Name _____ Class _____ Date _____

Practice 12-1

Inverse Variation

Suppose y varies inversely with x. Write an equation for each inverse variation.

1. $x = 9$ when $y = 6$
2. $x = 3.6$ when $y = 5$
3. $x = \frac{3}{4}$ when $y = \frac{2}{9}$
4. $x = 7$ when $y = 13$
5. $x = 8$ when $y = 9$
6. $x = 4.9$ when $y = 0.8$
7. $x = 11$ when $y = 44$
8. $y = 8$ when $x = 9.5$
9. $y = 12$ when $x = \frac{5}{6}$

Each pair of points is on the graph of an inverse variation. Find the missing value.

10. $(5, 8)$ and $(4, m)$
11. $(16, 5)$ and $(10, h)$
12. $(14, 8)$ and $(c, 7)$
13. $(3, 18)$ and $(a, 27)$
14. $(4, 28)$ and $(3, p)$
15. $(100, 25)$ and $(4, a)$
16. $(x, 7)$ and $(2, 14)$
17. $\left(\frac{2}{5}, \frac{3}{2}\right)$ and $\left(k, \frac{5}{2}\right)$
18. $(16, 3)$ and $(g, 24)$
19. $(2.4, 19.8)$ and $(h, 13.2)$
20. $(12.4, 6.6)$ and $(f, 8.8)$
21. $(3.2, k)$ and $(9.2, 0.8)$
22. $(18, 24)$ and $(72, v)$
23. $(17, 0.9)$ and $(5.1, x)$
24. $\left(\frac{3}{4}, y\right)$ and $\left(\frac{2}{3}, 18\right)$

Explain whether each situation represents a direct variation or an inverse variation.

25. The cost of a \$50 birthday gift is split among some friends.
26. You purchase some peaches at \$1.29/lb.

Tell whether the data in each table is a *direct variation*, or an *inverse variation*. Write an equation to model the data.

27.

x	2	7	10
y	35	10	7

28.

x	3	6	24
y	16	8	2

29.

x	5	6	8
y	55	66	88

30.

x	2	8	16
y	9	36	72

31.

x	2	3	9
y	18	12	4

32

x	2	6	10
y	4.2	12.6	21

33.

x	2	5	12
y	12.8	32	76.8

34.

x	1.2	1.5	2.4
y	5	4	2.5

35.

x	6	9	36
y	3	2	0.5

36. The volume V of a gas in a closed container varies inversely with the pressure p, in atmospheres, that is applied to that gas.
    a. If $V = 20\ m^3$ when $p = 1$ atm, find V when $p = 4$ atm.
    b. If $V = 24\ m^3$ when $p = 3$ atm, find p when $V = 36\ m^3$.
    c. If $V = 48\ m^3$ when $p = 2$ atm, find V when $p = 5$ atm.
37. The time t to travel a fixed distance varies inversely with the rate r of travel.
    a. If $t = 3$ h and $r = 25$ mi/h, find t when $r = 50$ mi/h.
    b. If $t = 120$ s and $r = 40$ ft/s, find r when $t = 25$ s.

Name _____ Class _____ Date _____

Practice 12-2

Graphing Rational Functions

Describe the graph of each function.

1. $f(x) = x^2 - 4$
2. $y = \frac{5}{x} - 1$
3. $y = \frac{3}{x}$
4. $g(x) = \sqrt{x + 2} - 1$
5. $y = -8x + 2$
6. $h(x) = 3x^2 - 4x + 1$
7. $h(x) = |2x + 7|$
8. $y = 0.2^x$
9. $y = \frac{x}{4}$

10. In an electric circuit the resistance R, in ohms, increases when the current I, in amps, in the circuit decreases. The function $R = \frac{1000}{I^2}$ relates the resistance to the current.
    a. What is the resistance when the current is 4 amps?
    b. What is the resistance when the current is 20 amps?
    c. What is the resistance when the current is 10 amps?

11. Light intensity decreases as you move farther away from the source of light. The function $I = \frac{12{,}000}{d^2}$ relates the light intensity I, in lumens, to the distance d, in feet, from the light source.
    a. What is the light intensity 2 ft away from the light source?
    b. What is the light intensity 8 ft away from the light source?
    c. What is the light intensity 25 ft away from the light source?

12. In a cylinder of constant volume, the height increases as the radius decreases. The function $h = \frac{360}{r^2}$ relates the height of the cylinder to the radius of the cylinder.
    a. What is the height of the cylinder when the radius is 5 m?
    b. What is the height of the cylinder when the radius is 12 m?

What value of x makes the denominator of each function equal to zero?

13. $y = \frac{5}{2x - 8}$
14. $y = \frac{12}{x}$
15. $y = \frac{5}{x + 7}$
16. $y = \frac{5x}{4x - 10}$
17. $y = \frac{-7x}{x + 3}$
18. $y = \frac{3}{x - 8}$
19. $y = \frac{6}{5x - 6}$
20. $y = \frac{9x}{3x + 5}$

Graph each function. Include a dashed line for each asymptote.

21. $y = \frac{2}{x}$
22. $y = \frac{2}{x - 1}$
23. $y = \frac{1}{x + 4}$
24. $y = \frac{2}{x} + 3$
25. $y = \frac{-2}{x + 6}$
26. $y = \frac{2x}{x - 6}$
27. $y = \frac{x + 3}{x - 2}$
28. $y = \frac{3}{x - 1} - 3$

Name _____ Class _____ Date _____

Practice 12-3

Simplifying Rational Expressions

Simplify each expression.

1. $\frac{6x^4}{18x^2}$
2. $\frac{15a^2}{25a^4}$
3. $\frac{32h^3}{48h^2}$
4. $\frac{12n^4}{21n^6}$
5. $\frac{3x - 6}{6}$
6. $\frac{x^2 - 2x}{x}$
7. $\frac{4t^2 - 2t}{2t}$
8. $\frac{a^3 - 2a^2}{2a^2 - 4a}$
9. $\frac{21x^2y}{14xy^2}$
10. $\frac{32x^3y^2}{24xy^4}$
11. $\frac{x^2 + 3x}{3x + 9}$
12. $\frac{x^2 - 5x}{5x - 25}$
13. $\frac{x^2 + 13x + 12}{x^2 - 144}$
14. $\frac{x^2 - 9}{x^3 - 3x^2}$
15. $\frac{x^3 + x^2}{x + 1}$
16. $\frac{3x - 2y}{2y - 3x}$
17. $\frac{x^2 + x - 6}{x^2 - x - 2}$
18. $\frac{x^2 + 3x + 2}{x^3 + x^2}$
19. $\frac{2x^2 - 8}{x^2 - 3x + 2}$
20. $\frac{2x^2 - 5x + 3}{x^2 - 1}$
21. $\frac{3x + 3y}{x^2 + xy}$
22. $\frac{10 + 3x - x^2}{x^2 - 4x - 5}$
23. $\frac{9 - x^2}{x^2 + x - 12}$
24. $\frac{x^2 + 2x - 15}{x^2 - 7x + 12}$
25. $\frac{x^2 + 7x - 8}{x^2 + 6x - 7}$
26. $\frac{x^2 + 3x - 10}{25 - x^2}$

27. Write and simplify the ratio $\frac{\text{perimeter of rectangle}}{\text{area of rectangle}}$. The perimeter of the rectangle is $10w$ and the area of the rectangle is $4w^2$.

28. The ratio $\frac{3 \cdot \text{volume of cone}}{\text{area of base}}$ determines the height of a cone. Find the height when the volume is $4r^3 + 2r^2$ and the area of the base is $6r^2$.

29. The ratio $\frac{2 \cdot \text{area of triangle}}{\text{height of triangle}}$ determines the length of the base of a triangle. Find the length of the base when the area is $3n^2 + 6n$ and the height is $2n + 4$.

30. The ratio $\frac{\text{volume of rectangular solid}}{\text{area of rectangular base}}$ determines the height of a rectangular solid. Find the height when the volume is $5s^3 + 10s^2$ and the area is $5s^2$.

