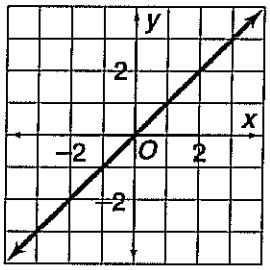
Quadratic Equations and Functions

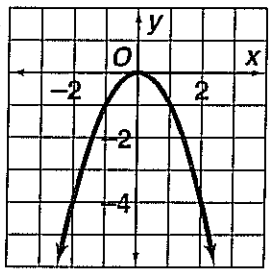
DIAGNOSING READINESS *page 508*

1. $2a - b^2 + c$ for $a = -1, b = 3, c = -2$: $2(-1) - 3^2 + (-2) = -2 - 9 - 2 = -13$ **2.** $\frac{c^2 - ab}{2a}$ for $a = -1, b = 3, c = -2$: $\frac{(-2)^2 - (-1)(3)}{2(-1)} = \frac{4 - (-3)}{-2} = -\frac{7}{2}$ **3.** $bc - 3a^2$ for $a = -1, b = 3, c = -2$: $3(-2) - 3(-1)^2 = -6 - 3(1) = -6 - 3 = -9$ **4.** $\frac{b^2 - 4ac}{2a}$ for $a = -1, b = 3, c = -2$: $\frac{3^2 - 4(-1)(-2)}{2(-1)} = \frac{9 - 8}{-2} = -\frac{1}{2}$ **5.** $5a + 2b(c - 1)$ for $a = -1, b = 3, c = -2$: $5(-1) + 2(3)(-2 - 1) = -5 + 6(-3) = -5 - 18 = -23$ **6.** $c^2 + 2ab - 1$ for $a = -1, b = 3, c = -2$: $(-2)^2 + 2(-1)(3) - 1 = 4 - 6 - 1 = -3$ **7.** $f(x) = -3x^2$; $f(-6) = -3(-6)^2 = -3(36) = -108$ **8.** $y = x^2 - 10$ for $x = -6$: $y = (-6)^2 - 10 = 36 - 10 = 26$ **9.** $h(x) = x^2 + 6x$; $h(-6) = (-6)^2 + 6(-6) = 36 - 36 = 0$ **10.** $y = (x - 1)^2$ for $x = -6$: $y = (-6 - 1)^2 = (-7)^2 = 49$ **11.** $y = 5 - 2x^2$ for $x = -6$: $5 - 2(-6)^2 = 5 - 2(36) = 5 - 72 = -67$ **12.** $y = (1 + x)^2$ for $x = -6$: $y = (1 + (-6))^2 = (-5)^2 = 25$ **13.** $g(x) = \frac{2}{3}x^2$; $g(-6) = \frac{2}{3}(-6)^2 = \frac{2}{3}(36) = 24$ **14.** $y = (2x)^2$ for $x = -6$: $y = (2(-6))^2 = (-12)^2 = 144$

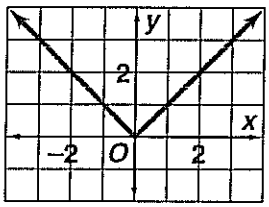
15. $y = x$



16. $y = -x^2$



17. $y = |x|$



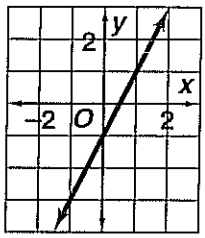
18. $(x + 2)(x - 3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$ **19.** $(2y + 1)(2y + 3) = (2y)^2 + 3(2y) + 1(2y) + 3 = 4y^2 + 6y + 2y + 3 = 4y^2 + 8y + 3$ **20.** $(3x - 7)(x + 4) = 3x^2 + 4(3x) - 7x - 28 = 3x^2 + 12x - 7x - 28 = 3x^2 + 5x - 28$ **21.** $4x^2 + 4x + 1 = (2x)^2 + 2(2x)(1) + 1^2 = (2x + 1)^2$ **22.** $5x^2 + 32x - 21 = (5x - 3)(x + 7)$ **23.** $8x^2 - 10x + 3 = (4x - 3)(2x - 1)$ **24.** $m^2 - 7m - 18 = (m - 9)(m + 2)$ **25.** $12y^2 + 8y - 15 = (6y - 5)(2y + 3)$ **26.** $x^2 - 18x + 81 = x^2 - 2(9)x + 9^2 = (x - 9)^2$

10-1 Exploring Quadratic Graphs *pages 510–516*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1. −24 **2.** 18 **3.** 12 **4.** 35

5.



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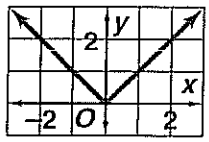

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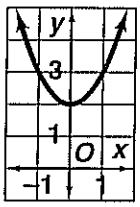
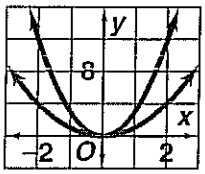


Investigation 1. $y = x^2$, $y = 3x^2$



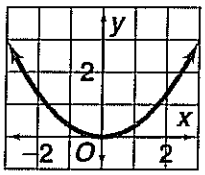
2–4. Answers may vary. Samples are given.

2a. Both graphs are U-shaped with the lowest points at the origin.

2b. $y = 3x^2$ is narrower than $y = x^2$.

3. $y = \frac{1}{3}x^2$ will be U-shaped with its lowest point at the origin. It will be wider than $y = x^2$.

4.

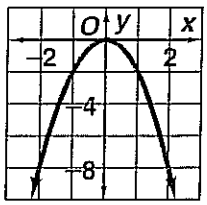


Yes; the graph is wider than $y = x^2$.

Check Understanding 1a. The vertex is (4, 3); it is a maximum. **1b.** The vertex is (−3, −3); it is a minimum.

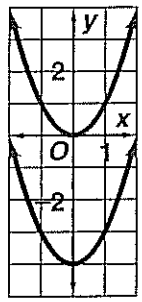
2.

x	$f(x) = -2x^2$	(x, y)
0	$-2(0)^2 = 0$	(0, 0)
1	$-2(1)^2 = -2$	(1, −2)
2	$-2(2)^2 = -8$	(2, −8)



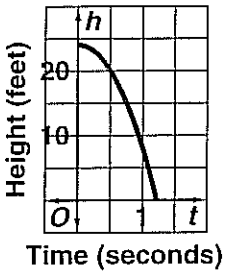
3. widest to narrowest graph: $y = \frac{1}{2}x^2$, $y = x^2$, $y = -2x^2$

4a. $y = x^2$, $y = x^2 - 4$
The graph of $y = x^2 - 4$ has the same shape as the graph of $y = x^2$, but it is shifted down 4 units.



4b. Positive values of c shift the vertex up. Negative values of c shift the vertex down.

5a. $h = -16t^2 + 24$

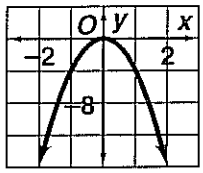


5b. Time must be nonnegative.

Exercises **1.** The vertex is $(2, 5)$; it is a maximum.
2. The vertex is $(-3, -2)$; it is a minimum.
3. The vertex is $(2, 1)$; it is a minimum.

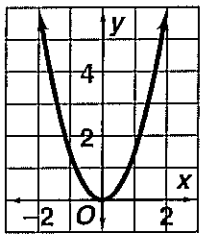
4.

x	$y = -4x^2$	(x, y)
0	$-4(0)^2 = 0$	$(0, 0)$
1	$-4(1)^2 = -4$	$(1, -4)$
2	$-4(2)^2 = -16$	$(2, -16)$



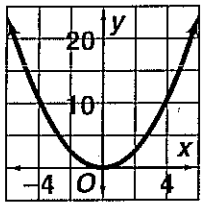
5.

x	$f(x) = 1.5x^2$	(x, y)
0	$1.5(0)^2 = 0$	$(0, 0)$
1	$1.5(1)^2 = 1.5$	$(1, 1.5)$
2	$1.5(2)^2 = 6$	$(2, 6)$



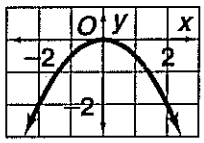
6.

x	$y = \frac{2}{3}x^2$	(x, y)
0	$\frac{2}{3}(0)^2 = 0$	$(0, 0)$
2	$\frac{2}{3}(2)^2 = \frac{8}{3}$	$\left(2, \frac{8}{3}\right)$
4	$\frac{2}{3}(4)^2 = \frac{32}{3}$	$\left(4, \frac{32}{3}\right)$
6	$\frac{2}{3}(6)^2 = 24$	$(6, 24)$



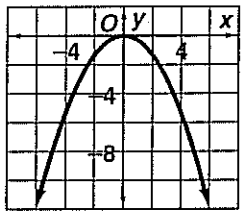
7.

x	$f(x) = -\frac{1}{2}x^2$	(x, y)
0	$-\frac{1}{2}(0)^2 = 0$	$(0, 0)$
1	$-\frac{1}{2}(1)^2 = -\frac{1}{2}$	$\left(1, -\frac{1}{2}\right)$
2	$-\frac{1}{2}(2)^2 = -2$	$(2, -2)$



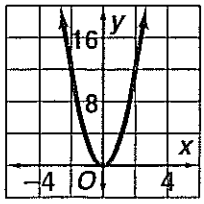
8.

x	$y = -\frac{1}{3}x^2$	(x, y)
0	$-\frac{1}{3}(0)^2 = 0$	$(0, 0)$
2	$-\frac{1}{3}(2)^2 = -\frac{4}{3}$	$\left(2, -\frac{4}{3}\right)$
4	$-\frac{1}{3}(4)^2 = -\frac{16}{3}$	$\left(4, -\frac{16}{3}\right)$
6	$-\frac{1}{3}(6)^2 = -12$	$(6, -12)$



9.

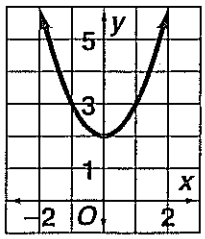
x	$f(x) = 3x^2$	(x, y)
0	$3(0)^2 = 0$	$(0, 0)$
1	$3(1)^2 = 3$	$(1, 3)$
2	$3(2)^2 = 12$	$(2, 12)$



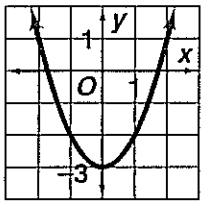
10. widest to narrowest graph: $y = \frac{1}{2}x^2$, $y = 3x^2$, $y = 4x^2$

11. widest to narrowest graph: $f(x) = \frac{1}{3}x^2$, $f(x) = x^2$, $f(x) = 5x^2$ **12.** widest to narrowest graph: $y = -\frac{1}{4}x^2$, $y = -\frac{1}{2}x^2$, $y = 5x^2$ **13.** widest to narrowest graph: $f(x) = -\frac{2}{3}x^2$, $f(x) = -2x^2$, $f(x) = -4x^2$

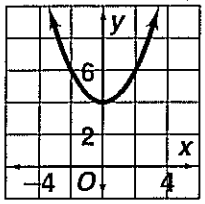
14. $f(x) = x^2 + 2$; same as $f(x) = x^2$, but shifted up 2 units



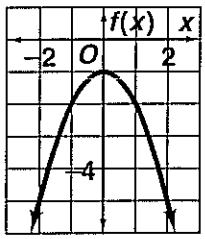
15. $y = x^2 - 3$; same as $y = x^2$, but shifted down 3 units



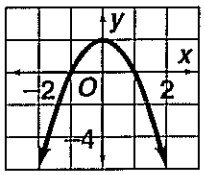
16. $y = \frac{1}{2}x^2 + 4$; same as $y = \frac{1}{2}x^2$, but shifted up 4 units



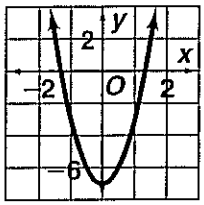
17. $f(x) = -x^2 - 1$; same as $f(x) = -x^2$, but shifted down 1 unit



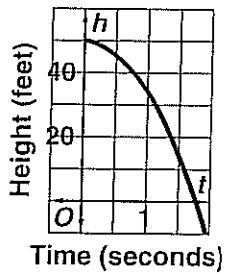
18. $y = -2x^2 + 2$; same as $y = -2x^2$, but shifted up 2 units



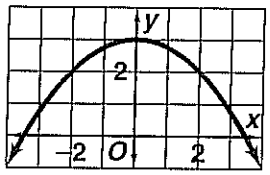
19. $f(x) = 4x^2 - 7$; same as $f(x) = 4x^2$, but shifted down 7 units



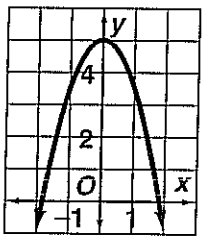
20. $h = -16t^2 + 50$



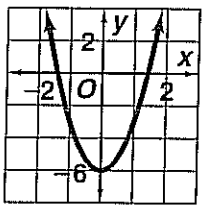
21. The parabola opens downward, so choose C, E, or F. The graph is shifted up 8 units, so choose E.
22. The parabola opens upward, so choose A, B, or D. The graph is shifted down 1 unit, so choose A.
23. The parabola opens downward, so choose C, E, or F. The graph is shifted up 5 units, so choose F.
24. The parabola opens upward, so choose A, B, or D. The graph is shifted up 4 units, so choose B.
25. The parabola opens downward, so choose C, E, or F. The graph is shifted up 2 units, so choose C.
26. The parabola opens upward, so choose A, B, or D. The graph is shifted down 5 units, so choose D.
27. The graph of $y = 2x^2$ is narrower.
28. The graph of $y = -x^2$ opens downward.
29. The graph of $y = 1.5x^2$ is narrower.
30. The graph of $y = \frac{1}{2}x^2$ is wider.
31. $y = -\frac{1}{4}x^2 + 3$; same as $y = -\frac{1}{4}x^2$, but shifted up 3 units



32. $f(x) = -1.5x^2 + 5$; same as $f(x) = -1.5x^2$, but shifted up 5 units



33. $y = 3x^2 - 6$; same as $y = 3x^2$, but shifted down 6 units



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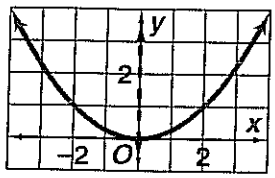

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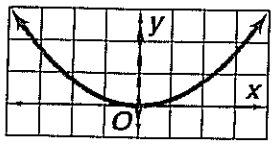

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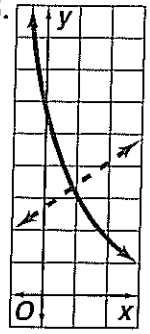
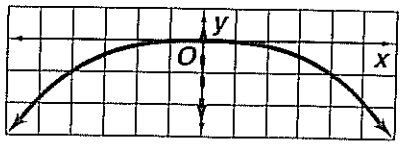
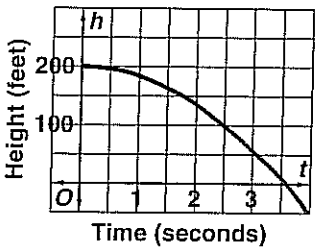


37.

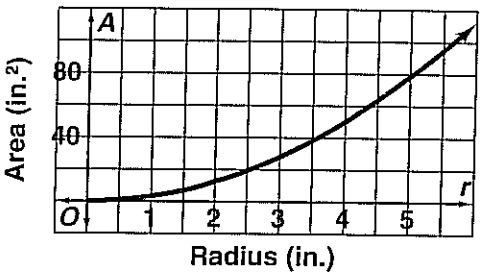


38a. $h = -16t^2 + 200$

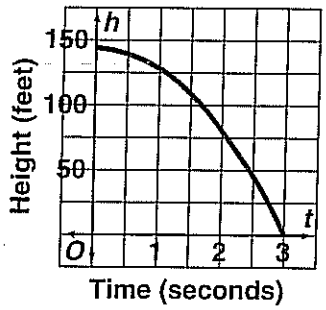


38b. $h = -16(1)^2 + 200 = 184$; 184 ft
38c. $h = -16(3)^2 + 200 = 56$; 56 ft

39a. The diameter of the pizza must be less than 12 in., so the radius must be less than 6 in.; $0 < r < 6$
39b. If $r = 6$, $A = \pi \cdot 6^2 = 36\pi$, so A must be less than 36π; $0 < A < 36\pi \approx 113.1$
39c. $A = \pi r^2$



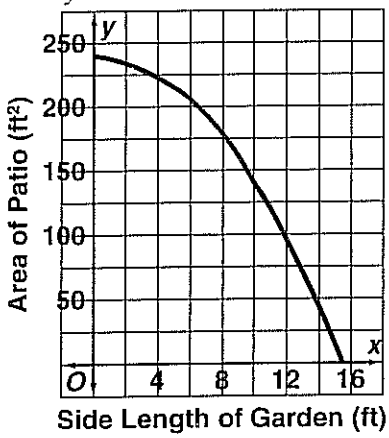
40. $a > 0$, so the parabola opens upward; K, L
41. $a < 0$, so the parabola opens downward; M
42. $|a|$ has the greatest value, so the graph is the narrowest; K **43.** $|a|$ has the least value, so the graph is the widest; M **44.** Answers may vary. Samples are given.
44a. $y = 5x^2$ **44b.** $y = -5x^2$ **44c.** $y = 3x^2$
45a. $h = -16t^2 + 144$



45b. 16 ft
45c. No; the apple falls 48 ft from $t = 1$ to $t = 2$, because it is accelerating.

46a. The graph of $y = ax^2 + c$ intersects the x-axis in two places when $c \neq 0$ and a and c have opposite signs.
46b. The graph of $y = ax^2 + c$ does not intersect the x-axis when $c \neq 0$ and a and c have the same signs.

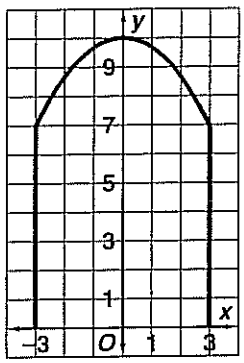
47a. $y = 240 - x^2$



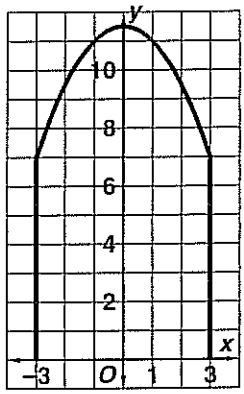
47b. $0 < x < 12$; the side length of the square garden must be less than the width of the patio.
47c. $96 < A < 240$; as the side length of the garden increases from 0 to 12, the area of the patio decreases from 240 to 96.
47d. about 6 ft

48a. $a > 0$ **48b.** $|a| > 1$

49a.



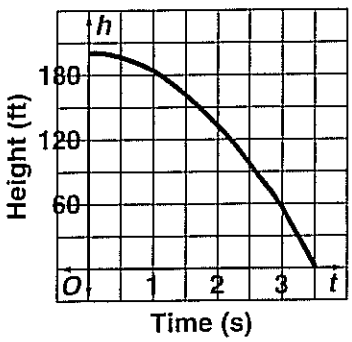
49b.



50. $|-3| > |2|$, so the answer is B. **51.** The graph of $y = -2x^2 - 1$ crosses the y-axis at -1; look for a c-value ($y = ax^2 + c$) that is less than -1; $y = 3x^2 - 3$, so the answer is G.
52. Test each point in the equation $y = 4x^2 + 3$.
A. $(0, 3); 3 \stackrel{?}{=} 4(0)^2 + 3, 3 = 3$
B. $(1, 7); 7 \stackrel{?}{=} 4(1)^2 + 3, 7 = 7$
C. $(-1, 7): 7 \stackrel{?}{=} 4(-1)^2 + 3, 7 = 7$
D. $(3, 27): 27 \stackrel{?}{=} 4(3)^2 + 3, 27 \neq 39$; the answer is D.

53. [4] **53a.**

t	$h = -16t^2 + 200$	(t, h)
0	$-16(0)^2 + 200 = 200$	$(0, 200)$
1	$-16(1)^2 + 200 = 184$	$(1, 184)$
2	$-16(2)^2 + 200 = 136$	$(2, 136)$
3	$-16(3)^2 + 200 = 56$	$(3, 56)$
4	$-16(4)^2 + 200 = -56$	$(4, -56)$



53b. When $h = 0, t \approx 3.5$; about 3.5 seconds
[3] estimate incorrect or missing
[2] error in table or graph
[1] table OR reasonable graph only

54. $x^3 - 4x^2 + 2x - 8 = x^2(x - 4) + 2(x - 4) = (x^2 + 2)(x - 4)$

55. $15a^3 - 18a^2 - 10a + 12 = 3a^2(5a - 6) - 2(5a - 6) = (3a^2 - 2)(5a - 6)$ **56.** $7b^3 + 14b^2 + b + 2 = 7b^2(b + 2) + 1(b + 2) = (7b^2 + 1)(b + 2)$
57. $y^3 + 3y^2 - 4y - 12 = y^2(y + 3) - 4(y + 3) = (y^2 - 4)(y + 3) = (y + 2)(y - 2)(y + 3)$
58. $2n^3 - 2n^2 - 24n = 2n(n^2 - n - 12) = 2n(n + 3)(n - 4)$ **59.** $30m^3 + 51m^2 + 9m = 3m(10m^2 + 17m + 3) = 3m(2m + 3)(5m + 1)$
60. $5x(3x - 4) = 15x^2 - 20x$ **61.** $(n - 7)9n = 9n^2 - 63n$ **62.** $-2t^2(6t - 11) = -12t^3 + 22t^2$
63. $4m^2(3m^4 - m^3 + 5) = 12m^6 - 4m^5 + 20m^2$
64. $-5y(3y^5 + 2y^3 - 4) = -15y^6 - 10y^4 + 20y$
65. $3c^3(-4c^2 + 7c - 8) = -12c^5 + 21c^4 - 24c^3$
66. $(2 - 0.2)x = 27; 1.8x = 27; x = 15$; 15 balloons

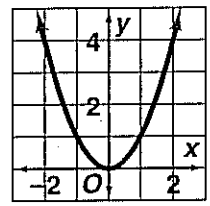
10-2 Quadratic Functions

pages 517–523

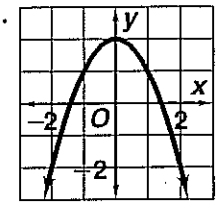
Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1. $\frac{1}{3}$ **2.** $-\frac{2}{3}$ **3.** $-3\frac{1}{2}$ **4.** 6

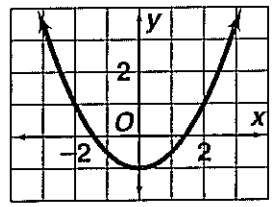
5.



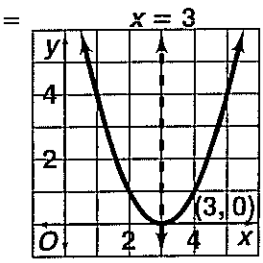
6.



7.

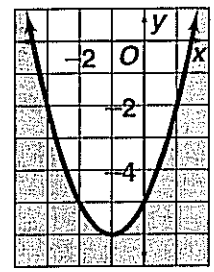


Check Understanding 1. $f(x) = x^2 - 6x + 9$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{-6}{2(1)} = 3, x = 3$; y-coordinate of vertex: $y = 3^2 - 6(3) + 9 = 0$; vertex $(3, 0)$; points $(1, 4), (5, 4)$

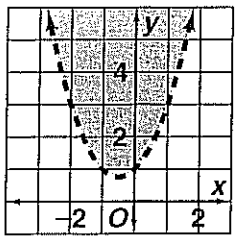


2a. $h = -16t^2 + 48t + 4; t = -\frac{b}{2a} = -\frac{48}{2(-16)} = 1.5$; 1.5 seconds **2b.** $h = -16(1.5)^2 + 48(1.5) + 4 = 40$; 40 ft

3a. $y \leq x^2 + 2x - 5$; boundary curve $y = x^2 + 2x - 5$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{2}{2(1)} = -1, x = -1$; y-coordinate of vertex: $y = (-1)^2 + 2(-1) - 5 = -6$; vertex $(-1, -6)$; solid line and shade below



3b. $y > x^2 + x + 1$; boundary curve $y = x^2 + x + 1$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{1}{2(1)} = -\frac{1}{2}, x = -\frac{1}{2}$; y-coordinate of vertex: $y = \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) + 1 = \frac{3}{4}$; vertex $\left(-\frac{1}{2}, \frac{3}{4}\right)$; dashed line and shade above

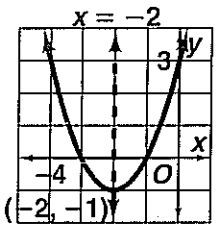


Exercises 1. $y = 2x^2 + 4$, or $y = 2x^2 + 0x + 4$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{0}{2(2)} = 0, x = 0$; y-coordinate of vertex: $y = 2(0)^2 + 4 = 4$; vertex $(0, 4)$ **2.** $f(x) = 2x^2 + 4x - 5$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{4}{2(2)} = -1, x = -1$; y-coordinate of vertex: $y = 2(-1)^2 + 4(-1) - 5 = -7$; vertex $(-1, -7)$ **3.** $y = x^2 - 8x - 9$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{-8}{2(1)} = 4, x = 4$; y-coordinate of vertex: $y = 4^2 - 8(4) - 9 = -25$; vertex $(4, -25)$ **4.** $y = 3x^2 - 9x + 5$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{-9}{2(3)} = 1.5, x = 1.5$; y-coordinate of vertex: $y = 3(1.5)^2 - 9(1.5) + 5 = -1.75$; vertex $(1.5, -1.75)$

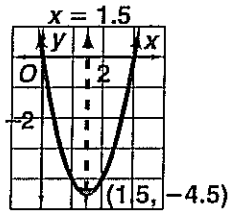
5–10. A. $y = x^2 - 6x$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{-6}{2(1)} = 3, x = 3$, and parabola opens upward, so Ex. 9. B. $y = x^2 + 6x$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{6}{2(1)} = -3, x = -3$, and parabola opens upward, so Ex. 5. C. $y = -x^2 - 6x$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{-6}{2(-1)} = -3, x = -3$, and parabola opens downward, so Ex. 7. D. $y = -x^2 + 6x$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{6}{2(-1)} = 3, x = 3$, and parabola opens downward, so Ex. 10. E. $y = -x^2 + 6$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{0}{2(-1)} = 0, x = 0$, and parabola opens downward, so Ex. 6. F. $y = x^2 - 6$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{0}{2(1)} = 0, x = 0$, and parabola opens upward, so Ex. 8.

