac{(x+4)(x-3)}{(x+8)(x+7)} =$$

$$\frac{(x+5)(x+4)^2}{(x+7)(x+8)^2} \quad 62. d = \sqrt{(4-1)^2 + (6-2)^2} =$$

$$\sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \quad 63. d =$$

$$\sqrt{(-1-4)^2 + (2-(-6))^2} = \sqrt{(-5)^2 + 8^2} =$$

$$\sqrt{25 + 64} = \sqrt{89} \approx 9.4$$

$$64. d = \sqrt{(-8-7)^2 + (5-5)^2} = \sqrt{(-15)^2 + 0^2} =$$

$$\sqrt{225} = 15 \quad 65. d = \sqrt{(-9-7)^2 + (-3-5)^2} =$$

$$\sqrt{(-16)^2 + (-8)^2} = \sqrt{256 + 64} = \sqrt{320} \approx 17.9$$

$$66. d = \sqrt{(-2 - (-1))^2 + (-7 - 5)^2} =$$

$$\sqrt{(-1)^2 + (-12)^2} = \sqrt{1 + 144} = \sqrt{145} \approx 12.0$$

$$67. d = \sqrt{(-6 - 0)^2 + (10 - (-5))^2} =$$

$$\sqrt{(-6)^2 + 15^2} = \sqrt{36 + 225} = \sqrt{261} \approx 16.2$$

$$68. \sqrt{28.9} \approx 5.38 \quad 69. \sqrt{289} = 17 \quad 70. -\sqrt{161.29} =$$

$$-12.7 \quad 71. \sqrt{4000} \approx 63.25 \quad 72. \sqrt{40} \approx 6.32$$

## 12.6 Adding and Subtracting Rational Expressions

pages 667-671

**Check Skills You'll Need** For complete solutions see Daily Skills Check and Lesson Quiz Transparencies or Presentation Pro CD-ROM.

$$1. \frac{2}{3} \quad 2. -\frac{2}{7} \quad 3. -2 \quad 4. 1\frac{1}{18} \quad 5. -\frac{1}{12} \quad 6. -\frac{1}{3} \quad 7. \frac{2x}{3} \quad 8. \frac{x}{2} \quad 9. \frac{1}{2y}$$

$$10. (x+2)(x+1) \quad 11. (y+3)(y+4) \quad 12. (t-2)^2$$

$$\text{Check Understanding} \quad 1a. \frac{3}{x+2} + \frac{2}{x+2} = \frac{3+2}{x+2} =$$

$$\frac{5}{x+2} \quad 1b. \frac{y}{y-5} + \frac{3y}{y-5} = \frac{y+3y}{y-5} = \frac{4y}{y-5}$$

$$1c. \frac{5n}{n+1} + \frac{2n}{n+1} = \frac{5n+2n}{n+1} = \frac{7n}{n+1}$$

$$2a. \frac{4}{t-2} - \frac{5}{t-2} = \frac{4-5}{t-2} = -\frac{1}{t-2}$$

$$2b. \frac{7b-2}{3b+6} - \frac{b+7}{3b+6} = \frac{7b-2-(b+7)}{3b+6} =$$

$$\frac{7b-2-b-7}{3b+6} = \frac{6b-9}{3b+6} = \frac{3(2b-3)}{3(b+2)} = \frac{2b-3}{b+2}$$

$$2c. \frac{2c+1}{5m+2} - \frac{3c-4}{5m+2} = \frac{2c+1-(3c-4)}{5m+2} =$$

$$\frac{2c+1-3c+4}{5m+2} = \frac{-c+5}{5m+2}$$

3a.  $\frac{3}{7y^4} + \frac{2}{3y^2} = \frac{3 \cdot 3}{7y^4 \cdot 3} + \frac{2 \cdot 7y^2}{3y^2 \cdot 7y^2} = \frac{9}{21y^4} + \frac{14y^2}{21y^4} = \frac{9 + 14y^2}{21y^4}$

3b.  $\frac{4}{25x} - \frac{49}{100} = \frac{4 \cdot 4}{25x \cdot 4} - \frac{49 \cdot x}{100 \cdot x} = \frac{16}{100x} - \frac{49x}{100x} = \frac{16 - 49x}{100x}$

3c.  $\frac{5}{12b} + \frac{15}{36b^2} = \frac{5 \cdot 3b}{12b \cdot 3b} + \frac{15}{36b^2} = \frac{15b}{36b^2} + \frac{15}{36b^2} = \frac{15b + 15}{36b^2} = \frac{3(5b + 5)}{3(12b^2)} = \frac{5b + 5}{12b^2}$

4a.  $\frac{5}{t+4} + \frac{3}{t-1} = \frac{5(t-1)}{(t+4)(t-1)} + \frac{3(t+4)}{(t+4)(t-1)} = \frac{5t-5+3t+12}{(t+4)(t-1)} = \frac{8t+7}{(t+4)(t-1)}$

4b.  $\frac{m}{2m+1} + \frac{3}{m-1} = \frac{m(m-1)}{(2m+1)(m-1)} + \frac{3(2m+1)}{(2m+1)(m-1)} = \frac{m^2-m+6m+3}{(2m+1)(m-1)} = \frac{m^2+5m+3}{(2m+1)(m-1)}$

4c.  $\frac{-2}{a+2} + \frac{3a}{2a-1} = \frac{-2(2a-1)}{(a+2)(2a-1)} + \frac{3a(a+2)}{(a+2)(2a-1)} = \frac{-4a+2+3a^2+6a}{(a+2)(2a-1)} = \frac{3a^2+2a+2}{(a+2)(2a-1)}$

5.  $\frac{1270}{r} + \frac{1270}{1.12r} = \frac{1422.4}{1.12r} + \frac{1270}{1.12r} = \frac{2692.4}{1.12r} \approx \frac{2404}{r}$

**Exercises** 1.  $\frac{5}{2m} + \frac{4}{2m} = \frac{5+4}{2m} = \frac{9}{2m}$

2.  $\frac{4}{6t-1} + \frac{3}{6t-1} = \frac{4+3}{6t-1} = \frac{7}{6t-1}$

3.  $\frac{n}{n+3} + \frac{n}{n+3} = \frac{n+n}{n+3} = \frac{2n}{n+3}$

4.  $\frac{5}{c-5} + \frac{9}{c-5} = \frac{5+9}{c-5} = \frac{14}{c-5}$

5.  $\frac{s^2+3}{4s^2+2} + \frac{s^2-2}{4s^2+2} = \frac{s^2+3+s^2-2}{4s^2+2} = \frac{2s^2+1}{4s^2+2}$

6.  $\frac{5c}{2c+7} + \frac{c-28}{2c+7} = \frac{5c+c-28}{2c+7} = \frac{6c-28}{2c+7}$

7.  $\frac{1}{2-b} - \frac{4}{2-b} = \frac{1-4}{2-b} = \frac{-3}{2-b}$

8.  $\frac{5}{t^2+1} - \frac{6}{t^2+1} = \frac{5-6}{t^2+1} = \frac{-1}{t^2+1}$

9.  $\frac{3t}{2t-3} - \frac{5t}{2t-3} = \frac{3t-5t}{2t-3} = \frac{-2t}{2t-3}$

10.  $\frac{2y+1}{y-1} - \frac{y+2}{y-1} = \frac{2y+1-y-2}{y-1} = \frac{y-1}{y-1} = 1$

11.  $\frac{3n+2}{n+4} - \frac{n-6}{n+4} = \frac{3n+2-n+6}{n+4} = \frac{2n+8}{n+4} = \frac{2(n+4)}{n+4} = 2$

12.  $\frac{3}{b-3} - \frac{b}{b-3} = \frac{3-b}{b-3} = \frac{-1(b-3)}{b-3} = -1$

13.  $2 = 2 \cdot x^2 = x \cdot x$ , LCD =  $2 \cdot x \cdot x = 2x^2$

14.  $6 = 2 \cdot 3, 9 = 3 \cdot 3$ , LCD =  $2 \cdot 3 \cdot 3 = 18$

15.  $z = z, 7z = 7 \cdot z$ , LCD =  $7 \cdot z = 7z$

16.  $5b = 5 \cdot b, 7b^3c = 7 \cdot b \cdot b \cdot b \cdot c$ , LCD =  $5 \cdot 7 \cdot b \cdot b \cdot b \cdot c = 35b^3c$

17.  $\frac{7}{3a} + \frac{2}{5} = \frac{7 \cdot 5}{3a \cdot 5} + \frac{2 \cdot 3a}{5 \cdot 3a} = \frac{35}{15a} + \frac{6a}{15a} = \frac{35+6a}{15a}$

18.  $\frac{4}{x} - \frac{2}{3} = \frac{4 \cdot 3}{x \cdot 3} - \frac{2 \cdot x}{3 \cdot x} = \frac{12}{3x} - \frac{2x}{3x} = \frac{12-2x}{3x}$

19.  $\frac{6}{5x^8} + \frac{4}{3x^6} = \frac{6 \cdot 3}{5x^8 \cdot 3} + \frac{4 \cdot 5x^2}{3x^6 \cdot 5x^2} = \frac{18}{15x^8} + \frac{20x^2}{15x^8} = \frac{18+20x^2}{15x^8}$

20.  $\frac{3}{15x^8} + \frac{1}{8m^3} + \frac{1}{12m^2} = \frac{3 \cdot 3}{8m^3 \cdot 3} + \frac{1 \cdot 2m}{12m^2 \cdot 2m} = \frac{9}{24m^3} + \frac{2m}{24m^3} = \frac{9+2m}{24m^3}$

21.  $\frac{27}{n^3} - \frac{9}{7n^2} = \frac{27 \cdot 7}{n^3 \cdot 7} - \frac{9 \cdot n}{7n^2 \cdot n} = \frac{189}{7n^3} - \frac{9n}{7n^3} = \frac{189-9n}{7n^3}$

22.  $\frac{9}{4x^2} + \frac{9}{5} = \frac{9 \cdot 5}{4x^2 \cdot 5} + \frac{9 \cdot 4x^2}{5 \cdot 4x^2} = \frac{45}{20x^2} + \frac{36x^2}{20x^2} = \frac{45+36x^2}{20x^2}$

23.  $\frac{9}{m+2} + \frac{8}{m-7} = \frac{9(m-7)}{(m+2)(m-7)} + \frac{8(m+2)}{(m+2)(m-7)} = \frac{9m-63+8m+16}{(m+2)(m-7)} = \frac{17m-47}{(m+2)(m-7)}$

24.  $\frac{a}{a+3} + \frac{4}{a+5} = \frac{a(a+5)}{(a+3)(a+5)} + \frac{4(a+3)}{(a+5)(a+3)} = \frac{a^2+5a+4a+12}{(a+3)(a+5)} = \frac{a^2+9a+12}{(a+3)(a+5)}$

25.  $\frac{a}{a+3} + \frac{a+5}{4} = \frac{a}{a+3} + \frac{a+5}{4} =$

26.  $\frac{a(4)}{(a+3)(4)} + \frac{(a+5)(a+3)}{4(a+3)} = \frac{4a+a^2+8a+15}{4(a+3)} = \frac{a^2+12a+15}{4(a+3)}$

27.  $\frac{c}{c+5} + \frac{4}{c+3} = \frac{c(c+3)}{(c+5)(c+3)} + \frac{4(c+5)}{(c+3)(c+5)} = \frac{c^2+3c+4c+20}{(c+5)(c+3)} = \frac{c^2+7c+20}{(c+5)(c+3)}$

28.  $\frac{3}{2a+1} + \frac{6}{2a-1} = \frac{3(2a-1)}{(2a+1)(2a-1)} + \frac{6(2a+1)}{(2a+1)(2a-1)} = \frac{6a-3+12a+6}{(2a+1)(2a-1)} = \frac{18a+3}{(2a+1)(2a-1)}$

29a.  $\frac{1}{r} + \frac{1}{0.7r} = \frac{0.7+1}{0.7r} = \frac{1.7}{0.7r} = \frac{17}{7r}$

29c.  $\frac{17}{7(3)} = \frac{17}{21} \approx 0.8$ ; about 0.8 h

30.  $\frac{y^2+2y-1}{3y+1} - \frac{2y^2-3}{3y+1} = \frac{y^2+2y-1-2y^2+3}{3y+1} = \frac{-y^2+2y+2}{3y+1}$

31.  $\frac{h^2+1}{2t^2-7} + \frac{h}{2t^2-7} = \frac{h^2+1+h}{2t^2-7}$

32.  $\frac{r-5}{9+p^3} - \frac{2k+1}{9+p^3} = \frac{r-5-2k-1}{9+p^3}$

33.  $\frac{2-x}{xy^2z} - \frac{5+z}{xy^2z} = \frac{2-x-5-z}{xy^2z}$

34.  $\frac{k}{xy^2z} + \frac{3k}{2m} = \frac{k}{2m^2} + \frac{3km}{2m^2} = \frac{k+3km}{2m^2}$

35.  $\frac{12}{ab} - \frac{15}{bc} = \frac{12c}{abc} - \frac{15a}{abc} = \frac{12c-15a}{abc}$

36.  $\frac{c^2}{ab} - \frac{a^2}{bc} = \frac{c^3-a^3}{abc} = \frac{c^3-a^3}{abc}$

37.  $9 + \frac{x-3}{x+2} = \frac{9(x+2)}{x+2} + \frac{x-3}{x+2} = \frac{9x+18+x-3}{x+2} = \frac{10x+15}{x+2}$

38.  $\frac{t}{2t-3} - 11 = \frac{t-22t+33}{2t-3} = \frac{-21t+33}{2t-3}$

39.  $\frac{x}{x^2-9} - \frac{x}{x^2+6x+9} = \frac{x}{(x-3)(x+3)} - \frac{x}{(x+3)^2} = \frac{x(x+3)}{(x-3)(x+3)^2} - \frac{x(x-3)}{(x+3)^2(x-3)} = \frac{x^2+3x-x^2+3x}{(x-3)(x+3)^2} = \frac{6x}{(x-3)(x+3)^2}$

40.  $\frac{k-24}{k^2-3k-18} - \frac{3}{k+3} + \frac{k+1}{k-6} = \frac{k-24}{(k+3)(k-6)} - \frac{3(k-6)}{(k+3)(k-6)} + \frac{(k+1)(k+3)}{(k+3)(k-6)} = \frac{k-24-3k+18+k^2+4k+3}{(k+3)(k-6)} = \frac{k^2+2k-3}{(k+3)(k-6)} =$

41. The student added the terms in the denominators.

42a.  $\frac{2}{r} + \frac{2}{1.25r} = \frac{2(1.25)}{1.25r} + \frac{2}{1.25r} = \frac{4.5}{1.25r} = \frac{18}{5r}$

42b.  $\frac{2}{d} + \frac{2}{0.8d} = \frac{2(0.8)}{0.8d} + \frac{2}{0.8d} = \frac{3.6}{0.8d} = \frac{9}{2d}$

42c. Yes; they both represent the time it takes to make a round trip.