Name _____ Class _____ Date _____

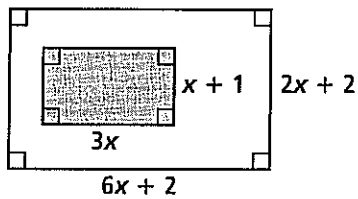
Practice 12-4

Multiplying and Dividing Rational Expressions

Find each product or quotient.

1. $\frac{5}{9} \cdot \frac{6}{15}$
2. $\frac{8}{3} \div \frac{16}{27}$
3. $\left(-\frac{3}{4}\right) \div \frac{16}{21}$
4. $\frac{2}{9} \div \left(-\frac{10}{3}\right)$
5. $\frac{18m}{4m^2} \div \frac{9m}{8}$
6. $\frac{8x}{12} \cdot \frac{4x}{6}$
7. $\frac{9}{15x} \cdot \frac{25x}{27}$
8. $\frac{12x^3}{25} \div \frac{16x}{5}$
9. $\frac{6x^3}{18x} \div \frac{9x^2}{10x^4}$
10. $\frac{4r^3}{10} \cdot \frac{25}{16r^2}$
11. $\frac{8n^2}{3} \div \frac{20n}{9}$
12. $\frac{14x^2}{5} \div 7x^4$
13. $\frac{4n^3}{11} \cdot \frac{33n}{36n^2}$
14. $\frac{24r^3}{35r^2} \div \frac{12r}{14r^3}$
15. $\frac{a^2 - 4}{3} \cdot \frac{9}{a + 2}$
16. $\frac{4b - 12}{5b^2} \cdot \frac{6b}{b - 3}$
17. $\frac{2b}{5} \cdot \frac{10}{b^2}$
18. $\frac{2b}{b + 3} \div \frac{b}{b + 3}$
19. $\frac{5y^3}{7} \cdot \frac{14y}{30y^2}$
20. $\frac{4p + 16}{5p} \div \frac{p + 4}{15p^3}$
21. $\frac{3(h + 2)}{h + 3} \div \frac{h + 2}{h + 3}$
22. $\frac{a^3 - a^2}{a^3} \cdot \frac{a^2}{a - 1}$
23. $\frac{h^2 + 6h}{h + 3} \cdot \frac{4h + 12}{h + 6}$
24. $\frac{n^2 - 1}{n + 2} \cdot \frac{n^2 - 4}{n + 1}$
25. $\frac{x^2 - x}{x} \cdot \frac{3x - 6}{3x - 3}$
26. $\frac{5x - 10}{x + 2} \cdot \frac{3}{3x - 6}$
27. $\frac{x^2 - 16}{x - 4} \div \frac{3x + 12}{x}$
28. $\frac{x^2 - 1}{3x - 3} \div \frac{x + 1}{3}$
29. $\frac{x^2 - 2x - 24}{x^2 - 5x - 6} \cdot \frac{x^2 + 5x + 6}{x^2 + 6x + 8}$
30. $\frac{x^2 + 2x - 35}{x^2 + 4x - 21} \cdot \frac{x^2 + 3x - 18}{x^2 + 9x + 18}$
31. $\frac{3x^2 + 14x + 8}{2x^2 + 7x - 4} \cdot \frac{2x^2 + 9x - 5}{3x^2 + 16x + 5}$
32. $\frac{8 + 2x - x^2}{x^2 + 7x + 10} \div \frac{x^2 - 11x + 28}{x^2 - x - 42}$
33. $\frac{x^2 - x - 6}{3x - 9} \cdot \frac{x^2 - 9}{x^2 + 6x + 9}$
34. $\frac{6x^2 + 13x + 6}{4x^2 - 9} \div \frac{6x^2 + x - 2}{4x^2 - 1}$
35. $\frac{x^2 - 2x - 35}{3x^2 + 27x} \div \frac{x^2 + 7x + 10}{6x^2 + 12x}$
36. $\frac{x^2 - x - 6}{2x^2 + 9x + 10} \div \frac{x^2 - 25}{2x^2 + 15x + 25}$
37. $\frac{15 - 14x - 8x^2}{4x^2 + 4x - 15} \div \frac{4x^2 + 13x - 12}{3x^2 + 13x + 4}$
38. $\frac{x^2 - 4x - 32}{x^2 - 8x - 48} \cdot \frac{3x^2 + 17x + 10}{3x^2 - 22x - 16}$
39. $\frac{9x^2 - 16}{6x^2 - 11x + 4} \div \frac{6x^2 + 11x + 4}{8x^2 + 10x + 3}$

40. Two darts are thrown at random onto the large rectangular region shown. Find the probability that both darts will land in the shaded region.



Name _____ Class _____ Date _____

Practice 12-5

Dividing Polynomials

Divide.

1. $\frac{10x - 25}{5}$
2. $\frac{4x^3 - 3x}{x}$
3. $(3x^2 - 6x) \div 3x$
4. $(10x^2 - 6x) \div 2x$
5. $(-8x^5 + 16x^4 - 24x^3 + 32x^2) \div 8x^2$
6. $(15x^2 - 30x) \div 5x$
7. $(x^2 - 14x + 49) \div (x - 7)$
8. $(2x^2 - 13x + 21) \div (x - 3)$
9. $(4x^2 - 16) \div (2x + 4)$
10. $(x^2 + 4x - 12) \div (x - 2)$
11. $(x^2 + 10x + 16) \div (x + 2)$
12. $(12x^2 - 5x - 2) \div (3x - 2)$
13. $(x^2 + 5x + 10) \div (x + 2)$
14. $(x^2 - 8x - 9) \div (x - 3)$
15. $(3x^2 - 2x - 13) \div (x - 2)$
16. $(x^3 + 3x^2 + 5x + 3) \div (x + 1)$
17. $(5 - 23x + 12x^2) \div (4x - 1)$
18. $(24 + 6x^2 + 25x) \div (3x - 1)$
19. $(2x^2 + 11x - 5) \div (x + 6)$
20. $(x^2 + 5x - 10) \div (x + 2)$
21. $(8x + 3 + 4x^2) \div (2x - 1)$
22. $(3x^2 + 11x - 4) \div (3x - 1)$
23. $(x^3 + x - x^2 - 1) \div (x - 1)$
24. $(10 + 21x + 10x^2) \div (2x + 3)$
25. $(6x^2 - 35x + 36) \div (3x - 4)$
26. $(-2x^2 - 33x + x^3 - 7) \div (x - 7)$

27. The volume of a rectangular prism is $15x^3 + 38x^2 - 23x - 6$. The height of the prism is $5x + 1$, and the width of the prism is $x + 3$. Find the length of the prism.

28. The width of a rectangle is $x + 1$, and the area is $x^3 + 2x^2 - 5x - 6$ cm. What is the length of the rectangle?

Name _____ Class _____ Date _____

Practice 12-6

Adding and Subtracting Rational Expressions

Simplify.