5. B **6.** E **7.** C **8.** F **9.** A **10.** D

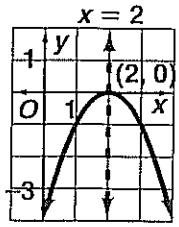
11. $f(x) = x^2 + 4x + 3$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{4}{2(1)} = -2$, $x = -2$; y-coordinate of vertex: $y = (-2)^2 + 4(-2) + 3 = -1$; vertex $(-2, -1)$; points $(0, 3), (-4, 3)$



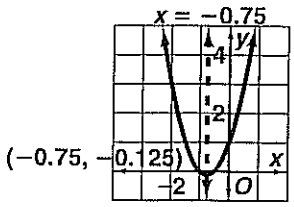
12. $y = 2x^2 - 6x$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{-6}{2(2)} = 1.5, x = 1.5$; y-coordinate of vertex: $y = 2(1.5)^2 - 6(1.5) = -4.5$; vertex $(1.5, -4.5)$; points $(0, 0), (4, 0)$



13. $y = -x^2 + 4x - 4$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{4}{2(-1)} = 2, x = 2$; y-coordinate of vertex: $y = -2^2 + 4(2) - 4 = 0$; vertex $(2, 0)$; points $(0, -4), (4, -4)$



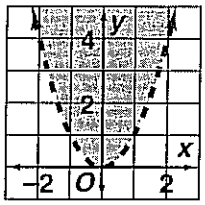
14. $y = 2x^2 + 3x + 1$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{3}{2(2)} = -0.75, x = -0.75$; y-coordinate of vertex: $y = 2(-0.75)^2 + 3(-0.75) + 1 = -0.125$; vertex $(-0.75, -0.125)$; points $(0, 1), (-2, 3)$



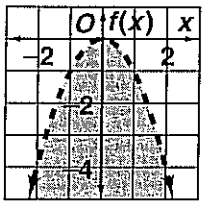
15a. $A = 40x - x^2$, or $A = -x^2 + 40x$; $x = -\frac{b}{2a} = -\frac{40}{2(-1)} = 20$; 20 ft **15b.** $A = 40(20) - 20^2 = 400$; 400 ft^2

16a. $h = -16t^2 + 40t + 6$; $t = -\frac{b}{2a} = -\frac{40}{2(-16)} = 1.25$; 1.25 s **16b.** $h = -16(1.25)^2 + 40(1.25) + 6 = 31$; 31 ft

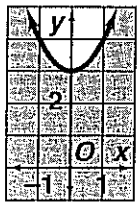
17. $y > x^2$; boundary curve $y = x^2$; dashed line and shade above



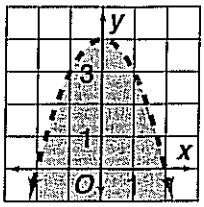
18. $f(x) < -x^2$; boundary curve $y = -x^2$; dashed line and shade below



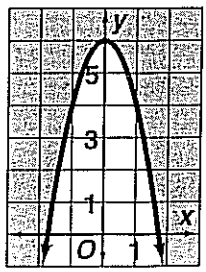
19. $y \le x^2 + 3$; boundary curve $y = x^2 + 3$, same as $y = x^2$ but shifted up 3 units; solid line and shade below



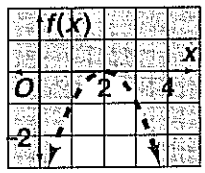
20. $y < -x^2 + 4$; boundary curve $y = -x^2 + 4$, same as $y = -x^2$ but shifted up 4 units; dashed line and shade below



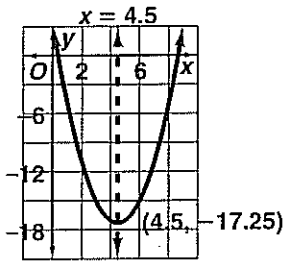
21. $y \ge -2x^2 + 6$; boundary curve $y = -2x^2 + 6$, same as $y = -2x^2$ but shifted up 6 units; solid line and shade above



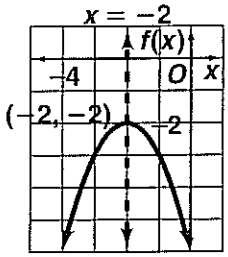
22. $f(x) > -x^2 + 4x - 4$; boundary curve $y = -x^2 + 4x - 4$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{4}{2(-1)} = 2$, $x = 2$; y-coordinate of vertex: $y = -2^2 + 4(2) - 4 = 0$; vertex $(2, 0)$; points $(3, -1)$, $(1, -1)$; dashed line and shade above



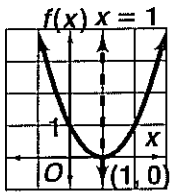
23. $y = x^2 - 9x + 3$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{-9}{2(1)} = 4.5$, $x = 4.5$; y-coordinate of vertex: $y = 4.5^2 - 9(4.5) + 3 = -17.25$; vertex $(4.5, -17.25)$; points $(0, 3)$, $(6, -15)$



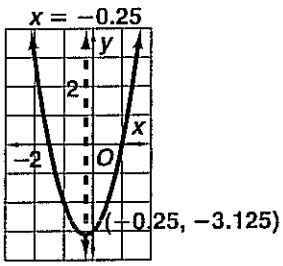
24. $f(x) = -x^2 - 4x - 6$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{-4}{2(-1)} = -2$, $x = -2$; y-coordinate of vertex: $y = -(-2)^2 - 4(-2) - 6 = -2$; vertex $(-2, -2)$; points $(0, -6)$, $(-4, -6)$



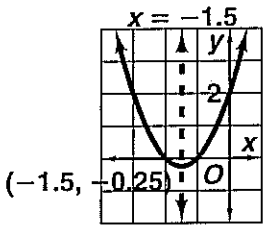
25. $f(x) = x^2 - 2x + 1$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{-2}{2(1)} = 1$, $x = 1$; y-coordinate of vertex: $y = 1^2 - 2(1) + 1 = 0$; vertex $(1, 0)$; points $(0, 1)$, $(2, 1)$



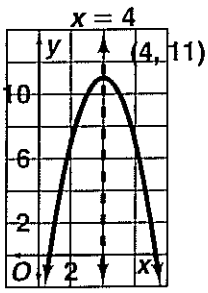
26. $y = 2x^2 + x - 3$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{1}{2(2)} = -0.25$, $x = -0.25$; y-coordinate of vertex: $y = 2(-0.25)^2 + (-0.25) - 3 = -3.125$; vertex $(-0.25, -3.125)$; points $(1, 0)$, $(-1, -2)$



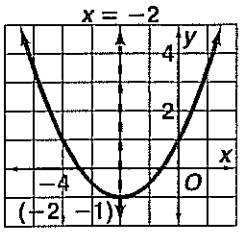
27. $y = x^2 + 3x + 2$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{3}{2(1)} = -1.5$, $x = -1.5$; y-coordinate of vertex: $y = (-1.5)^2 + 3(-1.5) + 2 = -0.25$; vertex $(-1.5, -0.25)$; points $(0, 2)$, $(-3, 2)$



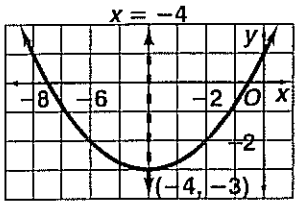
28. $y = -x^2 + 8x - 5$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{8}{2(-1)} = 4$, $x = 4$; y-coordinate of vertex: $y = -4^2 + 8(4) - 5 = 11$; vertex $(4, 11)$; points $(1, 2)$, $(5, 10)$



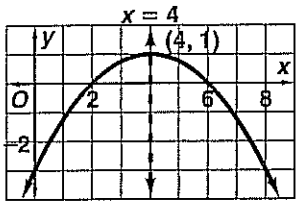
29. $y = \frac{1}{2}x^2 + 2x + 1$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{2}{2\left(\frac{1}{2}\right)} = -2$, $x = -2$; y-coordinate of vertex: $y = \frac{1}{2}(-2)^2 + 2(-2) + 1 = -1$; vertex $(-2, -1)$; points $(0, 1)$, $(-4, 1)$



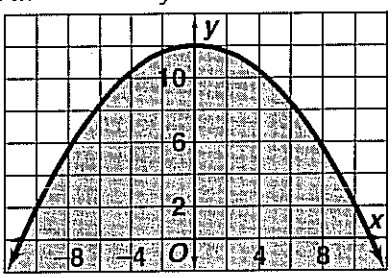
30. $y = \frac{1}{4}x^2 + 2x + 1$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{2}{2\left(\frac{1}{4}\right)} = -4$, $x = -4$; y-coordinate of vertex: $y = \frac{1}{4}(-4)^2 + 2(-4) + 1 = -3$; vertex $(-4, -3)$; points $(0, 1)$, $(-8, 1)$



31. $y = -\frac{1}{4}x^2 + 2x - 3$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{2}{2\left(-\frac{1}{4}\right)} = 4$, $x = 4$; y-coordinate of vertex: $y = -\frac{1}{4}(4)^2 + 2(4) - 3 = 1$; vertex $(4, 1)$; points $(0, -3)$, $(8, -3)$



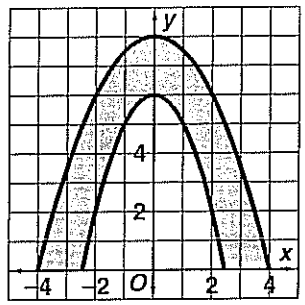
32–34. Answers may vary. Samples are given.
32. $y = 2x^2 - 8x + 1$ **33.** $y = -3x^2$ **34.** $y = 2x^2 + 4$
35a. $y = -2.2x^2 + 5.3x + 4$; $x = -\frac{b}{2a} = -\frac{5.3}{2(-2.2)} \approx 1.2$; about 1.2 meters **35b.** $y = -2.2(1.2)^2 + 5.3(1.2) + 4 = 7.192 \approx 7.2$; about 7.2 meters **36a.** $y \le -0.1x^2 + 12$
36b. boundary curve $y = -0.1x^2 + 12$; points $(0, 12)$, $(6, 8.4)$, $(10, 2)$, $(-6, 8.4)$, $(-10, 2)$



36c. Yes; when $x = 6$, $y = 8.4$, so the camper will fit.
37a. $S = -64p^2 + 1600p$; $p = -\frac{b}{2a} = -\frac{1600}{2(-64)} = 12.5$; \$12.50 **37b.** $S = -64(12.5)^2 + 1600(12.5) = 10{,}000$; \$10,000 **38.** Area ≈ 32 units^2 **39.** Area ≈ 26 units^2
40. Answers may vary. Sample: If the coefficient of the squared term is positive, the vertex is a minimum; if it is negative, the vertex is a maximum. **41.** Answers may vary. Sample: a affects whether the parabola opens up or down, b affects the axis of symmetry, and c affects the y-intercept.
42a. $2(w + \ell) = 26$; $w + \ell = 13$; $w = 13 - \ell$
42b. $A = \ell w = \ell(13 - \ell) = 13\ell - \ell^2 = -\ell^2 + 13\ell$; $A = -\ell^2 + 13\ell$ **42c.** $\ell = -\frac{b}{2a} = -\frac{13}{2(-1)} = 6.5$; $A = -6.5^2 + 13(6.5) = 42.25$, vertex $(6.5, 42.25)$
42d. $\ell = 6.5$, so $w = 13 - 6.5 = 6.5$; 6.5 ft by 6.5 ft

43a. $h = -4.9t^2 + 3.82t + 1.7; t = -\frac{b}{2a} = -\frac{3.82}{2(-4.9)} \approx 0.4$; about 0.4 seconds **43b.** $h = -4.9(0.6)^2 + 3.82(0.6) + 1.7 \approx 2.23$; no, because at 0.6 seconds the ball has a height of about 2.23 m, but the net has a height of 2.43 m

44. ① $y \geq -x^2 + 6$; boundary curve $y = -x^2 + 6$, parabola that opens downward with vertex (0, 6) and points (2, 2), (−2, 2)



② $y \leq -\frac{1}{2}x^2 + 8$; boundary curve $y = -\frac{1}{2}x^2 + 8$, parabola that opens downward with vertex (0, 8) and points (2, 6), (−2, 6), (4, 0), (−4, 0)

③ $y \geq 0$

45a. $h = -4.9t^2 + 3.8t + 0.5; t = -\frac{b}{2a} = -\frac{3.8}{2(-4.9)} \approx 0.4$; about 0.4 seconds **45b.** Use $t = 2\left(-\frac{3.8}{2(-4.9)}\right) \approx 0.7755$: $h \approx -4.9\,(0.7755)^2 + 3(0.7755) + 0.5 \approx 0.5$. No; the height of the ball is about 0.5 m, so it will take more time to reach the ground. **46a.** y-intercept: (0, 2) **46b.** axis of symmetry: $x = -2.5$ **46c.** $x = -\frac{b}{2a}; -2.5 = -\frac{b}{2(1)}; -5 = -b; b = 5$ **46d.** $y = x^2 + 5x + 2$ **46e.** Answers may vary. Sample: Test (−4, −2). $-2 \stackrel{?}{=} (-4)^2 + 5(-4) + 2; -2 \stackrel{?}{=} 16 - 20 + 2; -2 = -2$ ✓ **46f.** No; you would not be able to determine the b value using the vertex formula. **47.** $y = x^2 - 2x - 1$; $x = -\frac{b}{2a} = -\frac{-2}{2(1)} = 1; y = 1^2 - 2(1) - 1 = -2$; vertex (1, −2); A

48. F. $y = -x^2 - 2x; b = -2$ G. $y = -x^2 - 3x; b = -3$ H. $y = -x^2 + 2x; b = 2$ I. $y = -x^2 + 3x; b = 3$; the answer is I. **49.** $y = 0.5x^2 - 2x + 1; x = -\frac{b}{2a} = -\frac{-2}{2(0.5)} = 2; y = 0.5(2)^2 - 2(2) + 1 = -1$; vertex (2, −1); B

50. [2] axis of symmetry: $x = \frac{-b}{2a} = \frac{-0.3}{2(-0.009)} \approx 16.7$; maximum height: $y \approx -0.009(16.7)^2 + 0.3(16.7) + 4.5$; $y \approx 7$ ft [1] appropriate methods, but with a minor computational error **51.** The parabola opens downward, so choose C, E, or F; vertex (0, −2), so choose C or F; same shape as $y = -x^2$, so choose C. **52.** The parabola opens upward, so choose A, B, or D; the graph is wider than $y = x^2$ and $y = \frac{1}{2}x^2$, so choose A. **53.** The parabola opens downward, so choose C, E, or F; vertex (0, −2), so choose C or F; the graph is wider than $y = -x^2$, so choose F. **54.** The parabola opens upward, so choose A, B, or D; same shape as $y = x^2$ but shifted up 2 units, so choose D. **55.** The parabola opens upward, so choose A, B, or D; the graph is wider than $y = x^2$, so choose B. **56.** The parabola opens downward, so choose C, E, or F; vertex (0, 2), so choose E. **57.** $(c + 4)(c - 9) = c^2 - 9c + 4c - 36 = c^2 - 5c - 36$ **58.** $(2x - 5)(x + 6) = 2x^2 + 12x - 5x - 30 = 2x^2 + 7x - 30$ **59.** $(4t + 1)(5t + 3) = 20t^2 + 12t + 5t + 3 = 20t^2 + 17t + 3$ **60.** $(7n^2 - 2)(3n^2 - 8) = 21n^4 - 56n^2 - 6n^2 + 16 = 21n^4 - 62n^2 + 16$ **61.** $(a + 5)(2a^2 - a + 4) = 2a^3 - a^2 + 4a + 10a^2 - 5a + 20 = 2a^3 + 9a^2 - a + 20$ **62.** $(3r^2 + 6r - 7)(2r - 1) = 6r^3 - 3r^2 + 12r^2 - 6r - 14r + 7 = 6r^3 + 9r^2 - 20r + 7$

10-3 Finding and Estimating Square Roots pages 524–528

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

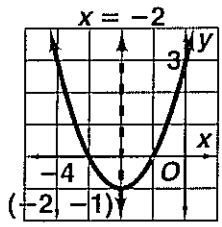
1. 121 **2.** 144 **3.** −144 **4.** 2.25 **5.** 0.36 **6.** $\frac{1}{4}$ **7.** $\frac{4}{9}$ **8.** $\frac{16}{25}$

Check Understanding **1a.** $\sqrt{49} = 7$ **1b.** $\pm\sqrt{36} = \pm 6$ **1c.** $-\sqrt{121} = -11$ **1d.** $\sqrt{\frac{1}{25}} = \frac{1}{5}$ **2a.** $\sqrt{8}$, irrational **2b.** $\pm\sqrt{225} = \pm 15$, rational **2c.** $-\sqrt{75}$, irrational **2d.** $\sqrt{\frac{1}{4}} = \frac{1}{2}$, rational **3.** $-\sqrt{121} < -\sqrt{105} < -\sqrt{100}$, so $-11 < -\sqrt{105} < -10$; between −11 and −10 **4.** $\sqrt{17.81} \approx 4.22$ **5.** $2x = 140$, so $x = 70$; $d = \sqrt{x^2 + (2x)^2} = \sqrt{70^2 + 140^2} = \sqrt{4900 + 19{,}600} = \sqrt{24{,}500} \approx 156.5$; about 156.5 ft

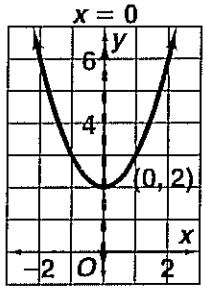
Exercises **1.** $\sqrt{169} = 13$ **2.** $\sqrt{400} = 20$ **3.** $\sqrt{\frac{1}{9}} = \frac{1}{3}$ **4.** $\sqrt{900} = 30$ **5.** $\sqrt{0.25} = 0.5$ **6.** $\sqrt{\frac{36}{49}} = \frac{6}{7}$ **7.** $-\sqrt{1.21} = -1.1$ **8.** $\sqrt{1.96} = 1.4$ **9.** $\sqrt{0.36} = 0.6$ **10.** $-\sqrt{144} = -12$ **11.** $\sqrt{\frac{25}{16}} = \frac{5}{4}$ **12.** $\pm\sqrt{0.01} = \pm 0.1$ **13.** $\sqrt{37}$, irrational **14.** $-\sqrt{0.04} = -0.2$, rational **15.** $\pm\sqrt{\frac{1}{5}}$, irrational **16.** $-\sqrt{\frac{16}{121}} = -\frac{4}{11}$, rational **17.** $\sqrt{25} < \sqrt{35} < \sqrt{36}$, so $5 < \sqrt{35} < 6$; between 5 and 6 **18.** $\sqrt{25} < \sqrt{27} < \sqrt{36}$, so $5 < \sqrt{27} < 6$; between 5 and 6 **19.** $-\sqrt{144} < -\sqrt{130} < -\sqrt{121}$, so $-12 < -\sqrt{130} < -11$; between −12 and −11 **20.** $\sqrt{169} < \sqrt{170} < \sqrt{196}$, so $13 < \sqrt{170} < 14$; between 13 and 14 **21.** $\sqrt{12} \approx 3.46$ **22.** $-\sqrt{203} \approx -14.25$ **23.** $\sqrt{11{,}550} \approx 107.47$ **24.** $-\sqrt{150} \approx -12.25$ **25.** $e = \sqrt{\frac{r}{h}} = \sqrt{\frac{3}{3.5}} \approx 0.93$ **26.** square roots of 400: ±20 **27.** square root of 0: 0 **28.** square roots of 625: ±25 **29.** square roots of $\frac{9}{4a}$: $\pm\frac{3}{7}$ **30.** square roots of 1.69: ±1.3 **31.** square roots of $\frac{1}{81}$: $\pm\frac{1}{9}$ **32.** square roots of 729: ±27 **33.** square roots of 2.25: ±1.5 **34.** square roots of 256: ±16 **35.** square roots of 0.01: ±0.1 **36.** square roots of $\frac{64}{121}$: $\pm\frac{8}{11}$ **37.** square roots of 40,804: ±202 **38.** 1 **39a.** $d = \sqrt{12{,}800h + h^2} = \sqrt{12{,}800(4200) + (4200)^2} = \sqrt{71{,}400{,}000} \approx 8450$; 8450 km **39b.** $d = \sqrt{12{,}800h + h^2} = \sqrt{12{,}800(3600) + (3600)^2} = \sqrt{59{,}040{,}000} \approx 7684$; 7684 km **40.** $\sqrt{441} = 21$ **41.** $-\sqrt{\frac{4}{25}} = -\frac{2}{5}$ **42.** $\sqrt{2} \approx 1.41$ **43.** $\sqrt{1.6} \approx 1.26$ **44.** $-\sqrt{30} \approx -5.48$ **45.** $-\sqrt{1089} = -33$

46. $-\sqrt{0.64} = -0.8$ **47.** $\sqrt{41} \approx 6.40$ **48.** $\sqrt{75} \approx 8.66$
49. Answers may vary. Sample: The first expression means the negative square root of 1, and the second expression means the positive square root of 1.
50. Answers may vary. Sample: 3 and 4 (since $3^2 + 4^2 = 9 + 16 = 25$, a perfect square) **51.** $\sqrt{16} = 4$
52a. $t = \sqrt{\frac{d}{16}} = \sqrt{\frac{400}{16}} = \sqrt{25} = 5$; 5 s **52b.** $t = \sqrt{\frac{d}{16}} = \sqrt{\frac{1600}{16}} = \sqrt{100} = 10$; 10 s **52c.** No; the object takes only twice as long to fall. **53.** False; zero has only one square root. **54.** False; $\sqrt{1} = 1$. **55.** True
56. True **57.** False; answers may vary. Sample: $\sqrt{4} + \sqrt{9} \neq \sqrt{4 + 9}$. **58.** False; answers may vary. Sample: $\sqrt{12}$ and $\sqrt{3}$ are irrational, but $\sqrt{36}$ is rational.
59a. Area $= 2(2) = 4$ units^2 **59b.** $\frac{1}{2}$ units^2
59c. $4\left(\frac{1}{2}\text{ units}^2\right) = 2$ units^2 **59d.** $\sqrt{2}$ units **60.** $\sqrt{1.69} = 1.3$ **61.** $\sqrt{4^2 + 3^2 + 11} = \sqrt{36} = 6$ **62.** $\sqrt{\frac{4}{81}} = \frac{2}{9}$
63. $\sqrt{80} \approx 9$ **64.** $\ell = \sqrt{\frac{A}{6}} = \sqrt{\frac{726}{6}} = \sqrt{121} = 11$
65. $\sqrt{100} \cdot \sqrt{256} - \sqrt{16} \cdot \sqrt{64} = 10(16) - 4(8) = 160 - 32 = 128$

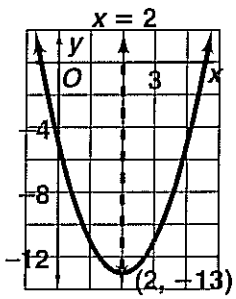
66. $y = x^2 + 4x + 3$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{4}{2(1)} = -2$, $x = -2$; y-coordinate of vertex: $y = (-2)^2 + 4(-2) + 3 = -1$; vertex $(-2, -1)$; points $(0, 3)$, $(-4, 3)$



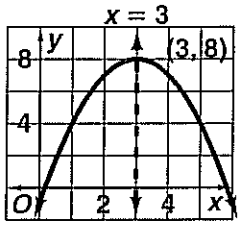
67. $y = x^2 + 2$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{0}{2(1)} = 0$, $x = 0$; y-coordinate of vertex: $y = 0^2 + 2 = 2$; vertex $(0, 2)$; points $(2, 6)$, $(-2, 6)$



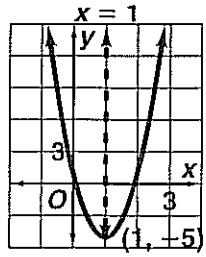
68. $y = 2x^2 - 8x - 5$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{-8}{2(2)} = 2$, $x = 2$; y-coordinate of vertex: $y = 2(2)^2 - 8(2) - 5 = -13$; vertex $(2, -13)$; points $(0, -5)$, $(4, -5)$



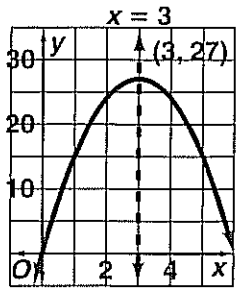
69. $y = -x^2 + 6x - 1$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{6}{2(-1)} = 3$, $x = 3$; y-coordinate of vertex: $y = -3^2 + 6(3) - 1 = 8$; vertex $(3, 8)$; points $(1, 4)$, $(5, 4)$



70. $y = 6x^2 - 12x + 1$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{-12}{2(6)} = 1$, $x = 1$; y-coordinate of vertex: $y = 6(1)^2 - 12(1) + 1 = -5$; vertex $(1, -5)$; points $(0, 1)$, $(2, 1)$



71. $y = -3x^2 + 18x$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{18}{2(-3)} = 3$, $x = 3$; y-coordinate of vertex: $y = -3(3)^2 + 18(3) = 27$; vertex $(3, 27)$; points $(0, 0)$, $(6, 0)$, $(1, 15)$, $(5, 15)$



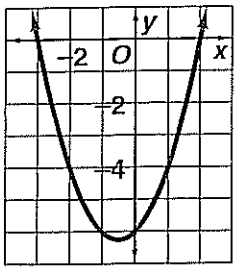
72. $(d + 9)(d - 9) = d^2 - 9d + 9d - 81 = d^2 - 81$
73. $(3t - 5)(3t + 5) = 9t^2 + 15t - 15t - 25 = 9t^2 - 25$ **74.** $85 \cdot 95 = (90 - 5)(90 + 5) = 90^2 - 5^2 = 8100 - 25 = 8075$
75. $(x + 13)^2 = (x + 13)(x + 13) = x^2 + 13x + 13x + 169 = x^2 + 26x + 169$
76. $(4y - 7)^2 = (4y - 7)(4y - 7) = 16y^2 - 28y - 28y + 49 = 16y^2 - 56y + 49$ **77.** $101^2 = (100 + 1)^2 = 100^2 + 2(100 \cdot 1) + 1^2 = 10{,}000 + 200 + 1 = 10{,}201$ **78.** $902^2 = (900 + 2)^2 = 900^2 + 2(900 \cdot 2) + 2^2 = 810{,}000 + 3600 + 4 = 813{,}604$ **79.** $(6k - 7)(6k + 7) = 36k^2 + 42k - 42k - 49 = 36k^2 - 49$ **80.** $(12b - 7)^2 = (12b - 7)(12b - 7) = 144b^2 - 84b - 84b + 49 = 144b^2 - 168b + 49$ **81.** $x^{-2}y^{-1}z^2 = \frac{z^2}{x^2y} = \frac{6^2}{3^2(-2)} = \frac{36}{-18} = -2$ **82.** $\frac{y^{-2}}{x^{-3}z^2} = \frac{x^3}{y^2z^2} = \frac{3^3}{(-2)^2 \cdot 6^2} = \frac{27}{144} = \frac{3}{16}$
83. $\frac{x^0y^{-3}}{z^{-1}} = \frac{z}{y^3} = \frac{6}{(-2)^3} = \frac{6}{-8} = -\frac{3}{4}$
84. $\frac{x^2y^{-1}}{z^{-3}} = \frac{x^2z^3}{y} = \frac{3^2 \cdot 6^3}{-2} = \frac{1944}{-2} = -972$
85. $x^{-1}y^{-2}z^0 = \frac{1}{xy^2} = \frac{1}{3(-2)^2} = \frac{1}{12}$
86. $x^2y^{-4}z^2 = \frac{x^2z^2}{y^4} = \frac{3^2 \cdot 6^2}{(-2)^4} = \frac{324}{16} = \frac{81}{4}$

10-4 Solving Quadratic Equations *pages 529–534*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1. 6 **2.** -9 **3.** ± 11 **4.** 1.2 **5.** 0.5 **6.** ± 1.1 **7.** $\frac{1}{2}$ **8.** $\pm\frac{1}{3}$ **9.** $\frac{7}{10}$

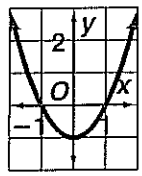
Investigation 1a. $1\frac{1}{2}$ **1b.** -4 and 1 **2a.** $2x - 3 = 0$; $2x = 3$; $x = \frac{3}{2} = 1\frac{1}{2}$ **2b.** yes **3.** Check $x = -4$: $(-4)^2 + 3(-4) - 4 = 16 - 12 - 4 = 0$, yes; check $x = 1$: $1^2 + 3(1) - 4 = 1 + 3 - 4 = 0$, yes. **4a.** $y = x^2 + x - 6$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{1}{2(1)} = -\frac{1}{2}$, y-coordinate of vertex: $y = \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) - 6 = -6\frac{1}{4}$; vertex $\left(-\frac{1}{2}, -6\frac{1}{4}\right)$; points $(1, -4)$, $(2, 0)$



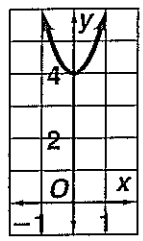
4b. −3 and 2 **4c.** Check $x = -3$: $(-3)^2 + (-3) - 6 = 9 - 3 - 6 = 0$; yes; check $x = 2$: $2^2 + 2 - 6 = 4 + 2 - 6 = 0$; yes.