43. Answers may vary. Sample: Not always; the numerator may contain a factor of the LCD.

44. Answers may vary. Sample:  $\frac{2w}{w+3}, \frac{3w^2}{w-3}$ :

45.  $f(x) - g(x) = 8x - \frac{1}{x} = \frac{8x(x)}{x} - \frac{1}{x} = \frac{8x^2-1}{x}$

46.  $f(x) \cdot g(x) = 8x \cdot \frac{1}{x} = \frac{8x}{x} = 8$

47.  $g(x) - h(x) = \frac{1}{x} - \frac{4}{x-5} = \frac{1(x-5)}{x(x-5)} - \frac{4x}{x(x-5)} = \frac{x-5-4x}{x(x-5)} = \frac{-3x-5}{x(x-5)}$

48.  $f(x) \cdot h(x) = 8x \cdot \frac{4}{(x-5)} = \frac{32x}{x-5}$

49.  $g(x) \div h(x) = \frac{1}{x} \div \frac{4}{(x-5)} = \frac{1}{x} \cdot \frac{x-5}{4} = \frac{x-5}{4x}$   
 50.  $h(x) \div f(x) = \frac{4}{(x-5)} \div 8x = \frac{4}{x-5} \cdot \frac{1}{8x} = \frac{1}{2x(x-5)}$   
 51.  $\frac{7d-2}{d^2+2d-8} - \frac{4}{d+4} - \frac{d}{d-2} =$   
 $\frac{7d-2}{(d+4)(d-2)} - \frac{4(d-2)}{(d+4)(d-2)} - \frac{d(d+4)}{(d+4)(d-2)} =$   
 $\frac{7d-2-4d+8-d^2-4d}{(d+4)(d-2)} = \frac{-d^2-d+6}{(d+4)(d-2)} =$   
 $\frac{-(d+3)(d-2)}{(d+4)(d-2)} = -\frac{d+3}{d+4}$

52.  $\frac{7x-10}{x^3+x^2-10x} - \frac{1}{x+5} = \frac{7x-10}{x(x^2+x-10)} - \frac{1}{x+5} =$   
 $\frac{(7x-10)(x+5)}{x(x+5)(x^2+x-10)} - \frac{1x(x^2+x-10)}{x(x+5)(x^2+x-10)} =$

$\frac{7x^2+25x-50-x^3-x^2+10x}{x(x+5)(x^2+x-10)} = \frac{-x^3+6x^2+35x-50}{x(x+5)(x^2+x-10)}$

53.  $\frac{x^2}{x^2+x-12} - \frac{x}{x+4} \cdot \frac{3}{x-3} =$   
 $\frac{x^2}{(x+4)(x-3)} - \frac{3x}{(x+4)(x-3)} = \frac{x^2-3x}{(x+4)(x-3)} =$   
 $\frac{x(x-3)}{(x+4)(x-3)} = \frac{x}{x+4}$

54.  $\frac{\frac{2}{a-5} \cdot \frac{a-1}{a+2}}{\frac{2(a-1)}{(a-5)(a+2)}} - \frac{\frac{a-2}{2+a} \cdot \frac{3}{5-a}}{\frac{3(a-2)}{-1(a-5)(a+2)}} = \frac{\frac{2a-2+3a-6}{(a-5)(a+2)}}{\frac{5a-8}{(a-5)(a+2)}} =$   
 55.  $\frac{5x}{3x-2} - \frac{2x}{3x-2} = \frac{5x-2x}{3x-2} = \frac{3x}{3x-2};$

D 56.  $x^2-1=(x-1)(x+1); x-1=x-1; \text{LCD}=(x-1)(x+1)=x^2-1; \text{H } 57. 4=2 \cdot 2, 5=5, 8=2 \cdot 2 \cdot 2; \text{LCM}=2 \cdot 2 \cdot 2 \cdot 5=40; \text{D}$

58. [2] 58a.  $\frac{10}{r} + \frac{10}{r+3} = \text{total time}; \frac{10(r+3) + 10r}{r(r+3)} =$   
 $\frac{10r+30+10r}{r(r+3)} = \frac{20r+30}{r(r+3)} = \frac{10(2r+3)}{r(r+3)}$

58b.  $\frac{10(2(12)+3)}{12(12+3)} = \frac{10(24+3)}{12(15)} = \frac{10(27)}{180} = \frac{270}{180} = \frac{3}{2}$

The ride took  $1\frac{1}{2}$  hours. [1] one computational error OR no work shown 59.  $(2x^4+8x^3-4x^2) \div 4x^2 =$   
 $(2x^4+8x^3-4x^2) \cdot \frac{1}{4x^2} = \frac{2x^4}{4x^2} + \frac{8x^3}{4x^2} - \frac{4x^2}{4x^2} = \frac{x^2}{2} + 2x - 1$

60.  $5b^2-10b+30$

$b+2 \overline{)5b^3+0b^2+10b+0}$

$5b^3+10b^2$

$-10b^2+10b$

$-10b^2-20b$

$30b+0$

$30b+60$

$-60$

quotient:  $5b^2-10b+30-\frac{60}{b+2}$

61.  $x=\sqrt{5x+6}; x^2=(\sqrt{5x+6})^2; x^2=5x+6; x^2-5x-6=0; (x-6)(x+1)=0; x-6=0 \text{ or } x+1=0;$   
 $x=6 \text{ or } x=-1 \text{ (extraneous); } x=6$  62.  $n=\sqrt{24-5n}; n^2=(\sqrt{24-5n})^2; n^2=24-5n; n^2+5n-24=0;$   
 $(n+8)(n-3)=0; n+8=0 \text{ or } n-3=0; n=-8$  (extraneous) or  $n=3; n=3$  63.  $\sqrt{16y}=-8$ ; no solution 64.  $a^2-48=0; a^2=48; a=\pm\sqrt{48}\approx\pm6.9$   
 65.  $2n^2=30; n^2=15; n=\pm\sqrt{15}\approx\pm3.9$   
 66.  $3p^2+60=0; 3p^2=-60; p^2=-20$ ; no solution

## 12-7 Solving Rational Equations

pages 672-677

**Check Skills You'll Need** For complete solutions see Daily Skills Check and Lesson Quiz Transparencies or Presentation Pro CD-ROM.

1.  $1\frac{2}{3}$  2.  $1\frac{1}{5}$  3.  $-9, 9$  4.  $4n$  5.  $15x$  6.  $24y^2$

**Check Understanding** 1a.  $\frac{1}{3} + \frac{1}{3x} = \frac{1}{6}; 6x\left(\frac{1}{3} + \frac{1}{3x}\right) =$

$6x\left(\frac{1}{6}\right); 2x+2=x; x=-2; \text{Check: } \frac{1}{3} + \frac{1}{3(-2)} \stackrel{?}{=} \frac{1}{6},$

$\frac{2}{6} - \frac{1}{6} = \frac{1}{6}; x=-2$  1b.  $\frac{4}{c} = \frac{3}{2c} - \frac{1}{5}; 10c\left(\frac{4}{c}\right) =$

$10c\left(\frac{3}{2c} - \frac{1}{5}\right); 40=15-2c; 25=-2c; c=-\frac{25}{2}$

Check:  $\frac{4}{-\frac{25}{2}} \stackrel{?}{=} \frac{3}{2\left(-\frac{25}{2}\right)} - \frac{1}{5}, -\frac{8}{25} = -\frac{3}{25} - \frac{5}{25}; c=-\frac{25}{2}$

2a.  $\frac{5}{m} = \frac{2}{m^2} + 2; m^2\left(\frac{5}{m}\right) = m^2\left(\frac{2}{m^2} + 2\right); 5m=2+2m^2;$

$2m^2-5m+2=0; (2m-1)(m-2)=0; 2m-1=0 \text{ or } m-2=0; m=\frac{1}{2} \text{ or } m=2; \text{Check: } \frac{5}{\frac{1}{2}} \stackrel{?}{=} \frac{2}{\left(\frac{1}{2}\right)^2} + 2, 10=$

$8+2; \frac{5}{2} \stackrel{?}{=} \frac{2}{2^2} + 2, \frac{5}{2} = \frac{1}{2} + \frac{4}{2}; m=\frac{1}{2} \text{ or } m=2$

2b.  $t-2=\frac{8-2t}{t-1}; (t-1)(t-2)=(t-1)\left(\frac{8-2t}{t-1}\right);$

$t^2-3t+2=8-2t; t^2-t-6=0; (t-3)(t+2)=0;$

$t-3=0 \text{ or } t+2=0; t=3 \text{ or } t=-2;$

Check:  $3-2 \stackrel{?}{=} \frac{8-2(3)}{3-1}, 1=\frac{2}{2}; -2-2 \stackrel{?}{=} \frac{8-2(-2)}{-2-1},$

$-4=\frac{12}{-3}; t=3 \text{ or } t=-2$  3.  $\frac{1}{45} + \frac{1}{75} = \frac{1}{n};$

$225n\left(\frac{1}{45} + \frac{1}{75}\right) = 225n\left(\frac{1}{n}\right); 5n+3n=225; 8n=225;$

$n=28.125=28\frac{1}{8}; 28\frac{1}{8} \text{ min } 4a. \frac{3}{a} = \frac{5}{a-2}; 3(a-2)=5a;$

$3a-6=5a; -2a=6; a=-3; \text{Check: } \frac{3}{-3} \stackrel{?}{=} \frac{5}{-3-2},$

$-1=-1; a=-3$  4b.  $\frac{n}{5} = \frac{4}{n+1}; n(n+1)=5(4);$

$n^2+n=20; n^2+n-20=0; (n+5)(n-4)=0;$

$n+5=0 \text{ or } n-4=0; n=-5 \text{ or } n=4;$

Check:  $\frac{-5}{5} \stackrel{?}{=} \frac{4}{-5+1}, -1=-1; \frac{4}{5} \stackrel{?}{=} \frac{4}{4+1}, \frac{4}{5}=\frac{4}{5}; n=-5$

or  $n=4$  5a.  $\frac{2}{c^2} = \frac{2}{c^2+1}; 2(c^2+1)=2c^2; 2c^2+2=2c^2;$

$2=0, \text{false; no solution}$  5b.  $\frac{w^2}{w-1} = \frac{1}{w-1}; w^2(w-1)=\frac{1}{w-1}; w^2-1=0 \text{ or } w-1=0; w^2=1 \text{ or }$

$w=1; w=\pm 1; \text{Check: } \frac{1^2}{1-1} \stackrel{?}{=} \frac{1}{1-1}, \frac{1}{0} \stackrel{?}{=} \frac{1}{0},$

undefined;  $\frac{(-1)^2}{-1-1} \stackrel{?}{=} \frac{1}{-1-1}, -\frac{1}{2}=-\frac{1}{2}; w=-1$

**Exercises** 1.  $\frac{1}{2} + \frac{2}{x} = \frac{1}{x}; 2x\left(\frac{1}{2} + \frac{2}{x}\right) = 2x\left(\frac{1}{x}\right); x+4=2;$

$x=-2$  2.  $5 + \frac{2}{p} = \frac{17}{p}; p\left(5 + \frac{2}{p}\right) = p\left(\frac{17}{p}\right); 5p+2=17;$

$5p=15; p=3$  3.  $\frac{3}{a} - \frac{5}{a} = 2; a\left(\frac{3}{a} - \frac{5}{a}\right) = a(2); 3-5=$

$2a; -2=2a; a=-1$  4.  $y - \frac{6}{y} = 5; y\left(y - \frac{6}{y}\right) = y(5);$

$y^2-6=5y; y^2-5y-6=0; (y+1)(y-6)=0;$

$y+1=0 \text{ or } y-6=0; y=-1 \text{ or } y=6$  5.  $\frac{5}{2s} + \frac{3}{4} = \frac{9}{4s}; 4s\left(\frac{5}{2s} + \frac{3}{4}\right) = 4s\left(\frac{9}{4s}\right); 10+3s=9; 3s=-1; s=-\frac{1}{3}$

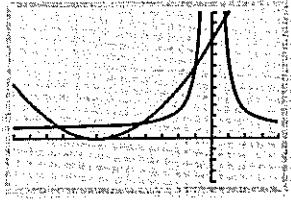
6.  $\frac{1}{t-2} = \frac{t}{8}$ ;  $1(8) = t(t-2)$ ;  $8 = t^2 - 2t$ ;  $t^2 - 2t - 8 = 0$ ;  
 $(t-4)(t+2) = 0$ ;  $t-4 = 0$  or  $t+2 = 0$ ;  $t = 4$  or  $t = -2$

7.  $\frac{2}{c-2} = 2 - \frac{4}{c}$ ;  $c(c-2)\left(\frac{2}{c-2}\right) = c(c-2)\left(2 - \frac{4}{c}\right)$ ;  
 $2c = 2c(c-2) - 4(c-2)$ ;  $2c = 2c^2 - 4c - 4c + 8$ ;  
 $2c^2 - 10c + 8 = 0$ ;  $c^2 - 5c + 4 = 0$ ;  $(c-4)(c-1) = 0$ ;  
 $c-4 = 0$  or  $c-1 = 0$ ;  $c = 4$  or  $c = 1$  8.  $\frac{5}{3p} + \frac{2}{3} = \frac{5+p}{2p}$ ;  
 $6p\left(\frac{5}{3p} + \frac{2}{3}\right) = 6p\left(\frac{5+p}{2p}\right)$ ;  $10 + 4p = 3(5+p)$ ;  $10 + 4p = 15 + 3p$ ;  $p = 5$  9.  $\frac{8}{x+3} = \frac{1}{x} + 1$ ;  $x(x+3)\left(\frac{8}{x+3}\right) = x(x+3)\left(\frac{1}{x} + 1\right)$ ;  $8x = x+3 + x(x+3)$ ;  $8x = x+3 + x^2 + 3x$ ;  $x^2 - 4x + 3 = 0$ ;  $(x-3)(x-1) = 0$ ;  
 $x-3 = 0$  or  $x-1 = 0$ ;  $x = 3$  or  $x = 1$  10.  $\frac{v+2}{v} + \frac{4}{3v} = 11$ ;  $3v\left(\frac{v+2}{v} + \frac{4}{3v}\right) = 3v(11)$ ;  $3(v+2) + 4 = 33v$ ;  
 $3v + 6 + 4 = 33v$ ;  $-30v = -10$ ;  $v = \frac{1}{3}$  11.  $\frac{4}{3(c+4)} + 1 = \frac{2c}{c+4}$ ;  $3(c+4)\left(\frac{4}{3(c+4)} + 1\right) = 3(c+4)\left(\frac{2c}{c+4}\right)$ ;  
 $4 + 3(c+4) = 3(2c)$ ;  $4 + 3c + 12 = 6c$ ;  $-3c = -16$ ;  
 $c = \frac{16}{3}$  12.  $\frac{a}{a+4} = 3 - \frac{4}{a+4}$ ;  $(a+4)\left(\frac{a}{a+4}\right) = (a+4)\left(3 - \frac{4}{a+4}\right)$ ;  
 $a = 3(a+4) - 4$ ;  $a = 3a + 12 - 4$ ;  
 $-2a = 8$ ;  $a = -4$  13.  $\frac{z}{z+2} = 3 - \frac{2}{z+2}$ ;  $(z+2)\left(\frac{z}{z+2}\right) = (z+2)\left(3 - \frac{2}{z+2}\right)$ ;  
 $z = 3(z+2) - 2$ ;  $z = 3z + 6 - 2$ ;  
 $-2z = 4$ ;  $z = -2$  14.  $\frac{a}{a+3} = \frac{2a}{a-3} - 1$ ;  
 $(a+3)(a-3)\left(\frac{a}{a+3}\right) = (a+3)(a-3)\left(\frac{2a}{a-3} - 1\right)$ ;  
 $a(a-3) = 2a(a+3) - (a+3)(a-3)$ ;  $a^2 - 3a = 2a^2 + 6a - a^2 + 9$ ;  $-9a = 9$ ;  $a = -1$  15.  $\frac{z}{z+2} - \frac{1}{z} = 1$ ;  
 $z(z+2)\left(\frac{z}{z+2} - \frac{1}{z}\right) = z(z+2)(1)$ ;  $z^2 - (z+2) = z^2 + 2z$ ;  $z^2 - z - 2 - z^2 - 2z = 0$ ;  $-3z = 2$ ;  $z = -\frac{2}{3}$  16.  $\frac{1}{3} + \frac{1}{4} = \frac{1}{n}$ ;  $12n\left(\frac{1}{3} + \frac{1}{4}\right) = 12n\left(\frac{1}{n}\right)$ ;  $4n + 3n = 12$ ;  
 $7n = 12$ ;  $n = \frac{12}{7} = 1\frac{5}{7}$  h 17.  $\frac{1}{20} + \frac{1}{35} = \frac{1}{n}$ ;  
 $140n\left(\frac{1}{20} + \frac{1}{35}\right) = 140n\left(\frac{1}{n}\right)$ ;  $7n + 4n = 140$ ;  $11n = 140$ ;  
 $n = \frac{140}{11} \approx 12.7$ ; about 12.7 min 18.  $\frac{5}{x+1} = \frac{x+2}{x+1}$ ;  
 $5(x+1) = (x+1)(x+2)$ ;  $5x + 5 = x^2 + 3x + 2$ ;  
 $x^2 - 2x - 3 = 0$ ;  $(x-3)(x+1) = 0$ ;  $x-3 = 0$  or  
 $x+1 = 0$ ;  $x = 3$  or  $x = -1$  (extraneous);  $x = 3$  19.  $\frac{4}{c+4} = \frac{c}{c+25}$ ;  $4(c+25) = c(c+4)$ ;  $4c + 100 = c^2 + 4c$ ;  $c^2 = 100$ ;  $c = \pm\sqrt{100}$ ;  $c = 10$  or  $c = -10$  20.  $\frac{3}{m-1} = \frac{2m}{m+4}$ ;  $3(m+4) = 2m(m-1)$ ;  $3m + 12 = 2m^2 - 2m$ ;  $2m^2 - 5m - 12 = 0$ ;  $(2m+3)(m-4) = 0$ ;  
 $2m+3 = 0$  or  $m-4 = 0$ ;  $m = -\frac{3}{2}$  or  $m = 4$  21.  $\frac{2x+4}{x-3} = \frac{3x}{x-3}$ ;  $(2x+4)(x-3) = 3x(x-3)$ ;  
 $2x^2 - 2x - 12 = 3x^2 - 9x$ ;  $x^2 - 7x + 12 = 0$ ;  
 $(x-3)(x-4) = 0$ ;  $x-3 = 0$  or  $x-4 = 0$ ;  $x = 3$  (extraneous) or  $x = 4$ ;  $x = 4$  22.  $\frac{30}{x+3} = \frac{30}{x-3}$ ;  
 $30(x-3) = 30(x+3)$ ;  $x-3 = x+3$ ;  $-3 = 3$ , false;  
no solution 23.  $\frac{x+2}{x+4} = \frac{x-2}{x-1}$ ;  $(x+2)(x-1) =$