1. $\frac{3x}{4} - \frac{x}{4}$
2. $\frac{3}{x} + \frac{5}{x}$
3. $\frac{5x}{6} - \frac{2x}{3}$
4. $\frac{x}{3} + \frac{x}{5}$
5. $\frac{3m}{4} + \frac{5m}{12}$
6. $\frac{4x}{7} - \frac{3x}{14}$
7. $\frac{6}{7t} - \frac{3}{7t}$
8. $\frac{d}{3} + \frac{4d}{3}$
9. $\frac{7}{2d} - \frac{3}{2d}$
10. $\frac{3}{2d^2} + \frac{4}{3d}$
11. $\frac{9}{m+1} - \frac{6}{m-1}$
12. $\frac{3}{x} - \frac{7}{x}$
13. $\frac{7a}{6} + \frac{a}{6}$
14. $\frac{4}{k+3} - \frac{8}{k+3}$
15. $\frac{3}{4z^2} + \frac{7}{4z^2}$
16. $\frac{6}{x^2-1} + \frac{7}{x-1}$
17. $\frac{2x}{x^2-1} - \frac{3}{x+1}$
18. $\frac{3t}{8} + \frac{3t}{8}$
19. $\frac{4}{3a^2} - \frac{1}{2a^3}$
20. $\frac{4}{a+4} + \frac{6}{a+4}$
21. $\frac{4}{x+3} + \frac{6}{x-2}$
22. $\frac{6}{7t^3} - \frac{8}{3t}$
23. $\frac{3}{2x+6} + \frac{4}{6x+18}$
24. $\frac{5}{8a} - \frac{3}{8a}$
25. $\frac{5}{r^2-4} + \frac{7}{r+2}$
26. $\frac{6}{a^2-2} + \frac{9}{a^2-2}$
27. $\frac{5x}{4} - \frac{x}{4}$
28. $\frac{4}{3x+6} - \frac{3}{2x+4}$
29. $\frac{4}{c^2+4c+3} + \frac{1}{c+3}$
30. $\frac{6}{x^2-3x+2} - \frac{4}{x-2}$

31. Brian rode his bike 2 mi to his friend's house. Brian's bike had a flat tire, so he had to walk home. His walking rate is 25% of his biking rate.

    a. Write an expression for the amounts of time Brian spent walking and riding his bike.

    b. If Brian's biking rate is 12 mi/h, how much time did he spend walking and riding his bike?

32. Trudi and Sean are on a river canoeing. Because of the current of the river, their downstream rate is 250% of their upstream rate. They canoe 3 mi upstream and then return to their starting point.

    a. Write an expression for the amount of time Trudi and Sean spend canoeing.

    b. If their upstream rate is 2 mi/h, how much time do Trudi and Sean spend canoeing?

    c. If their upstream rate is 3 mi/h, how much time do Trudi and Sean spend canoeing?

Name _____ Class _____ Date _____

Practice 12-7

Solving Rational Equations

Solve each equation. Check your solution.

1. $\frac{1}{x} + \frac{1}{2x} = \frac{1}{6}$
2. $\frac{x}{x+2} + \frac{4}{x-2} = 1$
3. $\frac{1}{3s} = \frac{s}{2} - \frac{1}{6s}$
4. $\frac{x+2}{x+8} = \frac{x-2}{x+4}$
5. $1 - \frac{3}{x} = \frac{4}{x^2}$
6. $\frac{7}{3(a-2)} - \frac{1}{a-2} = \frac{2}{3}$
7. $\frac{n}{n-4} = \frac{2n}{n+4}$
8. $x + \frac{6}{x} = -7$
9. $\frac{2}{r^2 - r} - 1 = \frac{2}{r-1}$
10. $\frac{y}{y+3} = \frac{6}{y+9}$
11. $\frac{d}{3} + \frac{1}{2} = \frac{1}{3d}$
12. $\frac{2m}{m-5} = \frac{2m+16}{m+3}$
13. $\frac{1}{m-4} + \frac{1}{m+4} = \frac{8}{m^2 - 16}$
14. $\frac{5}{x-2} = \frac{5x+10}{x^2}$
15. $\frac{k^2}{k+3} = \frac{9}{k+3}$
16. $\frac{h-3}{h+6} = \frac{2h+3}{h+6}$
17. $\frac{h}{6} - \frac{3}{2h} = \frac{8}{3h}$
18. $4 - \frac{3}{y} = \frac{5}{y}$
19. $\frac{1}{b-3} = \frac{b}{4}$
20. $\frac{1}{t^2} - \frac{2}{t} = \frac{3}{t^2}$
21. $\frac{2}{3n} + \frac{3}{4} = \frac{2}{3}$

22. David and Fiona have a house painting business. It takes Fiona 3 days to paint a certain house. David could paint the same house in 4 days. How long would it take them to paint the house if David and Fiona worked together?

23. Suppose the Williams Spring Water Company has two machines that bottle the spring water. Machine X fills the bottles twice as fast as Machine Y. Working together, it takes them 20 min to fill 450 bottles. How long would it take each machine working alone to fill the 450 bottles?

24. Chao, who is an experienced architect, can draw a certain set of plans in 6 h. It takes Carl, who is a new architect, 10 h to draw the same set of plans. How long would it take them working together to draw the set of plans?

25. For exercise, Joseph likes to walk and Vincent likes to ride his bike. Vincent rides his bike 12 km/h faster than Joseph walks. Joseph walks 20 km in the same amount of time that Vincent rides 44 km. Find the rate that each of them travels.

26. The Ryan Publishing Company has two printing presses. It takes the new printing press 45 min to print 10,000 fliers. Together the two presses can print the 10,000 fliers in 30 min. How long does it take the older printing press by itself to print the 10,000 fliers?

Name _____ Class _____ Date _____

Practice 12-8

Counting Methods and Permutations

Simplify each expression.

1. ${}_7P_2$
2. ${}_{12}P_6$
3. ${}_{11}P_3$
4. ${}_{10}P_3$
5. ${}_9P_8$
6. ${}_{12}P_7$
7. ${}_{20}P_7$
8. ${}_{15}P_3$
9. ${}_{16}P_4$
10. ${}_{25}P_3$
11. ${}_{17}P_2$
12. ${}_{15}P_2$

13. Suppose a license plate consists of five different letters.
    a. How many five-letter license plates are possible?
    b. In how many ways can a five-letter license plate be made with the letters from APRIL if none of the letters are repeated?
    c. Suppose a license plate is assigned randomly. What is the probability that it will contain the letters from APRIL?
14. In how many ways can nine mopeds be parked in a row?
15. Suppose there are three different ways in which you could go from your house to a friend's house. From your friend's house, there are four different ways in which you could go to the library. In how many different ways can you go from your house to the library after meeting your friend?
16. A sports card collection contains 20 baseball players, 15 basketball players, and 25 football players. In how many ways can you select one of each?
17. Suppose you are electing student council officers. The student council contains 24 students. In how many ways can a president, a vice-president, and a secretary be elected?
18. Suppose the code to a lock consists of three different numbers from the numbers 1 to 20, inclusive.
    a. How many three-number codes are possible?
    b. How many of the codes contain the numbers 6, 13, and 17?
19. A car dealer sells four different models of cars. Each of the cars can come in six different colors. For each of the cars, there are two different option packages available. In how many different ways can you select a car?
20. Teams in a math competition consist of six students. In how many ways can the six students be selected to work a problem on the board?

Name _____ Class _____ Date _____

Practice 12-9

Combinations

Simplify each expression.

1. ${}_9C_4$
2. ${}_{12}C_8$
3. ${}_9C_6$
4. ${}_{15}C_9$
5. ${}_{10}C_8$
6. ${}_{13}C_6$
7. ${}_{18}C_5$
8. ${}_{16}C_3$
9. ${}_{17}C_7$
10. ${}_9C_5$
11. ${}_{17}C_{13}$
12. ${}_{14}C_7$

13. A group of six tourists arrive at the airport 15 min before flight time. At the gate, they learn that only three seats are left on the airplane. How many different groups of three could get on the airplane?

14. In how many ways can you select 5 greeting cards from a choice of 12 cards at a store?

15. A committee of 4 students is to be formed from members of the student council. The student council contains 13 girls and 12 boys.

    a. How many different committees of four students are possible?

    b. How many committees will contain only boys?

    c. What is the probability that the committee will contain only boys?

16. Suppose your math class consists of 24 students. In how many ways can a group of 5 students be selected to form a math team?

17. A jar of marbles contains 6 yellow and 8 red marbles. Three marbles are selected at random.

    a. How many different groups of three marbles are possible?

    b. How many groups of three marbles will contain only red ones?

    c. What is the probability that the group of marbles will contain only red ones?

18. Suppose two members of your class need to be selected as members of the student council. Your class has 26 students in it. How many groups of two students can be selected?