Check Understanding

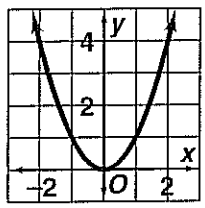
1a. $x^2 - 1 = 0$;
graph $y = x^2 - 1$;
$x = \pm 1$



1b. $2x^2 + 4 = 0$;
graph $y = 2x^2 + 4$;
no solution

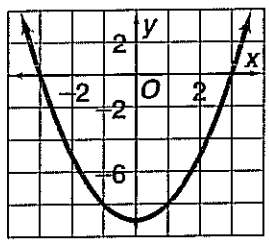


1c. $x^2 - 16 = -16$;
$x^2 = 0$;
graph $y = x^2$;
$x = 0$

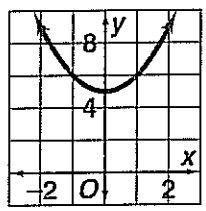


2a. $t^2 - 25 = 0$; $t^2 = 25$; $t = \pm\sqrt{25}$; $t = \pm 5$
2b. $3n^2 + 12 = 12$; $3n^2 = 0$; $n^2 = 0$; $n = 0$ **2c.** $2g^2 + 32 = 0$; $2g^2 = -32$; $g^2 = -16$; no solution **3.** $V = \pi r^2 h$; $1800 = \pi r^2(3)$; $r^2 = \frac{1800}{3\pi}$; $r = \sqrt{\frac{1800}{3\pi}} \approx 13.8$; about 14 ft

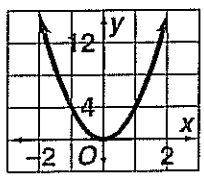
Exercises **1.** $x^2 - 9 = 0$;
graph $y = x^2 - 9$;
$x = \pm 3$



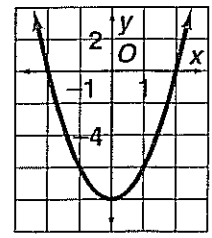
2. $x^2 + 5 = 0$;
graph $y = x^2 + 5$;
no solution



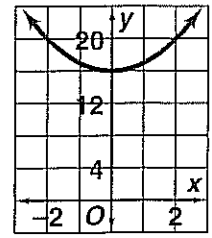
3. $4x^2 = 0$;
graph $y = 4x^2$;
$x = 0$



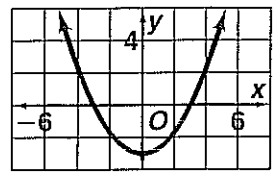
4. $2x^2 - 8 = 0$;
graph $y = 2x^2 - 8$;
$x = \pm 2$



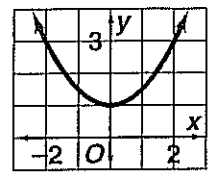
5. $x^2 + 16 = 0$;
graph $y = x^2 + 16$;
no solution



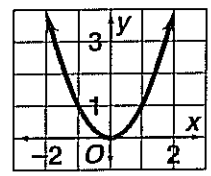
6. $\frac{1}{3}x^2 - 3 = 0$;
graph $y = \frac{1}{3}x^2 - 3$;
$x = \pm 3$



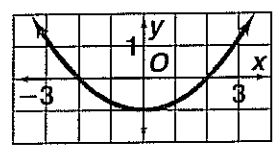
7. $\frac{1}{2}x^2 + 1 = 0$;
graph $y = \frac{1}{2}x^2 + 1$;
no solution



8. $x^2 + 7 = 7$;
$x^2 = 0$;
graph $y = x^2$;
$x = 0$



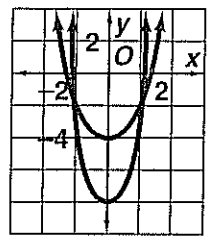
9. $\frac{1}{4}x^2 - 1 = 0$;
graph $y = \frac{1}{4}x^2 - 1$;
$x = \pm 2$



10. $k^2 = 49$; $k = \pm\sqrt{49}$; $k = \pm 7$ **11.** $b^2 = 441$; $b = \pm\sqrt{441}$; $b = \pm 21$ **12.** $m^2 - 225 = 0$; $m^2 = 225$; $m = \pm\sqrt{225}$; $m = \pm 15$ **13.** $c^2 + 25 = 25$; $c^2 = 0$; $c = 0$ **14.** $x^2 - 9 = -16$; $x^2 = -7$; no solution **15.** $4r^2 = 25$; $r^2 = \frac{25}{4}$; $r = \pm\sqrt{\frac{25}{4}}$; $r = \pm\frac{5}{2}$ **16.** $64p^2 = 4$; $p^2 = \frac{4}{64} = \frac{1}{16}$; $p = \pm\sqrt{\frac{1}{16}}$; $p = \pm\frac{1}{4}$ **17.** $6w^2 - 24 = 0$; $6w^2 = 24$; $w^2 = 4$; $w = \pm\sqrt{4}$; $w = \pm 2$ **18.** $27 - y^2 = 0$; $27 = y^2$; $y = \pm\sqrt{27}$ **19.** $x^2 = 256$; $x = \sqrt{256} = 16$; 16 m **20.** $x^2 = 90$; $x = \sqrt{90} \approx 9.5$; about 9.5 ft **21.** $\pi r^2 = 80$; $r^2 = \frac{80}{\pi}$; $r = \sqrt{\frac{80}{\pi}} \approx 5.0$; about 5.0 cm **22a.** $4\pi r^2 = 450$; $r^2 = \frac{450}{4\pi}$; $r = \sqrt{\frac{450}{4\pi}} \approx 6.0$; about 6.0 in. **22b.** The length of a radius cannot be negative. **23.** $y^2 = -36$; none **24.** $a^2 - 12 = 6$; $a^2 = 18$; two **25.** $n^2 - 15 = -15$; $n^2 = 0$; one **26.** $x^2 = 0.75(12^2) = 108$; $x = \sqrt{108} \approx 10.4$; 10.4 in. by 10.4 in. **27a.** $\pi r^2 = 400$; $r^2 = \frac{400}{\pi}$; $r = \sqrt{\frac{400}{\pi}} \approx 11.3$; 11.3 ft **27b.** $\pi r^2 = 800$; $r^2 = \frac{800}{\pi}$; $r = \sqrt{\frac{800}{\pi}} \approx 16.0$; 16.0 ft

27c. $16.0 \div 11.3 \approx 1.4 \neq 2$; no, the radius increases by about 1.4 times. **28.** $1.2q^2 - 7 = -34; 1.2q^2 = -27; q^2 = -\frac{27}{1.2}$; no solution **29.** $49t^2 - 16 = -7; 49t^2 = 9; t^2 = \frac{9}{49}$; $t = \pm\sqrt{\frac{9}{49}} = \pm\frac{3}{7}$ **30.** $3d^2 - \frac{1}{12} = 0; 3d^2 = \frac{1}{12}; d^2 = \frac{1}{36}$; $d = \pm\sqrt{\frac{1}{36}} = \pm\frac{1}{6}$ **31.** $\frac{1}{2}x^2 - 4 = 0; \frac{1}{2}x^2 = 4; x^2 = 8; x = \pm\sqrt{8} \approx \pm 2.8$ **32.** $7h^2 + 0.12 = 1.24; 7h^2 = 1.12; h^2 = \frac{1.12}{7} = 0.16; h = \pm\sqrt{0.16} = \pm 0.4$ **33.** $-\frac{1}{4}x^2 + 3 = 0$; $-\frac{1}{4}x^2 = -3; x^2 = 12; x = \pm\sqrt{12} \approx \pm 3.5$ **34.** $d = \frac{1}{2}at^2$; $12 = \frac{1}{2} \cdot 2t^2; t^2 = 12; t = \sqrt{12} \approx 3.5$; 3.5 s **35.** $x^2 - c = 0$; $x^2 = c$; $11^2 = c$ and $(-11)^2 = c$, so $c = 121$ **36a.** $x^2 = n$ has two solutions when $n > 0$. **36b.** $x^2 = n$ has exactly one solution when $n = 0$. **36c.** $x^2 = n$ has no solution when $n < 0$. **37.** Answers may vary. Sample: Michael subtracted 25 from the left side of the equation but added 25 to the right side.

38a. To solve $x^2 - 4 = 0$, graph $y = x^2 - 4$; to solve $2x^2 - 8 = 0$, graph $y = 2x^2 - 8$.



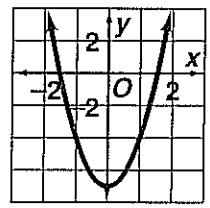
Both equations have the solutions 2 and −2.

38b. The first equation multiplied by 2 on both sides equals the second equation. **39a.** area of square: $(2r)^2$, or $4r^2$; area of circle: πr^2 **39b.** $4r^2 - \pi r^2 = 80$ **39c.** $r^2(4 - \pi) = 80; r^2 = \frac{80}{4 - \pi}; r = \sqrt{\frac{80}{4 - \pi}} \approx 9.7$; $2r = 2\sqrt{\frac{80}{4 - \pi}} \approx 19.3$; radius of circle ≈ 9.7 in., side of square ≈ 19.3 in. **40.** Answers may vary. Samples are given. **40a.** $5x^2 + 10 = 0; 5x^2 = -10; x^2 = -2$; no solution **40b.** $2x^2 + 0 = 0; 2x^2 = 0; x^2 = 0; x = 0$ **40c.** $-20x^2 + 80 = 0; -20x^2 = -80; x^2 = 4; x = \pm\sqrt{4} = \pm 2$ **41.** $A = \frac{1}{2}bh; 20 = \frac{1}{2}h^2; h^2 = 40; h = \sqrt{40} \approx 6.3$; 6.3 ft **42.** $A = \frac{1}{2}bh; 120 = \frac{1}{2} \cdot 2h^2; h^2 = 120$; $h = \sqrt{120} \approx 11.0$; 11.0 cm **43a.** $\ell = \frac{2.45t^2}{\pi^2} = \frac{2.45 \cdot 1^2}{\pi^2} \approx 0.2$; 0.2 m **43b.** $1.6 = \frac{2.45t^2}{\pi^2}; t^2 = \frac{1.6\pi^2}{2.45}; t = \sqrt{\frac{1.6\pi^2}{2.45}} \approx 2.5$; 2.5 s **43c.** $2.2 = \frac{2.45t^2}{\pi^2}; t^2 = \frac{2.2\pi^2}{2.45}; t = \sqrt{\frac{2.2\pi^2}{2.45}} \approx 3.0$; 3.0 s **43d.** You would shorten the pendulum, because as ℓ decreases, t decreases. **44a.** $(x + 7)^2 = 0; x + 7 = 0$; $x = -7$ **44b.** $y = (x + 7)^2$; vertex $(-7, 0)$ **44c.** Answers may vary. Sample: $h = 5$; solution to $y = (x + 5)^2$ is -5; vertex of $y = (x + 5)^2$ is $(-5, 0)$. **44d.** Vertex at $(4, 0)$, because the vertex is at $(-h, 0)$. **45.** $A = \frac{1}{2}h(b_1 + b_2)$; $1960 = \frac{1}{2}y(y + 4y); 1960 = \frac{1}{2}y(5y); 1960 = \frac{5}{2}y^2; y^2 = \frac{2}{5}(1960); y^2 = 784; y = \sqrt{784} = 28$; 28 cm **46.** $3x^2 - 48 = 0; 3x^2 = 48; x^2 = 16; x = \pm\sqrt{16} = \pm 4$; B

47. F. $5x^2 + 1 = 10, 5x^2 = 9, x^2 = \frac{9}{5}$, two solutions
G. $5x^2 + 5 = 10, 5x^2 = 5, x^2 = 1$, two solutions
H. $5x^2 + 9 = 10, 5x^2 = 1, x^2 = \frac{1}{5}$, two solutions
I. $5x^2 + 12 = 10, 5x^2 = -2, x^2 = -\frac{2}{5}$, no solution; the answer is I. **48.** The graph is shifted up 4 units from the origin, so the answer is $-0.25x^2 + 4 = 0$, B.

49. [2]

x	$y = 3x^2 - 7$	(x, y)
−2	$3(-2)^2 - 7 = 5$	$(-2, 5)$
−1	$3(-1)^2 - 7 = -4$	$(-1, -4)$
0	$3(0)^2 - 7 = -7$	$(0, -7)$
1	$3(1)^2 - 7 = -4$	$(1, -4)$
2	$3(2)^2 - 7 = 5$	$(2, 5)$

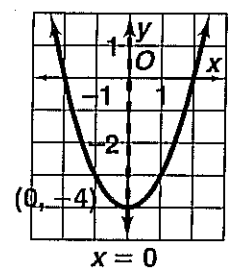


x-intercepts $\approx 1.5, \approx -1.5$
[1] minor error in table OR incorrect graph

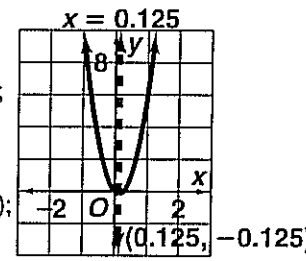
50. [4] **50a.** $96 = 6s^2; s^2 = 16; s = 4$, so edge is 4 ft. **50b.** $6(8)^2 = 6 \cdot 64 = 384$, so the surface area is 384 ft^2. And $96 \cdot 4 = 384$, so the surface area is quadrupled. [3] appropriate methods, but with one computational error [2] part (a) done correctly [1] no work shown **51.** $\sqrt{9} = 3$ **52.** $-\sqrt{169} = -13$ **53.** $\sqrt{1600} = 40$ **54.** $\sqrt{225} = 15$ **55.** $\sqrt{0.04} = 0.2$ **56.** $-\sqrt{2.56} = -1.6$ **57.** $\sqrt{\frac{25}{64}} = \frac{5}{8}$ **58.** $\sqrt{\frac{49}{81}} = \frac{7}{9}$ **59.** $x^2 + 5x + 4 = (x + 4)(x + 1)$ **60.** $y^2 - 15y + 26 = (y - 13)(y - 2)$ **61.** $a^2 + 3a - 10 = (a + 5)(a - 2)$ **62.** $z^2 - 6z - 72 = (z - 12)(z + 6)$ **63.** $c^2 - 14cd + 24d^2 = (c - 12d)(c - 2d)$ **64.** $t^2 + tu - 2u^2 = (t + 2u)(t - u)$ **65.** $3{,}613{,}500 = 3.6135 \times 10^6$ **66.** $0.0000348 = 3.48 \times 10^{-5}$ **67.** $-8.12 = -8.12 \times 10^0$ **68.** $3.1 \times 10^4 = 31{,}000$ **69.** $7.01 \times 10^5 = 701{,}000$ **70.** $6.2 \times 10^{-4} = 0.00062$

CHECKPOINT QUIZ 1 *page 534*

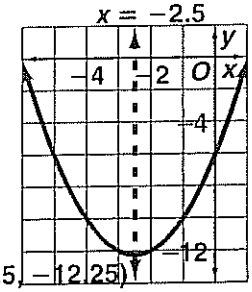
1. $y = x^2 - 4$, or $y = x^2 + 0x - 4$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{0}{2(1)} = 0, x = 0$; y-coordinate of vertex: $y = 0^2 - 4 = -4$; vertex $(0, -4)$; points $(2, 0), (-2, 0)$



2. $y = 8x^2 - 2x$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{-2}{2(8)} = \frac{1}{8} = 0.125, x = 0.125$; y-coordinate of vertex: $y = 8(0.125)^2 - 2(0.125) = -0.125$; vertex $(0.125, -0.125)$; points $(0, 0), (1, 6), (-1, 10)$



3. $f(x) = x^2 + 5x - 6$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{5}{2(1)} = -2.5$, $x = -2.5$; y-coordinate of vertex: $y = (-2.5)^2 + 5(-2.5) - 6 = -12.25$; vertex $(-2.5, -12.25)$; points $(0, -6), (1, 0)$



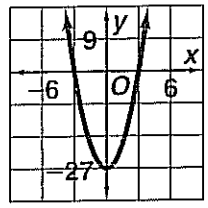
4a. $h = -16t^2 + 24t + 6$; $t = -\frac{b}{2a} = -\frac{24}{2(-16)} = 0.75$; 0.75 s **4b.** $h = -16(0.75)^2 + 24(0.75) + 6 = 15$; 15 ft

5. $\sqrt{100} = 10$ **6.** $\pm\sqrt{0.36} = \pm 0.6$ **7.** $-\sqrt{4} = -2$

8. $t^2 - 64 = 0$; $t^2 = 64$; $t = \pm\sqrt{64} = \pm 8$

9. $6m^2 - 150 = 0$; $6m^2 = 150$; $m^2 = 25$; $m = \pm\sqrt{25} = \pm 5$

10. $3x^2 - 27 = 0$; graph $y = 3x^2 - 27$; $x = \pm 3$



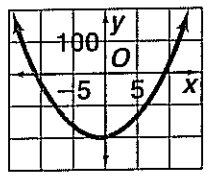
TECHNOLOGY *page 535*

1. 10.78, 13.22 **2.** −2, 8 **3.** −2, 1.5 **4.** −24.37, 0.37

5. 0.28, 17.72 **6.** −4.88, 36.88 **7.** −12, 6 **8a.** $A = \ell w$; $200 = x(2x + 3)$; $2x^2 + 3x - 200 = 0$

8b. graph $y = 2x^2 + 3x - 200$

8c. $x \approx 9.28$, $2x + 3 \approx 2(9.28) + 3 = 21.56$; 9.28 cm by 21.56 cm



10-5 Factoring to Solve Quadratic Equations *pages 536–540*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1. −1 **2.** 104 **3.** $-2\frac{5}{7}$ **4.** $(2c + 1)(c + 14)$

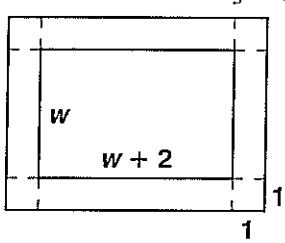
5. $(3p + 2)(p + 10)$ **6.** $(4x + 3)(x - 6)$

Check Understanding 1a. $(x + 7)(x - 4) = 0$; $x + 7 = 0$ or $x - 4 = 0$; $x = -7$ or $x = 4$ **1b.** $(3y - 5)(y - 2) = 0$; $3y - 5 = 0$ or $y - 2 = 0$; $3y = 5$ or $y = 2$; $y = \frac{5}{3}$ or $y = 2$ **1c.** $(6k + 9)(4k - 11) = 0$; $6k + 9 = 0$ or $4k - 11 = 0$; $6k = -9$ or $4k = 11$; $k = -\frac{9}{6} = -\frac{3}{2}$ or $k = \frac{11}{4}$ **2.** $x^2 + x - 12 = 0$; $(x + 4)(x - 3) = 0$; $x + 4 = 0$ or $x - 3 = 0$; $x = -4$ or $x = 3$ **3.** $x^2 - 12x = -36$; $x^2 - 12x + 36 = 0$; $(x - 6)(x - 6) = 0$; $x - 6 = 0$ or $x - 6 = 0$; $x = 6$ **4.** width of material $= x + 2 + 2 = x + 4$; length of material $= x + 1 + 2 + 2 = x + 5$; $(x + 4)(x + 5) = 182$; $x^2 + 9x + 20 = 182$; $x^2 + 9x - 162 = 0$; $(x + 18)(x - 9) = 0$; $x + 18 = 0$ or $x - 9 = 0$; $x = -18$ or $x = 9$, use $x = 9$; 9 in. × 10 in. × 2 in.