$$(x-2)(x+4); x^2 + x - 2 = x^2 + 2x - 8$$
;  $-x = -6$ ;  
 $x = 6$  24.  $\frac{2r}{r-4} - 2 = \frac{4}{r+5}$ ;  $(r-4)(r+5)\left(\frac{2r}{r-4} - 2\right) = (r-4)(r+5)\left(\frac{4}{r+5}\right)$ ;  $2r(r+5) - 2(r-4)(r+5) = 4(r-4)$ ;  $2r^2 + 10r - 2(r^2 + r - 20) = 4r - 16$ ;  
 $2r^2 + 10r - 2r^2 - 2r + 40 - 4r + 16 = 0$ ;  $4r = -56$ ;  
 $r = -14$  25.  $6 - \frac{2}{b} = \frac{-5}{b-3}$ ;  $b(b-3)\left(6 - \frac{2}{b}\right) = b(b-3)\left(\frac{-5}{b-3}\right)$ ;  $6b(b-3) - 2(b-3) = -5b$ ;  
 $6b^2 - 18b - 2b + 6 + 5b = 0$ ;  $6b^2 - 15b + 6 = 0$ ;  
 $3(2b^2 - 5b + 2) = 0$ ;  $(2b-1)(b-2) = 0$ ;  $2b-1 = 0$  or  $b-2 = 0$ ;  $b = \frac{1}{2}$  or  $b = 2$  26.  $\frac{r+1}{r-1} = \frac{r}{3} + \frac{2}{r-1}$ ;  
 $3(r-1)\left(\frac{r+1}{r-1}\right) = 3(r-1)\left(\frac{r}{3} + \frac{2}{r-1}\right)$ ;  $3(r+1) = r(r-1) + 3(2)$ ;  $3r + 3 = r^2 - r + 6$ ;  $r^2 - 4r + 3 = 0$ ;  
 $(r-3)(r-1) = 0$ ;  $r-3 = 0$  or  $r-1 = 0$ ;  $r = 3$  or  $r = 1$  (extraneous);  $r = 3$  27.  $\frac{3}{s-1} + 1 = \frac{12}{s^2-1}$ ;  $\frac{3}{s-1} + 1 = \frac{12}{(s-1)(s+1)}$ ;  $(s-1)(s+1)\left(\frac{3}{s-1} + 1\right) = (s-1)(s+1)\left(\frac{12}{(s-1)(s+1)}\right)$ ;  $3(s+1) + (s-1)(s+1) = 12$ ;  $3s + 3 + s^2 - 1 = 12$ ;  $s^2 + 3s - 10 = 0$ ;  $(s+5)(s-2) = 0$ ;  $s+5 = 0$  or  $s-2 = 0$ ;  $s = -5$  or  $s = 2$  28.  $\frac{d}{d+2} - \frac{2}{2-d} = \frac{d+6}{d^2-4}$ ;  $\frac{d}{d+2} - \frac{2}{-1(d-2)} = \frac{d+6}{(d-2)(d+2)}$ ;  $(d-2)(d+2)\left(\frac{d}{d+2} + \frac{2}{d-2}\right) = (d-2)(d+2)\left(\frac{d+6}{(d-2)(d+2)}\right)$ ;  $d(d-2) + 2(d+2) = d+6$ ;  $d^2 - 2d + 2d + 4 - d - 6 = 0$ ;  $d^2 - d - 2 = 0$ ;  
 $(d-2)(d+1) = 0$ ;  $d-2 = 0$  or  $d+1 = 0$ ;  $d = 2$  (extraneous) or  $d = -1$ ;  $d = -1$  29.  $\frac{u+1}{u} + \frac{1}{2u} = 4$ ;  
 $2u\left(\frac{u+1}{u} + \frac{1}{2u}\right) = 2u(4)$ ;  $2(u+1) + 1 = 8u$ ;  $2u + 2 + 1 = 8u$ ;  $-6u = -3$ ;  $u = \frac{1}{2}$  30.  $\frac{s}{3s+2} + \frac{s+3}{2s-4} = \frac{-2s}{3s^2-4s-4}$ ;  
 $\frac{s}{3s+2} + \frac{s+3}{2(s-2)} = \frac{-2s}{(3s+2)(s-2)}$ ;  $2(3s+2)(s-2) \cdot \left(\frac{s}{3s+2} + \frac{s+3}{2(s-2)}\right) = 2(3s+2)(s-2)\left(\frac{-2s}{(3s+2)(s-2)}\right)$ ;  $2s(s-2) + (3s+2)(s+3) = 2(-2s)$ ;  
 $2s^2 - 4s + 3s^2 + 11s + 6 = -4s$ ;  $5s^2 + 11s + 6 = 0$ ;  
 $(5s+6)(s+1) = 0$ ;  $5s+6 = 0$  or  $s+1 = 0$ ;  $s = -\frac{6}{5}$  or  $s = -1$  31.  $\frac{u+1}{u+2} = \frac{-1}{u-3} + \frac{u-1}{u^2-u-6}$ ;  $\frac{u+1}{u+2} = \frac{-1}{u-3} + \frac{u-1}{(u-3)(u+2)}$ ;  $(u+2)(u-3)\left(\frac{u+1}{u+2}\right) = (u+2)(u-3)\left(\frac{-1}{u-3} + \frac{u-1}{(u-3)(u+2)}\right)$ ;  $(u-3)(u+1) = -1(u+2) + u-1$ ;  $u^2 - 2u - 3 = -u - 2 + u - 1$ ;  
 $u^2 - 2u = 0$ ;  $u(u-2) = 0$ ;  $u = 0$  or  $u-2 = 0$ ;  $u = 0$  or  $u = 2$  32.  $\frac{1}{n} + \frac{1}{3n} = \frac{1}{9}$ ;  $9n\left(\frac{1}{n} + \frac{1}{3n}\right) = 9n\left(\frac{1}{9}\right)$ ;  $9+3=n$ ;  
 $n = 12$ ; 12 h 33a. Carlos's method:  $\frac{40}{x} = \frac{15}{x-20}$ ;  
 $x(x-20)\left(\frac{40}{x}\right) = x(x-20)\left(\frac{15}{x-20}\right)$ ;  $40(x-20) = 15x$ ;  
 $40x - 800 = 15x$ ;  $25x = 800$ ;  $x = 32$ . Ingrid's method:  
 $\frac{40}{x} = \frac{15}{x-20}$ ;  $40(x-20) = 15x$ ;  $40x - 800 = 15x$ ;  $25x = 800$ ;  $x = 32$ . 33b. Answers may vary. Sample: Cross-multiplying; I think it's quicker. 33c. No; it only works

for rational equations that are proportions.

$$34. \frac{5a}{3} = 2 + \frac{7a}{6}; 6\left(\frac{5a}{3}\right) = 6\left(2 + \frac{7a}{6}\right); 10a = 12 + 7a; 3a = 12; a = 4; \frac{7}{3b} = 9; 7 = 27b; b = \frac{7}{27}; \frac{2c - 15}{35c} = \frac{1}{5c}; 5c(2c - 15) = 35c; 10c^2 - 75c = 35c; 10c^2 - 110c = 0; 10c(c - 11) = 0; 10c = 0 \text{ or } c - 11 = 0; c = 0 \\ (\text{extraneous}) \text{ or } c = 11; c = 11; \frac{5}{2d} + \frac{3}{4} = \frac{9}{4d}; 4d\left(\frac{5}{2d} + \frac{3}{4}\right) = 4d\left(\frac{9}{4d}\right); 10 + 3d = 9; 3d = -1; d = -\frac{1}{3}$$



Xmin=-12 Ymin=-4  
Xmax=4 Ymax=12  
XscI=1 YscI=1

$$36. \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{100} + \frac{1}{200} = \frac{2}{200} + \frac{1}{200} = \frac{3}{200}; R_T = \frac{200}{3} = 66.\bar{6} \Omega \quad 37. \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}; \frac{1}{12} = \frac{1}{30} + \frac{1}{R_2}; \frac{1}{R_2} = \frac{1}{12} - \frac{1}{30} = \frac{5}{60} - \frac{2}{60} = \frac{3}{60} = \frac{1}{20}; R_2 = \frac{20}{1} = 20; 20 \Omega \quad 38. \frac{1}{R_T} = \frac{1}{R_1 + R_2} + \frac{1}{R_3} = \frac{1}{6+9} + \frac{1}{5} = \frac{1}{15} + \frac{3}{15} = \frac{4}{15}; R_T = \frac{15}{4} = 3.75; 3.75 \Omega \quad 39. \frac{1}{R_T} = \frac{1}{R_1 + R_2} + \frac{1}{R_3}; \frac{1}{10} = \frac{1}{10 + R_2} + \frac{1}{15}; \frac{1}{10 + R_2} = \frac{1}{10} - \frac{1}{15} = \frac{3}{30} - \frac{2}{30} = \frac{1}{30}; 10 + R_2 = 30; R_2 = 20; 20 \Omega$$

$$40. \text{Answers may vary. Sample: } \frac{2b}{b+2} = \frac{6b}{4b+3}$$

$$41. \frac{980}{450+s} = \frac{820}{450-s}; 980(450-s) = 820(450+s); 441,000 - 980s = 369,000 + 820s; -1800s = -72,000; s = 40; 40 \text{ mi/h} \quad 42. \frac{x-6}{x+3} + \frac{2x}{x-3} = \frac{4x+3}{x+3}; (x+3)(x-3)\left(\frac{x-6}{x+3} + \frac{2x}{x-3}\right) = (x+3)(x-3)\left(\frac{4x+3}{x+3}\right); (x-3)(x-6) + 2x(x+3) = (4x+3)(x-3); x^2 - 9x + 18 + 2x^2 + 6x = 4x^2 - 9x - 9; x^2 + 2x^2 - 4x^2 - 9x + 6x + 9x + 18 + 9 = 0; -x^2 + 6x + 27 = 0; x^2 - 6x - 27 = 0; (x-9)(x+3) = 0; x-9 = 0 \text{ or } x+3 = 0; x = 9 \text{ or } x = -3 \text{ (extraneous)}; x = 9 \quad 43. \frac{n}{n-2} + \frac{n}{n+2} = \frac{n}{n^2-4}; \frac{n}{n-2} + \frac{n}{n+2} = \frac{n}{(n-2)(n+2)}; (n-2)(n+2)\left(\frac{n}{n-2} + \frac{n}{n+2}\right) = (n-2)(n+2)\left(\frac{n}{(n-2)(n+2)}\right); n(n+2) + n(n-2) = n; n^2 + 2n + n^2 - 2n = n; 2n^2 - n = 0; n(2n-1) = 0; n = 0 \text{ or } 2n-1 = 0; n = 0 \text{ or } n = \frac{1}{2}$$

$$44. \frac{2}{r} + \frac{1}{r^2} + \frac{r^2+r}{r^3} = \frac{1}{r}; r^3\left(\frac{2}{r} + \frac{1}{r^2} + \frac{r^2+r}{r^3}\right) = r^3\left(\frac{1}{r}\right); 2r^2 + r + r^2 + r = r^2; 2r^2 + 2r = 0; 2r(r+1) = 0; 2r = 0 \text{ or } r+1 = 0; r = 0 \text{ (extraneous)} \text{ or } r = -1; r = -1 \quad 45. \frac{3}{t} - \frac{t^2-2t}{t^3} = \frac{4}{t^2}; t^3\left(\frac{3}{t} - \frac{t^2-2t}{t^3}\right) = t^3\left(\frac{4}{t^2}\right); 3t^2 - t^2 + 2t = 4t; 2t^2 + 2t = 0; 2(t^2 + t) = 0; (t-1)(t+1) = 0; t-1 = 0; t = 1 \quad 46. \frac{1}{75} + \frac{1}{60} + \frac{1}{80} = \frac{1}{n}; 1200n\left(\frac{1}{75} + \frac{1}{60} + \frac{1}{80}\right) = 1200n\left(\frac{1}{n}\right); 16n + 20n + 15n =$$

$$1200; 51n = 1200; n \approx 23.5; \text{about 23.5 min} \quad 47a. 0.80s \\ 47b. 50 - s \quad 47c. 0.30(50 - s) \quad 47d. 0.80s + 0.30(50 - s) = 0.62(50) \quad 47e. 0.8s + 15 - 0.3s = 31; 0.5s = 16; s = 32 \\ 47f. 50 - 32 = 18; 32 \text{ L of 80% solution and 18 L of 30% solution} \quad 48. \text{Let } x = \text{the time (in minutes) it would take the apprentice to do the part of the work they did together: } \frac{1}{x} + \frac{1}{0.75x} = \frac{1}{136}; 408x\left(\frac{1}{x} + \frac{1}{0.75x}\right) = 408x\left(\frac{1}{136}\right); 408 + 544 = 3x; 3x = 952; x = 317.\bar{3}. \text{ Now let } y = \text{the time (in minutes) it would take the apprentice to do the part of the work that Sumi did alone: } \frac{0.75}{272} = \frac{1}{y}; 0.75y = 272; y = 362.\bar{6}. \text{ Total time (in minutes) for the apprentice to wash all the windows alone} = x + y = 317.\bar{3} + 362.\bar{6} = 680; 680 \div 60 = 11\frac{1}{3}; 11\frac{1}{3} \text{ h, or 11 h 20 min.}$$

$$49. \frac{2}{n} + \frac{1}{2} = \frac{1}{n}; 2n\left(\frac{2}{n} + \frac{1}{2}\right) = 2n\left(\frac{1}{n}\right); 4 + n = 2; n = -2; B$$

$$50. x = \frac{1}{2} + \frac{3}{x}; 2x(x) = 2x\left(\frac{1}{2} + \frac{3}{x}\right); 2x^2 = x + 6;$$

$$2x^2 - x - 6 = 0; (2x+3)(x-2) = 0; 2x+3 = 0 \text{ or}$$

$$x-2 = 0; x = -\frac{3}{2} \text{ or } x = 2; G \quad 51. \text{LCD} = x \cdot 3 \cdot 2 = 6x; C$$

$$52. [2]\frac{3+x}{16+x} = \frac{1}{2}; 2(3+x) = 16+x; 6+2x =$$

$$16+x; x = 10; \text{Check: } \frac{3+10}{16+10} = \frac{1}{2}; \frac{13}{26} = \frac{1}{2} \checkmark$$