19. The letters of the alphabet are written on slips of paper and placed in a hat. Three letters are selected at random.

    a. How many different combinations of three letters are possible?

    b. How many combinations consist only of the letters A, C, H, I, K, or Y?

    c. What is the probability that the letters selected consist only of the letters A, C, H, I, K, or Y?

20. Three boys and four girls are running for president and vice-president of the student council. What is the probability that a boy will be elected president and a girl will be elected vice-president?

21. A lottery requires that you match three numbers in order. The three numbers are chosen from the numbers 1–20. What is the probability that you will win this lottery if numbers can be chosen only once?

Name _____ Class _____ Date _____

Reteaching 12-1

Inverse Variation

OBJECTIVE: Solving inverse variations **MATERIALS:** None

- The relationship shown by the equation $xy = k$, where $k \neq 0$, is called an inverse variation.
- The two quantities x and y multiplied together result in a constant k. As one quantity increases, the other decreases.

Example

Two young graduates from the business school of the University of Texas decided to open a music store. On the first day of business, they charged $12.00 for a CD. They sold 138 CDs. On the basis of their research, they believe there is an inverse variation between sales and price. If they are correct and the price of the CDs is lowered, the number sold should increase. They decide to lower the price of the CDs to $11.50.
How many CDs can they expect to sell?

Price	No. Sold
$12.00	138
$11.50	x

← **Put the data into a table.**

← **Let x represent the missing quantity.**

(Price 1)(No. sold at price 1) = (Price 2)(No. sold at price 2) ← **Write an equation showing the relationship of the variables.**

$(\$12.00)(138) = (\$11.50)(x)$ ← **Substitute the values.**

$1656 = 11.5x$

$x = 144$ ← **Solve for x.**

Dropping the price by $.50 will result in selling 144 CDs. This is an increase of 6 CDs.

Exercises

Refer to the example to answer each question.

1. How many CDs can the owners expect to sell if they drop the price to $11.00? What if they raise the price to $12.50? (Round your answers to the nearest whole number.)
2. If the owners want to sell 162 CDs, what price should they charge for each?
3. Make a table with the data from Exercises 1 and 2.

Name _____ Class _____ Date _____

Reteaching 12-2

Graphing Rational Functions

OBJECTIVE: Graphing rational functions **MATERIALS:** None

A vertical asymptote occurs at a value for which the function is not defined. The function $y = \frac{2}{x+1}$ is undefined at $x = -1$. Therefore, the vertical asymptote is $x = -1$.

Example

Graph the rational function $y = \frac{4}{x+2} - 1$.

Step 1 Find the vertical asymptotes.

$x + 2 = 0$ ← **Set the denominator equal to zero.**

$x = -2$ ← **Solve for x.**

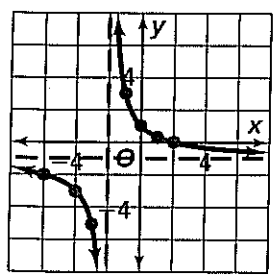
Step 2 Find the horizontal asymptotes.

From the form of the function ($y = \frac{a}{x-b} + c$) we know there is a horizontal asymptote at $y = -1$.

Step 3 Make a table of values using values of x near -2.

x	−6	−4	−3	−1	1	2	0
y	−2	−3	−5	3	$\frac{1}{3}$	0	1

Step 4 Graph the data.



Exercises

Graph each rational function.

1. $y = \frac{4}{x+2}$
2. $y = \frac{2}{x-3} + 2$
3. $y = \frac{2}{x} + 5$
4. $y = \frac{1}{x} - 5$
5. $y = \frac{3}{x} + 2$
6. $y = \frac{2}{x+2} + 1$

Name _____ Class _____ Date _____

Reteaching 12-3

Simplifying Rational Expressions

OBJECTIVE: Simplifying rational expressions **MATERIALS:** None

Example

Simplify $\frac{3x + 6}{2x + 4}$.

$3x + 6 = 3(x + 2)$ ← **Factor the numerator.**

$2x + 4 = 2(x + 2)$ ← **Factor the denominator.**

$= \frac{3(x + 2)}{2(x + 2)}$ ← **Rewrite the expression in terms of the factors.**

$= \frac{3(\cancel{x + 2})}{2(\cancel{x + 2})}$ ← **Mark through common factors in the numerator and denominator. These two factors cancel because any number divided by itself equals 1.**

$= \frac{3}{2}$ ← **Simplify.**

Example

Simplify $\frac{4x - 24}{x^2 - 9x + 18}$.

$4x - 24 = 4(x - 6)$ ← **Factor the numerator.**

$x^2 - 9x + 18 = (x - 6)(x - 3)$ ← **Factor the denominator.**

$= \frac{4(x - 6)}{(x - 6)(x - 3)}$ ← **Rewrite the expression in terms of the factors.**

$= \frac{4(\cancel{x - 6})}{(\cancel{x - 6})(x - 3)}$ ← **Mark through common factors in the numerator and denominator. These two factors cancel because any number divided by itself is 1.**

$= \frac{4}{x - 3}$ ← **Simplify.**

Exercises

Simplify each expression.

1. $\frac{5x - 15}{3x - 9}$
2. $\frac{x + 7}{2x + 14}$
3. $\frac{2x - 2}{x - 1}$
4. $\frac{5x - 20}{x^2 - 16}$
5. $\frac{x^2 - 6x - 16}{x^2 - x - 6}$
6. $\frac{6x^2 + 3x}{2x^2 + 11x + 5}$

Name _____ Class _____ Date _____

Reteaching 12-4

Multiplying and Dividing Rational Expressions

OBJECTIVE: Multiplying and dividing rational expressions

MATERIALS: None

When multiplying rational expressions, look for common factors.

Example

Multiply $\frac{3x-6}{5x-20} \cdot \frac{10x-40}{27x-54}$.

$$\frac{3x-6}{5x-20} \cdot \frac{10x-40}{27x-54} = \frac{3(x-2)}{5(x-4)} \cdot \frac{10(x-4)}{27(x-2)}$$ ← **Factor each expression.**

$$= \frac{3(x-2)}{5(x-4)} \cdot \frac{10^2(x-4)}{27_9(x-2)}$$ ← **Divide out common factors and reduce fractions.**

$$= \frac{2}{9}$$ ← **Simplify.**

When dividing rational expressions, multiply by the reciprocal. The reciprocal of a fraction is the fraction with the numerator and denominator interchanged.

Example

Divide $\frac{x^2+x}{3x-15} \div \frac{x^2+2x+1}{6x-30}$.

$$\frac{x^2+x}{3x-15} \div \frac{x^2+2x+1}{6x-30} = \frac{x^2+x}{3x-15} \cdot \frac{6x-30}{x^2+2x+1}$$ ← **Multiply by the reciprocal.**

$$= \frac{x(x+1)}{3(x-5)} \cdot \frac{6(x-5)}{(x+1)(x+1)}$$ ← **Factor the numerators and denominators.**

$$= \frac{x(x+1)}{3(x-5)} \cdot \frac{6^2(x-5)}{(x+1)(x+1)}$$ ← **Divide out common factors.**

$$= \frac{2x}{x+1}$$ ← **Simplify.**

Exercises

Simplify.

1. $\frac{x^2-x}{2x+4} \cdot \frac{x+2}{x}$

2. $\frac{x^2+x}{x^2+8x+7} \cdot (x+7)$

3. $\frac{x^2-1}{x^2+4x+3} \div \frac{x-1}{x^2+2x-3}$

4. $\frac{x^2-9}{5x+15} \div \frac{x-3}{x+3}$

5. $\frac{x^2-x-30}{6x-36} \div \frac{5x+25}{x}$

6. $\frac{x^2-9}{x^2+4x-12} \div \frac{x^2+2x-3}{x^2+5x-6}$

Name _____ Class _____ Date _____

Reteaching 12-5

Dividing Polynomials

OBJECTIVE: Dividing polynomials

MATERIALS: None

The procedure for dividing two polynomials is similar to the one for dividing whole numbers.

If the dividend or the divisor has missing terms, remember to insert these terms with zero coefficients.