Exercises 1. $(x - 3)(x - 7) = 0$; $x - 3 = 0$ or $x - 7 = 0$; $x = 3$ or $x = 7$ **2.** $(x + 4)(2x - 9) = 0$; $x + 4 = 0$ or $2x - 9 = 0$; $x = -4$ or $2x = 9$; $x = -4$ or $x = 4.5$ **3.** $t(t + 1) = 0$; $t = 0$ or $t + 1 = 0$; $t = 0$ or $t = -1$ **4.** $-3n(2n - 5) = 0$; $-3n = 0$ or $2n - 5 = 0$; $n = 0$ or $2n = 5$; $n = 0$ or $n = 2.5$ **5.** $(7x + 2)(5x + 4) = 0$; $7x + 2 = 0$ or $5x + 4 = 0$; $7x = -2$ or $5x = -4$; $x = -\frac{2}{7}$ or $x = -\frac{4}{5}$ **6.** $(4a - 7)(3a + 8) = 0$; $4a - 7 = 0$ or $3a + 8 = 0$; $4a = 7$ or $3a = -8$; $a = \frac{7}{4}$ or $a = -\frac{8}{3}$ **7.** $b^2 + 3b - 4 = 0$; $(b - 1)(b + 4) = 0$; $b - 1 = 0$ or $b + 4 = 0$; $b = 1$ or $b = -4$ **8.** $m^2 - 5m - 14 = 0$; $(m + 2)(m - 7) = 0$; $m + 2 = 0$ or $m - 7 = 0$; $m = -2$ or $m = 7$ **9.** $w^2 - 8w = 0$; $w(w - 8) = 0$; $w = 0$ or $w - 8 = 0$; $w = 0$ or $w = 8$ **10.** $x^2 - 16x + 55 = 0$; $(x - 5)(x - 11) = 0$; $x - 5 = 0$ or $x - 11 = 0$; $x = 5$ or $x = 11$ **11.** $k^2 - 3k - 10 = 0$; $(k + 2)(k - 5) = 0$; $k + 2 = 0$ or $k - 5 = 0$; $k = -2$ or $k = 5$ **12.** $n^2 + n - 12 = 0$; $(n - 3)(n + 4) = 0$; $n - 3 = 0$ or $n + 4 = 0$; $n = 3$ or $n = -4$ **13.** $x^2 + 8x = -15$; $x^2 + 8x + 15 = 0$; $(x + 3)(x + 5) = 0$; $x + 3 = 0$ or $x + 5 = 0$; $x = -3$ or $x = -5$ **14.** $t^2 - 3t = 28$; $t^2 - 3t - 28 = 0$; $(t + 4)(t - 7) = 0$; $t + 4 = 0$ or $t - 7 = 0$; $t = -4$ or $t = 7$ **15.** $n^2 = 6n$; $n^2 - 6n = 0$; $n(n - 6) = 0$; $n = 0$ or $n - 6 = 0$; $n = 0$ or $n = 6$ **16.** $2c^2 - 7c = -5$; $2c^2 - 7c + 5 = 0$; $(2c - 5)(c - 1) = 0$; $2c - 5 = 0$ or $c - 1 = 0$; $2c = 5$ or $c = 1$; $c = 2.5$ or $c = 1$ **17.** $3q^2 + 16q = -5$; $3q^2 + 16q + 5 = 0$; $(3q + 1)(q + 5) = 0$; $3q + 1 = 0$ or $q + 5 = 0$; $3q = -1$ or $q = -5$; $q = -\frac{1}{3}$ or $q = -5$ **18.** $4y^2 = 25$; $4y^2 - 25 = 0$; $(2y - 5)(2y + 5) = 0$; $2y - 5 = 0$ or $2y + 5 = 0$; $2y = 5$ or $2y = -5$; $y = 2.5$ or $y = -2.5$ **19.** $5q^2 + 18q = 8$; $5q^2 + 18q - 8 = 0$; $(5q - 2)(q + 4) = 0$; $5q - 2 = 0$ or $q + 4 = 0$; $5q = 2$ or $q = -4$; $q = \frac{2}{5}$ or $q = -4$ **20.** $2z^2 - 10z = -12$; $2z^2 - 10z + 12 = 0$; $(2z - 6)(z - 2) = 0$; $2z - 6 = 0$ or $z - 2 = 0$; $2z = 6$ or $z = 2$; $z = 3$ or $z = 2$ **21.** $12 = 2x^2 + 5x$; $2x^2 + 5x - 12 = 0$; $(2x - 3)(x + 4) = 0$; $2x - 3 = 0$ or $x + 4 = 0$; $2x = 3$ or $x = -4$; $x = \frac{3}{2}$ or $x = -4$ **22.** $(x + 3)^2 = 64$; $x^2 + 6x + 9 = 64$; $x^2 + 6x - 55 = 0$; $(x - 5)(x + 11) = 0$; $x - 5 = 0$ or $x + 11 = 0$; $x = 5$ or $x = -11$, use $x = 5$; 5 cm **23.** $V = \ell wh$; $280 = 4(n + 2)(n + 5)$; $70 = (n + 2)(n + 5)$; $n^2 + 7n + 10 = 70$; $n^2 + 7n - 60 = 0$; $(n + 12)(n - 5) = 0$; $n + 12 = 0$ or $n - 5 = 0$; $n = -12$ or $n = 5$, use $n = 5$ **24.** $w(2w + 3) = 90$; $2w^2 + 3w - 90 = 0$; $(2w + 15)(w - 6) = 0$; $2w + 15 = 0$ or $w - 6 = 0$; $2w = -15$ or $w = 6$; $w = -7.5$ or $w = 6$, use $w = 6$; $2w + 3 = 2(6) + 3 = 15$; 6 ft × 15 ft

25. $A = \frac{1}{2}bh$; $110 = \frac{1}{2}x(2x + 2)$; $110 = x^2 + x$; $x^2 + x - 110 = 0$; $(x - 10)(x + 11) = 0$; $x - 10 = 0$ or $x + 11 = 0$; $x = 10$ or $x = -11$, use $x = 10$; $2x + 2 = 2(10) + 2 = 22$; the base is 10 ft, and the height is 22 ft. **26.** $x(x + 1) = 10x - 14$; $x^2 + x = 10x - 14$; $x^2 - 9x + 14 = 0$; $(x - 2)(x - 7) = 0$; $x - 2 = 0$ or $x - 7 = 0$; $x = 2$ or $x = 7$; the numbers are 2 and 3, or 7 and 8. **27.** $2q^2 + 22q = -60$; $2q^2 + 22q + 60 = 0$; $q^2 + 11q + 30 = 0$; $(q + 6)(q + 5) = 0$; $q + 6 = 0$ or

$q + 5 = 0; q = -6$ or $q = -5$ **28.** $4 = -5n + 6n^2$; $6n^2 - 5n - 4 = 0; (3n - 4)(2n + 1) = 0; 3n - 4 = 0$ or $2n + 1 = 0; 3n = 4$ or $2n = -1; n = \frac{4}{3}$ or $n = -\frac{1}{2}$
29. $6y^2 + 12y + 13 = 2y^2 + 4; 4y^2 + 12y + 9 = 0$; $(2y + 3)(2y + 3) = 0; 2y + 3 = 0; 2y = -3; y = -\frac{3}{2}$
30. $3a^2 + 4a = 2a^2 - 2a - 9; a^2 + 6a + 9 = 0$; $(a + 3)(a + 3) = 0; a + 3 = 0; a = -3$
31. $3t^2 + 8t = t^2 - 3t - 12; 2t^2 + 11t + 12 = 0$; $(2t + 3)(t + 4) = 0; 2t + 3 = 0$ or $t + 4 = 0; 2t = -3$ or $t = -4; t = -1.5$ or $t = -4$ **32.** $4x^2 + 20 = 10x + 3x^2 - 4; x^2 - 10x + 24 = 0; (x - 4)(x - 6) = 0$; $x - 4 = 0$ or $x - 6 = 0; x = 4$ or $x = 6$
33. $2k^2 - 3 + 12k = k + 60; 2k^2 + 11k - 63 = 0$; $(2k - 7)(k + 9) = 0; 2k - 7 = 0$ or $k + 9 = 0$; $2k = 7$ or $k = -9; k = \frac{7}{2}$ or $k = -9$
34. $15y^2 + 45y - 9 = 4y - 5y^2; 20y^2 + 41y - 9 = 0$; $(5y - 1)(4y + 9) = 0; 5y - 1 = 0$ or $4y + 9 = 0$; $5y = 1$ or $4y = -9; y = \frac{1}{5}$ or $y = -\frac{9}{4}$
35.



width of material $= w + 1 + 1 = w + 2$; length of material $= w + 2 + 1 + 1 = w + 4$; $(w + 2)(w + 4) = 80; w^2 + 6w + 8 = 80$; $w^2 + 6w - 72 = 0; (w - 6)(w + 12) = 0; w - 6 = 0$ or $w + 12 = 0; w = 6$ or $w = -12$, use $w = 6; w + 2 = 8$, $w + 4 = 10$; 8 in. × 10 in.
36a. $-16t^2 + 29t + 6 = 0; 16t^2 - 29t - 6 = 0$; $(16t + 3)(t - 2) = 0; 16t + 3 = 0$ or $t - 2 = 0; 16t = -3$ or $t = 2; t = -\frac{3}{16}$ or $t = 2$, use $t = 2$; 2 s **36b.** Vertex of graph is about (0.9, 19.14); about 19 ft. **37.** Answers may vary. Sample: To solve a quadratic equation, write the equation in standard form, factor the quadratic expression, use the Zero-Product Property, and solve for the variable. $x^2 + 8x = -15; x^2 + 8x + 15 = 0$; $(x + 3)(x + 5) = 0; x + 3 = 0$ or $x + 5 = 0; x = -3$ or $x = -5$ **38.** Answers may vary. Sample: $A = 56 = 8 \cdot 7$, so $x = 6, a = 2$, and $b = 1; A = 56 = 4 \cdot 14$, so $x = 3$, $a = 1$, and $b = 11$. **39.** Answers may vary. Sample: $x^2 - 2x - 8 = 0; (x - 4)(x + 2) = 0; x - 4 = 0$ or $x + 2 = 0; x = 4$ or $x = -2$ **40a.** ① $x^2 = x; x^2 - x = 0$; $x(x - 1) = 0; x = 0$ or $x - 1 = 0; x = 0$ or $x = 1$ ② $x^2 = -x; x^2 + x = 0; x(x + 1) = 0; x = 0$ or $x + 1 = 0$; $x = 0$ or $x = -1$ **40b.** 0 **41.** $x^3 - 10x^2 + 24x = 0$; $x(x^2 - 10x + 24) = 0; x(x - 4)(x - 6) = 0; x = 0$ or $x - 4 = 0$ or $x - 6 = 0; x = 0$ or $x = 4$ or $x = 6$
42. $x^3 - 5x^2 + 4x = 0; x(x^2 - 5x + 4) = 0$; $x(x - 1)(x - 4) = 0; x = 0$ or $x - 1 = 0$ or $x - 4 = 0$; $x = 0$ or $x = 1$ or $x = 4$ **43.** $3x^3 - 9x^2 = 0$; $3x^2(x - 3) = 0; 3x^2 = 0$ or $x - 3 = 0; x = 0$ or $x = 3$
44. $x^3 + 3x^2 - 70x = 0; x(x^2 + 3x - 70) = 0$; $x(x - 7)(x + 10) = 0; x = 0$ or $x - 7 = 0$ or $x + 10 = 0$; $x = 0$ or $x = 7$ or $x = -10$ **45.** $3x^3 - 30x^2 + 27x = 0$; $3x(x^2 - 10x + 9) = 0; 3x(x - 1)(x - 9) = 0; 3x = 0$ or $x - 1 = 0$ or $x - 9 = 0; x = 0$ or $x = 1$ or $x = 9$
46. $2x^3 = -2x^2 + 40x; 2x^3 + 2x^2 - 40x = 0$; $2x(x^2 + x - 20) = 0; 2x(x - 4)(x + 5) = 0; 2x = 0$ or $x - 4 = 0$ or $x + 5 = 0; x = 0$ or $x = 4$ or $x = -5$
47. length $= 5 + x + 5 = x + 10$; width $= 3 + x + 3 = x + 6; 124 = (x + 10)(x + 6) - 2x - 2x$; $x^2 + 16x + 60 - 4x - 124 = 0; x^2 + 12x - 64 = 0$; $(x - 4)(x + 16) = 0; x - 4 = 0$ or $x + 16 = 0; x = 4$ or $x = -16$, use $x = 4$ **48.** Answers may vary. Samples are given. **48a.** $(x + 5)(x - 8) = 0; x^2 - 3x - 40 = 0$
48b. $(x - 3)(x + 2) = 0; x^2 - x - 6 = 0$
48c. $(x - \frac{1}{2})(x + 10) = 0; x^2 + \frac{19}{2}x - 5 = 0$; $2x^2 + 19x - 10 = 0$ **48d.** $(3x - 2)(7x + 5) = 0$; $21x^2 + x - 10 = 0$ **49.** $x^3 + 5x^2 - x - 5 = 0$; $x^2(x + 5) - 1(x + 5) = 0; (x^2 - 1)(x + 5) = 0$; $(x - 1)(x + 1)(x + 5) = 0; x - 1 = 0$ or $x + 1 = 0$ or $x + 5 = 0; x = 1$ or $x = -1$ or $x = -5$
50. $x^3 + x^2 - 4x - 4 = 0; x^2(x + 1) - 4(x + 1) = 0$; $(x^2 - 4)(x + 1) = 0; (x - 2)(x + 2)(x + 1) = 0$; $x - 2 = 0$ or $x + 2 = 0$ or $x + 1 = 0; x = 2$ or $x = -2$ or $x = -1$ **51.** $(x - 7)(2x + 8) = 0; x - 7 = 0$ or $2x + 8 = 0; x = 7$ or $2x = -8; x = 7$ or $x = -4$; D
52. $4x^2 - 35x - 9 = 0; (4x + 1)(x - 9) = 0; 4x + 1 = 0$ or $x - 9 = 0; 4x = -1$ or $x = 9; x = -\frac{1}{4}$ or $x = 9$; sum of solutions: $-\frac{1}{4} + 9 = 8\frac{3}{4} = 8.75$; H **53.** A. Test -3: $-3(2(-3) - 5)(10(-3) + 3) = -3(-11)(-27) = -891 \neq 0$. B. Test -0.3: $(-0.3)(2(-0.3) - 5)(10(-0.3) + 3) = (-0.3)(-5.6)(0) = 0$. C. Test 0: $0(2(0) - 5)(10(0) + 3) = 0(-5)(3) = 0$. D. Test 2.5: $2.5(2(2.5) - 5)(10(2.5) + 3) = 2.5(0)(28) = 0$. The answer is A. **54.** $3x^2 = 11$ has two solutions, and $5y^2 = 12$ has two solutions, so the answer is C. **55.** $2n^2$ and $(2n)^2$; the answer is D.
56. [2] $3x^2 + 20x + 1 = 8; 3x^2 + 20x - 7 = 0$; $(3x - 1)(x + 7) = 0; x = \frac{1}{3}, x = -7$ [1] appropriate methods, but with one computational error
57. $x^2 = 320; x = \sqrt{320} \approx 17.9$; 17.9 ft **58.** $38 = \pi r^2$; $r^2 = \frac{38}{\pi}; r = \sqrt{\frac{38}{\pi}} \approx 3.5$; 3.5 ft **59.** $2x^2 + 13x + 15 = (2x + 3)(x + 5)$ **60.** $3y^2 - 10y + 3 = (3y - 1)(y - 3)$
61. $4t^2 + 5t - 6 = (4t - 3)(t + 2)$ **62.** $6n^2 + 7n - 3 = (3n - 1)(2n + 3)$ **63.** $15a^3 - 50a^2 - 40a = 5a(3a^2 - 10a - 8) = 5a(3a + 2)(a - 4)$
64. $-18b^3 + 42b^2 - 20b = -2b(9b^2 - 21b + 10) = -2b(3b - 2)(3b - 5)$

10-6 Completing the Square
pages 541–546

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1. $d^2 - 8d + 16$ **2.** $x^2 + 22x + 121$ **3.** $k^2 - 16k + 64$ **4.** $(b + 5)^2$ **5.** $(t + 7)^2$ **6.** $(n - 9)^2$

Check Understanding 1. $\left(-\frac{22}{2}\right)^2 = (-11)^2 = 121$

2. $m^2 - 6m = 247; m^2 - 6m + 9 = 247 + 9; (m - 3)^2 = 256; m - 3 = \pm\sqrt{256} = \pm 16; m - 3 = 16$ or

$m - 3 = -16$; $m = 19$ or $m = -13$ **3a.** $x^2 + 5x + 3 = 0$; $x^2 + 5x = -3$; $x^2 + 5x + \frac{25}{4} = -3 + \frac{25}{4}$; $\left(x + \frac{5}{2}\right)^2 = \frac{13}{4}$; $x + \frac{5}{2} = \pm\sqrt{\frac{13}{4}} \approx \pm 1.80$; $x \approx -\frac{5}{2} + 1.80$ or $x \approx -\frac{5}{2} - 1.80$; $x \approx -0.70$ or $x \approx -4.30$ **3b.** $x^2 - 14x + 16 = 0$; $x^2 - 14x = -16$; $x^2 - 14x + 49 = -16 + 49$; $(x - 7)^2 = 33$; $x - 7 = \pm\sqrt{33} \approx \pm 5.74$; $x \approx 7 + 5.74$ or $x \approx 7 - 5.74$; $x \approx 12.74$ or $x \approx 1.26$ **4a.** $4a^2 - 8a = 24$; $a^2 - 2a = 6$; $a^2 - 2a + 1 = 6 + 1$; $(a - 1)^2 = 7$; $a - 1 = \pm\sqrt{7} \approx \pm 2.65$; $a \approx 1 + \sqrt{7}$ or $a \approx 1 - \sqrt{7}$; $a \approx 3.65$ or $a \approx -1.65$ **4b.** $5n^2 - 3n - 15 = 10$; $5n^2 - 3n = 25$; $n^2 - \frac{3}{5}n = 5$; $n^2 - \frac{3}{5}n + \frac{9}{100} = 5 + \frac{9}{100}$; $\left(n - \frac{3}{10}\right)^2 = \frac{509}{100}$; $n - \frac{3}{10} = \pm\sqrt{\frac{509}{100}} \approx \pm 2.26$; $n \approx \frac{3}{10} + 2.26$ or $n \approx \frac{3}{10} - 2.26$; $n \approx 2.56$ or $n \approx -1.96$

Exercises **1.** $\left(\frac{14}{2}\right)^2 = 7^2 = 49$ **2.** $\left(-\frac{8}{2}\right)^2 = (-4)^2 = 16$ **3.** $\left(-\frac{40}{2}\right)^2 = (-20)^2 = 400$ **4.** $\left(-\frac{6}{2}\right)^2 = (-3)^2 = 9$ **5.** $\left(\frac{24}{2}\right)^2 = 12^2 = 144$ **6.** $\left(-\frac{36}{2}\right)^2 = (-18)^2 = 324$ **7.** $r^2 + 8r = 48$; $r^2 + 8r + 16 = 48 + 16$; $(r + 4)^2 = 64$; $r + 4 = \pm\sqrt{64} = \pm 8$; $r + 4 = 8$ or $r + 4 = -8$; $r = 4$ or $r = -12$ **8.** $x^2 - 10x = 40$; $x^2 - 10x + 25 = 40 + 25$; $(x - 5)^2 = 65$; $x - 5 = \pm\sqrt{65}$; $x - 5 = \sqrt{65}$ or $x - 5 = -\sqrt{65}$; $x = 5 + \sqrt{65} \approx 13.06$ or $x = 5 - \sqrt{65} \approx -3.06$ **9.** $q^2 + 22q = -85$; $q^2 + 22q + 121 = -85 + 121$; $(q + 11)^2 = 36$; $q + 11 = \pm\sqrt{36} = \pm 6$; $q + 11 = 6$ or $q + 11 = -6$; $q = -5$ or $q = -17$ **10.** $m^2 + 6m = 9$; $m^2 + 6m + 9 = 9 + 9$; $(m + 3)^2 = 18$; $m + 3 = \pm\sqrt{18}$; $m + 3 = \sqrt{18}$ or $m + 3 = -\sqrt{18}$; $m = -3 + \sqrt{18} \approx 1.24$ or $m = -3 - \sqrt{18} \approx -7.24$ **11.** $r^2 + 20r = 261$; $r^2 + 20r + 100 = 261 + 100$; $(r + 10)^2 = 361$; $r + 10 = \pm\sqrt{361} = \pm 19$; $r + 10 = 19$ or $r + 10 = -19$; $r = 9$ or $r = -29$ **12.** $g^2 - 2g = 323$; $g^2 - 2g + 1 = 323 + 1$; $(g - 1)^2 = 324$; $g - 1 = \pm\sqrt{324} = \pm 18$; $g - 1 = 18$ or $g - 1 = -18$; $g = 19$ or $g = -17$ **13.** $r^2 - 2r - 35 = 0$; $r^2 - 2r = 35$; $r^2 - 2r + 1 = 35 + 1$; $(r - 1)^2 = 36$; $r - 1 = \pm\sqrt{36} = \pm 6$; $r - 1 = 6$ or $r - 1 = -6$; $r = 7$ or $r = -5$ **14.** $x^2 + 10x + 17 = 0$; $x^2 + 10x = -17$; $x^2 + 10x + 25 = -17 + 25$; $(x + 5)^2 = 8$; $x + 5 = \pm\sqrt{8} \approx \pm 2.83$; $x + 5 \approx 2.83$ or $x + 5 \approx -2.83$; $x \approx -5 + 2.83 = -2.17$ or $x \approx -5 - 2.83 = -7.83$ **15.** $p^2 - 12p + 11 = 0$; $p^2 - 12p = -11$; $p^2 - 12p + 36 = -11 + 36$; $(p - 6)^2 = 25$; $p - 6 = \pm\sqrt{25} = \pm 5$; $p - 6 = 5$ or $p - 6 = -5$; $p = 11$ or $p = 1$ **16.** $w^2 + 3w - 5 = 0$; $w^2 + 3w = 5$; $w^2 + 3w + \frac{9}{4} = 5 + \frac{9}{4}$; $\left(w + \frac{3}{2}\right)^2 = \frac{29}{4}$; $w + \frac{3}{2} = \pm\sqrt{\frac{29}{4}} \approx \pm 2.69$; $w + \frac{3}{2} \approx 2.69$ or $w + \frac{3}{2} \approx -2.69$; $w \approx 2.69 - 1.5 = 1.19$ or $w \approx -2.69 - 1.5 = -4.19$ **17.** $m^2 + m - 28 = 0$; $m^2 + m = 28$; $m^2 + m + \frac{1}{4} = 28 + \frac{1}{4}$; $\left(m + \frac{1}{2}\right)^2 = \frac{113}{4}$; $m + \frac{1}{2} = \pm\sqrt{\frac{113}{4}} \approx \pm 5.32$; $m + \frac{1}{2} \approx 5.32$ or $m + \frac{1}{2} \approx -5.32$; $m \approx -0.5 + 5.32 = 4.82$ or $m \approx -0.5 - 5.32 = -5.82$ **18.** $a^2 + 9a - 682 = 0$; $a^2 + 9a = 682$; $a^2 + 9a + \frac{81}{4} = 682 + \frac{81}{4}$; $\left(a + \frac{9}{2}\right)^2 = \frac{2809}{4}$; $a + \frac{9}{2} = \pm\sqrt{\frac{2809}{4}} = \pm 26.5$; $a + \frac{9}{2} = 26.5$ or $a + \frac{9}{2} = -26.5$; $a = -4.5 + 26.5 = 22$ or $a = -4.5 - 26.5 = -31$ **19.** $2k^2 + 4k = 10$; $k^2 + 2k = 5$; $k^2 + 2k + 1 = 5 + 1$; add 1. **20.** $3x^2 + 12x = 24$; $x^2 + 4x = 8$; $x^2 + 4x + 4 = 8 + 4$; add 4. **21.** $5t^2 + 9t = 15$; $t^2 + \frac{9}{5}t = 3$; $t^2 + \frac{9}{5}t + \frac{81}{100} = 3 + \frac{81}{100}$; add $\frac{81}{100}$. **22.** $4y^2 + 8y - 36 = 0$; $y^2 + 2y = 9$; $y^2 + 2y + 1 = 9 + 1$; $(y + 1)^2 = 10$; $y + 1 = \pm\sqrt{10} \approx \pm 3.16$; $y + 1 \approx 3.16$ or $y + 1 \approx -3.16$; $y \approx -1 + 3.16 = 2.16$ or $y \approx -1 - 3.16 = -4.16$ **23.** $3q^2 - 12q = 15$; $q^2 - 4q = 5$; $q^2 - 4q + 4 = 5 + 4$; $(q - 2)^2 = 9$; $q - 2 = \pm\sqrt{9} = \pm 3$; $q - 2 = 3$ or $q - 2 = -3$; $q = 5$ or $q = -1$ **24.** $2x^2 - 10x - 20 = 8$; $2x^2 - 10x = 28$; $x^2 - 5x = 14$; $x^2 - 5x + \frac{25}{4} = 14 + \frac{25}{4}$; $\left(x - \frac{5}{2}\right)^2 = \frac{81}{4}$; $x - \frac{5}{2} = \pm\sqrt{\frac{81}{4}} = \pm\frac{9}{2}$; $x - \frac{5}{2} = \frac{9}{2}$ or $x - \frac{5}{2} = -\frac{9}{2}$; $x = \frac{14}{2} = 7$ or $x = -\frac{4}{2} = -2$ **25a.** length: $1 + x + x = 2x + 1$; width: $x + 1$; area: $(2x + 1)(x + 1)$ **25b.** $(2x + 1)(x + 1) = 28$; $2x^2 + 3x + 1 = 28$ **25c.** $2x^2 + 3x = 27$; $x^2 + \frac{3}{2}x = \frac{27}{2}$; $x^2 + \frac{3}{2}x + \frac{9}{16} = \frac{27}{2} + \frac{9}{16}$; $\left(x + \frac{3}{4}\right)^2 = \frac{225}{16}$; $x + \frac{3}{4} = \pm\sqrt{\frac{225}{16}} = \pm\frac{15}{4}$; $x + \frac{3}{4} = \frac{15}{4}$ or $x + \frac{3}{4} = -\frac{15}{4}$; $x = \frac{12}{4} = 3$ or $x = -\frac{18}{4} = -4\frac{1}{2}$, use $x = 3$ **26.** $b^2 + 4b + 1 = 0$; $b^2 + 4b = -1$; $b^2 + 4b + 4 = -1 + 4$; $(b + 2)^2 = 3$; $b + 2 = \pm\sqrt{3} \approx \pm 1.73$; $b + 2 \approx 1.73$ or $b + 2 \approx -1.73$; $b \approx -2 + 1.73 = -0.27$ or $b \approx -2 - 1.73 = -3.73$ **27.** $c^2 + 7c = -12$; $c^2 + 7c + \frac{49}{4} = -12 + \frac{49}{4}$; $\left(c + \frac{7}{2}\right)^2 = \frac{1}{4}$; $c + \frac{7}{2} = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$; $c + \frac{7}{2} = \frac{1}{2}$ or $c + \frac{7}{2} = -\frac{1}{2}$; $c = -\frac{6}{2} = -3$ or $c = -\frac{8}{2} = -4$ **28.** $h^2 + 6h - 40 = 0$; $h^2 + 6h = 40$; $h^2 + 6h + 9 = 40 + 9$; $(h + 3)^2 = 49$; $h + 3 = \pm\sqrt{49} = \pm 7$; $h + 3 = 7$ or $h + 3 = -7$; $h = 4$ or $h = -10$ **29.** $y^2 - 8y = -12$; $y^2 - 8y + 16 = -12 + 16$; $(y - 4)^2 = 4$; $y - 4 = \pm\sqrt{4} = \pm 2$; $y - 4 = 2$ or $y - 4 = -2$; $y = 6$ or $y = 2$ **30.** $4m^2 - 40m + 56 = 0$; $4m^2 - 40m = -56$; $m^2 - 10m = -14$; $m^2 - 10m + 25 = -14 + 25$; $(m - 5)^2 = 11$; $m - 5 = \pm\sqrt{11} \approx \pm 3.32$; $m - 5 \approx 3.32$ or $m - 5 \approx -3.32$; $m \approx 5 + 3.32 = 8.32$ or $m \approx 5 - 3.32 = 1.68$ **31.** $k^2 + 4k + 11 = -10$; $k^2 + 4k = -21$; $k^2 + 4k + 4 = -21 + 4$; $(k + 2)^2 = -17$; no solution **32.** $2x^2 - 15x + 6 = 41$; $2x^2 - 15x = 35$; $x^2 - \frac{15}{2}x = \frac{35}{2}$; $x^2 - \frac{15}{2}x + \frac{225}{16} = \frac{35}{2} + \frac{225}{16}$; $\left(x - \frac{15}{4}\right)^2 = \frac{505}{16}$; $x - \frac{15}{4} = \pm\sqrt{\frac{505}{16}} \approx \pm 5.62$; $x - 3.75 \approx 5.62$ or $x - 3.75 \approx -5.62$; $x \approx 3.75 + 5.62 = 9.37$ or $x \approx 3.75 - 5.62 = -1.87$ **33.** $3d^2 - 24d = 3$; $d^2 - 8d = 1$;

$d^2 - 8d + 16 = 1 + 16; (d-4)^2 = 17; d - 4 = \pm\sqrt{17} \approx \pm 4.12; d - 4 \approx 4.12$ or $d - 4 \approx -4.12; d \approx 4 + 4.12 = 8.12$ or $d \approx 4 - 4.12 = -0.12$ **34.** $x^2 + 9x + 20 = 0; x^2 + 9x = -20; x^2 + 9x + \frac{81}{4} = -20 + \frac{81}{4}; \left(x + \frac{9}{2}\right)^2 = \frac{1}{4}; x + \frac{9}{2} = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}; x + \frac{9}{2} = \frac{1}{2}$ or $x + \frac{9}{2} = -\frac{1}{2}; x = -\frac{8}{2} = -4$ or $x = -\frac{10}{2} = -5$ **35a.** $\ell + 2w = 50; \ell = 50 - 2w$ **35b.** $w\ell = 150; w(50 - 2w) = 150; -2w^2 + 50w = 150; w^2 - 25w = -75; w^2 - 25w + \frac{625}{4} = -75 + \frac{625}{4}; \left(w - \frac{25}{2}\right)^2 = \frac{325}{4}; w - \frac{25}{2} = \pm\sqrt{\frac{325}{4}} \approx \pm 9.0; w - 12.5 \approx 9.0$ or $w - 12.5 \approx -9.0; w \approx 12.5 + 9.0 = 21.5$ or $w \approx 12.5 - 9.0 = 3.5$ **35c.** If $w = 21.5$, then $\ell = 50 - 2(21.5) = 7$, so 7 ft × 21.5 ft. Or, if $w = 3.5$, then $\ell = 50 - 2(3.5) = 43$, so 43 ft × 3.5 ft. **35d.** $7(21.5) = 150.5$, and $43(3.5) = 150.5$; no, because the answers in part (b) were rounded. **36.** The student did not divide each side of the equation by 4.