[1] appropriate method, but with one computational error

$$53. \frac{5}{x^2y^2z} - \frac{8}{x^2y^2z} = \frac{5-8}{x^2y^2z} = -\frac{3}{x^2y^2z}$$

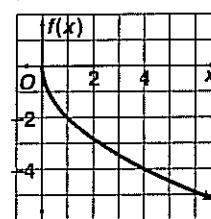
$$54. \frac{3h^2}{2t^2-8} + \frac{h}{t-2} = \frac{3h^2}{2(t^2-4)} + \frac{h}{t-2} =$$

$$\frac{3h^2}{2(t-2)(t+2)} + \frac{h}{t-2} =$$

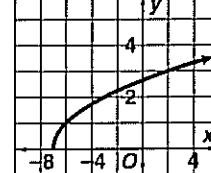
$$\frac{3h^2}{2(t-2)(t+2)} + \frac{2h(t+2)}{2(t-2)(t+2)} = \frac{3h^2+2ht+4h}{2(t-2)(t+2)}$$

$$55. \frac{k-11}{k^2+6k-40} - \frac{5}{k-4} = \frac{k-11}{(k-4)(k+10)} - \frac{5}{k-4} = \frac{k-11}{(k-4)(k+10)} - \frac{5(k+10)}{(k-4)(k+10)} = \frac{k-11-5k-50}{(k-4)(k+10)} = \frac{-4k-61}{(k-4)(k+10)}$$

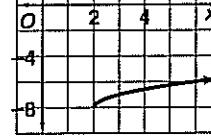
$$56. f(x) = -2\sqrt{x}; \text{points: } (0, 0), (1, -2), (4, -4), (7, -5.3)$$

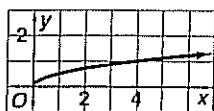


$$57. y = \sqrt{x+7}; \text{translate } y = \sqrt{x}, 7 \text{ units left}$$

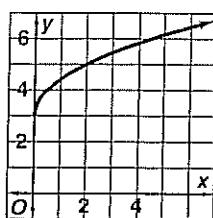


$$58. f(x) = \sqrt{x-2} - 8; \text{translate } y = \sqrt{x}, 2 \text{ units right and 8 units down}$$

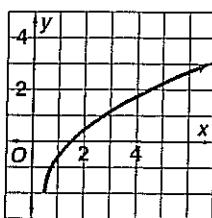




59.  $y = \sqrt{0.25x}$ ; points:  $(0, 0)$ ,  $(1, 0.5)$ ,  $(4, 1)$ ,  $(6, 1.2)$



60.  $y = \sqrt{2x} + 3$ ; points:  $(0, 3)$ ,  $(2, 5)$ ,  $(4.5, 6)$ ,  $(6, 6.5)$



61.  $y = \sqrt{4x - 2} - 2$ ; points:  $(0.5, -2)$ ,  $(1.5, 0)$ ,  $(2.75, 1)$ ,  $(4.5, 2)$ ,  $(6.75, 3)$

62.  $x^2 + 23x + 90 = 0$ ;  $(x + 18)(x + 5) = 0$ ;  $x + 18 = 0$  or  $x + 5 = 0$ ;  $x = -18$  or  $x = -5$  63.  $x^2 - 19x + 88 = 0$ ;  $(x - 8)(x - 11) = 0$ ;  $x - 8 = 0$  or  $x - 11 = 0$ ;  $x = 8$  or  $x = 11$  64.  $x^2 + 22x - 23 = 0$ ;  $(x + 23)(x - 1) = 0$ ;  $x + 23 = 0$  or  $x - 1 = 0$ ;  $x = -23$  or  $x = 1$  65.  $x^2 + 2x - 48 = 0$ ;  $(x + 8)(x - 6) = 0$ ;  $x + 8 = 0$  or  $x - 6 = 0$ ;  $x = -8$  or  $x = 6$  66.  $x^2 + 52 = -17x$ ;  $x^2 + 17x + 52 = 0$ ;  $(x + 13)(x + 4) = 0$ ;  $x + 13 = 0$  or  $x + 4 = 0$ ;  $x = -13$  or  $x = -4$  67.  $x^2 + 92x - 9 = -100$ ;  $x^2 + 92x + 91 = 0$ ;  $(x + 91)(x + 1) = 0$ ;  $x + 91 = 0$  or  $x + 1 = 0$ ;  $x = -91$  or  $x = -1$

## READING MATH

page 678

1a. Dawn:  $\frac{1}{15}$ ; Phil:  $\frac{1}{10}$  1b.  $\left(\frac{1}{15} + \frac{1}{10}\right)t = 1$ ;  $30\left(\frac{1}{15} + \frac{1}{10}\right)t = 30(1)$ ;  $(2 + 3)t = 30$ ;  $5t = 30$ ;  $t = 6$ ; 6 minutes 2.  $\frac{1}{10} + \frac{1}{n} = \frac{1}{6}$ ;  $30n\left(\frac{1}{10} + \frac{1}{n}\right) = 30n\left(\frac{1}{6}\right)$ ;  $3n + 30 = 5n$ ;  $30 = 2n$ ;  $n = 15$ ; 15 hours

## 12-3 Counting Methods and Permutations pages 679-685

**Check Skills You'll Need** For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies or Presentation Pro CD-ROM*.

1.  $\frac{1}{2}$  2.  $\frac{1}{2}$  3.  $\frac{1}{6}$  4. 0 5.  $\frac{1}{36}$  6.  $\frac{1}{4}$

**Investigation 1.** Assign a letter to each book: *A*, *B*, *C*. List all possible orders: *ABC*, *ACB*, *BAC*, *BCA*, *CAB*, *CBA*; 6. **2.** Answers may vary. Sample: You can make a list of all the possibilities. 3.  $\frac{1}{6}, \frac{1}{6}$

## Check Understanding

1a.



There are eight possible outfits.

1b. Answers may vary. Sample: No; the diagram would take up too much space.  $2 \cdot 5 \cdot 3 = 15$ ; 15 pizzas

$$3 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 40,320$$

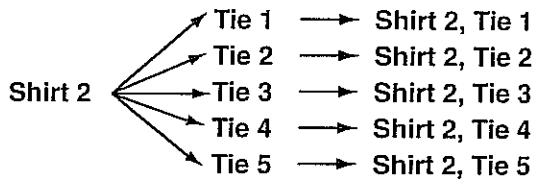
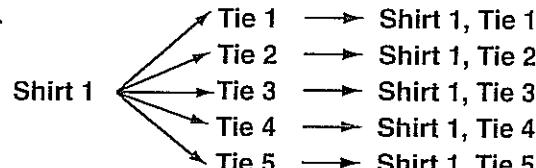
40,320 ways

4a.  ${}_9P_3 = 9 \cdot 8 \cdot 7 = 504$  4b.  ${}_7P_3 = 7 \cdot 6 \cdot 5 = 210$

4c.  ${}_5P_2 = 5 \cdot 4 = 20$  5a.  $10 \cdot 9 \cdot 8 \cdot 7 = 5040$  5b. A six-letter password; there are more possible, since  $26 > 10$ .

## Exercises

1.



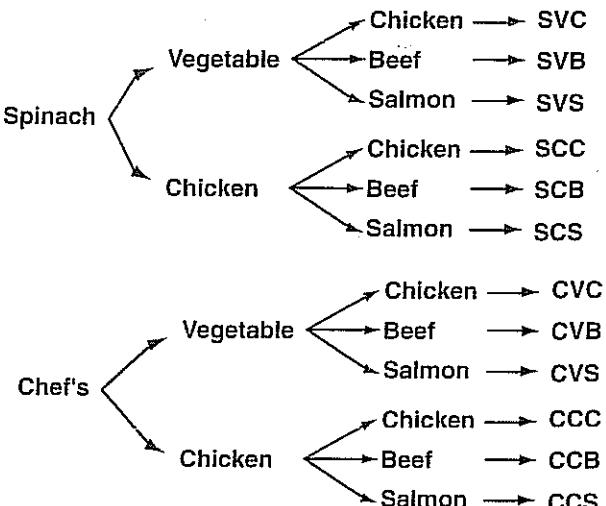
He has 10 shirt-tie choices.

2. Salad

Soup

Main Course

Menu



There are 12 possible menus. 3a. first digit: 8; second digit: 10; third digit: 10; fourth digit: 10

3b.  $8 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 8,000,000$ ; 8,000,000 telephone numbers 4a.  $2 \cdot 3 = 6$ ; 6 routes 4b.  $2 \cdot 3 \cdot 2 = 12$ ; 12 routes 5.  $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 3,628,800$ ; 3,628,800 orders

6.  $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ ; 120 arrangements

7.  ${}_8P_4 = 8 \cdot 7 \cdot 6 \cdot 5 = 1680$  8.  ${}_9P_4 = 9 \cdot 8 \cdot 7 \cdot 6 = 3024$

9.  ${}_6P_4 = 6 \cdot 5 \cdot 4 \cdot 3 = 360$  10.  ${}_5P_4 = 5 \cdot 4 \cdot 3 \cdot 2 = 120$

11.  ${}_7P_7 = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$  12.  ${}_7P_6 = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 5040$  13.  ${}_7P_5 = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = 2520$

14.  ${}_7P_2 = 7 \cdot 6 = 42$  15.  $10 \cdot 9 \cdot 8 \cdot 7 = 5040$

16.  $24 \cdot 23 \cdot 22 = 12,144$

17.  ${}_8P_6 = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3$ ;  ${}_6P_2 = 6 \cdot 5$ ;  ${}_8P_6$  is greater.

18.  ${}_9P_7 = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3$ ;  ${}_9P_2 = 9 \cdot 8$ ;  ${}_9P_7$  is greater.

19.  ${}_{10}P_3 = 10 \cdot 9 \cdot 8 = 720$ ;  ${}_8P_4 = 8 \cdot 7 \cdot 6 \cdot 5 = 1680$ ;  ${}_8P_4$

is greater. 20a. 2 vowels; 2 consonants 20b.  $2 \cdot 2 = 4$ ; 4 ways 20c. No; number of consonants · number of vowels = number of vowels · number of consonants.

21a.  $4 \cdot 3 \cdot 2 = 24$ ; 24 arrangements 21b.  $\frac{1}{24}$

22a.  $4 \cdot 3 \cdot 2 \cdot 1 = 24$ ; 24 permutations

22b.  $26 \cdot 25 \cdot 24 \cdot 23 = 358,800$ ;  $\frac{24}{358,800} = \frac{1}{14,950}$

22c. Answers may vary. Sample: No; if someone tries to guess your password, they'll probably try your name or initials first.

23.  $\frac{{}_5P_3}{{}_5P_2} = \frac{5 \cdot 4 \cdot 3}{5 \cdot 4} = 3$

24.  $\frac{{}_4P_3}{{}_4P_2} = \frac{4 \cdot 3 \cdot 2}{4 \cdot 3} = 2$  25.  $\frac{{}_7P_3}{{}_7P_2} = \frac{7 \cdot 6 \cdot 5}{7 \cdot 6} = 5$

26a.  $10 \cdot 10 \cdot 10 \cdot 10 = 10,000$  26b. With repetition; there are more permutations when repetition is allowed.

27a.  $26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 260,000$ ; 260,000 license plates 27b.  $92(260,000) = 23,920,000$ ; 23,920,000 license plates 28. Check students' work. 29a.  $26 \cdot 26 \cdot 26 = 17,576$ ; 17,576 codes 29b.  $17,576 - 50 = 17,526$ ; 17,526 codes 30a.  $2 \cdot 26 \cdot 26 \cdot 26 = 35,152$ ; 35,152 call letters

30b.  $2 \cdot 26 \cdot 26 \cdot 26 \cdot 26 = 913,952$ ; 913,952 call letters

31a.  $26 + 26 + 26 + 26 + 26 = 18,278$ ; 18,278

companies 31b.  $26 \cdot 26 \cdot 26 \cdot 26 + 26 \cdot 26 \cdot 26 \cdot 26 = 12,338,352$ ; 12,338,352 companies 31c. NASDAQ;

$12,338,352 - 18,278 = 12,320,074$ ; 12,320,074 more companies 32a. ABC, ACB; 2 arrangements

32b. ABCD, ABDC, ACBD, ACDB, ADBC, ADCB; 6 arrangements 32c.  $(n - 1)!$  33.  ${}_nP_r = 210$  and  $r = 3$ ;

${}_7P_3 = 7 \cdot 6 \cdot 5 = 210$ ;  $n = 7$  34a.  $5 \cdot 4 \cdot 3 = 60$ ; 60 numbers

34b.  $3 \cdot 2 \cdot 1 = 6$ ; 6 numbers 34c.  $\frac{6}{60} = \frac{1}{10}$

34d.  $1 - \frac{1}{10} = \frac{9}{10}$  35. False; if  $a = 3$  and  $b = 2$ , then

$(a - b)! = 1! = 1$ , but  $a! - b! = 3! - 2! = 4$ .