Example

$(x^2 - 5x + 8) \div (x - 3)$

$$\begin{array}{r} x \\ x - 3\overline{)x^2 - 5x + 8} \\ \underline{x^2 - 3x} \\ -2x + 8 \end{array}$$

← **Think** $x\overline{)x^2} = \frac{x^2}{x} = x$.

← **Multiply** $x(x - 3) = x^2 - 3x$.

← **Subtract** $(x^2 - 5x) - (x^2 - 3x) = -2x$, **and bring down the 8.**

Repeat the process.

$$\begin{array}{r} x - 2 \\ x - 3\overline{)x^2 - 5x + 8} \\ \underline{x^2 - 3x} \\ -2x + 8 \\ \underline{-2x + 6} \\ 2 \end{array}$$

← **Think** $x\overline{)-2x} = \frac{-2x}{x} = -2$.

← **Multiply** $-2(x - 3) = -2x + 6$.

← **Subtract** $(-2x + 8) - (-2x + 6) = 2$. **The remainder is 2.**

The answer is $x - 2 + \frac{2}{x - 3}$.

Exercises

Divide.

1. $(x^2 + 5x + 6) \div (x + 3)$
2. $(2x^2 + 5x - 1) \div (2x - 1)$
3. $(x^3 - 8) \div (x + 2)$
4. $(x^3 - 2x + 1) \div (x - 1)$
5. $(x^2 - 8x + 16) \div (x - 4)$
6. $(6x^2 + 42x + 60) \div (x + 4)$
7. $(2x^2 - 2x - 24) \div (x + 3)$
8. $(2x^3 + 17x^2 + 38x + 15) \div (x + 5)$
9. $(x^3 + 7x^2 + 8x - 16) \div (x - 2)$
10. $(4x^3 + 22x^2 + 36x + 18) \div (x + 3)$

Name _____ Class _____ Date _____

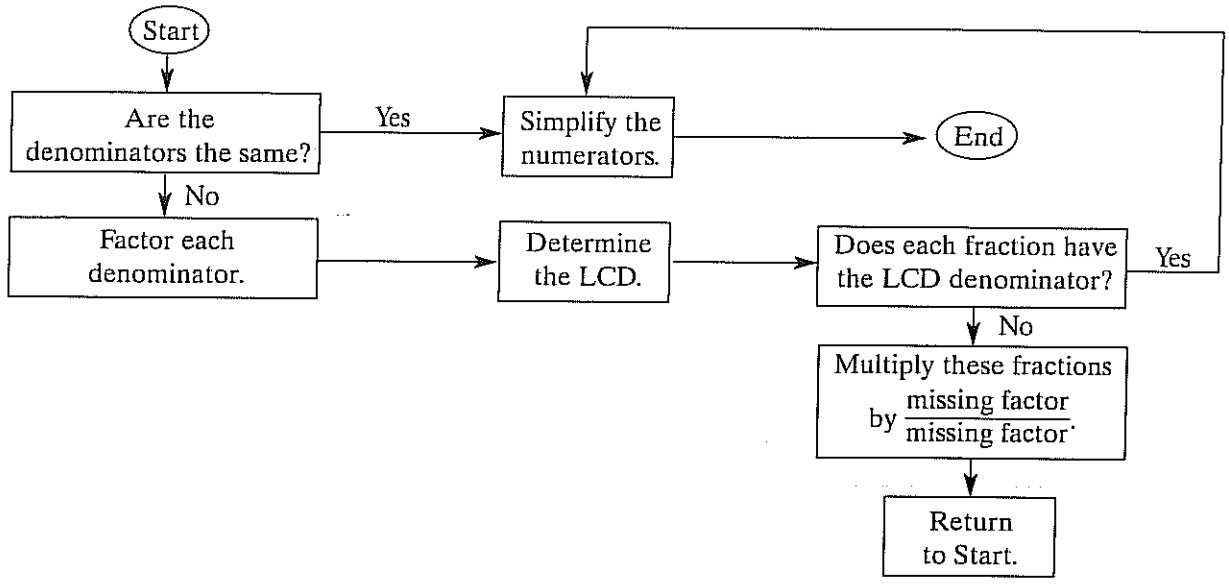
Reteaching 12-6

Adding and Subtracting Rational Expressions

OBJECTIVE: Adding and subtracting rational expressions

MATERIALS: None

Use the flowchart to add and subtract rational expressions.



Example

Simplify $\frac{2x}{x^2 - 1} + \frac{3}{x - 1}$ by following the flowchart.

$x^2 - 1 = (x - 1)(x + 1)$ ← **Factor each denominator. If a denominator is already simplified, rewrite it in a set of parentheses.**

$x - 1 = (x - 1)$

$(x - 1)(x + 1)$ ← **Determine the LCD.**

$\frac{3}{x - 1} \cdot \frac{(x + 1)}{(x + 1)} = \frac{3x + 3}{x^2 - 1}$ ← **The first fraction already has the LCD for a denominator, but the second one does not. Multiply the second fraction by the factor it is missing.**

$\frac{2x}{x^2 - 1} + \frac{3x + 3}{x^2 - 1} = \frac{2x + 3x + 3}{x^2 - 1}$ ← **Simplify the numerators.**

$= \frac{5x + 3}{x^2 - 1}$

Exercises

Simplify the following using the flowchart.

1. $\frac{x + 1}{x^2 - 4} + 3$
2. $\frac{2x - 5}{x^2 + 3x + 2} + \frac{4}{x + 2}$
3. $\frac{3z + 2}{16 - z^2} + \frac{3}{z - 4}$
4. $\frac{x + 1}{x + 5} - \frac{5}{x^2 + 6x + 5}$
5. $\frac{1}{x^2 - 9} + \frac{4}{x + 3}$
6. $\frac{x + 1}{x + 3x + 2} + \frac{2}{x + 2}$

Name _____ Class _____ Date _____

Reteaching 12-7

Solving Rational Equations

OBJECTIVE: Solving equations involving rational expressions

MATERIALS: None

Example

Solve the equation $\frac{4}{3x} + \frac{3}{4x} = \frac{5}{2x^2}$.

Step 1 Find the LCD.

$3x = \textcircled{3} \cdot x$ ← **Factor the denominators. Find where each factor appears the most times and circle it.**

$4x = \textcircled{2 \cdot 2} \cdot x$

$2x^2 = 2 \cdot \textcircled{x \cdot x}$

$3 \cdot 2 \cdot 2 \cdot x \cdot x = 12x^2$ ← **Multiply the circled terms to find the LCD.**

Step 2 Solve the equation.

$12x^2\left(\frac{4}{3x}\right) + 12x^2\left(\frac{3}{4x}\right) = 12x^2\left(\frac{5}{2x^2}\right)$ ← **Multiply each term by the LCD.**

$16x + 9 = 30$ ← **Simplify.**

$25x = 30$ ← **Solve.**

$x = \frac{6}{5}$

Exercises

Solve each equation.

1. $\frac{3}{2x} + \frac{5}{6x} = \frac{4}{5x^2}$
2. $\frac{2}{3x} + \frac{4}{5} = \frac{3}{2x}$
3. $\frac{6x}{5} - \frac{1}{2} = \frac{2x}{3}$
4. $\frac{2}{5x} - \frac{5}{2x} = \frac{3}{5x^2}$
5. $\frac{3}{y-3} = \frac{3}{y^2-9}$
6. $\frac{5}{2x-2} = \frac{15}{x^2-1}$
7. $\frac{1}{m-1} = \frac{3}{m^2-1}$
8. $\frac{x}{x-2} = \frac{3x}{x+2}$
9. $\frac{4}{x} + 1 = \frac{6}{x}$
10. $\frac{x+2}{3} = x - 2$
11. $\frac{5}{x} - \frac{4}{x} = 8 + \frac{1}{x}$
12. $\frac{11}{x} + \frac{13}{x} = 12$
13. $\frac{x}{2x} + \frac{2}{4x} = \frac{5x}{x}$
14. $\frac{9}{x} + \frac{6}{5x} = \frac{6}{2x^2}$
15. $\frac{5}{3x} - \frac{x}{x^2} = \frac{1}{6x^2}$
16. $\frac{x}{x+1} + 2 = 5$
17. $\frac{2}{x-5} + 1 = \frac{5}{x-5}$
18. $\frac{21}{x^2} - \frac{10}{x} = \frac{15}{x^2}$
19. $\frac{3}{2x} - \frac{2}{x^2} = \frac{2}{x}$
20. $\frac{3}{x^2-9} = \frac{2}{x-3}$
21. $\frac{5}{x^2-4} + \frac{2}{x-2} = \frac{3}{x+2}$

Name _____ Class _____ Date _____

Reteaching 12-8

Counting Methods and Permutations

OBJECTIVE: Using permutations to count outcomes

MATERIALS: Calculator

Think of the Multiplication Counting Principle as choices · choices · choices . . .