37. Answers may vary. Sample: Add 1 to each side of the equation, and then complete the square by adding 225 to each side of the equation. Write $x^2 + 30x + 225$ as the square $(x + 15)^2$ and add 1 and 225 to get 226. Then take square roots and solve the resulting equations.

38. Answers may vary. Sample: $x^2 + 10x - 50 = 0; x^2 + 10x = 50; x^2 + 10x + 25 = 50 + 25; (x + 5)^2 = 75; x + 5 = \pm\sqrt{75} \approx \pm 8.7; x + 5 \approx 8.7$ or $x + 5 \approx -8.7; x \approx 3.7$ or $x \approx -13.7$ **39.** Estimates may vary. Sample: Locate $\approx (-1, 5)$ and $\approx (5, 5)$, so the values of x are about -1 and 5. Equation: $x^2 - 4x - 1 = 5; x^2 - 4x = 6; x^2 - 4x + 4 = 6 + 4; (x - 2)^2 = 10; x - 2 = \pm\sqrt{10} \approx \pm 3.16; x - 2 \approx 3.16$ or $x - 2 \approx -3.16; x \approx 5.16$ or $x \approx -1.16$ **40.** Estimates may vary. Sample: Locate $\approx (1, 5)$ and $\approx (7, 5)$, so the values of x are about 1 and 7. Equation: $-\frac{1}{2}x^2 + 4x + 1 = 5; -\frac{1}{2}x^2 + 4x = 4; x^2 - 8x = -8; x^2 - 8x + 16 = -8 + 16; (x - 4)^2 = 8; x - 4 = \pm\sqrt{8} \approx \pm 2.83; x - 4 \approx 2.83$ or $x - 4 \approx -2.83; x \approx 6.83$ or $x \approx 1.17$ **41.** length $= x$, width $= 2x + 3$; Area $= \ell w$; $x(2x + 3) = 80; 2x^2 + 3x = 80; x^2 + \frac{3}{2}x = 40; x^2 + \frac{3}{2}x + \frac{9}{16} = 40 + \frac{9}{16}; \left(x + \frac{3}{4}\right)^2 = \frac{649}{16}; x + \frac{3}{4} = \pm\sqrt{\frac{649}{16}} \approx \pm 6.37; x + 0.75 \approx 6.37$ or $x + 0.75 \approx -6.37; x \approx 5.62$ or $x \approx -7.12$, use $x \approx 5.62 \approx 5.6$; width $= x = 5.6$, and length $= 2x + 3 = 2(5.6) + 3 = 14.2$; 5.6 ft by 14.2 ft **42a.** $2[x(x + 7) + x^2 + x(x + 7)] = 2(x^2 + 7x + x^2 + x^2 + 7x) = 2(3x^2 + 14x) = 6x^2 + 28x$ **42b.** $6x^2 + 28x = 6 \cdot 8^2; 6x^2 + 28x = 384$ **42c.** $x^2 + \frac{28}{6}x = \frac{384}{6}; x^2 + \frac{14}{3}x = 64; x^2 + \frac{14}{3}x + \frac{196}{36} = 64 + \frac{196}{36}; \left(x + \frac{7}{3}\right)^2 = \frac{2500}{36}; x + \frac{7}{3} = \pm\sqrt{\frac{2500}{36}} = \pm\frac{50}{6} = \pm\frac{25}{3}; x + \frac{7}{3} = \frac{25}{3}$ or $x + \frac{7}{3} = -\frac{25}{3}; x = \frac{18}{3} = 6$ or $x = -\frac{32}{3}$, use $x = 6; x + 7 = 6 + 7 = 13$; 13 in. × 6 in. × 6 in. **43a.** length $= 1 + x + x + x + x = 4x + 1$, width $= x + 1$, area to subtract out $= \frac{1}{2}x^2; A = (4x + 1)(x + 1) - \frac{1}{2}x^2 = 4x^2 + 5x + 1 - \frac{1}{2}x^2 = \frac{7}{2}x^2 + 5x + 1$ **43b.** $\frac{7}{2}x^2 + 5x + 1 = 200; \frac{7}{2}x^2 + 5x = 199; x^2 + \frac{10}{7}x = \frac{398}{7}; x^2 + \frac{10}{7}x + \frac{100}{196} = \frac{398}{7} + \frac{100}{196}; \left(x + \frac{5}{7}\right)^2 = \frac{11{,}244}{196}; x + \frac{5}{7} = \pm\sqrt{\frac{11{,}244}{196}} \approx \pm 7.57; x + \frac{5}{7} \approx 7.57$ or $x + \frac{5}{7} \approx -7.57; x \approx 6.86$ or $x \approx -8.28$, use $x \approx 6.86$ **43c.** $x \approx 6.86 \approx 7; A \approx \frac{7}{2} \cdot 7^2 + 5 \cdot 7 + 1 = 207.5$; 207.5 ft^2 **44a.** $x^2 - 6x + 4 = 0; x^2 - 6x = -4; x^2 - 6x + 9 = -4 + 9; (x - 3)^2 = 5; x - 3 = \pm\sqrt{5}; x = 3 \pm \sqrt{5}$ **44b.** $y = x^2 - 6x + 4; x = -\frac{b}{2a} = -\frac{-6}{2(1)} = 3; y = 3^2 - 6(3) + 4 = -5$; vertex $(3, -5)$ **44c.** Answers may vary. Sample: p is the x-coordinate of the vertex. **45.** $x^2 + bx + 100; 2\sqrt{100} = 2(10) = 20$; B **46.** F. $t^2 - 14t + 49 = (t - 7)^2$
G. $9b^2 + 66b + 121 = (3b + 11)^2$
H. $4m^2 - 24m + 36 = (2m - 6)^2$
I. $81k^2 - 120k + 100 \neq (9k - 10)^2$; the answer is I.
47. $x^2 + 6x - 11 = 0; x^2 + 6x = 11; x^2 + 6x + 9 = 11 + 9; (x + 3)^2 = 20; x + 3 = t\sqrt{20} \approx \pm 4.47; x + 3 \approx 4.47$ or $x + 3 \approx -4.47; x \approx 1.47$ or $x \approx -7.47$; D
48. [2] $\frac{1}{2}(x)(x + x + 4) = 200; \frac{1}{2}(x)(2x + 4) = 200; (x)(x + 2) = 200; x^2 + 2x = 200; x^2 + 2x + 1 = 201; (x + 1)^2 = 201; x + 1 \approx \pm 14.18; x = 13.18$ or $x = -15.18$; The value of x is about 13.18 cm. [1] appropriate methods, but with one computational error
49. [4] **49a.** $(8 + x)(12 + x) = 2 \cdot (8 \cdot 12); x^2 + 20x - 96 = 0$ **49b.** $x^2 + 20x = 96; x^2 + 20x + 100 = 196; (x + 10)^2 = 196; x + 10 = \pm 14; x = 4$ **49c.** 12 ft by 16 ft [3] appropriate methods, but with one computational error [2] part (c) not done [1] no work shown
50. $x^2 - 4x - 21 = 0; (x + 3)(x - 7) = 0; x + 3 = 0$ or $x - 7 = 0; x = -3$ or $x = 7$ **51.** $n^2 + 11n + 30 = 0; (n + 6)(n + 5) = 0; n + 6 = 0$ or $n + 5 = 0; n = -6$ or $n = -5$ **52.** $t^2 - 5t = 0; t(t - 5) = 0; t = 0$ or $t - 5 = 0; t = 0$ or $t = 5$ **53.** $9v^2 - 64 = 0; (3v - 8)(3v + 8) = 0; 3v - 8 = 0$ or $3v + 8 = 0; 3v = 8$ or $3v = -8; v = \frac{8}{3}$ or $v = -\frac{8}{3}$ **54.** $4c^2 + 12c = -9; 4c^2 + 12c + 9 = 0; (2c + 3)(2c + 3) = 0; 2c + 3 = 0; 2c = -3; c = -\frac{3}{2}$ **55.** $12w^2 = 28w + 5; 12w^2 - 28w - 5 = 0; (6w + 1)(2w - 5) = 0; 6w + 1 = 0$ or $2w - 5 = 0; 6w = -1$ or $2w = 5; w = -\frac{1}{6}$ or $w = \frac{5}{2}$ **56.** $x^2 + 4x + 4 = x^2 + 2(2x) + 2^2 = (x + 2)^2$ **57.** $t^2 - 22t + 121 = t^2 - 2(11t) + 11^2 = (t - 11)^2$ **58.** $b^2 - 25 = b^2 - 5^2 = (b + 5)(b - 5)$ **59.** $16c^2 + 24c + 9 = (4c)^2 + 2(4c)(3) + 3^2 = (4c + 3)^2$ **60.** $49s^2 - 169 = (7s)^2 - 13^2 = (7s + 13)(7s - 13)$ **61.** $8m^3 - 18m = 2m(4m^2 - 9) = 2m((2m)^2 - 3^2) = 2m(2m + 3)(2m - 3)$ **62.** $25m^2 + 120m + 144 = (5m)^2 + 2(5m)(12) + 12^2 = (5m + 12)^2$ **63.** $400k^2 - 9 = (20k)^2 - 3^2 = (20k + 3)(20k - 3)$ **64.** $256g^2 - 121 = (16g)^2 - 11^2 = (16g + 11)(16g - 11)$ **65.** $(r^3)^4 = r^{3 \cdot 4} = r^{12}$
66. $p(p^2)^6 = p(p^{2 \cdot 6}) = p(p^{12}) = p^{1 + 12} = p^{13}$
67. $-y^3(y^{-1})^2 = -y^3y^{-2} = -\frac{y^3}{y^2} = -y$ **68.** $(m^5)^{-8} = m^{-40} = \frac{1}{m^{40}}$ **69.** $-w^7(w^8)^{-1} = -w^7w^{-8} = -\frac{w^7}{w^8} = -\frac{1}{w}$
70. $t^8(t^{-7})^{-3} = t^8t^{21} = t^{8+21} = t^{29}$

10.7 Using the Quadratic Formula pages 547–552

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1. 9 **2.** $\frac{49}{4}$ **3.** $\frac{81}{4}$ **4.** 6, 4 **5.** 2, −18 **6.** 1, −5 **7.** −7, 8

Check Understanding **1a.** $x^2 - 2x - 8 = 0; a = 1, b = -2, c = -8; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-8)}}{2(1)} = \frac{2 \pm \sqrt{36}}{2} = \frac{2 \pm 6}{2} = 1 \pm 3;$ $x = 1 + 3 = 4$ or $x = 1 - 3 = -2$ **1b.** $x^2 - 4x = 117;$ $x^2 - 4x - 117 = 0; a = 1, b = -4, c = -117; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-117)}}{2(1)} = \frac{4 \pm \sqrt{484}}{2} = \frac{4 \pm 22}{2} = 2 \pm 11; x = 2 + 11 = 13$ or $x = 2 - 11 = -9$ **2a.** $-3x^2 + 5x - 2 = 0; a = -3, b = 5, c = -2; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{5^2 - 4(-3)(-2)}}{2(-3)} = \frac{-5 \pm \sqrt{1}}{-6} = \frac{-5 \pm 1}{-6}; x = \frac{-5 + 1}{-6} = \frac{2}{3} \approx 0.67$ or $x = \frac{-5 - 1}{-6} = 1$ **2b.** $7x^2 - 2x - 8 = 0; a = 7, b = -2, c = -8;$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(7)(-8)}}{2(7)} = \frac{2 \pm \sqrt{228}}{14}; x \approx \frac{2 + 15.10}{14} \approx 1.22$ or $x \approx \frac{2 - 15.10}{14} \approx -0.94$ **3a.** $0 = -16t^2 + 38.4t + 3.5$ **3b.** $a = -16, b = 38.4, c = 3.5; t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-38.4 \pm \sqrt{38.4^2 - 4(-16)(3.5)}}{2(-16)} = \frac{-38.4 \pm \sqrt{1698.56}}{-32}; t \approx \frac{-38.4 + 41.21}{-32} \approx -0.1$ or $t \approx \frac{-38.4 - 41.21}{-32} \approx 2.5$; 2.5 s

4a. Quadratic formula; the equation cannot be factored easily. **4b.** Factoring; the equation is easily factorable. **4c.** Square roots; there is no x term.

Exercises **1.** $2x^2 + 5x + 3 = 0; a = 2, b = 5, c = 3;$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{5^2 - 4(2)(3)}}{2(2)} = \frac{-5 \pm \sqrt{1}}{4} = \frac{-5 \pm 1}{4}; x = \frac{-5 + 1}{4} = -1$ or $x = \frac{-5 - 1}{4} = -1.5$ **2.** $5x^2 + 16x - 84 = 0; a = 5, b = 16, c = -84;$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-16 \pm \sqrt{16^2 - 4(5)(-84)}}{2(5)} = \frac{-16 \pm \sqrt{1936}}{10} = \frac{-16 \pm 44}{10}; x = \frac{-16 + 44}{10} = 2.8$ or $x = \frac{-16 - 44}{10} = -6$ **3.** $4x^2 - 12x + 9 = 0; a = 4, b = -12, c = 9; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(9)}}{2(4)} = \frac{12 \pm \sqrt{0}}{8} = \frac{12}{8} = 1.5$ **4.** $3x^2 + 47x = -30;$ $3x^2 + 47x + 30 = 0; a = 3, b = 47, c = 30;$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-47 \pm \sqrt{47^2 - 4(3)(30)}}{2(3)} = \frac{-47 \pm \sqrt{1849}}{6} = \frac{-47 \pm 43}{6}; x = \frac{-47 + 43}{6} \approx -0.67$ or $x = \frac{-47 - 43}{6} = -15$ **5.** $12x^2 - 77x - 20 = 0; a = 12, b = -77, c = -20; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-77) \pm \sqrt{(-77)^2 - 4(12)(-20)}}{2(12)} = \frac{77 \pm \sqrt{6889}}{24} = \frac{77 \pm 83}{24};$ $x = \frac{77 + 83}{24} \approx 6.67$ or $x = \frac{77 - 83}{24} = -0.25$

6. $3x^2 + 39x + 108 = 0; a = 3, b = 39, c = 108;$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-39 \pm \sqrt{39^2 - 4(3)(108)}}{2(3)} = \frac{-39 \pm \sqrt{225}}{6} = \frac{-39 \pm 15}{6}; x = \frac{-39 + 15}{6} = -4$ or $x = \frac{-39 - 15}{6} = -9$ **7.** $3x^2 + 40x - 128 = 0; a = 3, b = 40, c = -128; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-40 \pm \sqrt{40^2 - 4(3)(-128)}}{2(3)} = \frac{-40 \pm \sqrt{3136}}{6} = \frac{-40 \pm 56}{6};$ $x = \frac{-40 + 56}{6} \approx 2.67$ or $x = \frac{-40 - 56}{6} = -16$

8. $2x^2 - 9x - 221 = 0; a = 2, b = -9, c = -221; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(-221)}}{2(2)} = \frac{9 \pm \sqrt{1849}}{4} = \frac{9 \pm 43}{4}; x = \frac{9 + 43}{4} = 13$ or $x = \frac{9 - 43}{4} = -8.5$

9. $5x^2 - 68x = 192; 5x^2 - 68x - 192 = 0; a = 5, b = -68, c = -192; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-68) \pm \sqrt{(-68)^2 - 4(5)(-192)}}{2(5)} = \frac{68 \pm \sqrt{8464}}{10} = \frac{68 \pm 92}{10}; x = \frac{68 + 92}{10} = 16$ or $x = \frac{68 - 92}{10} = -2.4$

10. $5x^2 + 13x - 1 = 0; a = 5, b = 13, c = -1; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-13 \pm \sqrt{13^2 - 4(5)(-1)}}{2(5)} = \frac{-13 \pm \sqrt{189}}{10};$ $x = \frac{-13 + \sqrt{189}}{10} \approx 0.07$ or $x = \frac{-13 - \sqrt{189}}{10} \approx -2.67$

11. $2x^2 - 24x + 33 = 0; a = 2, b = -24, c = 33; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-24) \pm \sqrt{(-24)^2 - 4(2)(33)}}{2(2)} = \frac{24 \pm \sqrt{312}}{4}; x = \frac{24 + \sqrt{312}}{4} \approx 10.42$ or $x = \frac{24 - \sqrt{312}}{4} \approx 1.58$ **12.** $7x^2 + 100x - 4 = 0; a = 7, b = 100, c = -4;$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-100 \pm \sqrt{100^2 - 4(7)(-4)}}{2(7)} = \frac{-100 \pm \sqrt{10{,}112}}{14}; x = \frac{-100 + \sqrt{10{,}112}}{14} \approx 0.04$ or $x = \frac{-100 - \sqrt{10{,}112}}{14} \approx -14.33$ **13.** $8x^2 - 3x - 7 = 0;$ $a = 8, b = -3, c = -7; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(8)(-7)}}{2(8)} = \frac{3 \pm \sqrt{233}}{16}; x = \frac{3 + \sqrt{233}}{16} \approx 1.14$ or $x = \frac{3 - \sqrt{233}}{16} \approx -0.77$

14. $6x^2 + 5x - 40 = 0; a = 6, b = 5, c = -40; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{5^2 - 4(6)(-40)}}{2(6)} = \frac{-5 \pm \sqrt{985}}{12};$ $x = \frac{-5 + \sqrt{985}}{12} \approx 2.20$ or $x = \frac{-5 - \sqrt{985}}{12} \approx -3.03$

15. $3x^2 - 11x - 2 = 0; a = 3, b = -11, c = -2; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(3)(-2)}}{2(3)} = \frac{11 \pm \sqrt{145}}{6}; x = \frac{11 \pm \sqrt{145}}{6} \approx 3.84$ or $x = \frac{11 - \sqrt{145}}{6} \approx -0.17$ **16a.** $0 = -16t^2 + 10t + 3$ **16b.** $a = -16, b = 10, c = 3; t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-10 \pm \sqrt{10^2 - 4(-16)(3)}}{2(-16)} = \frac{-10 \pm \sqrt{292}}{-32}; t = \frac{-10 + \sqrt{292}}{-32} \approx -0.22$ or

$t = \frac{-10 - \sqrt{292}}{-32} \approx 0.8; 0.8$ s **17a.** $0 = -16t^2 + 50t + 3.5$ **17b.** $a = -16, b = 50, c = 3.5; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-50 \pm \sqrt{50^2 - 4(-16)(3.5)}}{2(-16)} = \frac{-50 \pm \sqrt{2724}}{-32}$; $x = \frac{-50 + \sqrt{2724}}{-32} \approx -0.1$ or $x = \frac{-50 - \sqrt{2724}}{-32} \approx 3.2; 3.2$ s **18.** Completing the square or graphing; the x^2 term is 1 but the equation is not factorable. **19.** Factoring or square roots; the equation is easily factorable and there is no x term. **20.** Quadratic formula; the equation cannot be factored. **21.** Quadratic formula; the equation cannot be factored. **22.** Factoring; the equation is easily factorable. **23.** Quadratic formula; the equation cannot be factored. **24.** $2t^2 = 72; t^2 = 36$; $t = \pm\sqrt{36} = \pm 6; t = 6$ or $t = -6$ **25.** $3x^2 + 2x - 4 = 0$; $a = 3, b = 2, c = -4; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(3)(-4)}}{2(3)} = \frac{-2 \pm \sqrt{52}}{6}; x = \frac{-2 + \sqrt{52}}{6} \approx 0.87$ or $x = \frac{-2 - \sqrt{52}}{6} \approx -1.54$ **26.** $5b^2 - 10 = 0; 5(b^2 - 2) = 0; b^2 - 2 = 0; b^2 = 2; b = \pm\sqrt{2}; b = \sqrt{2} \approx 1.41$ or $b = -\sqrt{2} \approx -1.41$ **27.** $3x^2 + 4x = 10; 3x^2 + 4x - 10 = 0$; $a = 3, b = 4, c = -10; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(3)(-10)}}{2(3)} = \frac{-4 \pm \sqrt{136}}{6}; x = \frac{-4 + \sqrt{136}}{6} \approx 1.28$ or $x = \frac{-4 - \sqrt{136}}{6} \approx -2.61$ **28.** $m^2 - 4m = -4$; $m^2 - 4m + 4 = 0; (m - 2)^2 = 0; m - 2 = 0; m = 2$ **29.** $13n^2 - 117 = 0; 13(n^2 - 9) = 0; 13(n - 3)(n + 3) = 0; n - 3 = 0$ or $n + 3 = 0; n = 3$ or $n = -3$ **30.** $3s^2 - 4s = 2; 3s^2 - 4s - 2 = 0; a = 3, b = -4$, $c = -2; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-2)}}{2(3)} = \frac{4 \pm \sqrt{40}}{6}; x = \frac{4 + \sqrt{40}}{6} \approx 1.72$ or $x = \frac{4 - \sqrt{40}}{6} \approx -0.39$ **31.** $5b^2 - 2b - 7 = 0$; $(5b - 7)(b + 1) = 0; 5b - 7 = 0$ or $b + 1 = 0; 5b = 7$ or $b = -1; b = \frac{7}{5} = 1.4$ or $b = -1$ **32.** $15x^2 - 12x - 48 = 0$; $3(5x^2 - 4x - 16) = 0; 5x^2 - 4x - 16 = 0; a = 5$, $b = -4, c = -16; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(5)(-16)}}{2(5)} = \frac{4 \pm \sqrt{336}}{10}; x = \frac{4 + \sqrt{336}}{10} \approx 2.23$ or $x = \frac{4 - \sqrt{336}}{10} \approx -1.43$ **33a.** 7 ft × 8 ft **33b.** $x(x + 1) = 60; x^2 + x - 60 = 0; a = 1, b = 1$, $c = -60; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4(1)(-60)}}{2(1)} = \frac{-1 \pm \sqrt{241}}{2}; x = \frac{-1 + \sqrt{241}}{2} \approx 7.26$ or $x = \frac{-1 - \sqrt{241}}{2} \approx -8.26$; use $x \approx 7.26$ and $x + 1 \approx 7.26 + 1 = 8.26$; 7.26 ft × 8.26 ft **34.** $0 = -16t^2 + 30t + 6; a = -16, b = 30$, $c = 6; t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-30 \pm \sqrt{30^2 - 4(-16)(6)}}{2(-16)} = \frac{-30 \pm \sqrt{1284}}{-32}$; $t = \frac{-30 + \sqrt{1284}}{-32} \approx -0.2$ or $t = \frac{-30 - \sqrt{1284}}{-32} \approx 2.1$; about 2.1 s **35.** Answers may vary. Sample: You solve the linear equation using transformations and you solve the quadratic equation using the quadratic formula. **36.** $A = \frac{1}{2}bh$; $20 = \frac{1}{2}x(x + 2); 40 = x^2 + 2x; x^2 + 2x - 40 = 0$; $a = 1, b = 2, c = -40; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(1)(-40)}}{2(1)} = \frac{-2 \pm \sqrt{164}}{2}; x = \frac{-2 + \sqrt{164}}{2} \approx 5.40$ or $x = \frac{-2 - \sqrt{164}}{2} \approx 7.40$; use $x \approx 5.40$ and $x + 2 \approx 5.40 + 2 = 7.40$; 7.40 ft and 5.40 ft **37.** $A = \frac{1}{2}bh$; $50 = \frac{1}{2}x(x + 6); 100 = x^2 + 6x; x^2 + 6x - 100 = 0$; $a = 1, b = 6, c = -100; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{6^2 - 4(1)(-100)}}{2(1)} = \frac{-6 \pm \sqrt{436}}{2}; x = \frac{-6 + \sqrt{436}}{2} \approx 7.44$ or $x = \frac{-6 - \sqrt{436}}{2} \approx -13.44$; use $x \approx 7.44$ and $x + 6 \approx 7.44 + 6 = 13.44$; 13.44 cm and 7.44 cm **38.** Answers may vary. Sample: A rectangle has length x. Its width is 5 ft longer than three times the length. Find the dimensions if its area is 182 ft^2.