36.  $2 \cdot 3 \cdot 3 \cdot 4 = 72$  37.  $4 \cdot 2 \cdot 2 = 16$  38.  $2 \cdot 1 \cdot 2 = 4$

39.  ${}_4P_3 = 4 \cdot 3 \cdot 2 = 24$  40.  $\frac{1}{x} + \frac{1}{2x} = 5$ ;  $2x\left(\frac{1}{x} + \frac{1}{2x}\right) =$

$2x(5); 2 + 1 = 10x$ ;  $10x = 3$ ;  $x = \frac{3}{10}$  41.  $\frac{2}{n} + \frac{1}{n+1} =$

$\frac{11}{n^2+n}$ ;  $\frac{2}{n} + \frac{1}{n+1} = \frac{11}{n(n+1)}$ ;  $n(n+1)\left(\frac{2}{n} + \frac{1}{n+1}\right) =$

$n(n+1)\left(\frac{11}{n(n+1)}\right)$ ;  $2(n+1) + n = 11$ ;  $2n + 2 + n = 11$ ;

$3n = 9$ ;  $n = 3$  42.  $\frac{m}{2} = \frac{24-m}{m}$ ;  $m^2 = 2(24 - m)$ ;

$m^2 = 48 - 2m$ ;  $m^2 + 2m - 48 = 0$ ;  $(m + 8)(m - 6) = 0$ ;

$m + 8 = 0$  or  $m - 6 = 0$ ;  $m = -8$  or  $m = 6$

43.  $\frac{10}{3v+6} = \frac{3v}{v+2} + \frac{v^2}{3v+6}$ ;  $\frac{10}{3(v+2)} =$

$\frac{3v}{v+2} + \frac{v^2}{3(v+2)}$ ;  $3(v+2)\left(\frac{10}{3(v+2)}\right) =$

$3(v+2)\left(\frac{3v}{v+2} + \frac{v^2}{3(v+2)}\right)$ ;  $10 = 9v + v^2$ ;  $v^2 + 9v - 10 =$

$0$ ;  $(v+10)(v-1) = 0$ ;  $v + 10 = 0$  or  $v - 1 = 0$ ;  $v = -10$

or  $w = 1$  44.  $\frac{3w}{w-1} - \frac{2w}{w+3} = \frac{8w+40}{w^2+2w-3}$ ,

$\frac{3w}{w-1} - \frac{2w}{w+3} = \frac{8w+40}{(w+3)(w-1)}$ ;

$(w+3)(w-1)\left(\frac{3w}{w-1} - \frac{2w}{w+3}\right) =$

$(w+3)(w-1)\left(\frac{8w+40}{(w+3)(w-1)}\right)$ ;

$3w(w+3) - 2w(w-1) = 8w+40$ ;

$3w^2 + 9w - 2w^2 + 2w - 8w - 40 = 0$ ;  $w^2 + 3w - 40 = 0$ ;

$(w+8)(w-5) = 0$ ;  $w + 8 = 0$  or  $w - 5 = 0$ ;  $w = -8$  or  $w = 5$  45.  $\frac{h-5}{h+4} + \frac{h+1}{h+3} = \frac{-6h-6}{h^2+7h+12}$ ,

$\frac{h-5}{h+4} + \frac{h+1}{h+3} = \frac{-6(h+1)}{(h+4)(h+3)}$ ;

$(h+4)(h+3)\left(\frac{h-5}{h+4} + \frac{h+1}{h+3}\right) =$

$(h+4)(h+3)\left(\frac{-6h-6}{(h+4)(h+3)}\right)$ ;

$(h+3)(h-5) + (h+4)(h+1) = -6h-6$ ;

$h^2 - 2h - 15 + h^2 + 5h + 4 = -6h - 6$ ;  $2h^2 + 9h - 5 = 0$ ;

$(2h-1)(h+5) = 0$ ;  $2h-1 = 0$  or  $h+5 = 0$ ;  $h = \frac{1}{2}$  or  $h = -5$  46.  $m\angle A = 45^\circ$ , so  $m\angle B = 45^\circ$ ;  $AC = BC = 5$ ;

$AB = 5\sqrt{2} \approx 7$  47.  $\sin 32^\circ = \frac{BC}{64}$ ,  $BC = 64 \sin 32^\circ \approx 34$ ;

$\cos 32^\circ = \frac{AC}{64}$ ,  $AC = 64 \cos 32^\circ \approx 54$  48.  $\sin 75^\circ = \frac{20}{AB}$ ,

$AB \sin 75^\circ = 20$ ,  $AB = \frac{20}{\sin 75^\circ} \approx 21$ ;  $\tan 75^\circ = \frac{20}{BC}$ ,

$BC \tan 75^\circ = 20$ ,  $BC = \frac{20}{\tan 75^\circ} \approx 5$  49.  $\cos 8^\circ = \frac{48}{AB}$ ,

$AB \cos 8^\circ = 48$ ,  $AB = \frac{48}{\cos 8^\circ} \approx 48$ ;  $\tan 8^\circ = \frac{AC}{48}$ ,  $AC =$

$48 \tan 8^\circ \approx 7$  50.  $x^2 + 12x + 1 = 0$ ;  $x^2 + 12x = -1$ ;

$x^2 + 12x + 36 = -1 + 36$ ;  $(x+6)^2 = 35$ ;  $x+6 =$

$\pm\sqrt{35}$ ;  $x = -6 \pm \sqrt{35}$  51.  $x^2 + 6x + 20 = 0$ ;  $x^2 + 6x = -20$ ;

$x^2 + 6x + 9 = -20 + 9$ ;  $(x+3)^2 = -11$ ; no solution

52.  $x^2 + 8x = 11$ ;  $x^2 + 8x + 16 = 11 + 16$ ;  $(x+4)^2 = 27$ ;

$x+4 = \pm\sqrt{27}$ ;  $x = -4 \pm 3\sqrt{3}$  53.  $x^2 + 7x + 1 = 13$ ;

$x^2 + 7x = 12$ ;  $x^2 + 7x + \frac{49}{4} = 12 + \frac{49}{4}$ ;  $\left(x + \frac{7}{2}\right)^2 = \frac{97}{4}$ ;

$x + \frac{7}{2} = \pm\sqrt{\frac{97}{4}}$ ;  $x = -\frac{7}{2} \pm \frac{\sqrt{97}}{2}$  54.  $2x^2 + 8x + 7 =$

$9$ ;  $2x^2 + 8x = 2$ ;  $x^2 + 4x = 1$ ;  $x^2 + 4x + 4 = 1 + 4$ ;

$(x+2)^2 = 5$ ;  $x+2 = \pm\sqrt{5}$ ;  $x = -2 \pm \sqrt{5}$

55.  $x^2 - 18x + 65 = 0$ ;  $x^2 - 18x = -65$ ;  $x^2 - 18x + 81 =$

$-65 + 81$ ;  $(x-9)^2 = 16$ ;  $x-9 = \pm\sqrt{16}$ ;  $x = 9 \pm 4$ ;

$x = 9 + 4 = 13$  or  $x = 9 - 4 = 5$

## CHECKPOINT QUIZ 2

page 685

1.  $\frac{x^2 - 4}{x+3} \cdot \frac{x^2 + 7x + 12}{x-2} =$

$\frac{(x-2)(x+2)}{x+3} \cdot \frac{(x+4)(x+3)}{x-2} = (x+2)(x+4)$

2.  $\frac{z+5}{z} \div \frac{3z+15}{4z} = \frac{z+5}{z} \cdot \frac{4z}{3z+15} =$

$\frac{z+5}{z} \cdot \frac{4z}{3(z+5)} = \frac{4z(z+5)}{3z(z+5)} = \frac{4}{3}$

$$3. \quad a - 3 \frac{a^2 + 5a + 15}{a^3 + 2a^2 + 0a - 1}$$

$$\begin{array}{r} a^3 - 3a^2 \\ \hline 5a^2 + 0a \\ 5a^2 - 15a \\ \hline 15a - 1 \\ 15a - 45 \\ \hline 44 \end{array}$$

quotient:  $a^2 + 5a + 15 + \frac{44}{a-3}$

$$4. \quad \frac{9}{x-3} - \frac{4}{x-3} = \frac{9-4}{x-3} = \frac{5}{x-3} \quad 5. \quad \frac{8}{m+2} - \frac{6}{3-m} =$$

$$\frac{8}{m+2} - \frac{6}{-1(m-3)} = \frac{8}{m+2} + \frac{6}{m-3} =$$

$$\frac{8(m-3)}{(m+2)(m-3)} + \frac{6(m+2)}{(m+2)(m-3)} = \frac{8m-24+6m+12}{(m+2)(m-3)} =$$

$$\frac{14m-12}{(m+2)(m-3)} \quad 6. \quad \frac{6}{t} + \frac{3}{t^2} = \frac{6t}{t^2} + \frac{3}{t^2} = \frac{6t+3}{t^2} \quad 7. \quad \frac{9}{t} + \frac{3}{2} =$$

$$12; 2t\left(\frac{9}{t} + \frac{3}{2}\right) = 2t(12); 18 + 3t = 24t; 18 = 21t; t = \frac{18}{21} = \frac{6}{7}$$

$$8. \quad \frac{10}{z+4} = \frac{30}{2z+3}; 10(2z+3) = 30(z+4); 20z+30 =$$

$$30z+120; -90 = 10z; z = -9 \quad 9. \quad c - \frac{8}{c} = -7; c\left(c - \frac{8}{c}\right) = c(-7); c^2 - 8 = -7c; c^2 + 7c - 8 = 0;$$

$$(c+8)(c-1) = 0; c+8 = 0 \text{ or } c-1 = 0; c = -8 \text{ or } c = 1 \quad 10a. \quad 4 \cdot 4 = 16 \quad 10b. \quad 4 \cdot 3 = 12$$

## 12.9 Combinations pages 686–697

**Check Skills You'll Need** For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies or Presentation Pro CD-ROM*.

1. 60 2. 120 3. 210 4. 840 5.  $\frac{1}{4}$  6.  $\frac{5}{72}$  7.  $\frac{3}{4}$  8. 0.07

**Check Understanding** 1a.  ${}_4C_2 = \frac{4P_2}{2P_2} = \frac{4 \cdot 3}{2 \cdot 1} = 6$

1b.  ${}_7C_3 = \frac{7P_3}{3P_3} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 35 \quad$  1c.  ${}_{10}C_4 = \frac{10P_4}{4P_4} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} = 210 \quad$  2a.  ${}_5C_2 \quad$  2b.  ${}_5C_2 = \frac{5P_2}{2P_2} = \frac{5 \cdot 4}{2 \cdot 1} = 10$

2c. Order does not matter. 3.  $P(\text{novel, then history book}) = \frac{3 \cdot 2}{5P_2} = \frac{3 \cdot 2}{5 \cdot 4} = \frac{3}{10} \quad$  4a.  ${}_{10}C_4 = \frac{10P_4}{4P_4} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} = 210 \quad$  4b. 1 4c.  $\frac{1}{210}$

**Exercises** 1.  ${}_6C_6 = \frac{6P_6}{6P_6} = 1 \quad$  2.  ${}_6C_5 = \frac{6P_5}{5P_5} =$

$\frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 6 \quad$  3.  ${}_6C_4 = \frac{6P_4}{4P_4} = \frac{6 \cdot 5 \cdot 4 \cdot 3}{4 \cdot 3 \cdot 2 \cdot 1} = 15$

4.  ${}_6C_3 = \frac{6P_3}{3P_3} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20 \quad$  5.  ${}_6C_2 = \frac{6P_2}{2P_2} = \frac{6 \cdot 5}{2 \cdot 1} = 15$

6.  ${}_6C_1 = \frac{6P_1}{1P_1} = \frac{6}{1} = 6 \quad$  7.  ${}_8C_6 = \frac{8P_6}{6P_6} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} =$

28 8.  ${}_8C_2 = \frac{8P_2}{2P_2} = \frac{8 \cdot 7}{2 \cdot 1} = 28 \quad$  9.  ${}_7C_5 = \frac{7P_5}{5P_5} =$

$\frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 21 \quad$  10.  ${}_7C_2 = \frac{7P_2}{2P_2} = \frac{7 \cdot 6}{2 \cdot 1} = 21$

11.  ${}_{12}C_9 = \frac{12P_9}{9P_9} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 220$

12.  ${}_8C_3 = \frac{8P_3}{3P_3} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56 \quad$  13.  $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 =$

$5040; P(\text{rainbow}) = \frac{1}{5040} \quad$  14a.  ${}_{10}C_2 = \frac{10P_2}{2P_2} = \frac{10 \cdot 9}{2 \cdot 1} = 45$

14b. AE, AI, or EI; 3 combinations 14c.  $P(\text{only vowels}) = \frac{3}{45} = \frac{1}{15} \quad$  14d.  ${}_4C_2 = \frac{4P_2}{2P_2} = \frac{4 \cdot 3}{2 \cdot 1} = 6; P(B, C, D, \text{ or } F) =$

$\frac{6}{45} = \frac{2}{15} \quad$  15a.  ${}_8C_3 = \frac{8P_3}{3P_3} = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56$

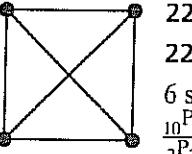
15b. 1 committee 15c.  $P(\text{all boys}) = \frac{1}{56} \quad$  15d.  ${}_5C_3 = \frac{5P_3}{3P_3} = \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = 10; P(\text{no boys}) = P(\text{all girls}) = \frac{10}{56} = \frac{5}{28}$

16.  ${}_5C_3 = \frac{5P_3}{3P_3} = \frac{5 \cdot 4 \cdot 3}{3 \cdot 2 \cdot 1} = 10 \quad$  17.  ${}_3C_3 = \frac{3P_3}{3P_3} = \frac{3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 1$

18.  ${}_7C_3 = \frac{7P_3}{3P_3} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2 \cdot 1} = 35 \quad$  19.  ${}_4C_3 = \frac{4P_3}{3P_3} = \frac{4 \cdot 3 \cdot 2}{3 \cdot 2 \cdot 1} =$

4 20. combination, since order is not important

21. permutation, since order is important

22a. 

22b. 6 segments

22c.  ${}_4C_2 = \frac{4P_2}{2P_2} = \frac{4 \cdot 3}{2 \cdot 1} = 6;$

6 segments 22d.  ${}_{10}C_2 = \frac{10P_2}{2P_2} = \frac{10 \cdot 9}{2 \cdot 1} = 45; 45 \text{ segments}$

23a. Answers may vary. Sample: It is a combination problem because order is not important. 23b.  ${}_{10}C_2 = \frac{10P_2}{2P_2} = \frac{10 \cdot 9}{2 \cdot 1} = 45; 45 \text{ handshakes}$

23c. Yes; each line segment joins two points, and each handshake connects two people. 24a.  $40 \cdot 39 \cdot 38 = 59,280; 59,280 \text{ sequences}$  24b.  $59,280 \div 40 = 1482; 1482 \text{ sequences}$

24c.  $\frac{1482}{59,280} = \frac{1}{40} \quad$  24d. Answers may vary. Sample: It is unlikely someone will guess the right sequence with more than 59,000 possibilities. 25. Answers may vary.

Sample: Both permutations and combinations are arrangements of some or all of a group of objects. However, permutations take into account order, and combinations do not. 26.  ${}_2C_2 + {}_2C_1 + {}_2C_0 = 1 + 2 + 1 = 4 \quad$  27.  ${}_3C_3 + {}_3C_2 + {}_3C_1 + {}_3C_0 = 1 + 3 + 3 + 1 = 8 \quad$  28a.  $4 \cdot 3 \cdot 2 = 24; 24 \text{ numbers}$

28b.  $P(\text{even}) = \frac{1}{4} \quad$  29. Check students' work.