Example

You are hosting a New Year's party at which a total of seven people are present. You decide to distribute gag gifts in the following way: Guests will pick a number from a jar and will open a gift in order of their numbers. In how many possible ways can the gifts be distributed?

The number of possible ways in which the gifts can be distributed is $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 7! = 5040$

Person	No. of Choices
1	7
2	6
3	5
4	4
5	3
6	2
7	1

The next year, you decide to make your party more interesting. Now, Person 1 will open a present. Person 2 will choose between opening a new present or taking the present that person 1 opened. If Person 2 takes Person 1's gift, Person 1 gets to open another. Person 3 can open a new present or can take an already opened gift. In how many ways can the gifts be distributed now?

Person	No. of Choices
1	7
2	7
3	7
4	7
5	7
6	7
7	7

Now each person will have seven gifts to choose from, counting both opened and unopened gifts. The number of possible distributions of gifts is now

$7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 = 7^7 = 823{,}543.$

Exercises

For each situation, make a table and calculate the number of possible gift distributions.

1. There are 11 guests. Each chooses in turn from the unopened gifts.
2. There are 11 guests. Each chooses any gift, opened or unopened.
3. There are 14 guests. Each chooses in turn from the unopened gifts.
4. There are 14 guests. Each chooses any gift, opened or unopened.

Name _____ Class _____ Date _____

Reteaching 12-9

Combinations

OBJECTIVE: Finding combinations

MATERIALS: Calculator

The expression ${}_nC_r$ represents the number of combinations of n objects arranged r at a time. Your calculator allows you to calculate ${}_nC_r$ quickly.

Example

You have 12 CDs in your collection. You select 4 CDs at random to take to a party. How many different sets of CDs could you select?

${}_{12}C_4$ ← **Write the expression.**

= 12 MATH ◀ ▼ ▼ ENTER 4 ENTER ← **Use these calculator keystrokes to find ${}_{12}C_4$.**

= 495 ← **Write the answer.**

There are 495 different combinations possible.

Exercises

Use a calculator to solve each combination problem.

1. You are hosting a dinner party. You are making a salad for each guest from lettuce and three other ingredients. The other ingredients that the guests may choose from are tomatoes, cucumbers, mushrooms, croutons, and bacon bits. How many different salads can be made?

2. You have 12 players on your volleyball team. How many different combinations of 6 players can the coach choose?

3. There are 11 girls and 9 boys trying out for the cheerleading squad at the high school. The squad will contain 5 boys and 5 girls.

   a. How many different combinations of girls are possible?

   b. How many different combinations of boys are possible?

4. You have a list of 20 errands to do by the end of the day. By 8:00 P.M., you have completed 6 of the 20 errands. How many different combinations of errands could you have completed?

5. Five students are running for student council. Only three students can be elected. How many different combinations of students are possible?

Chapter 12 Answers

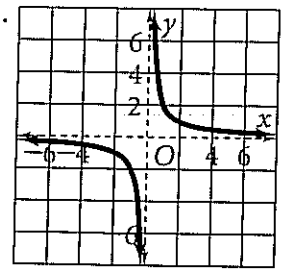
Practice 12-1

1. $xy = 54$ **2.** $xy = 18$ **3.** $xy = \frac{1}{6}$ **4.** $xy = 91$ **5.** $xy = 72$
6. $xy = 3.92$ **7.** $xy = 484$ **8.** $xy = 76$ **9.** $xy = 10$ **10.** 10
11. 8 **12.** 16 **13.** 2 **14.** $37\frac{1}{3}$ **15.** 625 **16.** 4 **17.** $\frac{6}{25}$ **18.** 2
19. 3.6 **20.** 9.3 **21.** 2.3 **22.** 6 **23.** 3 **24.** 16
25. Inverse **26.** Direct **27.** Inverse variation; $xy = 70$
28. Inverse variation; $xy = 48$ **29.** Direct variation; $y = 11x$
30. Direct variation; $y = 4.5x$ **31.** Inverse variation; $xy = 36$
32. Direct variation; $y = 2.1x$ **33.** Direct variation; $y = 6.4x$
34. Inverse variation; $xy = 6$ **35.** Inverse variation; $xy = 18$
36a. 5 m^3 **36b.** 2 atm **36c.** 19.2 m^3 **37a.** 1.5 h **37b.** 192 ft/s

Practice 12-2

1. Parabola **2.** Rational function **3.** Rational function
4. Radical function **5.** Line **6.** Parabola **7.** Absolute value
8. Exponential growth **9.** Line **10a.** 62.5 ohms **10b.** 2.5 ohms
10c. 10 ohms **11a.** 3000 lumens **11b.** 187.5 lumens
11c. 19.2 lumens **12a.** 14.4 m **12b.** 2.5 m **13.** 4 **14.** 0
15. -7 **16.** 2.5 **17.** -3 **18.** 8 **19.** $\frac{6}{5}$ **20.** $-\frac{5}{3}$

21.



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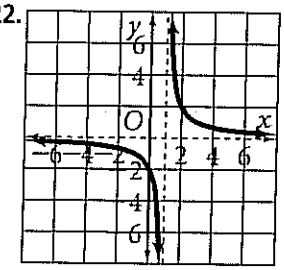

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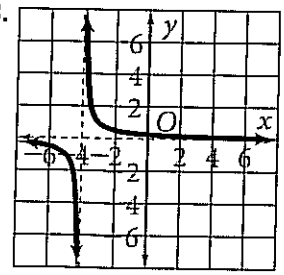

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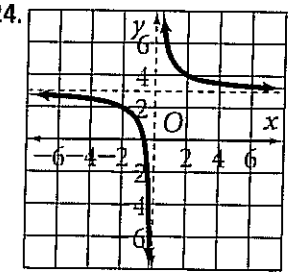

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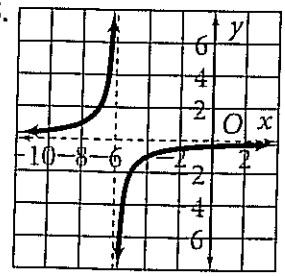

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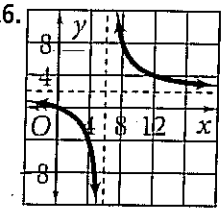

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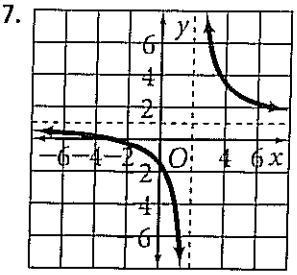

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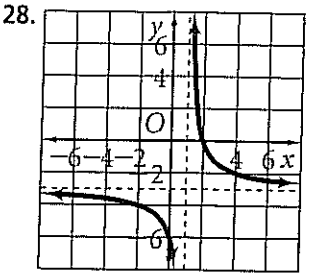