$x(3x + 5) = 182; 3x^2 + 5x - 182 = 0$; $a = 3, b = 5, c = -182$; $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{5^2 - 4(3)(-182)}}{2(3)} = \frac{-5 \pm \sqrt{2209}}{6} = \frac{-5 \pm 47}{6}; x = \frac{-5 + 47}{6} = 7$ or $x = \frac{-5 - 47}{6} \approx -8.7$; use $x = 7$ and $3x + 5 = 3(7) + 5 = 26$; 7 ft × 26 ft **39.** if the expression $b^2 - 4ac$ equals zero **40.** $0 = -16t^2 + 5t + 50; a = -16, b = 5, c = 50$; $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{5^2 - 4(-16)(50)}}{2(-16)} = \frac{-5 \pm \sqrt{3225}}{-32}; t = \frac{-5 + \sqrt{3225}}{-32} \approx -1.6$ or $t = \frac{-5 - \sqrt{3225}}{-32} \approx 1.9$; about 1.9 s **41a.** Check students' work. **41b.** $2025 - 1900 = 125; P = 0.0089(125)^2 + 1.1149(125) + 78.4491 \approx 356.9$; about 356.9 million **41c.** $300 = 0.0089t^2 + 1.1149t + 78.4491; 0.0089t^2 + 1.1149t - 221.5509$; $t = \frac{-1.1149 \pm \sqrt{1.1149^2 - 4(0.0089)(-221.5509)}}{2(0.0089)}$; $t \approx 107$ or $t \approx -232$; use $t \approx 107; 1900 + 107 = 2007$ **42a.** $s = \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-b + \sqrt{b^2 - 4ac} - b - \sqrt{b^2 - 4ac}}{2a} = \frac{-2b}{2a} = -\frac{b}{a}; s = -\frac{b}{a}$ **42b.** $x - 8 = -\frac{3}{2}; x = 6.5$ **43a.** 38,100 tons $\cdot \frac{2000 \text{ pounds}}{1 \text{ ton}} =$ 76,200,000 pounds $= 7.62 \times 10^7$ pounds **43b.** 38,100 tons − 5100 tons = 33,000 tons = 3.3×10^4 tons **43c.** $0 = -16t^2 + 0t + 630; 16t^2 = 630$; $t^2 = \frac{630}{16}$; $t = \pm\sqrt{\frac{630}{16}} \approx \pm 6.27$; about 6.27 s **44.** $\frac{9 \pm \sqrt{(-9)^2 - 4(5)(-7)}}{2(5)}; a = 5, b = -9, c = -7$; $5x^2 - 9x - 7 = 0$, or $5x^2 - 9x = 7$; D **45.** $0 = 15x^2 - 59x - 112; a = 15, b = -59, c = -112; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-59) \pm \sqrt{(-59)^2 - 4(15)(-112)}}{2(15)} =$

$\frac{59 \pm \sqrt{10{,}201}}{30} = \frac{59 \pm 101}{30}$; $x = \frac{59 + 101}{30} \approx 5.3$ or $x = \frac{59 - 101}{30} = -1.4$; F **46.** [2] $6x^2 - 40 = 11x$; $6x^2 - 11x - 40 = 0$; $x = \frac{11 \pm \sqrt{(-11)^2 - 4(6)(-40)}}{2(6)}$; $x \approx 1.8$ or $x \approx 3.7$ [1] correct substitution into quadratic formula, with one calculation error **47.** $d^2 - 10d + 13 = 0$; $d^2 - 10d = -13$; $d^2 - 10d + 25 = -13 + 25$; $(d - 5)^2 = 12$; $d - 5 = \pm\sqrt{12} \approx \pm 3.46$; $d - 5 \approx 3.46$ or $d - 5 \approx -3.46$; $d \approx 8.46$ or $d \approx 1.54$ **48.** $z^2 + 3z = -2$; $z^2 + 3z + \frac{9}{4} = -2 + \frac{9}{4}$; $\left(z + \frac{3}{2}\right)^2 = \frac{1}{4}$; $z + \frac{3}{2} = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$; $z + \frac{3}{2} = \frac{1}{2}$ or $z + \frac{3}{2} = -\frac{1}{2}$; $z = -1$ or $z = -2$ **49.** $3x^2 + 18x - 1 = 0$; $x^2 + 6x - \frac{1}{3} = 0$; $x^2 + 6x = \frac{1}{3}$; $x^2 + 6x + 9 = \frac{1}{3} + 9$; $(x + 3)^2 = \frac{28}{3}$; $x + 3 = \pm\sqrt{\frac{28}{3}} \approx \pm 3.06$; $x + 3 \approx 3.06$ or $x + 3 \approx -3.06$; $x \approx 0.06$ or $x \approx -6.06$ **50.** $2c^2 + 11c + 15$; $2(15) = 30$; factors with product 30 and sum 11: 5 and 6; $2c^2 + 11c + 15 = 2c^2 + (6 + 5)c + 15 = 2c^2 + 6c + 5c + 15 = 2c(c + 3) + 5(c + 3) = (2c + 5)(c + 3)$ **51.** $3z^2 + 10z - 8$; $3(-8) = -24$; factors with product -24 and sum 10: 12 and -2; $3z^2 + 10z - 8 = 3z^2 + (12 - 2)z - 8 = 3z^2 + 12z - 2z - 8 = 3z(z + 4) - 2(z + 4) = (3z - 2)(z + 4)$ **52.** $5n^2 - 33n - 14$; $5(-14) = -70$; factors with product -70 and sum -33: -35 and 2; $5n^2 - 33n - 14 = 5n^2 + (-35 + 2)n - 14 = 5n^2 - 35n + 2n - 14 = 5n(n - 7) + 2(n - 7) = (5n + 2)(n - 7)$ **53.** $12v^2 + 32v - 35$; $12(-35) = -420$; factors with product -420 and sum 32: 42 and -10; $12v^2 + 32v - 35 = 12v^2 + (-10 + 42)v - 35 = 12v^2 - 10v + 42v - 35 = 2v(6v - 5) + 7(6v - 5) = (6v - 5)(2v + 7)$ **54.** $6x^2 - 13x + 5$; $6(5) = 30$; factors with product 30 and sum -13: -3 and -10; $6x^2 - 13x + 5 = 6x^2 + (-3 - 10)x + 5 = 6x^2 - 3x - 10x + 5 = 3x(2x - 1) - 5(2x - 1) = (2x - 1)(3x - 5)$ **55.** $15t^2 + 19t + 6$; $15(6) = 90$; factors with product 90 and sum 19: 10 and 9; $15t^2 + 19t + 6 = 15t^2 + (10 + 9)t + 6 = 15t^2 + 10t + 9t + 6 = 5t(3t + 2) + 3(3t + 2) = (5t + 3)(3t + 2)$

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a. $4x^2 - 8x - 45 = 0$ **b.** $a = 4, b = -8, c = -45$ **c.** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(4)(-45)}}{2(4)} = \frac{8 \pm \sqrt{784}}{8} = \frac{8 \pm 28}{8}$; $x = \frac{8 + 28}{8} = 4.5$ or $x = \frac{8 - 28}{8} = -2.5$

10-8 Using the Discriminant *pages 554–558*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1. -80 **2.** 72 **3.** -283 **4.** 2.18, 0.15 **5.** 0.39, -0.64 **6.** 7, 5

Check Understanding 1a. $x^2 = 2x - 3$; $x^2 - 2x + 3 = 0$; $a = 1, b = -2, c = 3$; $b^2 - 4ac = (-2)^2 - 4(1)(3) = 4 - 12 = -8 < 0$; 0 solutions **1b.** $3x^2 - 4x = 7$; $3x^2 - 4x - 7 = 0$; $a = 3, b = -4, c = -7$; $b^2 - 4ac = (-4)^2 - 4(3)(-7) = 16 + 84 = 100 > 0$; 2 solutions **1c.** $5x^2 + 8 = 2x$; $5x^2 - 2x + 8 = 0$; $a = 5, b = -2, c = 8$; $b^2 - 4ac = (-2)^2 - 4(5)(8) = 4 - 160 = -156 < 0$; 0 solutions **2.** $20 = -16t^2 + 32t + 5$; $0 = -16t^2 + 32t - 15$; $b^2 - 4ac = 32^2 - 4(-16)(-15) = 1024 - 960 = 64 > 0$; yes

Exercises 1. The graph has 2 x-intercepts, so the equation has 2 solutions, and the discriminant is positive; A. **2.** The graph has no x-intercepts, so the equation has no solutions, and the discriminant is negative; C. **3.** The graph has 1 x-intercept, so the equation has 1 solution, and the discriminant is zero; B. **4.** $x^2 - 3x + 4 = 0$; $b^2 - 4ac = (-3)^2 - 4(1)(4) = 9 - 16 = -7 < 0$; 0 solutions **5.** $x^2 - 6x + 9 = 0$; $b^2 - 4ac = (-6)^2 - 4(1)(9) = 36 - 36 = 0$; 1 solution **6.** $x^2 + 4x - 2 = 0$; $b^2 - 4ac = 4^2 - 4(1)(-2) = 16 + 8 = 24 > 0$; 2 solutions **7.** $x^2 - 1 = 0$; $b^2 - 4ac = 0^2 - 4(1)(-1) = 0 + 4 = 4 > 0$; 2 solutions **8.** $x^2 - 2x - 3 = 0$; $b^2 - 4ac = (-2)^2 - 4(1)(-3) = 4 + 12 = 16 > 0$; 2 solutions **9.** $x^2 + x = 0$; $b^2 - 4ac = 1^2 - 4(1)(0) = 1 - 0 = 1 > 0$; 2 solutions **10.** $2x^2 - 3x + 4 = 0$; $b^2 - 4ac = (-3)^2 - 4(2)(4) = 9 - 32 = -23 < 0$; 0 solutions **11.** $0 = x^2 - 6x + 5$; $b^2 - 4ac = (-6)^2 - 4(1)(5) = 36 - 20 = 16 > 0$; 2 solutions **12.** $x^2 - 7x + 6 = 0$; $b^2 - 4ac = (-7)^2 - 4(1)(6) = 49 - 24 = 25 > 0$; 2 solutions **13.** $x^2 + 2x + 1 = 0$; $b^2 - 4ac = 2^2 - 4(1)(1) = 4 - 4 = 0$; 1 solution **14.** $0 = 2x^2 + 4x - 3$; $b^2 - 4ac = 4^2 - 4(2)(-3) = 16 + 24 = 40 > 0$; 2 solutions **15.** $0 = x^2 + 2x + 9$; $b^2 - 4ac = 2^2 - 4(1)(9) = 4 - 36 = -32 < 0$; 0 solutions **16.** $280 = (18 - x)(15 + x)$; $280 = 270 + 3x - x^2$; $x^2 - 3x + 10 = 0$; $b^2 - 4ac = (-3)^2 - 4(1)(10) = 9 - 40 = -31 < 0$; none **17.** $7000 = 5400 + 300n - 50n^2$; $50n^2 - 300n + 1600 = 0$; $b^2 - 4ac = (-300)^2 - 4(50)(1600) = 90{,}000 - 320{,}000 = -230{,}000 < 0$; No, because the discriminant is negative. **18a.** $16 = -16t^2 + 35t$; $16t^2 - 35t + 16 = 0$; $b^2 - 4ac = (-35)^2 - 4(16)(16) = 1225 - 1024 = 201 > 0$; yes **18b.** $20 = -16t^2 + 35t$; $16t^2 - 35t + 20 = 0$; $b^2 - 4ac = (-35)^2 - 4(16)(20) = 1225 - 1280 = -55 < 0$; no **18c.** $30 = -16t^2 + 35t$; $16t^2 - 35t + 30 = 0$; $b^2 - 4ac = (-35)^2 - 4(16)(30) = 1225 - 1920 = -695 < 0$; no **18d.** $35 = -16t^2 + 35t$; $16t^2 - 35t + 35 = 0$; $b^2 - 4ac = (-35)^2 - 4(16)(35) = 1225 - 2240 = -1015 < 0$; no **19.** $2x^2 + 4x + 15 = 0$; $b^2 - 4ac = 4^2 - 4(2)(15) = 16 - 120 = -104 < 0$; 0 solutions, so 0 x-intercepts **20.** $4x^2 + 5x + 2 = 0$; $b^2 - 4ac = 5^2 - 4(4)(2) = 25 - 32 = -7 < 0$; 0 solutions, so 0 x-intercepts **21.** $x^2 - 8x + 12 = 0$; $b^2 - 4ac = (-8)^2 - 4(1)(12) = 64 - 48 = 16 > 0$; 2 solutions, so 2 x-intercepts **22.** $\frac{1}{2}x^2 + 4x - 7 = 0$; $b^2 - 4ac = 4^2 - 4\left(\frac{1}{2}\right)(-7) = 16 + 14 = 30 > 0$; 2 solutions, so 2 x-intercepts **23.** $0.25x^2 - 1.2x + 3.2 = 0$; $b^2 - 4ac = (-1.2)^2 - 4(0.25)(3.2) = 1.44 - 3.2 = -1.76 < 0$; 0 solutions, so 0 x-intercepts **24.** $5x^2 - 4.7x - 3.5 = 0$; $b^2 - 4ac = (-4.7)^2 - 4(5)(-3.5) = 22.09 + 70 = 92.09 > 0$; 2 solutions, so 2 x-intercepts **25a.** $S = p(54 - 0.75p)$; $S = 54p - 0.75p^2$; $S = -0.75p^2 + 54p$

25b. $1{,}000{,}000 = -0.75p^2 + 54p$; $0.75p^2 - 54p + 1{,}000{,}000 = 0$; $b^2 - 4ac = (-54)^2 - 4(0.75)(1{,}000{,}000) = 2916 - 3{,}000{,}000 = -2{,}997{,}084 < 0$; no **25c.** $-\frac{b}{2a} = -\frac{54}{2(-0.75)} = 36$; \$36 **25d.** If a product is too expensive, fewer people will buy it. **26.** $x^2 + 4x + k = 0$; $b^2 - 4ac = 4^2 - 4(1)(k) = 16 - 4k$ **26a.** $16 - 4k < 0$; $-4k < -16$; $k > 4$ **26b.** $16 - 4k = 0$; $4k = 16$; $k = 4$ **26c.** $16 - 4k > 0$; $-4k > -16$; $k < 4$ **27a.** cell B2: $x^2 + bx + 1 = 0$; $b^2 - 4ac = b^2 - 4(1)(1) = b^2 - 4$; A2 ^ 2 − 4; cell C2: $x^2 + bx + 2 = 0$; $b^2 - 4ac = b^2 - 4(1)(2) = b^2 - 8$; A2 ^ 2 − 8 **27b.** $x^2 + bx + 1 = 0$; $b^2 - 4ac = b^2 - 4(1)(1) = b^2 - 4$; for no solutions, $b^2 - 4 < 0$; $b^2 < 4$; $|b| < 2$ **28.** $0 = 3i^2 - 2i + 450$; $b^2 - 4ac = (-2)^2 - 4(3)(450) = 4 - 5400 = -5396 < 0$; no **29.** $2x^2 + 5x - 1 = 0$; $b^2 - 4ac = 5^2 - 4(2)(-1) = 25 + 8 = 33$; note $25 - 8 = 17$; answers may vary. Sample: Kenji used $c = 1$ instead of $c = -1$. **30a.** $x^2 - 6x + 5 = 0$; discriminant: $b^2 - 4ac = (-6)^2 - 4(1)(5) = 36 - 20 = 16$; solutions: $(x - 5)(x - 1) = 0$; $x - 5 = 0$ or $x - 1 = 0$; $x = 5$ or $x = 1$ **30b.** $x^2 + x - 20 = 0$; discriminant: $b^2 - 4ac = 1^2 - 4(1)(-20) = 1 + 80 = 81$; solutions: $(x - 4)(x + 5) = 0$; $x - 4 = 0$ or $x + 5 = 0$; $x = 4$ or $x = -5$ **30c.** $2x^2 - 7x - 3 = 0$; discriminant: $b^2 - 4ac = (-7)^2 - 4(2)(-3) = 49 + 24 = 73$; solutions: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-7) \pm \sqrt{73}}{2(2)} = \frac{7 \pm \sqrt{73}}{4}$; $x = \frac{7 + \sqrt{73}}{4} \approx 3.89$ or $x = \frac{7 - \sqrt{73}}{4} \approx -0.39$

30d. The solutions are rational, because the square root of a discriminant that is a perfect square is a positive integer. **31.** $x^2 - 2x + 5 = 0$; $b^2 - 4ac = (-2)^2 - 4(1)(5) = 4 - 20 = -16 < 0$; no **32.** $2x^2 - 4x + 3 = 0$; $b^2 - 4ac = (-4)^2 - 4(2)(3) = 16 - 24 = -8 < 0$; no **33.** $4x^2 + x - 5 = 0$; $b^2 - 4ac = 1^2 - 4(4)(-5) = 1 + 80 = 81 > 0$; yes; $(4x + 5)(x - 1) = 0$; $4x + 5 = 0$ or $x - 1 = 0$; $4x = -5$ or $x = 1$; $x = -1.25$ or $x = 1$ **34.** $-3x^2 - x + 2 = 0$; $b^2 - 4ac = (-1)^2 - 4(-3)(2) = 1 + 24 = 25 > 0$; yes; $(-3x + 2)(x + 1) = 0$; $-3x + 2 = 0$ or $x + 1 = 0$; $3x = 2$ or $x = -1$; $x = \frac{2}{3}$ or $x = -1$ **35.** $x^2 - 5x + 7 = 0$; $b^2 - 4ac = (-5)^2 - 4(1)(7) = 25 - 28 = -3 < 0$; no **36.** $2x^2 - 3x - 5 = 0$; $b^2 - 4ac = (-3)^2 - 4(2)(-5) = 9 + 40 = 49 > 0$; yes; $(2x - 5)(x + 1) = 0$; $2x - 5 = 0$ or $x + 1 = 0$; $2x = 5$ or $x = -1$; $x = 2.5$ or $x = -1$

37. Answers may vary. Sample: Use values for a, b, and c such that the discriminant is positive.

38–40. $ax^2 + bx + c = 0$ will have two solutions whenever $b^2 - 4ac > 0$, or $b^2 > 4ac$. **38.** $b^2 < 4ac$, so it is never true that $b^2 > 4ac$. **39.** $b^2 = 0$, so it is sometimes true that $b^2 > 4ac$. **40.** $ac < 0$, so $4ac < 0$, and it is always true that $b^2 > 4ac$ because $b^2 > 0$. **41.** The equation has 2 solutions; since the parabola crosses the x-axis once, it must cross it again. **42.** $y = 2x^2 + 8x + 10$ has a vertex closer to the x-axis; its discriminant is closer to zero.

43. $b^2 - 4ac = 7^2 - 4(1)(-4) = 49 + 16 = 65$; C **44.** F. $3x^2 - 5x + 1 = 0$; $b^2 - 4ac = (-5)^2 - 4(3)(1) = 25 - 12 = 13 > 0$; 2 solutions. G. $3x^2 - 5x + 4 = 0$; $b^2 - 4ac = (-5)^2 - 4(3)(4) = 25 - 48 = -23 < 0$, no solutions. H. $-3x^2 - 11x + 4 = 0$; $b^2 - 4ac = (-11)^2 - 4(-3)(4) = 121 + 48 = 169 > 0$, 2 solutions. I. $-2x^2 - 3x + 1 = 0$; $b^2 - 4ac = (-3)^2 - 4(-2)(1) = 9 + 8 = 17 > 0$, 2 solutions. The answer is G.

45. A. $35 = 20x^2 - 15x + 47$; $20x^2 - 15x + 12 = 0$; $b^2 - 4ac = (-15)^2 - 4(20)(12) = 225 - 960 = -735 < 0$, no solutions. B. $15x + 7 = 0$; $15x = -7$; $x = -\frac{7}{15}$, 1 solution; the answer is B.

46. A. $ax^2 + 5x - 3 = 0$; $b^2 - 4ac = 5^2 - 4a(-3) = 25 + 12a$. B. $5x^2 + bx - 3 = 0$; $b^2 - 4ac = b^2 - 4(5)(-3) = b^2 + 60$; the answer is D.

47. A. $4x^2 - x - 6 = 0$; $b^2 - 4ac = (-1)^2 - 4(4)(-6) = 1 + 96 = 97$. B. $4x^2 - x + 6 = 0$; $b^2 - 4ac = (-1)^2 - 4(4)(6) = 1 - 96 = -95$; the answer is A.

48. [2] $x(25 - x) = 136$; $x^2 - 25x + 136 = 0$; $x = \frac{-(-25) \pm \sqrt{(-25)^2 - 4(1)(136)}}{2(1)} = \frac{25 \pm 9}{2} = 17$ or 8; yes, 17 cm by 8 cm [1] no work shown or appropriate methods with one computational error

49. $4x^2 + 4x - 3 = 0$; $a = 4, b = 4, c = -3$; $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{4^2 - 4(4)(-3)}}{2(4)} = \frac{-4 \pm \sqrt{64}}{8} = \frac{-4 \pm 8}{8}$; $x = \frac{-4 + 8}{8} = 0.5$ or $x = \frac{-4 - 8}{8} = -1.5$ **50.** $x^2 + 2x - 7 = 0$; $a = 1, b = 2, c = -7$; $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(1)(-7)}}{2(1)} = \frac{-2 \pm \sqrt{32}}{2}$; $x = \frac{-2 + \sqrt{32}}{2} \approx 1.83$ or $x = \frac{-2 - \sqrt{32}}{2} \approx -3.83$

51. $6x^2 - 2x - 1 = 0$; $a = 6, b = -2, c = -1$; $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(6)(-1)}}{2(6)} = \frac{2 \pm \sqrt{28}}{12}$; $x = \frac{2 + \sqrt{28}}{12} \approx 0.61$ or $x = \frac{2 - \sqrt{28}}{12} \approx -0.27$

52. $x^2 + x = 5$; $x^2 + x - 5 = 0$; $a = 1, b = 1, c = -5$; $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4(1)(-5)}}{2(1)} = \frac{-1 \pm \sqrt{21}}{2}$; $x = \frac{-1 + \sqrt{21}}{2} \approx 1.79$ or $x = \frac{-1 - \sqrt{21}}{2} \approx -2.79$

53. $3x^2 - 8x + 1 = 0$; $a = 3, b = -8, c = 1$; $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(3)(1)}}{2(3)} = \frac{8 \pm \sqrt{52}}{6}$; $x = \frac{8 + \sqrt{52}}{6} \approx 2.54$ or $x = \frac{8 - \sqrt{52}}{6} \approx 0.13$

54. $2x^2 - 7x = -6$; $2x^2 - 7x + 6 = 0$; $a = 2, b = -7, c = 6$; $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(6)}}{2(2)} = \frac{7 \pm \sqrt{1}}{4} = \frac{7 \pm 1}{4}$; $x = \frac{7 + 1}{4} = 2$ or $x = \frac{7 - 1}{4} = 1.5$

55. $y = \$1000 \cdot 1.0075^{12} \approx \$1093.81$ **56.** $y = \$200 \cdot 1.01125^{40} \approx \$312.88$ **57.** $y = \$5000 \cdot 1.0041\overline{6}^{48} \approx \$6104.48$ **58.** $9 - 5 = 4$, $13 - 9 = 4$; arithmetic

59. $-16 - (-11) = -5$, $-21 - (-16) = -5$; arithmetic

60. $20 \div 10 = 2$, $40 \div 20 = 2$; geometric

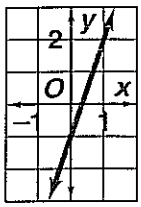
61. $6 - 3 = 3$, $9 - 6 = 3$; arithmetic

CHECKPOINT QUIZ 2 *page 558*

1. $(x + 3)(x - 7) = 0; x + 3 = 0$ or $x - 7 = 0; x = -3$ or $x = 7$ **2.** $x^2 + 12x + 27 = 0; (x + 3)(x + 9) = 0;$ $x + 3 = 0$ or $x + 9 = 0; x = -3$ or $x = -9$
3. $x^2 - 5x = 50; x^2 - 5x - 50 = 0; (x + 5)(x - 10) = 0;$ $x + 5 = 0$ or $x - 10 = 0; x = -5$ or $x = 10$
4. $x^2 + 2x - 1 = 0; a = 1, b = 2, c = -1; x =$ $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{2^2 - 4(1)(-1)}}{2(1)} = \frac{-2 \pm \sqrt{8}}{2};$ $x = \frac{-2 + \sqrt{8}}{2} \approx 0.4$ or $x = \frac{-2 - \sqrt{8}}{2} \approx -2.4$
5. $x^2 - 5x - 4 = 0; a = 1, b = -5, c = -4; x =$ $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-4)}}{2(1)} =$ $\frac{5 \pm \sqrt{41}}{2}; x = \frac{5 + \sqrt{41}}{2} \approx 5.7$ or $x = \frac{5 - \sqrt{41}}{2} \approx -0.7$
6. $x^2 - 8x - 33 = 0; (x - 11)(x + 3) = 0; x - 11 = 0$ or $x + 3 = 0; x = 11$ or $x = -3$ **7.** $4x^2 - x - 3 = 0;$ $(4x + 3)(x - 1) = 0; 4x + 3 = 0$ or $x - 1 = 0; 4x = -3$ or $x = 1; x = -\frac{3}{4}$ or $x = 1$ **8.** $4x^2 - x + 3 = 0; a = 4,$ $b = -1, c = 3; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$ $\frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(3)}}{2(4)} = \frac{1 \pm \sqrt{-47}}{8};$ discriminant < 0, no solution **9.** $2x^2 - 3x + 1 = 0;$ $(2x - 1)(x - 1) = 0; 2x - 1 = 0$ or $x - 1 = 0; 2x = 1$ or $x = 1; x = 0.5$ or $x = 1$ **10.** $4x^2 - 3x + 5 = 0;$ $b^2 - 4ac = (-3)^2 - 4(4)(5) = 9 - 80 = -71 < 0;$ 0 solutions

10-9 Choosing a Linear, Quadratic, or Exponential Model *pages 559–566*

Check Skills You'll Need For complete solutions see *Daily Skills Check and Lesson Quiz Transparencies* or *Presentation Pro CD-ROM.*

1.