30a.  ${}_{12}C_5 = \frac{12P_5}{5P_5} = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 792; 792 \text{ choices}$

30b. If the 3 defective ones are selected, then 2 others from the remaining 9 computers are selected;  ${}_9C_2 =$

$\frac{9P_2}{2P_2} = \frac{9 \cdot 8}{2 \cdot 1} = 36; 36 \text{ ways.}$  30c. Probability  $= \frac{36}{792} = \frac{1}{22}$

31.  ${}_nC_1 = \frac{nP_1}{1P_1} = \frac{n}{1} = n; \text{ always}$  32.  ${}_3C_x > x; \text{ try } x = 2:$

${}_3C_2 > 2, 3 > 2, \text{ true; try } x = 3: {}_3C_3 > 3, 1 > 3, \text{ false;}$

sometimes 33.  ${}_nC_{n-1} = n; \text{ try } n = 6: {}_6C_{6-1} = {}_6C_5 =$

$\frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 6 = n; \text{ always}$  34.  ${}_8C_3 = \frac{8 \cdot 7 \cdot 6}{3 \cdot 2 \cdot 1} = 56;$

2 favorable outcomes are Pythagorean triples: 3, 4, 5, and 5, 12, 13;  $P(\text{Pythagorean triple}) = \frac{2}{56} = \frac{1}{28}$  35a.  ${}_6C_4 =$

$\frac{6P_4}{4P_4} = \frac{6 \cdot 5 \cdot 4 \cdot 3}{4 \cdot 3 \cdot 2 \cdot 1} = 15 \quad$  35b. 6 questions are left; you

must still answer 3 questions. 35c.  ${}_6C_3 = \frac{6P_3}{3P_3} =$

$\frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20 \quad$  35d.  $15 \cdot 20 = 300$

| $f(x)$ |   |   |   |
|--------|---|---|---|
| 8      |   |   |   |
| 4      |   |   | x |
| 0      | 2 | 4 |   |

36a.  $f(x) = {}_xC_2; f(2) = {}_2C_2 = 1; f(3) = {}_3C_2 = 3; f(4) = {}_4C_2 = 6; f(5) = {}_5C_2 = 10$

36b. It is the function  $f(x) = \frac{x(x-1)}{2}, \text{ which uses the combination formula } {}_xC_2 = \frac{x!}{2!(x-2)!} \text{ for } x. \text{ This represents the}$

number of combination groups of 2. **36c.** Groups can only be made from sets of objects,

which means they must be integers. **37.**  ${}_{12}C_3 =$

$$\frac{12P_3}{3P_3} = \frac{12 \cdot 11 \cdot 10}{3 \cdot 2 \cdot 1} = 220; \text{B}$$

$${}_{12}P_2 = \frac{8P_2}{2P_2} = \frac{8 \cdot 7}{2 \cdot 1} = 28; 10 - 28 = -18; \text{F}$$

$${}_{12}P_2 = \frac{4 \cdot 3}{2 \cdot 1} = 6; \text{C}$$

**40.** [2] There are 10 possible pizzas; combination; order does not matter;  ${}_{10}C_2 = \frac{5P_2}{2P_2} = \frac{5 \cdot 4}{2 \cdot 1} =$

10 [1] Incorrect explanation OR minor error **41.**  ${}_{10}P_2 =$

$$4 \cdot 3 = 12$$

$${}_{10}P_6 = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 5040$$

$${}_{10}P_4 = 7 \cdot 6 \cdot 5 \cdot 4 = 840$$

$${}_{10}P_1 = 9$$

$${}_{23}P_1 = 23$$

$$46. 21 \cdot 21 \cdot 21 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 92,610,000;$$

92,610,000 license plates **47.**  $\frac{4}{x} + \frac{x}{x-4} = 1;$

$$x(x-4)\left(\frac{4}{x} + \frac{x}{x-4}\right) = x(x-4)(1); 4(x-4) + x^2 =$$

$$x^2 - 4x; 4x - 16 + x^2 = x^2 - 4x; 8x = 16; x = 2$$

$$48. \sqrt{r-7} = \sqrt{2r+4}; (\sqrt{r-7})^2 = (\sqrt{2r+4})^2;$$

$$r-7 = 2r+4; -11 = r; \text{Check: } \sqrt{-11-7} \stackrel{?}{=}$$

$$\sqrt{2(-11)+4}; \sqrt{-18} \stackrel{?}{=} \sqrt{-18}; \text{no solution}$$

$$49. \frac{2}{z} - \frac{3}{2z} = 5; 2z\left(\frac{2}{z} - \frac{3}{2z}\right) = 2z(5); 4 - 3 = 10z; 10z =$$

$$1; z = \frac{1}{10}$$

$$50. \sqrt{3v+10} = v; (\sqrt{3v+10})^2 = v^2;$$

$$3v+10 = v^2; v^2 - 3v - 10 = 0; (v-5)(v+2) = 0;$$

$$v-5 = 0 \text{ or } v+2 = 0; v = 5 \text{ or } v = -2 \text{ (extraneous);}$$

$$v = 5$$

$$51. \frac{1}{a+2} + \frac{1}{a-2} = \frac{10}{a-2};$$

$$(a-2)(a+2)\left(\frac{1}{a+2} + \frac{1}{a-2}\right) = (a-2)(a+2)\left(\frac{10}{a-2}\right);$$

$$a-2+a+2 = 10(a+2); 2a = 10a+20; -8a = 20;$$

$$a = -\frac{20}{8} = -2\frac{1}{2}$$

$$52. \sqrt{16m} = \sqrt{m^2}; (\sqrt{16m})^2 =$$

$$(\sqrt{m^2})^2; 16m = m^2; m^2 - 16m = 0; m(m-16) = 0;$$

$$m = 0 \text{ or } m-16 = 0; m = 0 \text{ or } m = 16$$

$$53. 2x^2 + 12x - 11 = 0; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$$

$$-\frac{12 \pm \sqrt{12^2 - 4(2)(-11)}}{2(2)} = \frac{-12 \pm \sqrt{232}}{4},$$

$$x = \frac{-12 + \sqrt{232}}{4} \approx 0.81 \text{ or } x = \frac{-12 - \sqrt{232}}{4} \approx -6.81$$

$$54. x^2 - 7x + 2 = 0; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$$

$$-\frac{(-7) \pm \sqrt{(-7)^2 - 4(1)(2)}}{2(1)} = \frac{7 \pm \sqrt{41}}{2}, x = \frac{7 + \sqrt{41}}{2} \approx$$

$$6.70 \text{ or } x = \frac{7 - \sqrt{41}}{2} \approx 0.30$$

$$55. x^2 + 4x - 8 = 0; x =$$

$$-\frac{b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(1)(-8)}}{2(1)} = \frac{-4 \pm \sqrt{48}}{2},$$

$$x = \frac{-4 + \sqrt{48}}{2} \approx 1.46 \text{ or } x = \frac{-4 - \sqrt{48}}{2} \approx -5.46$$

$$56. -9x^2 + x + 15 = 0; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$$

$$-\frac{1 \pm \sqrt{1^2 - 4(-9)(15)}}{2(-9)} = \frac{-1 \pm \sqrt{541}}{-18}; x =$$

$$-\frac{1 + \sqrt{541}}{-18} \approx -1.24 \text{ or } x = \frac{-1 - \sqrt{541}}{-18} \approx 1.35$$

$$57. -x^2 - 3x + 18 = 0; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$$

$$-\frac{(-3) \pm \sqrt{(-3)^2 - 4(-1)(18)}}{2(-1)} = \frac{3 \pm \sqrt{81}}{-2} = \frac{3 \pm 9}{-2}; x =$$

$$\frac{3+9}{-2} = -6 \text{ or } x = \frac{3-9}{-2} = 3$$

$$58. 1.5x^2 - 20x - 1 = 0;$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-20) \pm \sqrt{(-20)^2 - 4(1.5)(-1)}}{2(1.5)} =$$

$$\frac{20 \pm \sqrt{406}}{3}; x = \frac{20 + \sqrt{406}}{3} \approx 13.38 \text{ or } x = \frac{20 - \sqrt{406}}{3} \approx -0.05$$

## TEST-TAKING STRATEGIES

page 692

$$\begin{array}{r} 3x - 2 \\ x^2 - 2\sqrt{3}x^3 - 2x^2 + 0x - 4 \\ \underline{-3x^3} \quad \underline{-6x} \\ -2x^2 + 6x - 4 \\ \underline{-2x^2} \quad \underline{+4} \\ 6x - 8 \end{array}$$

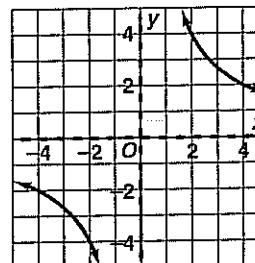
The remainder is  $6x - 8$ , so the answer is A.

2.  $y = \frac{3}{x+2} + 2$ ; the horizontal asymptote is  $y = 2$ , so the answer is G.  $3. \frac{4x}{3} + \frac{4x}{5} = 1; 15\left(\frac{4x}{3} + \frac{4x}{5}\right) = 15(1); 20x + 12x = 15; 32x = 15; x = \frac{15}{32}; 4x = 4\left(\frac{15}{32}\right) = \frac{15}{8}$ ; the answer is B.

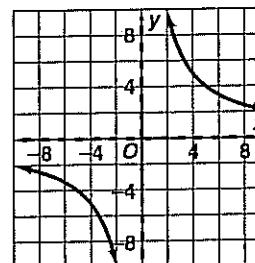
## CHAPTER REVIEW

pages 693–695

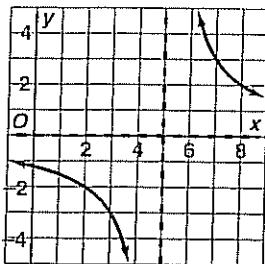
- A rational expression is in simplest form if the numerator and the denominator have no common factors other than 1.
- The y-axis is a vertical asymptote of the function  $y = \frac{1}{x}$ .
- A permutation is an arrangement of some or all of a set of objects in a specific order.
- The first step in solving a rational equation is to find the least common denominator.
- An equation of the form  $y = \frac{k}{x}$ , where  $k$  is a constant, is called an inverse variation.
- $xy = 6(1) = 6; xy = 6$
- $xy = 90(0.1) = 9; xy = 9$
- $xy = 88(0.05) = 4.4; xy = 4.4$
- $9x = 3(12); 9x = 36; x = 4$
- $10. 4(2.65) = 4.24y; 10.6 = 4.24y; y = 2.5$
- $11. 100r = 75(25); 100r = 1875; r = 18.75$
12. check products  $xy$ :  $2(35) = 70, 5(14) = 70, 10(7) = 70$ ; inverse variation;  $xy = 70$
13. check ratios  $\frac{y}{x} \cdot \frac{24.6}{3} = 8.2, \frac{41}{5} = 8.2, \frac{82}{10} = 8.2$ ; direct variation;  $y = 8.2x$
14. check products  $xy$ :  $1(3) = 3, 4\left(\frac{3}{4}\right) = 3, 9\left(\frac{1}{3}\right) = 3$ ; inverse variation;  $xy = 3$



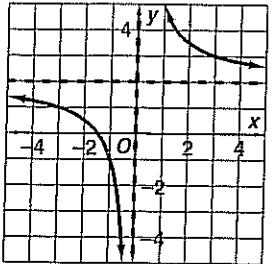
15.  $xy = 8$ ; vertical asymptote:  $x = 0$ ; horizontal asymptote:  $y = 0$ ; points:  $(-4, -2), (-2, -4), (2, 4), (4, 2)$



16.  $y = \frac{20}{x}$ ; vertical asymptote:  $x = 0$ ; horizontal asymptote:  $y = 0$ ; points:  $(-10, -2), (-5, -4), (-4, -5), (-2, -10), (2, 10), (4, 5), (5, 4), (10, 2)$



17.  $y = \frac{6}{x-5}$ ; vertical asymptote:  $x = 5$ ; horizontal asymptote:  $y = 0$ ; points:  $(-1, -1), (2, -2), (3, -3), (7, 3), (8, 2)$



18.  $y = \frac{3}{x} + 2$ ; vertical asymptote:  $x = 0$ ; horizontal asymptote:  $y = 2$ ; points:  $(-3, 1), (-1, -1), (1, 5), (3, 3)$

19. Answers may vary. Sample:  $y = \frac{1}{x+1}$  20. The graph of  $f(x) = \frac{5}{x+3}$  gets closer and closer to the lines  $x = -3$

$$\text{and } y = 0. 21. \frac{x^2 - 4}{x+2} = \frac{(x-2)(x+2)}{x+2} = x-2$$

$$22. \frac{5x}{20x+15} = \frac{5x}{5(4x+3)} = \frac{x}{4x+3} 23. \frac{6x-18}{x-3} =$$

$$\frac{6(x-3)}{x-3} = 6 24. \frac{-3t}{t^3-t^2} = \frac{-3t}{t^2(t-1)} = \frac{-3}{t(t-1)}$$

$$25. \frac{z+2}{2z^2+z-6} = \frac{z+2}{(2z-3)(z+2)} = \frac{1}{2z-3}$$

$$26. \frac{x^2-3x-10}{x^2-x-20} = \frac{(x-5)(x+2)}{(x-5)(x+4)} = \frac{x+2}{x+4}$$

$$27. \frac{8}{m-3} \cdot \frac{3m}{m+1} = \frac{24m}{(m-3)(m+1)}$$

$$28. \frac{4t-12}{t^2-9} \cdot (3t+9) = \frac{4(t-3) \cdot 3(t+3)}{(t+3)(t-3)} = 12$$

$$29. \frac{4n+8}{3n} \div \frac{4}{9n} = \frac{4n+8}{3n} \cdot \frac{9n}{4} = \frac{4(n+2)}{3n} \cdot \frac{3(3n)}{4} =$$

$$3(n+2) 30. \frac{2e+1}{8e-4} \div \frac{4e^2+4e+1}{4e-2} =$$

$$\frac{2e+1}{8e-4} \cdot \frac{4e-2}{4e^2+4e+1} = \frac{2e+1}{4(2e-1)} \cdot \frac{2(2e-1)}{(2e+1)(2e+1)} =$$

$$\frac{1}{2(2e+1)} 31. (14x^2 - 28x) \div 7x = (14x^2 - 28x) \cdot \frac{1}{7x} =$$

$$\frac{14x^2}{7x} - \frac{28x}{7x} = 2x - 4 32. (24x^6 + 32x^5 - 8x^2) \div 8x^2 =$$

$$(24x^6 + 32x^5 - 8x^2) \cdot \frac{1}{8x^2} = \frac{24x^6}{8x^2} + \frac{32x^5}{8x^2} - \frac{8x^2}{8x^2} =$$

$$3x^4 + 4x^3 - 1 33. (50x^5 - 7x^4 + x^2) \div x^3 =$$

$$(50x^5 - 7x^4 + x^2) \cdot \frac{1}{x^3} = \frac{50x^5}{x^3} - \frac{7x^4}{x^3} + \frac{x^2}{x^3} =$$

$$50x^2 - 7x + \frac{1}{x}$$

$$34. \begin{array}{r} x+7 \\ x+1 \overline{)x^2+8x+2} \\ \underline{x^2+x} \\ 7x+2 \\ \underline{7x+7} \\ -5 \end{array}$$

$$\text{quotient: } x+7 - \frac{5}{x+1}$$

$$35. \begin{array}{r} x+4 \\ x+4 \overline{)x^2+8x-16} \\ \underline{x^2+4x} \\ 4x-16 \\ \underline{4x+16} \\ -32 \end{array}$$

$$\text{quotient: } x+4 - \frac{32}{x+4}$$

$$36. \begin{array}{r} 2x^2 - 7x + 4 \\ 4x + 3 \overline{)8x^3 - 22x^2 - 5x + 12} \\ \underline{8x^3 + 6x^2} \\ -28x^2 - 5x \\ \underline{-28x^2 - 21x} \\ 16x + 12 \\ \underline{16x + 12} \\ 0 \end{array}$$

quotient:  $2x^2 - 7x + 4$

$$37. \frac{8x}{x-7} - \frac{4}{x-7} = \frac{8x-4}{x-7}$$

$$38. \frac{6}{7x} + \frac{1}{4} = \frac{6(4)}{7x(4)} + \frac{1(7x)}{4(7x)} = \frac{24}{28x} + \frac{7x}{28x} = \frac{7x+24}{28x}$$

$$39. \frac{9}{3x-1} + \frac{5x}{2x+3} =$$

$$\frac{9(2x+3)}{(3x-1)(2x+3)} + \frac{5x(3x-1)}{(3x-1)(2x+3)} =$$

$$\frac{18x+27+15x^2-5x}{(3x-1)(2x+3)} = \frac{15x^2+13x+27}{(3x-1)(2x+3)}$$

$$40. \frac{7m}{m^2-1} - \frac{10}{m+1} =$$

$$\frac{7m}{(m-1)(m+1)} - \frac{10(m-1)}{(m-1)(m+1)} = \frac{7m-10m+10}{(m-1)(m+1)} =$$

$$\frac{-3m+10}{(m-1)(m+1)} 41. \text{LCD} = 4 \cdot x \cdot (3x-2) \cdot 2 =$$

$$8x(3x-2) = 24x^2 - 16x; C 42. \frac{1}{2} + \frac{3}{t} = \frac{5}{8};$$

$$8t\left(\frac{1}{2} + \frac{3}{t}\right) = 8t\left(\frac{5}{8}\right); 4t + 24 = 5t; 24 = t; t = 24$$

$$43. 9 + \frac{1}{t} = \frac{1}{4}; 4t\left(9 + \frac{1}{t}\right) = 4t\left(\frac{1}{4}\right); 36t + 4 = t; 35t = -4;$$