Practice 12-3

1. $\frac{x^2}{3}$ **2.** $\frac{3}{5a^2}$ **3.** $\frac{2h}{3}$ **4.** $\frac{4}{7n^2}$ **5.** $\frac{x-2}{2}$ **6.** $x - 2$
7. $2t - 1$ **8.** $\frac{a}{2}$ **9.** $\frac{3x}{2y}$ **10.** $\frac{4x^2}{3y^2}$ **11.** $\frac{x}{3}$ **12.** $\frac{x}{5}$ **13.** $\frac{x+1}{x-12}$
14. $\frac{x+3}{x^2}$ **15.** x^2 **16.** -1 **17.** $\frac{x+3}{x+1}$ **18.** $\frac{x+2}{x^2}$
19. $\frac{2(x+2)}{x-1}$ **20.** $\frac{2x-3}{x+1}$ **21.** $\frac{3}{x}$ **22.** $-\frac{x+2}{x+1}$
23. $-\frac{x+3}{x+4}$ **24.** $\frac{x+5}{x-4}$ **25.** $\frac{x+8}{x+7}$ **26.** $-\frac{x-2}{x-5}$
27. $\frac{5}{2w}$ **28.** $2r + 1$ **29.** $3n$ **30.** $s + 2$

Practice 12-4

1. $\frac{2}{9}$ **2.** $\frac{9}{2}$ **3.** $-\frac{63}{64}$ **4.** $-\frac{1}{15}$ **5.** $\frac{4}{m^2}$ **6.** $\frac{4x^2}{9}$ **7.** $\frac{5}{9}$ **8.** $\frac{3x^2}{20}$
9. $\frac{10x^4}{27}$ **10.** $\frac{5r}{8}$ **11.** $\frac{6n}{5}$ **12.** $\frac{2}{5x^2}$ **13.** $\frac{n^2}{3}$ **14.** $\frac{4r^3}{5}$
15. $3a - 6$ **16.** $\frac{24}{5b}$ **17.** $\frac{4}{b}$ **18.** 2 **19.** $\frac{y^2}{3}$ **20.** $12p^2$
21. 3 **22.** a **23.** $4h$ **24.** $n^2 - 3n + 2$ **25.** $x - 2$
26. $\frac{5}{x+2}$ **27.** $\frac{x}{3}$ **28.** 1 **29.** $\frac{x+3}{x+1}$ **30.** $\frac{x-5}{x+3}$
31. $\frac{3x+2}{3x+1}$ **32.** $-\frac{x+6}{x+5}$ **33.** $\frac{x^2+5x+6}{3x+9}$
34. $\frac{2x+1}{2x-3}$ **35.** $\frac{2x-14}{x+9}$ **36.** $\frac{x-3}{x-5}$ **37.** $-\frac{3x+1}{2x-3}$
38. $\frac{x+5}{x-12}$ **39.** $\frac{4x+3}{2x-1}$ **40.** $\frac{9x^2}{14x^2+96x+16}$

Practice 12-5

1. $2x - 5$ **2.** $4x^2 - 3$ **3.** $x - 2$ **4.** $5x - 3$
5. $-x^3 + 2x^2 - 3x + 4$ **6.** $3x - 6$ **7.** $x - 7$ **8.** $2x - 7$
9. $2x - 4$ **10.** $x + 6$ **11.** $x + 8$ **12.** $4x + 1$
13. $x + 3 + \frac{4}{x+2}$ **14.** $x - 5 - \frac{24}{x-3}$
15. $3x + 4 - \frac{5}{x-2}$ **16.** $x^2 + 2x + 3$ **17.** $3x - 5$
18. $2x + 9 + \frac{33}{3x-1}$ **19.** $2x - 1 + \frac{1}{x+6}$
20. $x + 3 - \frac{16}{x+2}$ **21.** $2x + 5 + \frac{8}{2x-1}$
22. $x + 4$ **23.** $x^2 + 1$ **24.** $5x + 3 + \frac{1}{2x+3}$
25. $2x - 9$ **26.** $x^2 + 5x + 2 + \frac{7}{x-7}$ **27.** $3x - 2$
28. $x^2 + x - 6$

Chapter 12 Answers

Practice 12-6

1. $\frac{x}{2}$ 2. $\frac{8}{x}$ 3. $\frac{x}{6}$ 4. $\frac{8x}{15}$ 5. $\frac{7m}{6}$ 6. $\frac{5x}{14}$ 7. $\frac{3}{7t}$ 8. $\frac{5d}{3}$ 9. $\frac{2}{d}$
10. $\frac{9 + 8d}{6d^2}$ 11. $\frac{3m - 15}{(m + 1)(m - 1)}$ 12. $-\frac{4}{x}$ 13. $\frac{4a}{3}$
14. $-\frac{4}{k + 3}$ 15. $\frac{5}{2z^2}$ 16. $\frac{7x + 13}{x^2 - 1}$ 17. $\frac{-x + 3}{x^2 - 1}$ 18. $\frac{3t}{4}$
19. $\frac{8a - 3}{6a^3}$ 20. $\frac{10}{a + 4}$ 21. $\frac{10x + 10}{(x + 3)(x - 2)}$
22. $\frac{18 - 56t^2}{21t^3}$ 23. $\frac{13}{6(x + 3)}$ 24. $\frac{1}{4a}$ 25. $\frac{7r - 9}{r^2 - 4}$
26. $\frac{15}{a^2 - 2}$ 27. x 28. $-\frac{1}{6(x + 2)}$ 29. $\frac{c + 5}{(c + 3)(c + 1)}$
30. $\frac{-4x + 10}{(x - 2)(x - 1)}$ 31a. $\frac{10}{r}$ 31b. $\frac{5}{6}$ h or 50 min
32a. $\frac{21}{5r}$ 32b. 2.1 h or 2h 6 min 32c. 1.4 h or 1 h 24 min

Practice 12-7

1. 9 2. -6 3. $-1, 1$ 4. No solution 5. $-1, 4$ 6. 4
7. 0, 12 8. $-6, -1$ 9. -2 10. $-6, 3$ 11. $-2, 0.5$
12. No solution 13. no solution 14. No solution 15. 3
16. No solution 17. $-5, 5$ 18. 2 19. $4, -1$ 20. -1 21. -8
22. 1.71 days 23. Machine X, 30 min; Machine Y, 60 min
24. 3.75 h 25. Joseph, 10 km/h; Vincent, 22 km/h 26. 90 min

Practice 12-8

1. 42 2. 665,280 3. 990 4. 720 5. 362,880 6. 3,991,680
7. 390,700,800 8. 2730 9. 43,680 10. 13,800 11. 272 12. 210
13a. 7,893,600 13b. 120 13c. $\frac{1}{65780}$
14. 362,880 15. 12 16. 7500 17. 12,144
18a. 6840 18b. 6 19. 48 20. 720

Practice 12-9

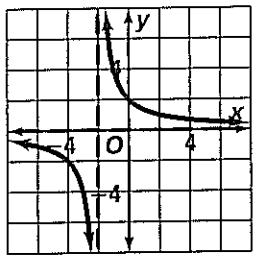
1. 126 2. 495 3. 84 4. 5005 5. 45 6. 1716 7. 8568
8. 560 9. 19,448 10. 126 11. 2380 12. 3432 13. 20
14. 792 15a. 12,650 15b. 495 15c. 0.039 16. 42,504
17a. 364 17b. 56 17c. $\frac{2}{13}$ 18. 325 19a. 2600
19b. 20 19c. $\frac{1}{130}$ 20. $\frac{2}{7}$ 21. $\frac{1}{6840}$

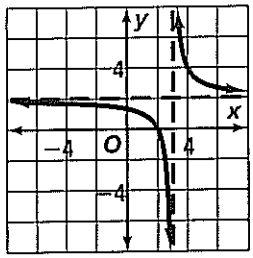
Reteaching 12-1

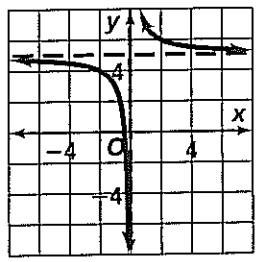
1. 151; 132 2. \$10.22
3.