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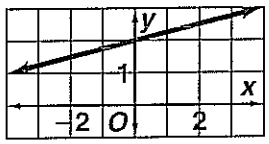

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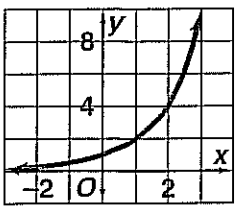

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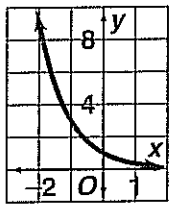

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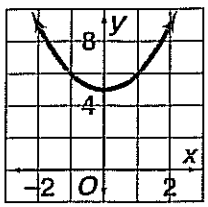

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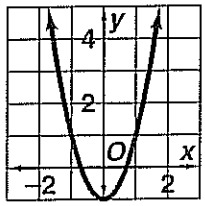
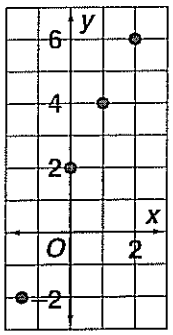
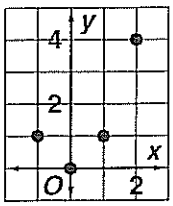
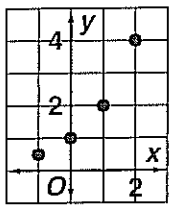
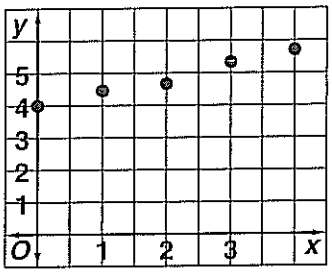


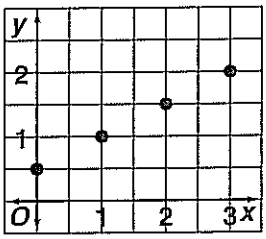
Check Understanding

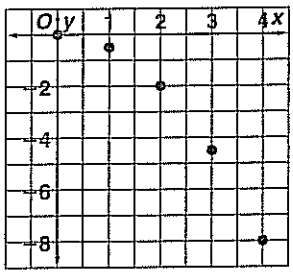
1a.

linear

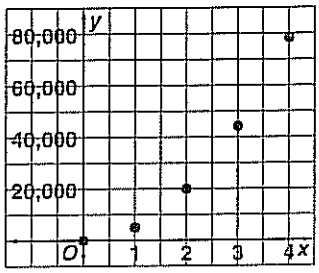
1b.

quadratic

1c.

exponential

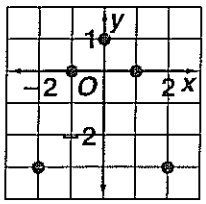
2a.

Test for a common ratio: $4.4 \div 4 = 1.1$, $4.84 \div 4.4 = 1.1$, $5.324 \div 4.84 = 1.1$, $5.8564 \div 5.324 = 1.1$; common ratio is 1.1, so write an exponential model: $y = 4 \cdot 1.1^x$.

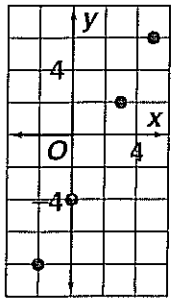
2b.

Test for a common difference: $1 - \frac{1}{2} = \frac{1}{2}$, $1\frac{1}{2} - 1 = \frac{1}{2}, 2 - 1\frac{1}{2} = \frac{1}{2}$; common difference is $\frac{1}{2}$, so write a linear model: $y = \frac{1}{2}x + \frac{1}{2}$.

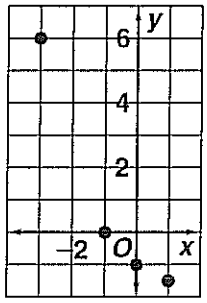
2c.

Test for a common second difference: $-0.5 - 0 = -0.5, -2 - (-0.5) = -1.5, -4.5 - (-2) = -2.5, -8 - (-4.5) = -3.5$, and $-1.5 - (-0.5) = -1, -2.5 - (-1.5) = -1, -3.5 - (-2.5) = -1$; common second difference is -1, so write a quadratic model: $y = ax^2$, $-2 = a \cdot 2^2, a = -\frac{1}{2}; y = -\frac{1}{2}x^2$.

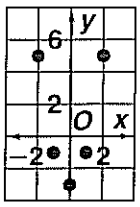
3a.

Test for a common second difference: $5000 - 0 = 5000$, $20{,}000 - 5000 = 15{,}000$, $44{,}000 - 20{,}000 = 24{,}000$, $79{,}000 - 44{,}000 = 35{,}000$, and $15{,}000 - 5000 = 10{,}000, 24{,}000 - 15{,}000 = 9000, 35{,}000 - 24{,}000 = 11{,}000$; common second difference is about 10,000, so the function is quadratic.

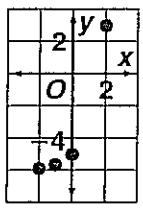
3b. $y = ax^2; 20{,}000 = a \cdot 2^2, a = 5000; y = 5000x^2$

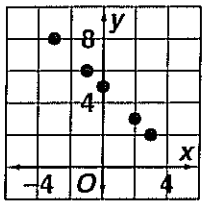
Exercises 1.  quadratic

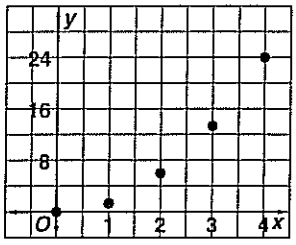
2. 
linear

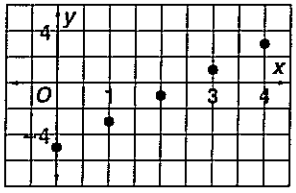
3. 
exponential

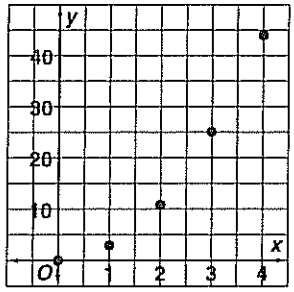
4.  quadratic

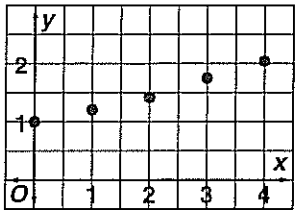
5.  exponential

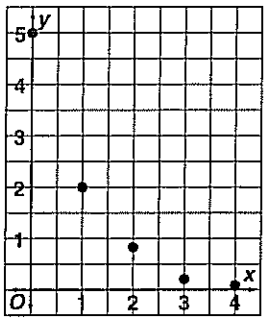
6.  linear

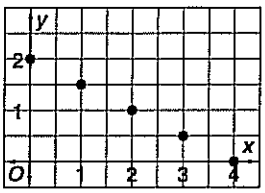
7. 
Test for a common second difference: $1.5 - 0 = 1.5$, $6 - 1.5 = 4.5$, $13.5 - 6 = 7.5$, $24 - 13.5 = 10.5$, and $4.5 - 1.5 = 3$, $7.5 - 4.5 = 3$, $10.5 - 7.5 = 3$; common second difference is 3, so the function is quadratic.
$y = ax^2, 6 = a \cdot 2^2, a = 1.5; y = 1.5x^2$

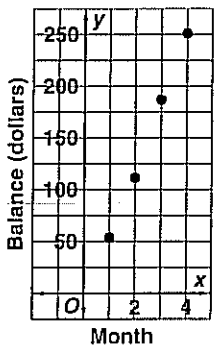
8. 
Test for a common difference: $-3 - (-5) = 2$, $-1 - (-3) = 2$, $1 - (-1) = 2, 3 - 1 = 2$; common difference is 2, so the function is linear. $y = 2x - 5$

9. 
Test for a common second difference: $2.8 - 0 = 2.8$, $11.2 - 2.8 = 8.4$, $25.2 - 11.2 = 14$, $44.8 - 25.2 = 19.6$, and $8.4 - 2.8 = 5.6, 14 - 8.4 = 5.6, 19.6 - 14 = 5.6$; common second difference is 5.6, so the function is quadratic.
$y = ax^2, 11.2 = a \cdot 2^2, a = \frac{11.2}{4} = 2.8; y = 2.8x^2$

10. 
Test for a common ratio: $1.2 \div 1 = 1.2$, $1.44 \div 1.2 = 1.2$, $1.728 \div 1.44 = 1.2$, $2.0736 \div 1.728 = 1.2$; common ratio is 1.2, so the function is exponential. $y = 1 \cdot 1.2^x$

11. 
Test for a common ratio: $2 \div 5 = 0.4, 0.8 \div 2 = 0.4$, $0.32 \div 0.8 = 0.4$, $0.128 \div 0.32 = 0.4$; common ratio is 0.4, so the function is exponential. $y = 5 \cdot 0.4^x$

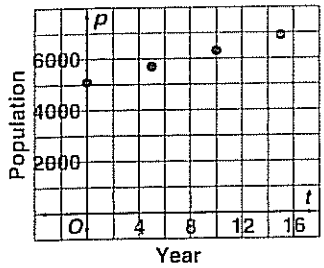
12. 
Test for a common difference: $1.5 - 2 = -0.5$, $1 - 1.5 = -0.5, 0.5 - 1 = -0.5, 0 - 0.5 = -0.5$; common difference is -0.5, so the function is linear. $y = -\frac{1}{2}x + 2$

13a. 

linear
13b. $123 - 58 = 65$, $187 - 123 = 64$, $251 - 187 = 64$; yes
13c. 64 **13d.** $y = 64x - 5$

14a. The data appears exponential, so test for a common ratio: $14{,}500 \div 16{,}500 \approx 0.88, 12{,}750 \div 14{,}500 \approx 0.88$, $11{,}200 \div 12{,}750 \approx 0.88, 9900 \div 11{,}200 \approx 0.88$; common ratio is about 0.88, so the function is exponential.
14b. $y = 16{,}500 \cdot 0.88^x$
15a. $41 - 0 = 41, 164 - 41 = 123, 370 - 164 = 206$
15b. $123 - 41 = 82, 206 - 123 = 83$
15c. $d = 41t^2$
15d. $d = 41 \cdot 2.5^2 = 256.25$; 256.25 cm

16a.



linear **16b.** $5 - 0 = 5$, $10 - 5 = 5, 15 - 10 = 5$; 5 years **16c.** differences of consecutive terms: $5700 - 5100 = 600$, $6300 - 5700 = 600$, $6900 - 6300 = 600$; possible common differences: $600 \div 5 = 120, 120, 120$ **16d.** $p = 120t + 5100$ **17a.** $5 - 0 = 5$; 5 years **17b.** differences of consecutive terms: $4855 - 4457 = 398, 5284 - 4855 = 429, 5691 - 5284 = 407$, $6080 - 5691 = 389$; possible common differences: $398 \div 5 = 79.6, 429 \div 5 = 85.8, 407 \div 5 = 81.4$, $389 \div 5 = 77.8$ **17c.** $(79.6 + 85.8 + 81.4 + 77.8) \div 4 = 81.15 \approx 81.2$ **17d.** $p = 81.2t + 4457$ **17e.** $2010 - 1980 = 30$; $p = 81.2(30) + 4457 = 6893$; 6893 million, or about 6.9 billion **18.** Answers may vary. Sample: Linear data have a common first difference, quadratic data have a common second difference, and exponential data have a common ratio. **19.** $y = 0.875x^2 - 0.435x + 1.515$ **20.** $y = 1.987 \cdot 0.770^x$ **21.** $y = 2.125x^2 - 4.145x + 2.955$ **22.** $y = -0.336x^2 - 0.219x + 4.666$ **23.** $y = -1.1x + 3.5$ **24.** $y = 0.102 \cdot 2.582^x$

25ai.

x	y		
1	−2		
		) 3	
2	1		) 2
		) 5	
3	6		) 2
		) 7	
4	13		) 2
		) 9	
5	22		

25aii.

x	y		
1	3		
		) 9	
2	12		) 6
		) 15	
3	27		) 6
		) 21	
4	48		) 6
		) 27	
5	75		

25aiii.

x	y		
1	−1		
		) 7	
2	6		) 8
		) 15	
3	21		) 8
		) 23	
4	44		) 8
		) 31	
5	75		

25b. The second common difference is twice the coefficient of x^2. **25c.** When second differences are the same, the data are quadratic. You can determine the coefficient of x^2 by dividing the second difference by 2.

26. Answers may vary. Sample:

x	y
0	5
2	13
4	29
6	53

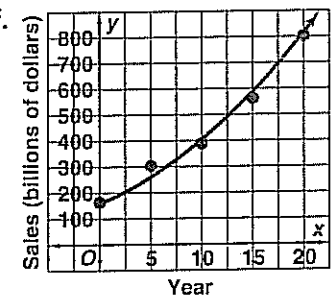
27a. The data appear quadratic, so test for a common second difference: $2.2 - 0.5 = 1.7, 4.9 - 2.2 = 2.7$, $8.7 - 4.9 = 3.8, 13.6 - 8.7 = 4.9$, and $2.7 - 1.7 = 1$, $3.8 - 2.7 = 1.1, 4.9 - 3.8 = 1.1$; common second difference is about 1.1, so the function is quadratic. **27b.** $8.7 = a \cdot 0.8^2, a = \frac{8.7}{0.8^2} \approx 13.6; d = 13.6t^2$ **27c.** $d = 13.6 \cdot 2^2 = 54.4$; about 54.4 ft **28a.** $y = ax^2 + bx + c$ with $(0, 4)$: $4 = a \cdot 0^2 + b \cdot 0 + c$, $4 = 0 + 0 + c$, $c = 4$ **28b.** Answers may vary. Sample:
① $13 = a \cdot 1^2 + b \cdot 1 + 4$, or $a + b = 9$
② $26 = a \cdot 2^2 + b \cdot 2 + 4$, or $4a + 2b = 22$

28c. Multiply ① by 2 and subtract ②:

$$\begin{array}{rrcr} & 2a + 2b & = & 18 \\ - & 4a + 2b & = & 22 \\ \hline & -2a & = & -4 \\ & a & = & 2 \end{array}$$

$a + b = 9, 2 + b = 9, b = 7; a = 2$ and $b = 7$
28d. $y = ax^2 + bx + c$ with $a = 2, b = 7$, and $c = 4$: $y = 2x^2 + 7x + 4$ **28e.** Answers may vary. Sample: Check $(2, 26)$. $26 \stackrel{?}{=} 2(2^2) + 7(2) + 4; 26 \stackrel{?}{=} 8 + 14 + 4; 26 = 26$ ✓ **29a.** $303 \div 164 \approx 1.85, 388 \div 303 \approx 1.28$, $562 \div 388 \approx 1.45, 802 \div 562 \approx 1.43$ **29b.** $303 - 164 = 139, 388 - 303 = 85, 562 - 388 = 174, 802 - 562 = 240$ **29c.** $85 - 139 = -54, 174 - 85 = 89, 240 - 174 = 66$ **29d.** 1.85; the ratio is much larger than the other ratios. **29e.** 85; the difference is much smaller than the other differences.

29f.

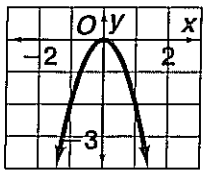


30. The data appear linear, so test for a common difference: $4 - 2 = 2, 6 - 4 = 2, 8 - 6 = 2, 10 - 8 = 2$; common difference is 2, so the data are linear with equation $y = 2x + 2$; B. **31.** Test for a common ratio. F. $4 \div 16 = 0.25, 2 \div 4 = 0.5, -2 \div 2 = -1, 4 \div -2 = -2$, no common ratio. G. $-4 \div -7 \approx 0.57, -1 \div -4 = 0.25, 2 \div -1 = -2, 5 \div 2 = 2.5$, no common ratio. H. $3 \div 1 = 3, 8 \div 3 = 2.\overline{6}, 14 \div 8 = 1.75$, close to a common ratio. I. $3 \div -4 = -0.75, 8 \div 3 = 2.\overline{6}, 14 \div 8 = 1.75, 12 \div 14 \approx 0.86$, no common ratio. The answer is H. **32.** [2] $p = 33{,}500(1.014)^n, 33{,}500(1.014)^{10} \approx 38{,}497$ [1] correct formula, inaccurate evaluation **33.** [4] **33a.** Test for a common difference: $41 - 43.5 = -2.5$, $38.5 - 41 = -2.5, 36 - 38.5 = -2.5, 33.5 - 36 = -2.5$,

$31 - 33.5 = -2.5$; common difference is -2.5, so the model is linear. **33b.** $d = -2.5n + 43.5$ **33c.** $0 = -2.5n + 43.5$; $2.5n = 43.5$; $n = 17.4$; 18 marbles
[3] appropriate methods, but with one computational error [2] part (c) not answered [1] no work shown
34. $y = -x^2$; $b^2 - 4ac = 0^2 - 4(-1)(0) = 0 - 0 = 0$; 1 x-intercept **35.** $y = x^2 + 3x + 4$; $b^2 - 4ac = 3^2 - 4(1)(4) = 9 - 16 = -7 < 0$; 0 x-intercepts
36. $y = 4x^2 - 10x + 3$; $b^2 - 4ac = (-10)^2 - 4(4)(3) = 100 - 48 = 52 > 0$; 2 x-intercepts

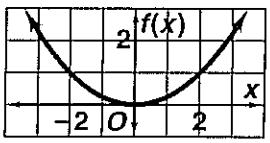
37.

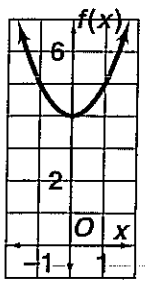
x	$y = -2x^2$	(x, y)
0	$-2(0)^2 = 0$	$(0, 0)$
1	$-2(1)^2 = -2$	$(1, -2)$
2	$-2(2)^2 = -8$	$(2, -8)$



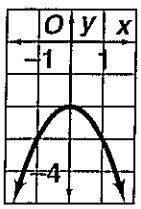
38.

x	$f(x) = \frac{1}{4}x^2$	(x, y)
0	$\frac{1}{4}(0)^2 = 0$	$(0, 0)$
2	$\frac{1}{4}(2)^2 = 1$	$(2, 1)$
3	$\frac{1}{4}(3)^2 = 2.25$	$(3, 2.25)$

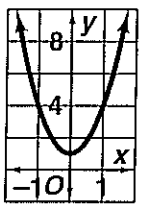


39. 

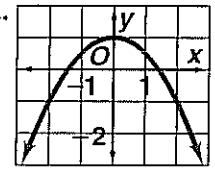
$f(x) = x^2 + 4$; same as $f(x) = x^2$, but shifted up 4 units

40. 

$y = -x^2 - 2$; same as $y = -x^2$, but shifted down 2 units

41. 

$y = 3x^2 + 1$; same as $y = 3x^2$, but shifted up 1 unit

42. 

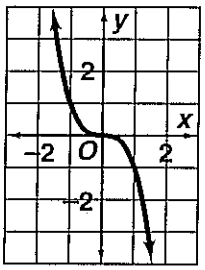
$y = -\frac{1}{2}x^2 + 1$; same as $y = -\frac{1}{2}x^2$, but shifted up 1 unit

43. $y = 2^x$ for $x = -3$: $y = 2^{-3} = \frac{1}{2^3} = \frac{1}{8} = 0.125$
44. $f(x) = -2^x$ for $x = 5$: $-2^5 = -32$
45. $g(t) = 2 \cdot 3^t$ for $t = -3$: $2 \cdot 3^{-3} = 2 \cdot \frac{1}{3^3} = \frac{2}{27}$
46. $f(t) = 10 \cdot 5^t$ for $t = 2$: $10 \cdot 5^2 = 10 \cdot 25 = 250$
47. $y = \left(\frac{1}{2}\right)^t$ for $t = -4$: $\left(\frac{1}{2}\right)^{-4} = \left(\frac{2}{1}\right)^4 = 2^4 = 16$
48. $y = 9 \cdot \left(\frac{3}{2}\right)^x$ for $x = 3$: $9 \cdot \left(\frac{3}{2}\right)^3 = 9 \cdot \frac{27}{8} = 30.375$

EXTENSION *page 567*

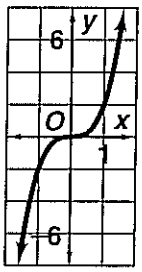
1. Both graphs have the same shape, go through the origin, and lie in Quadrants I and III. The graph of $y = x^3$ is narrower than the graph of $y = \frac{1}{3}x^3$.
2ai.

x	$y = -x^3$	(x, y)
-2	$-(-2)^3 = 8$	$(-2, 8)$
-1	$-(-1)^3 = 1$	$(-1, 1)$
0	$-0^3 = 0$	$(0, 0)$
1	$-1^3 = -1$	$(1, -1)$
2	$-2^3 = -8$	$(2, -8)$



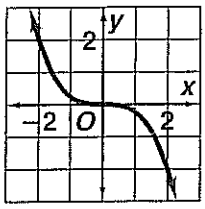
2aii.

x	$y = 2x^3$	(x, y)
-2	$2(-2)^3 = -16$	$(-2, -16)$
-1	$2(-1)^3 = -2$	$(-1, -2)$
0	$2(0)^3 = 0$	$(0, 0)$
1	$2(1)^3 = 2$	$(1, 2)$
2	$2(2)^3 = 16$	$(2, 16)$



2aiii.

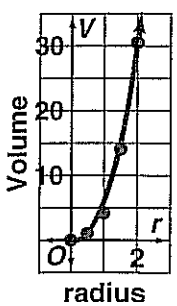
x	$y = -\frac{1}{4}x^3$	(x, y)
-2	$-\frac{1}{4}(-2)^3 = 2$	$(-2, 2)$
-1	$-\frac{1}{4}(-1)^3 = \frac{1}{4}$	$\left(-1, \frac{1}{4}\right)$
0	$-\frac{1}{4}(0)^3 = 0$	$(0, 0)$
1	$-\frac{1}{4}(1)^3 = -\frac{1}{4}$	$\left(1, -\frac{1}{4}\right)$
2	$-\frac{1}{4}(2)^3 = -2$	$(2, -2)$



2b. The graphs with negative coefficients of x^3 are in Quadrants II and IV, and the graph with a positive coefficient of x^3 is in Quadrants I and III. All of the graphs go through the origin. **3.** Yes; the sign of a changes which quadrants the graphs are in, and the larger $|a|$, the narrower the graph. **4a.** no
4b. Yes; half of the graph is reflected over the y-axis and then over the x-axis.

5a.

r	$V = \frac{4}{3}\pi r^3$	(V, r)
0	$\frac{4}{3}\pi(0)^3 = 0$	$(0, 0)$
0.5	$\frac{4}{3}\pi(0.5)^3 \approx 0.5$	$(0.5, 0.5)$
1	$\frac{4}{3}\pi(1)^3 \approx 4.2$	$(1, 4.2)$
1.5	$\frac{4}{3}\pi(1.5)^3 \approx 14.1$	$(1.5, 14.1)$
2	$\frac{4}{3}\pi(2)^3 \approx 33.5$	$(2, 33.5)$



5b. when $V = 10$, $r \approx 1.3$; about 1.3 ft

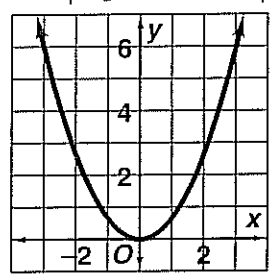
TEST-TAKING STRATEGIES *page 568*

1a. $x^2 + 4x - n^2 = 0$; $b^2 - 4ac = 4^2 - 4(1)(-n^2) = 16 + 4n^2$ **1b.** The smallest value of n^2 is 0, $16 + 0 > 0$.
1c. Since the discriminant is always positive, the equation has two solutions; C. **2.** $x^2 + 2xy + y^2 = 0$; $(x + y)^2 = 0$; $x + y = 0$; $x = -y$; $\frac{x}{y} = \frac{-1}{1}$; C

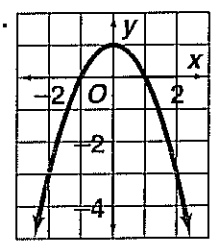
CHAPTER REVIEW *pages 569–571*

1. The U-shaped graph of a quadratic function is a parabola. **2.** If the quadratic expression $ax^2 + bx + c$ cannot be factored, one good way to solve the equation $ax^2 + bx + c = 0$ is to use completing the square.
3. If $a^2 = b$ and $a > 0$, then a is the principal square root of b. **4.** The vertex of a parabola is the point at which the parabola intersects the axis of symmetry.
5. The discriminant can be used to determine the number of solutions of a quadratic equation.
6–9. Answers may vary. Samples are given.
6. $y = -2x^2$ **7.** $y = 2x^2$ **8.** $y = x^2$ **9.** $y = \frac{1}{2}x^2$
10.

x	$y = \frac{2}{3}x^2$	(x, y)
0	$\frac{2}{3}(0)^2 = 0$	$(0, 0)$
1	$\frac{2}{3}(1)^2 = \frac{2}{3}$	$\left(1, \frac{2}{3}\right)$
2	$\frac{2}{3}(2)^2 = 2\frac{2}{3}$	$\left(2, 2\frac{2}{3}\right)$
3	$\frac{2}{3}(3)^2 = 6$	$(3, 6)$

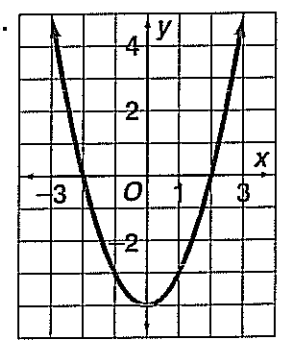


11.