$$t = -\frac{4}{35} 44. \frac{3}{m-4} + \frac{1}{3(m-4)} = \frac{6}{m};$$

$$3m(m-4)\left(\frac{3}{m-4} + \frac{1}{3(m-4)}\right) = 3m(m-4)\left(\frac{6}{m}\right);$$

$$9m + m = 18(m-4); 10m = 18m - 72; 8m = 72;$$

$$m = 9 45. \frac{2c}{c-4} - 2 = \frac{4}{c+5};$$

$$(c-4)(c+5)\left(\frac{2c}{c-4} - 2\right) = (c-4)(c+5)\left(\frac{4}{c+5}\right);$$

$$2c(c+5) - 2(c^2 + c - 20) = 4(c-4);$$

$$2c^2 + 10c - 2c^2 - 2c + 40 = 4c - 16; 8c + 40 =$$

$$4c - 16; 4c = -56; c = -14 46. \frac{5}{2x-3} = \frac{7}{3x}; 5(3x) =$$

$$7(2x-3); 15x = 14x - 21; x = -21 47. \frac{2}{x} = \frac{2}{x^2} + \frac{1}{2};$$

$$2x^2\left(\frac{2}{x}\right) = 2x^2\left(\frac{2}{x^2} + \frac{1}{2}\right); 2x(2) = 2(2) + 1(x^2);$$

$$4x = 4 + x^2; x^2 - 4x + 4 = 0; (x-2)^2 = 0; x-2 = 0;$$

$$x = 2 48. \frac{1}{2} + \frac{1}{n} = \frac{1}{1.5}; \frac{1}{2} + \frac{1}{n} = \frac{2}{3}; 6n\left(\frac{1}{2} + \frac{1}{n}\right) = 6n\left(\frac{2}{3}\right);$$

$$3n + 6 = 4n; 6 = n; 6 \text{ minutes} 49. {}_5P_3 = 5 \cdot 4 \cdot 3 = 60$$

$$50. {}_8P_4 = 8 \cdot 7 \cdot 6 \cdot 5 = 1680 51. {}_6P_4 = 6 \cdot 5 \cdot 4 \cdot 3 =$$

$$360 52. {}_5P_2 = 5 \cdot 4 = 20 53. {}_9P_4 = 9 \cdot 8 \cdot 7 \cdot 6 = 3024$$

$$54a. 8 \cdot 2 \cdot 10 = 160; 160 \text{ possible area codes}$$

$$54b. 8 \cdot 10 \cdot 10 = 800; 800 - 160 = 640; 640 \text{ new area codes} 55. {}_6C_5 = \frac{6P_5}{5P_5} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 6 56. {}_{10}C_4 =$$

$$\frac{10P_4}{4P_4} = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} = 210 57. {}_9C_2 = \frac{9P_2}{2P_2} = \frac{9 \cdot 8}{2 \cdot 1} = 36$$

$$58. {}_{11}C_3 = \frac{11P_3}{3P_3} = \frac{11 \cdot 10 \cdot 9}{3 \cdot 2 \cdot 1} = 165 59. {}_5C_4 = \frac{5P_4}{4P_4} =$$

$$\frac{5 \cdot 4 \cdot 3 \cdot 2}{4 \cdot 3 \cdot 2 \cdot 1} = 5 60. {}_4C_3 = \frac{4P_3}{3P_3} = \frac{4 \cdot 3 \cdot 2}{3 \cdot 2 \cdot 1} = 4;$$

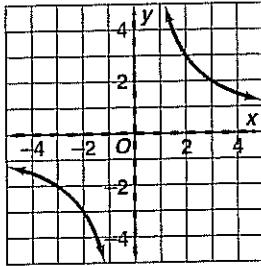
$$4 \text{ combinations} 61. {}_{15}C_{10} = \frac{15P_{10}}{10P_{10}} =$$

$$\frac{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 3003; 3003 \text{ combinations}$$

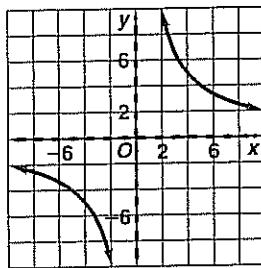
**CHAPTER TEST**

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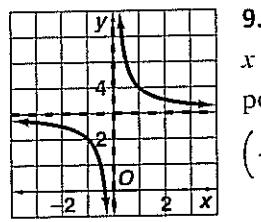
1.  $k = xy = 6(5) = 30$  2.  $k = xy = 0.1(78) = 7.8$  3.  $k = xy = -10(2.4) = -24$  4.  $k = xy = 9.1(5.3) = 48.23$   
 5. check products  $xy$ : 3(12) = 36,  $-9(-4) = 36$ , 6(6) = 36,  $-18(2) = -36$ ; D 6.  $1.5(40) = x(50)$ ;  $60 = 50x$ ;  $x = \frac{60}{50} = 1.2$ ; 1.2 h



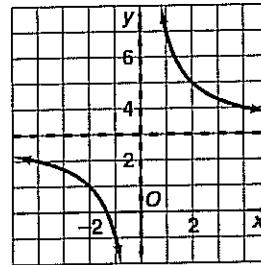
7.  $y = \frac{6}{x}$ ; vertical asymptote:  $x = 0$ ; horizontal asymptote:  $y = 0$ ; points:  $(-6, -1), (-3, -2), (-2, -3), (2, 3), (3, 2), (6, 1)$



8.  $xy = 20$ , or  $y = \frac{20}{x}$ ; vertical asymptote:  $x = 0$ ; horizontal asymptote:  $y = 0$ ; points:  $(-10, -2), (-5, -4), (-4, -5), (-2, -10), (2, 10), (4, 5), (5, 4), (10, 2)$



9.  $y = \frac{1}{x} + 3$ ; vertical asymptote:  $x = 0$ ; horizontal asymptote:  $y = 3$ ; points:  $(-4, 2\frac{3}{4}), (-2, 2\frac{1}{2}), (-1, 2), (-\frac{1}{2}, 1), (\frac{1}{2}, 5), (1, 4), (2, 3\frac{1}{2}), (4, 3\frac{1}{4})$



10.  $y = \frac{4}{x} + 3$ ; vertical asymptote:  $x = 0$ ; horizontal asymptote:  $y = 3$ ; points:  $(-4, 2), (-2, 1), (-1, -1), (1, 7), (2, 5), (4, 4)$

11. Answers may vary. Sample: In direct variation and in inverse variation, the variables are related to each other by a constant. But in direct variation, that number is the ratio of any corresponding pair of input and output values. Distance travelled varies directly with average speed. In inverse variation, that number is the product of any corresponding pair of input and output values. The cost per person, of splitting a \$14 pizza, varies inversely with the number of people

who are sharing it.

$$12. \frac{3}{x-2} \cdot \frac{x^2-4}{12} =$$

$$\frac{3}{x-2} \cdot \frac{(x-2)(x+2)}{3(4)} = \frac{x+2}{4} \quad 13. \frac{5x}{x^2+2x} \div \frac{30x^2}{x+2} =$$

$$\frac{5x}{x^2+2x} \cdot \frac{x+2}{30x^2} = \frac{5x}{x(x+2)} \cdot \frac{x+2}{5x(6x)} = \frac{1}{6x^2}$$

$$14. \frac{4w}{3w-5} \cdot \frac{7}{2w} = \frac{2(2w)}{3w-5} \cdot \frac{7}{2w} = \frac{14}{3w-5}$$

$$15. \frac{6c-2}{c+5} \div \frac{3c-9}{c} = \frac{6c-2}{c+5} \cdot \frac{c}{3c-9} =$$

$$\frac{2(3c-1)}{c+5} \cdot \frac{c}{3(c-3)} = \frac{c}{3(c+5)(c-3)}$$

16. Answers may vary. Sample:  $\frac{x+2}{(x-6)(x+3)}$

$$17. (12x^4 + 9x^3 - 10x^2) \div 3x^3 = \\ (12x^4 + 9x^3 - 10x^2) \cdot \frac{1}{3x^3} = \frac{12x^4}{3x^3} + \frac{9x^3}{3x^3} - \frac{10x^2}{3x^3} = \\ 4x + 3 - \frac{10}{3x}$$

$$18. \begin{array}{r} x^3 - 2x^2 + 4x - 8 \\ x + 2 \overline{)x^4 + 0x^3 + 0x^2 + 0x - 16} \\ \underline{x^4 + 2x^3} \\ -2x^3 + 0x^2 \\ \underline{-2x^3 - 4x^2} \\ 4x^2 + 0x \\ \underline{4x^2 + 8x} \\ -8x - 16 \\ \underline{-8x - 16} \\ 0 \end{array}$$

quotient:  $x^3 - 2x^2 + 4x - 8$

$$19. \begin{array}{r} 2x^3 - 2x^2 - x \\ 2x - 1 \overline{)4x^4 - 6x^3 + 0x^2 + x + 7} \\ \underline{4x^4 - 2x^3} \\ -4x^3 + 0x^2 \\ \underline{-4x^3 + 2x^2} \\ -2x^2 + x \\ \underline{-2x^2 + x} \\ 0 + 7 \end{array}$$

quotient:  $2x^3 - 2x^2 - x + \frac{7}{2x-1}$

$$20. \begin{array}{r} 2x^2 - 5x - 2 \\ 3x + 2 \overline{)6x^3 - 11x^2 - 16x + 13} \\ \underline{6x^3 + 4x^2} \\ -15x^2 - 16x \\ \underline{-15x^2 - 10x} \\ -6x + 13 \\ \underline{-6x - 4} \\ 17 \end{array}$$

quotient:  $2x^2 - 5x - 2 + \frac{17}{3x+2}$

$$21. 3(2) = 4x; 6 = 4x; x = 1.5; 1\frac{1}{2} \text{ h} \quad 22. \frac{v}{3} + \frac{v}{v+5} =$$

$$\frac{-4}{v+5}; 3(v+5)\left(\frac{v}{3} + \frac{v}{v+5}\right) = 3(v+5)\left(\frac{-4}{v+5}\right);$$

$$v(v+5) + 3v = 3(-4); v^2 + 5v + 3v = -12;$$

$$v^2 + 8v + 12 = 0; (v+6)(v+2) = 0; v+6 = 0$$

$$\text{or } v+2 = 0; v = -6 \text{ or } v = -2 \quad 23. \frac{16}{x+10} = \frac{8}{2x-1};$$

$$16(2x-1) = 8(x+10); 32x - 16 = 8x + 80; 24x = 96;$$

$$x = 4 \quad 24. \frac{2}{3} + \frac{t+6}{t-3} = \frac{18}{2(t-3)}; 6(t-3)\left(\frac{2}{3} + \frac{t+6}{t-3}\right) =$$

$$6(t-3)\left(\frac{18}{2(t-3)}\right); 2(t-3)(2) + 6(t+6) = 3(18);$$

$$4t - 12 + 6t + 36 = 54; 10t = 30; t = 3 \text{ (extraneous);}$$

$$\text{no solution} \quad 25. \frac{5}{t} + \frac{t}{t+1} = \frac{5(t+1)}{t(t+1)} + \frac{t(t)}{t(t+1)} =$$

$$\frac{5t+5+t^2}{t(t+1)} = \frac{t^2+5t+5}{t(t+1)} \quad 26. \frac{9}{n} - \frac{8}{n+1} =$$

$$\frac{9(n+1)}{n(n+1)} - \frac{8(n)}{n(n+1)} = \frac{9n+9-8n}{n(n+1)} = \frac{n+9}{n(n+1)}$$

$$27. \frac{2y}{y^2-9} - \frac{1}{y-3} = \frac{2y}{(y-3)(y+3)} - \frac{1(y+3)}{(y-3)(y+3)} =$$

$$\frac{2y-y-3}{(y-3)(y+3)} = \frac{y-3}{(y-3)(y+3)} = \frac{1}{y+3}$$

$$28. \frac{4b-2}{3b} + \frac{b}{b+2} = \frac{(4b-2)(b+2)}{3b(b+2)} + \frac{3b(b)}{3b(b+2)} =$$

$$\frac{4b^2+6b-4+3b^2}{3b(b+2)} = \frac{7b^2+6b-4}{3b(b+2)} \quad 29. \text{Permutation; order}$$

is important;  $30 \cdot 29 = 870$ ; 870 different pairs.

**30.** Combination; order is not important;  ${}_6C_2 = \frac{6P_2}{2P_2} = \frac{6 \cdot 5}{2 \cdot 1} = 15$ ; 15 different ways. **31.** Combination; order is not important;  ${}_6C_2 = \frac{6P_2}{2P_2} = \frac{6 \cdot 5}{2 \cdot 1} = 15$ ; 15 different pairs of toppings. **32.**  ${}_6C_4 = \frac{6P_4}{4P_4} = \frac{6 \cdot 5 \cdot 4 \cdot 3}{4 \cdot 3 \cdot 2 \cdot 1} = 15$  **33.**  ${}_8C_4 = \frac{8P_4}{4P_4} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1} = 70$  **34.**  ${}_5C_4 = \frac{5P_4}{4P_4} = \frac{5 \cdot 4 \cdot 3 \cdot 2}{4 \cdot 3 \cdot 2 \cdot 1} = 5$

**35.**  ${}_7C_4 = \frac{7P_4}{4P_4} = \frac{7 \cdot 6 \cdot 5 \cdot 4}{4 \cdot 3 \cdot 2 \cdot 1} = 35$  **36.**  ${}_4C_3 = \frac{4P_3}{3P_3} = \frac{4 \cdot 3 \cdot 2}{3 \cdot 2 \cdot 1} = 4$  **37.**  ${}_8P_6 = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = 20,160$

**38.**  ${}_{10}P_7 = 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 = 604,800$  **39.**  ${}_5C_2 = \frac{5P_2}{2P_2} = \frac{5 \cdot 4}{2 \cdot 1} = 10$  **40.** 5 · 4 = 20; 20 different combinations **41.** 15 books: 3 plays, 1 poetry, and 11 novels; number of ways to choose 2 novels from 11:

${}_{11}C_2 = \frac{11 \cdot 10}{2 \cdot 1} = 55$ ; number of ways to choose 1 play from 3:  ${}_3C_1 = 3$ ; number of ways to choose 2 novels and a play:  $55 \cdot 3 = 165$ ; number of ways to choose 3 books from 15:  ${}_{15}C_3 = \frac{15 \cdot 14 \cdot 13}{3 \cdot 2 \cdot 1} = 455$ ;  $P(\text{two novels and a play}) = \frac{165}{455} = \frac{33}{91}$

### STANDARDIZED TEST PREP pages 697–699

**1.**  $\frac{2x}{3} = 5$ ,  $2x = 15$ ,  $x = 7.5$ ;  $\frac{2y - 2}{4} = 3$ ,  $2y - 2 = 12$ ,

$$2y = 14$$
,  $y = 7$ ;  $\frac{z}{2} + \frac{z}{3} = 5$ ,  $6\left(\frac{z}{2} + \frac{z}{3}\right) = 6(5)$ ,  $3z + 2z = 30$ ,  $5z = 30$ ,  $z = 6$ ;  $x > y$ ; **A** **2.**  $1200 + 7x = 23x$ ;  $16x = 1200$ ;  $x = 75$ ; **G** **3.**  $2(w + (2w + 3)) = 72$ ;  $w + 2w + 3 = 36$ ;  $3w = 33$ ;  $w = 11$ ;  $2w + 3 = 2(11) + 3 = 25$ ; **C**

**4.** mean =  $(79 + 82 + 83 + 87 + 94) \div 5 = 425 \div 5 = 85$ ; **H** **5.**  $1575(1.08) = 1701$ ; **A** **6.**  $P(A \text{ and } B) = P(A) \cdot P(B)$ ;  $\frac{1}{8} = \frac{5}{6} \cdot P(B)$ ;  $P(B) = \frac{1}{8} \cdot \frac{6}{5} = \frac{6}{40} = \frac{3}{20}$ ; **G**