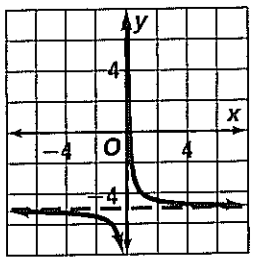
Price	No. Sold
\$12.50	132
\$12.00	138
\$11.50	144
\$11.00	151
\$10.22	162

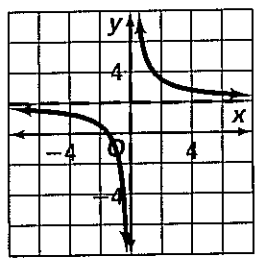
Reteaching 12-2

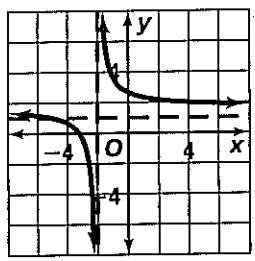
1. 

2. 

3. 

4. 

5. 

6. 

Reteaching 12-3

1. $\frac{5}{3}$ 2. $\frac{1}{2}$ 3. 2 4. $\frac{5}{x + 4}$ 5. $\frac{x - 8}{x - 3}$ 6. $\frac{3x}{x + 5}$

Reteaching 12-4

1. $\frac{x - 1}{2}$ 2. x 3. $x - 1$ 4. $\frac{x + 3}{5}$ 5. $\frac{x}{30}$ 6. $\frac{x - 3}{x - 2}$

Reteaching 12-5

1. $x + 2$ 2. $x + 3 + \frac{2}{2x - 1}$ 3. $x^2 - 2x + 4 - \frac{16}{x + 2}$
4. $x^2 + x - 1$ 5. $x - 4$ 6. $6x + 18 - \frac{12}{x + 4}$
7. $2x - 8$ 8. $2x^2 + 7x + 3$ 9. $x^2 + 9x + 26 + \frac{36}{x - 2}$
10. $4x^2 + 10x + 6$

Reteaching 12-6

1. $\frac{3x^2 + x - 11}{(x + 2)(x - 2)}$ 2. $\frac{6x - 1}{(x + 1)(x + 2)}$
3. $\frac{-10}{(z - 4)(-z - 4)}$ 4. $\frac{x^2 + 2x - 4}{(x + 5)(x + 1)}$
5. $\frac{4x - 11}{(x - 3)(x + 3)}$ 6. $\frac{3}{x + 2}$

Reteaching 12-7

1. $\frac{12}{35}$ 2. $\frac{25}{24}$ 3. $\frac{15}{16}$ 4. $-\frac{2}{7}$ 5. -2 6. 5 7. 2 8. 0, 4 9. 2

10. 4 11. No solution 12. 2 13. $\frac{1}{9}$ 14. $\frac{5}{17}$ 15. $\frac{1}{4}$

16. $-\frac{3}{2}$ 17. 8 18. $\frac{3}{5}$ 19. -4 20. $-\frac{3}{2}$ 21. 15

Reteaching 12-8

1. 11! 2. 11^{11} 3. 14! 4. 14^{14}

Reteaching 12-9

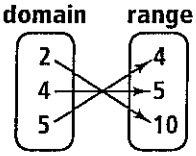
1. 10 2. 924 3a. 462 3b. 126 4. 38,760 5. 10

Enrichment 12-1

1. Answers may vary. Sample: (2, 10), (4, 5), (5, 4)

2. Answers may vary. Sample:

x	y
2	10
4	5
5	4



3. Yes; For every point, $xy = -12$ and $xy = 12$.
4. Answers may vary. Sample:

x	$f(x)$	x	$g(x)$	x	$h(x)$	x	$i(x)$
2	6	2	−6	−2	−6	−2	6
3	4	3	−4	−3	−4	−3	4
4	3	4	−3	−4	−3	−4	3

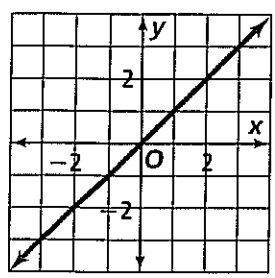
$f(x) = \frac{12}{x}$ for $x > 0$, $g(x) = -\frac{12}{x}$ for $x > 0$,
$h(x) = \frac{12}{x}$ for $x < 0$, $i(x) = -\frac{12}{x}$ for $x < 0$

Enrichment 12-2

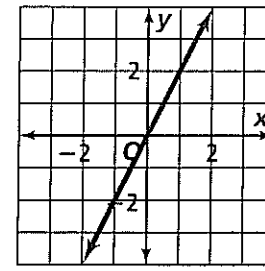
1. 100 m 2. There are two contour lines for every 200 m change.
3. C–7; 500 m 4. Begins at 800 m, flows downhill to 100m.
5. B–1; 500 m 6. Fox Hill

Enrichment 12-3

1. y-values for chart: -4, $-3\frac{1}{2}$, none, $-2\frac{1}{2}$, -2, 0, 2



2. y-values for chart: 4, 3, none, 1, 0, -2, -4



3. $x = 0$ 4. As the positive values of x get closer to 0, y becomes greater; as the negative values of x get closer to 0, y becomes smaller.

Enrichment 12-4

1. $\frac{a+2}{a-3} \div (a+2)$ 2. $\left(\frac{2t-6}{t+1} \div 2\right) - t$

3. $\frac{x^2 - 2x + 1}{3x^2 + 9x + 6} \div (x - 1)$

4. $\left(\frac{x^2 + 2x + 1}{2x^2 + 4x + 2} \div x\right) - 1$

5. $\left(\frac{9a - 3}{a + 4} \div 3\right) - a$ 6. $(4x^2 \div 2x) + 1$

7. $\left(\frac{16c^2 - 64}{c - 2} \div 4c\right) + 8 = \frac{4(c+2)}{c} + 8$ or

$\frac{16c^2 - 64}{c - 2} \div (4c + 8) = 4$

8. $(10n \div 5n) + 25 = 27$ or $10n \div (5n + 25) = \frac{2n}{n+5}$

9. Answers may vary. Sample: $a + b(c)$; $(a + b)c$.

Enrichment 12-5

1. 16, 8, 4, 2, 1; 128, 256, 512, 1024, 2048; product: 2048
2. Odd numbers that are halved discard the remainder of 1. At the end of the process, add all numbers in doubling column that appear opposite an odd number in the halving column. $13 + 26 + 208 = 247$ 3. Check students' work.

Enrichment 12-6

1. $\frac{1}{4} + \frac{1}{8}$ 2. $\frac{1}{5} + \frac{1}{3}$ 3. $\frac{1}{2} + \frac{1}{8}$ 4. $\frac{1}{5} + \frac{1}{10}$ 5. $\frac{1}{3} + \frac{1}{12}$

6. $\frac{2}{3} + \frac{1}{4}$ 7. $\frac{2}{3} + \frac{1}{15}$ 8. $\frac{2}{3} + \frac{1}{6}$ 9. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$

10. $\frac{1}{2} + \frac{1}{5}$ 11. $\frac{1}{2} + \frac{1}{12}$ 12. $\frac{1}{2} + \frac{1}{7}$ 13. $\frac{1}{2} + \frac{1}{4} + \frac{1}{16}$

14. $\frac{1}{4} + \frac{1}{8} + \frac{1}{16}$ 15. $\frac{1}{6} + \frac{1}{9}$ 16. $\frac{1}{4} + \frac{1}{16}$

17. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$ 18. $\frac{1}{2} + \frac{1}{9}$

19. $\frac{11}{12} = \frac{1}{2} + \frac{1}{4} + \frac{1}{6}$
$\frac{11}{15} = \frac{1}{2} + \frac{1}{6} + \frac{1}{15}$ or
$\frac{11}{15} = \frac{1}{3} + \frac{1}{3} + \frac{1}{15}$
$\frac{5}{6} = \frac{1}{2} + \frac{1}{3}$