**7.** I is not a function because of (1, 3) and (1, 7); II and III are functions; **C** **8.**  $f(x) = \frac{-3}{x-1}$ ;  $f(-1) = \frac{-3}{-1-1} = \frac{-3}{-2} = \frac{3}{2}$ ; **G** **9.** Test points in each function: **A.**  $y = 2x$ ;  $-5 = 2(-1)$ , false + **B.**  $y = 2x - 3$ ;  $-5 = 2(-1) - 3$ , true;  $-1 = 2(1) - 3$ , true;  $3 = 2(3) - 3$ , true;  $7 = 2(5) - 3$ , true; the answer is **B**. **10.**  $9 - 14 = -5$ ,  $4 - 9 = -5$ , so the sequence is arithmetic with common difference  $-5$ ;  $4 - 5 = -1$ ,  $-1 - 5 = -6$ ,  $-6 - 5 = -11$ ; **G**

**11.**  $a = 800 - 30t$ ; **B** **12.**  $3x + 2y = 7$ ;  $2y = -3x + 7$ ;  $y = -\frac{3}{2}x + 7$ ; slope of a perpendicular line is  $\frac{2}{3}$ ; **H**

**13.** **①**  $3y - 8 = -5x$ , or  $3y = -5x + 8$ , or  $y = -\frac{5}{3}x + \frac{8}{3}$  **②**  $3x = 2y - 18$ , or  $2y = 3x + 18$ , or  $y = \frac{3}{2}x + 9$ ; use substitution:  $-\frac{5}{3}x + \frac{8}{3} = \frac{3}{2}x + 9$

$$6\left(-\frac{5}{3}x + \frac{8}{3}\right) = 6\left(\frac{3}{2}x + 9\right)$$

$$-10x + 16 = 9x + 54$$

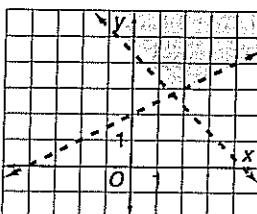
$$-19x = 38$$
;  $x = -2$ ; substitute  $-2$  for  $x$  in **②**:  $y =$

$$\frac{3}{2}(-2) + 9 = 6$$
; intersect at  $(-2, 6)$ ; **C** **14.** **①**  $-2x - 3y = -15$ , or  $2x + 3y = 15$  **②**  $3x + 2y = 0$ , or  $2y = -3x$ , or

$$y = -\frac{3}{2}x$$
; substitute  $-\frac{3}{2}x$  for  $y$  in **①**:  $2x + 3\left(-\frac{3}{2}x\right) =$

$$15$$
,  $2x - \frac{9}{2}x = 15$ ,  $-\frac{5}{2}x = 15$ ,  $x = 15\left(-\frac{2}{5}\right) = -6$ ; substitute

$-6$  for  $x$  in **②**:  $y = -\frac{3}{2}(-6) = 9$ ;  $(-6, 9)$ ; **F** **15.** Test each point in the inequality  $4y - 3x \leq 8$ ;  $4(2) - 3(0) \leq 8$ ,  $5 \leq 8$ , true for  $(0, 1)$ ;  $4\left(\frac{1}{4}\right) - 3(-3) \leq 8$ ,  $10 \leq 8$ , false for  $(-3, \frac{1}{4})$ ;  $4(-17.6) - 3(5) \leq 8$ ,  $-85.4 \leq 8$ , true for  $(5, -17.6)$ ;  $4\left(\frac{2}{5}\right) - 3(-4) \leq 8$ ,  $13.6 \leq 8$ , false for  $(-4, \frac{2}{5})$ ; I and III; **D**



- 16.** **①**  $2y > x + 4$ , or  $y > \frac{1}{2}x + 2$  **②**  $3y + 3x > 13$ , or  $3y > -3x + 13$ , or  $y > -x + \frac{13}{3}$  The only true statement is  $y > 1$ , so the answer is **I**.

**17.**  $(2.5 \times 10^4)(3.0 \times 10^{-15}) = 7.5 \times 10^{-11}$ ; **D**

**18.**  $2^2 \cdot (-1)^3 \cdot (-2)^{-1} = 4 \cdot (-1) \cdot \left(-\frac{1}{2}\right) = 2$ ; **G**

**19.**  $\frac{20x^2y^4}{30x^5y^2} = \frac{2y^2}{3x^3}$ ; **B** **20.**  $1000 \cdot 1.055^{10}$ ; **G**

**21.**  $(3x^2 - 7x - 2) - (8x - 3) = 3x^2 - 15x + 1$ ; **D**

**22.**  $12x^5 + 4x^3 - 16x^2$ ; GCF =  $4x^2$ ; **H**

**23.**  $(3x - 1)(5x + 3) = 15x^2 + 9x - 5x - 3 =$

$15x^2 + 4x - 3$ ; **D** **24.**  $(2x - 3)(5x + 4) =$

$10x^2 + 8x - 15x - 12 = 10x^2 - 7x - 12$ ; **G**

**25.**  $x^2 + 3x - 10 = (x - 2)(x + 5)$ ; **A** **26.**  $y =$

$-3x^2 - 6x - 1$ ;  $x$ -coordinate of vertex =

$-\frac{b}{2a} = -\frac{-6}{2(-3)} = -1$ ;  $y$ -coordinate of vertex =

$-3(-1)^2 - 6(-1) - 1 = -3 + 6 - 1 = 2$ ; **I**

**27.**  $2x^2 + 5x + 3 = 0$ ;  $(2x + 3)(x + 1) = 0$ ;  $2x + 3 = 0$

or  $x + 1 = 0$ ;  $x = -\frac{3}{2}$  or  $x = -1$ ; **D** **28.**  $x^2 + 4x - 5 = 0$

$(x + 5)(x - 1) = 0$ ;  $x + 5 = 0$  or  $x - 1 = 0$ ;  $x = -5$  or

$x = 1$ ; **H** **29.**  $2x^2 + 5x + 1 = 0$ ;  $b^2 - 4ac = 5^2 - 4(2)(1) =$

$25 - 8 = 17$ ,  $17 > 0$ , two solutions; **C** **30.**  $x^2 - 6x - 11 =$

$0$ ;  $x^2 - 6x = 11$ ;  $x^2 - 6x + 9 = 11 + 9$ ;  $(x - 3)^2 = 20$ ;

$x - 3 = \pm\sqrt{20} = \pm 2\sqrt{5}$ ;  $x = 3 \pm 2\sqrt{5}$ ; **I**

**31.**  $h^2 + 60^2 = 100^2$ ,  $h^2 + 3600 = 10,000$ ;  $h^2 = 6400$ ;

$h = 80$ ; **B** **32.**  $8\sqrt{2} \approx 11.3$ ; **I** **33.**  $4 + \sqrt{y - 3} = 11$ ;

$\sqrt{y - 3} = 7$ ;  $y - 3 = 49$ ;  $y = 52$ ; **A**

**34.**  $\sin D = \frac{EF}{DE}$  so **F** is true;

$\cos D = \frac{DF}{DE} \neq \frac{DE}{DF}$ , so **G** is

false;  $\sin E = \frac{DF}{DE}$ , so **H** is true;

$\cos E = \frac{EF}{DE}$ , so **I** is true; the answer is **G**.

**35.**  $2(5) = x(10)$ ;  $10 = 10x$ ;  $x = 1$ ; **B** **36.**  $3(50) = x(60)$ ;

$150 = 60x$ ;  $x = 2.5$ ; **H** **37.**  $\frac{2}{3} = \frac{x}{60}$ ;  $5x = 2(60)$ ;  $5x = 120$ ;

$x = 24$ ; **D** **38.**  $\frac{3x - 9}{x^2 - 6x + 9} = \frac{3(x - 3)}{(x - 3)(x - 3)} = \frac{3}{x - 3}$ ; **G**

**39.**  $y = \frac{8}{x - 10}$ ; vertical asymptote:  $x = 10$ ; **D**

**40.**  $\frac{x - 4}{x^2 + 2x} \div \frac{x^2 - 16}{x^2 - x} = \frac{x - 4}{x^2 + 2x} \cdot \frac{x^2 - x}{x^2 - 16} =$

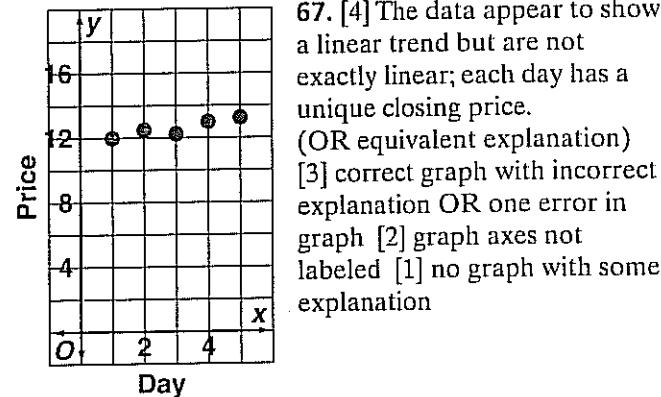
$\frac{x - 4}{x(x + 2)} \cdot \frac{x(x - 1)}{(x - 4)(x + 4)} = \frac{x - 1}{(x + 2)(x + 4)} = \frac{x - 1}{x^2 + 6x + 8}$ ; **F**

**41.**  $\frac{5}{m^3} \div \frac{10}{m^2} = \frac{5}{m^3} \cdot \frac{m^2}{10} = \frac{5m^2}{10m^3} = \frac{1}{2m}$ ; **D**

42.  $\frac{x-1}{x+3} \cdot \frac{x-3}{x^2-1} = \frac{x-1}{x+3} \cdot \frac{x-3}{(x-1)(x+1)} =$   
 $\frac{x-3}{(x+3)(x+1)} = \frac{x-3}{x^2+4x+3}; G$  43.  $a^2 - b^2$  and  $a^2 + b^2$ ;  
D 44.  $2x - y = 3$  and  $0.5(4x - 2y) = 3$  are linear equations; D 45. degree of  $4x^3 - 8x^2 - 7x$  is 3; GCF of  $6x^3 - 9x^2 + 12$  is 3; C 46.  $f(4) = 2(4)^2 - 23 = 9; f(3) = 3^2 + 5 = 14$ ; B 47.  $2x - 3y = 5, -3y = -2x + 5, y = \frac{2}{3}x - \frac{5}{3}$ , slope =  $\frac{2}{3}$ ; 4y - 2 = 7x, 4y = 7x + 2, y =  $\frac{7}{4}x + \frac{1}{2}$ , slope =  $\frac{7}{4}$ ; B 48. median = mean of two middle values =  $(1+1) \div 2 = 2 \div 2 = 1$ ; mode = most frequent value = 1; C 49.  $f(0) = 0^3 - 0 = 0; f(-2) = (-2)^3 - (-2) = -8 + 2 = -6$ ; A 50. [2]  ${}_{10}P_3 = 10 \cdot 9 \cdot 8 = 720$   
[1] appropriate methods, but with one computational error 51. [2]  $\frac{4x+10}{2x^2+7x+5} = \frac{2(2x+5)}{(2x+5)(x+1)} = \frac{2}{x+1}$   
[1] appropriate methods, but with one computational error 52. [2]  $4x^2 - 12x + 9 = (2x-3)(2x-3)$   
[1] incorrect answer, or no work shown 53. [2]  $6^2 + 10^2 = c^2; 136 = c^2; c = \sqrt{136} \approx 11.66$ ; about 11.66 cm [1]  
appropriate methods, but with one computational error 54. [2] Extraneous solutions are false solutions; they can occur when solving rational and radical equations. [1] definition without equation types  
55. [2] 55a.  $\frac{7}{12} \cdot \frac{5}{11} = \frac{35}{132}$  55b.  $\frac{7}{12} \cdot \frac{6}{11} = \frac{7}{22}$  [1] one part correct 56. [2]  $(-2)(-1 \cdot 4)^4 = -2(256) = -512$   
[1] appropriate methods, but with one computational error 57. [2]  $6x\left(\frac{x}{6} + \frac{1}{2}\right) = 6x\left(\frac{3}{x}\right); x^2 + 3x = 18$ ;  
 $x^2 + 3x - 18 = 0; (x+6)(x-3) = 0; x+6 = 0$  or  
 $x-3 = 0; x = -6$  or  $x = 3$  [1] appropriate methods, but with one computational error 58. [2]  $x(x-4) = 45$ ;  
 $x^2 - 4x - 45 = 0; (x-9)(x+5) = 0$ ; 9 and -5; -5 is an extraneous solution; the solution 9 gives the numbers 9 and  $9-4=5$ . [1] both numbers not given  
59. [2]  $2a - 3(-3) = 13; 2a = 4; a = 2; 2 + 3b = 8; 3b = 6; b = 2$  [1] appropriate methods, but with one computational error 60. [2]  $\frac{12,187 - 11,787}{11,787} \approx 0.034$ ; about 3.4% [1] appropriate methods, but with one computational error 61. [2] asymptotes:  $x = 2$ ,  $y = 1$ ;  $y = \frac{5}{x-2} + 1 = -\frac{3}{2}$ , so  $y$ -intercept is  $-\frac{3}{2}, 0 = \frac{5}{x-2} + 1, -1 = \frac{5}{x-2}, x-2 = -5, x = -3$ , so  $x$ -intercept is -3. [1] asymptotes or intercepts only  
62. [2]  $\frac{32-42}{42} \approx -0.2380$ ; 23.8% [1] appropriate methods, but with one computational error 63. [2] No; when  $\frac{x}{x-7} - \frac{2}{x-7} = \frac{5}{x-7}$  is multiplied by the LCD,  $x-7$ , the resulting equation is  $x-2=5$ . However, the solution 7 is extraneous. [1] appropriate answer but no explanation 64. [2]  $x - 472 = 1634; x = 2106; x + 472 = 2106 + 472 = 2578$  [1] value of  $x$  found correctly but  $x + 472$  not found 65. [4] 65a.  $t - 19 = \frac{19 - 40}{150 - 300} (\ell - 150); t - 19 = \frac{7}{50} (\ell - 150); t - 19 = \frac{7}{50} \ell - 21; t = \frac{7}{50} \ell - 2$  65b.  $t = \frac{7}{50} (200) - 2; t = 26$ ;  
26 mm 65c.  $61 = \frac{7}{50} \ell - 2; 63 = \frac{7}{50} \ell; 450 = \ell$ ; 450 mm

[3] appropriate methods, but with one computational error [2] incorrect equation used correctly OR correct equation used incorrectly [1] no work shown

66. [4]  $\cos 28^\circ = \frac{BC}{7}, BC = 7 \cos 28^\circ \approx 6.18; \sin 28^\circ = \frac{AC}{7}, AC = 7 \sin 28^\circ \approx 3.29$  (OR equivalent solution)  
[3] correct equations, but error using calculator  
[2] only one side found correctly [1] no work shown



### REAL-WORLD SNAPSHOTs pages 700-701

Activity 1 a.  $f = \frac{k}{L}; 220.0 = \frac{k}{70}; k = 15,400$

| b. Fret | Position (cm) |
|---------|---------------|
| 0       | 70.0          |
| 1       | 66.1          |
| 2       | 62.4          |
| 3       | 58.9          |
| 4       | 55.6          |
| 5       | 52.4          |
| 6       | 49.5          |
| 7       | 46.7          |
| 8       | 44.1          |
| 9       | 41.6          |
| 10      | 39.3          |
| 11      | 37.1          |
| 12      | 35.0          |

Activity 2 a.  $\frac{392.0}{261.6} \approx 1.5$

b. They are all close to 1.5.

c. octave:  $\frac{440}{220} = 2$ ; major third:  $\frac{329.6}{261.6} \approx 1.26$

d. Answers may vary. Sample: about 1.3;  $\frac{349.2}{261.6} \approx 1.33$