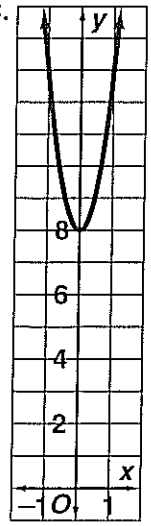


$y = -x^2 + 1$; same as $y = -x^2$, but shifted up 1 unit

12.

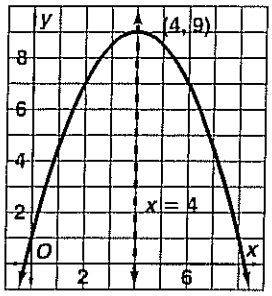


$y = x^2 - 4$; same as $y = x^2$, but shifted down 4 units

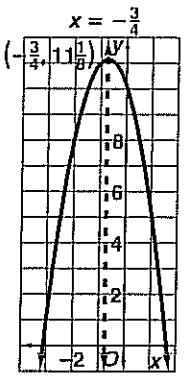
13. 

$y = 5x^2 + 8$; same as $y = 5x^2$, but shifted up 8 units

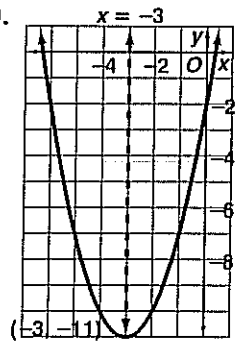
14. $y = 4x^2 + 1$; $a = 4 > 0$, so the parabola opens upward and has a minimum. **15.** $y = -3x^2 - 7$; $a = -3 < 0$, so the parabola opens downward and has a maximum. **16.** $y = \frac{1}{2}x^2 + 9$; $a = \frac{1}{2} > 0$, so the parabola opens upward and has a minimum. **17.** $y = -x^2 + 6$; $a = -1 < 0$, so the parabola opens downward and has a maximum.

18. 

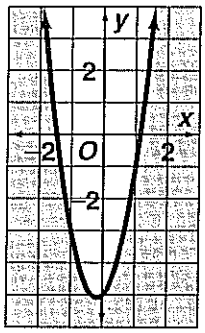
$y = -\frac{1}{2}x^2 + 4x + 1$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{4}{2\left(-\frac{1}{2}\right)} = 4$; y-coordinate of vertex: $y = -\frac{1}{2}(4)^2 + 4(4) + 1 = 9$; vertex $(4, 9)$; points $(0, 1)$, $(2, 7)$

19. 

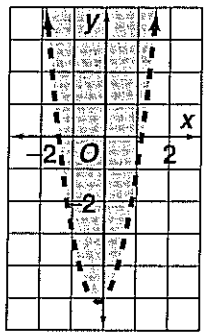
$y = -2x^2 - 3x + 10$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{-3}{2(-2)} = -\frac{3}{4}$; y-coordinate of vertex: $y = -2\left(-\frac{3}{4}\right)^2 - 3\left(-\frac{3}{4}\right) + 10 = 11\frac{1}{8}$; vertex $\left(-\frac{3}{4}, 11\frac{1}{8}\right)$; points $(0, 10)$, $(1, 5)$

20. 

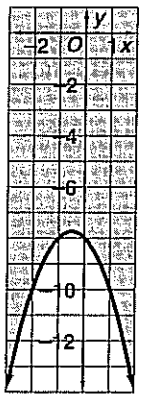
$y = x^2 + 6x - 2$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{6}{2(1)} = -3$; y-coordinate of vertex: $y = (-3)^2 + 6(-3) - 2 = -11$; vertex $(-3, -11)$; points $(0, -2)$, $(-1, -7)$

21. 

$y \le 3x^2 + x - 5$; boundary curve $y = 3x^2 + x - 5$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{1}{2(3)} = -\frac{1}{6}$; y-coordinate of vertex: $y = 3\left(-\frac{1}{6}\right)^2 + \left(-\frac{1}{6}\right) - 5 = -5\frac{1}{12}$; vertex $\left(-\frac{1}{6}, -5\frac{1}{12}\right)$; points $(0, -5)$, $(1, -1)$; solid line and shade below

22. 

$y > 3x^2 + x - 5$; boundary curve $y = 3x^2 + x - 5$, same as Exercise 21; dashed line and shade above

23. 

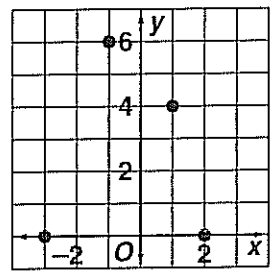
$y \ge -x^2 - x - 8$; boundary curve $y = -x^2 - x - 8$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{-1}{2(-1)} = -\frac{1}{2}$; y-coordinate of vertex: $y = -\left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) - 8 = -7\frac{3}{4}$; vertex $\left(-\frac{1}{2}, -7\frac{3}{4}\right)$; points $(0, -8)$, $(1, -10)$; solid line and shade above

24. $\sqrt{86}$, irrational; $\sqrt{86} \approx 9.27$ **25.** $-\sqrt{121}$, rational; $-\sqrt{121} = -11$ **26.** $\pm\sqrt{\frac{1}{2}}$, irrational; $\pm\sqrt{\frac{1}{2}} \approx \pm 0.71$ **27.** $\sqrt{2.55}$, irrational; $\sqrt{2.55} \approx 1.60$ **28.** $-\sqrt{\frac{4}{25}}$, rational; $-\sqrt{\frac{4}{25}} = -\frac{2}{5}$ **29.** $-\sqrt{47}$, irrational; $-\sqrt{47} \approx -6.86$ **30.** $\sqrt{0.36}$, rational; $\sqrt{0.36} = 0.6$ **31.** $\sqrt{140}$, irrational; $\sqrt{140} \approx 11.83$ **32.** $-\sqrt{1}$, rational; $-\sqrt{1} = -1$ **33.** $\sqrt{196}$, rational; $\sqrt{196} = 14$ **34.** $6(x^2 - 2) = 12$; $x^2 - 2 = 2$; $x^2 = 4$; $x = \pm 2$; $x = 2$ or $x = -2$ **35.** $-5m^2 = -125$; $m^2 = 25$; $m = \pm 5$; $m = 5$ or $m = -5$ **36.** $9(w^2 + 1) = 9$; $w^2 + 1 = 1$; $w^2 = 0$; $w = 0$ **37.** $3r^2 + 27 = 0$; $3r^2 = -27$; $r^2 = -9$; no solution **38.** $x^2 + 7x + 12 = 0$; $(x + 4)(x + 3) = 0$; $x + 4 = 0$ or $x + 3 = 0$; $x = -4$ or $x = -3$ **39.** $5x^2 - 10x = 0$; $5x^2 - 10x + 0 = 0$; $5x(x - 2) = 0$; $5x = 0$ or $x - 2 = 0$; $x = 0$ or $x = 2$ **40.** $2x^2 - 9x = x^2 - 20$; $x^2 - 9x + 20 = 0$; $(x - 4)(x - 5) = 0$; $x - 4 = 0$ or $x - 5 = 0$; $x = 4$ or $x = 5$ **41.** $2x^2 + 5x = 3$; $2x^2 + 5x - 3 = 0$; $(2x - 1)(x + 3) = 0$; $2x - 1 = 0$ or $x + 3 = 0$; $2x = 1$ or $x = -3$; $x = \frac{1}{2}$ or $x = -3$ **42.** $3x^2 - 5x = -3x^2 + 6$; $6x^2 - 5x - 6 = 0$; $(3x + 2)(2x - 3) = 0$; $3x + 2 = 0$ or $2x - 3 = 0$; $3x = -2$ or $2x = 3$; $x = -\frac{2}{3}$ or $x = \frac{3}{2} = 1\frac{1}{2}$

43. $x^2 - 5x + 4 = 0; (x - 1)(x - 4) = 0; x - 1 = 0$ or $x - 4 = 0; x = 1$ or $x = 4$ **44.** $x^2 + 6x - 5 = 0; x^2 + 6x = 5; x^2 + 6x + 9 = 5 + 9; (x + 3)^2 = 14; x + 3 = \pm\sqrt{14} \approx \pm 3.74; x + 3 \approx 3.74$ or $x + 3 \approx -3.74; x \approx 0.74$ or $x \approx -6.74$ **45.** $x^2 = 3x - 1; x^2 - 3x = -1; x^2 - 3x + \frac{9}{4} = -1 + \frac{9}{4}; \left(x - \frac{3}{2}\right)^2 = \frac{5}{4}; x - \frac{3}{2} = \pm\sqrt{\frac{5}{4}} \approx \pm 1.12;$ $x - 1.5 \approx 1.12$ or $x - 1.5 \approx -1.12; x \approx 2.62$ or $x \approx 0.38$ **46.** $2x^2 + 7x = -6; x^2 + \frac{7}{2}x = -3; x^2 + \frac{7}{2}x + \frac{49}{16} = -3 + \frac{49}{16}; \left(x + \frac{7}{4}\right)^2 = \frac{1}{16}; x + \frac{7}{4} = \pm\sqrt{\frac{1}{16}} = \pm\frac{1}{4};$ $x + \frac{7}{4} = \frac{1}{4}$ or $x + \frac{7}{4} = -\frac{1}{4}; x = -\frac{6}{4} = -1\frac{1}{2}$ or $x = -\frac{8}{4} = -2$ **47.** $A = \pi r^2; 16 = \pi r^2; r^2 = \frac{16}{\pi}; r = \sqrt{\frac{16}{\pi}} \approx 2.3;$ 2.3 in. **48.** $A = \ell w; 170 = (2w - 3)w; 2w^2 - 3w - 170 = 0;$ $(2w + 17)(w - 10) = 0; 2w + 17 = 0$ or $w - 10 = 0;$ $2w = -17$ or $w = 10; w = -8.5$ or $w = 10$; use $w = 10$, then $2w - 3 = 2(10) - 3 = 17$; 10 ft × 17 ft **49.** $x^2 - 10 = 3; x^2 + 0x - 13 = 0; b^2 - 4ac = 0^2 - 4(1)(-13) = 52 > 0$; 2 solutions **50.** $3x^2 = 27;$ $3x^2 + 0x - 27 = 0; b^2 - 4ac = 0^2 - 4(3)(-27) = 324 > 0$; 2 solutions **51.** $x^2 + 3 = 2x; x^2 - 2x + 3 = 0;$ $b^2 - 4ac = (-2)^2 - 4(1)(3) = 4 - 12 = -8 < 0;$ 0 solutions **52.** $x^2 + 10x = -25; x^2 + 10x + 25 = 0;$ $b^2 - 4ac = 10^2 - 4(1)(25) = 100 - 100 = 0$; 1 solution **53.** $4x^2 + 3x - 8 = 0; a = 4, b = 3, c = -8;$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4(4)(-8)}}{2(4)} = \frac{-3 \pm \sqrt{137}}{8};$ $x = \frac{-3 + \sqrt{137}}{8} \approx 1.09$ or $x = \frac{-3 - \sqrt{137}}{8} \approx -1.84$ **54.** $2x^2 - 7x = -3; 2x^2 - 7x + 3 = 0; a = 2, b = -7, c = 3;$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(3)}}{2(2)} = \frac{7 \pm \sqrt{25}}{4} = \frac{7 \pm 5}{4}; x = \frac{7 + 5}{4} = 3$ or $x = \frac{7 - 5}{4} = 0.5$ **55.** $-x^2 + 8x + 4 = 5; -x^2 + 8x - 1 = 0; a = -1, b = 8,$ $c = -1; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-8 \pm \sqrt{8^2 - 4(-1)(-1)}}{2(-1)} = \frac{-8 \pm \sqrt{60}}{-2}; x = \frac{-8 + \sqrt{60}}{-2} \approx 0.13$ or $x = \frac{-8 - \sqrt{60}}{-2} \approx 7.87$ **56.** $9x^2 - 270 = 0; 9x^2 = 270; x^2 = 30; x = \pm\sqrt{30} \approx \pm 5.48; x = 5.48$ or $x = -5.48$ **57.** $5x^2 - 10 = x^2 + 90;$ $4x^2 - 100 = 0; (2x - 10)(2x + 10) = 0; 2x - 10 = 0$ or $2x + 10 = 0; 2x = 10$ or $2x = -10; x = 5$ or $x = -5$; use factoring, because the equation is easily factorable. **58.** $9x^2 + 30x - 29 = 0; a = 9, b = 30, c = -29;$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-30 \pm \sqrt{30^2 - 4(9)(-29)}}{2(9)} = \frac{-30 \pm \sqrt{1944}}{18}; x = \frac{-30 + \sqrt{1944}}{18} \approx 0.78$ or $x = \frac{-30 - \sqrt{1944}}{18} \approx -4.12$; use the quadratic formula, because the trinomial does not factor easily. **59.** $2x^2 - 9x = x^2 - 20; x^2 - 9x + 20 = 0;$ $(x - 4)(x - 5) = 0; x - 4 = 0$ or $x - 5 = 0; x = 4$ or $x = 5$; use factoring, because the equation is easily factorable. **60.** $x^2 - 6x + 9 = 0; (x - 3)^2 = 0; x - 3 = 0; x = 3$; use factoring, because the equation is easily factorable. **61.** $x^2 + 3x - 225 = 3x; x^2 - 225 = 0; x^2 = 225; x = \pm\sqrt{225} = \pm 15; x = 15$ or $x = -15$; use square roots, because the equation has no x term.

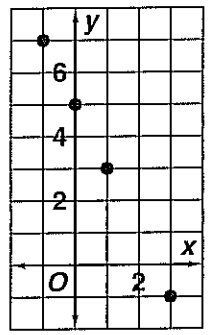
62. $x^2 + 8x = 4; x^2 + 8x + 16 = 4 + 16; (x + 4)^2 = 20;$ $x + 4 = \pm\sqrt{20} \approx \pm 4.47; x + 4 \approx 4.47$ or $x + 4 \approx -4.47;$ $x \approx 0.47$ or $x \approx -8.47$; complete the square, because the equation is in the form $x^2 + bx = c$. **63.** $(p + 2)^2 = 400;$ $p + 2 = \pm\sqrt{400} = \pm 20; p + 2 = 20$ or $p + 2 = -20;$ $p = 18$ or $p = -22$; use $p = 18$; length = 18 ft; area = $(18 \text{ ft})^2 = 324 \text{ ft}^2$ **64.** $0 = -16t^2 + 20t + 6; a = -16, b = 20, c = 6; x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-20 \pm \sqrt{20^2 - 4(-16)(6)}}{2(-16)} = \frac{-20 \pm \sqrt{784}}{-32} = \frac{-20 \pm 28}{-32}; x = \frac{-20 + 28}{-32} = -0.25$ or $x = \frac{-20 - 28}{-32} = 1.5$, use $x = 1.5$; 1.5 seconds

65.



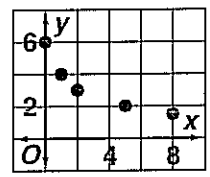
quadratic

66.



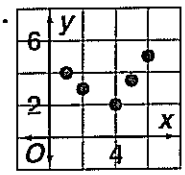
linear

67.



exponential

68.



quadratic

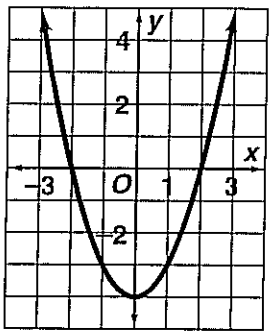
69. Test for a common ratio: $5 \div 2.5 = 2, 10 \div 5 = 2,$ $20 \div 10 = 2, 40 \div 20 = 2$; common ratio is 2, so write an exponential equation: $y = 5(2)^x$. **70.** Test for a common difference: $-2 - (-5) = 3, 1 - (-2) = 3, 4 - 1 = 3,$ $7 - 4 = 3$; common difference is 3, so write a linear equation: $y = 3x - 2$. **71.** Test for a common second difference: $1 - 4 = -3, 0 - 1 = -1, 1 - 0 = 1, 4 - 1 = 3$, and $-1 - (-3) = 2, -1 - (-3) = 2, 1 - (-1) = 2,$ $3 - 1 = 2$; common second difference is 2, so write a quadratic equation: $y = (x + 1)^2$. **72.** Test for a common ratio: $5 \div 0.5 = 10, 50 \div 5 = 10, 500 \div 50 = 10,$ $5000 \div 500 = 10$; common ratio is 10, so write an exponential equation: $y = \frac{1}{2}(10)^x$.

CHAPTER TEST *page 572*

1. The graph is the same as $y = x^2$, but shifted down 3 units, so $y = x^2 - 3$; D. **2.** The graph is a parabola through the origin and opening downward, so $y = ax^2$

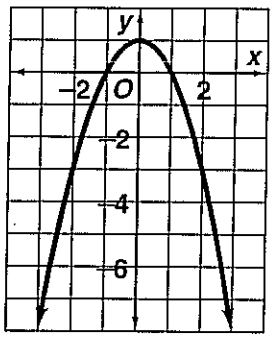
with $a < 0$; choose $y = -2x^2$; C. **3.** The graph is a parabola through the origin and opening upward, so $y = ax^2$ with $a > 0$; choose $y = 3x^2$; A. **4.** The graph is a parabola that opens downward and is shifted up 1 unit, so choose $y = -3x^2 + 1$; B. **5.** $y = 3x^2 - 7$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{0}{2(3)} = 0, x = 0$; y-coordinate of vertex: $y = 3(0)^2 - 7 = -7$; vertex $(0, -7)$; $a = 3 > 0$, so the parabola opens upward and the vertex is a minimum. **6.** $y = x^2 - 3x + 2$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{-3}{2(1)} = 1.5, x = 1.5$; y-coordinate of vertex: $y = (1.5)^2 - 3(1.5) + 2 = -0.25$; vertex $(1.5, -0.25)$; $a = 1 > 0$, so the parabola opens upward and the vertex is a minimum. **7.** $y = -2x^2 + 10x - 1$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{10}{2(-2)} = 2.5, x = 2.5$; y-coordinate of vertex: $-2(2.5)^2 + 10(2.5) - 1 = 11.5$; vertex $(2.5, 11.5)$; $a = -2 < 0$, so the parabola opens downward and the vertex is a maximum. **8.** $y = \frac{1}{2}x^2 + 6x$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{6}{2\left(\frac{1}{2}\right)} = -6$; y-coordinate of vertex: $y = \frac{1}{2}(-6)^2 + 6(-6) = -18$; vertex $(-6, -18)$; $a = \frac{1}{2} > 0$, so the parabola opens upward and the vertex is a minimum.

9.



$y = x^2 - 4$; same as $y = x^2$, but shifted down 4 units

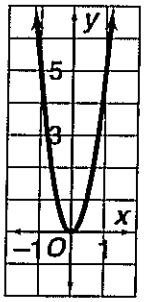
10.



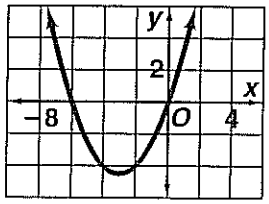
$y = -x^2 + 1$; same as $y = -x^2$, but shifted up 1 unit

11.

x	$y = 5x^2$	(x, y)
0	$5(0)^2 = 0$	$(0, 0)$
1	$5(1)^2 = 5$	$(1, 5)$
2	$5(2)^2 = 20$	$(2, 20)$

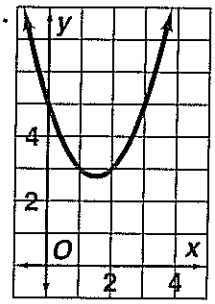


12.



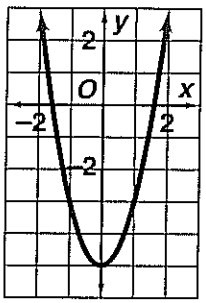
$y = \frac{1}{2}x^2 + 3x$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{3}{2\left(\frac{1}{2}\right)} = -3$; y-coordinate of vertex: $y = \frac{1}{2}(-3)^2 + 3(-3) = -4.5$; vertex $(-3, -4.5)$; points $(0, 0), (-1, -2.5)$

13.



$y = x^2 - 3x + 5$; axis of symmetry: $x = -\frac{b}{2a} = -\frac{-3}{2(1)} = 1.5, x = 1.5$; y-coordinate of vertex: $y = (1.5)^2 - 3(1.5) + 5 = 2.75$; vertex $(1.5, 2.75)$; points $(0, 5), (1, 3)$

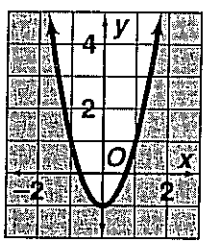
14.



$y = 2x^2 - 5$; same as $y = 2x^2$, but shifted down 5 units

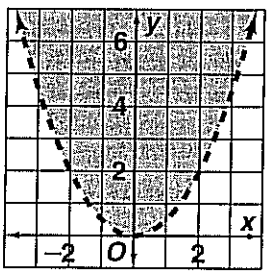
15. Answers may vary. Sample: You can tell how wide it is and whether it opens upward or downward.
16. $y = 5x^2$; $b^2 - 4ac = 0^2 - 4(5)(0) = 0$; 1 x-intercept
17. $y = 3x^2 + 10$; $b^2 - 4ac = 0^2 - 4(3)(10) = -120 < 0$; 0 x-intercepts **18.** $y = -2x^2 + x + 7$; $b^2 - 4ac = 1^2 - 4(-2)(7) = 57 > 0$; 2 x-intercepts **19.** $y = x^2 - 4x$; $b^2 - 4ac = (-4)^2 - 4(1)(0) = 16 > 0$; 2 x-intercepts

20.



$y \le 2x^2 - 1$; boundary curve $y = 2x^2 - 1$, same as $y = 2x^2$ but shifted down 1 unit; solid line and shade below

21.



$y > \frac{1}{2}x^2$; boundary curve $y = \frac{1}{2}x^2$; dashed line and shade above

22. $\sqrt{1.44} = 1.2$ **23.** $\sqrt{1600} = 40$
24. $\sqrt{\frac{4}{9}} = \frac{2}{3}$ **25.** $\sqrt{0.04} = 0.2$

26. $\sqrt{25} < \sqrt{28} < \sqrt{36}$, so $5 < \sqrt{28} < 6$; between 5 and 6 **27.** $\sqrt{121} < \sqrt{136} < \sqrt{144}$, so $11 < \sqrt{136} < 12$; between 11 and 12 **28.** $\sqrt{324} < \sqrt{332} < \sqrt{361}$, so $18 < \sqrt{332} < 19$; between 18 and 19

29. $-\sqrt{9} < -\sqrt{8.99} < -\sqrt{4}$, so $-3 < -\sqrt{8.99} < -2$; between -3 and -2 **30.** $x^2 + 4x = -4$; $x^2 + 4x + 4 = 0$; $b^2 - 4ac = 4^2 - 4(1)(4) = 16 - 16 = 0$; 1 solution

31. $x^2 + 8 = 0$; $x^2 = -8$; no solution **32.** $2x^2 + x = 0$; $b^2 - 4ac = 1^2 - 4(2)(0) = 1 > 0$; 2 solutions

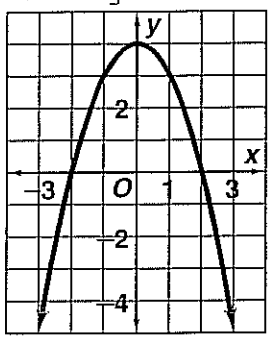
33. $3x^2 - 9x = -5$; $3x^2 - 9x + 5 = 0$; $b^2 - 4ac = (-9)^2 - 4(3)(5) = 81 - 60 = 21 > 0$; 2 solutions

34. $kx^2 - 10x + 25 = 0$; $b^2 - 4ac = 0$; $(-10)^2 - 4k(25) = 0$; $100 - 100k = 0$; $100 = 100k$; $k = 1$ **35.** $2x^2 = 50$; $x^2 = 25$; $x = \pm\sqrt{25} = \pm 5$; $x = 5$ or $x = -5$ **36.** $-3x^2 + 7x = -10$; $3x^2 - 7x - 10 = 0$; $(3x - 10)(x + 1) = 0$; $3x - 10 = 0$ or $x + 1 = 0$; $3x = 10$ or $x = -1$; $x = \frac{10}{3} \approx 3.33$ or $x = -1$

37. $x^2 + 6x + 9 = 25$; $(x + 3)^2 = 25$; $x + 3 = \pm\sqrt{25} = \pm 5$; $x + 3 = 5$ or $x + 3 = -5$; $x = 2$ or $x = -8$

38. $-x^2 - x + 2 = 0$; $(-x + 1)(x + 2) = 0$; $-x + 1 = 0$ or $x + 2 = 0$; $x = 1$ or $x = -2$ **39.** $x^2 + 4x = 1$; $x^2 + 4x + 4 = 1 + 4$; $(x + 2)^2 = 5$; $x + 2 = \pm\sqrt{5} \approx \pm 2.24$; $x + 2 \approx 2.24$ or $x + 2 \approx -2.24$; $x \approx 0.24$ or $x \approx -4.24$ **40.** $12x^2 + 16x - 28 = 0$; $3x^2 + 4x - 7 = 0$; $(3x + 7)(x - 1) = 0$; $3x + 7 = 0$ or $x - 1 = 0$; $3x = -7$ or $x = 1$; $x = -\frac{7}{3} \approx -2.33$ or $x = 1$

41.



Answers may vary. Sample: $y = -x^2 + 4$

42. $V = \pi r^2 h$; $140 = \pi r^2 \cdot 10$; $140 = 10\pi r^2$; $r^2 = \frac{140}{10\pi} = \frac{14}{\pi}$; $r = \sqrt{\frac{14}{\pi}} \approx 2.1$; 2.1 ft **43.** $A = \ell w$; $800 = (2w)w$; $800 = 2w^2$; $w^2 = 400$; $w = \sqrt{400} = 20$; $2w = 2(20) = 40$; $w = 20$ ft, $\ell = 40$ ft **44.** quadratic; $y = \frac{1}{2}x^2$

45. exponential; $y = \frac{1}{2}(2)^x$ **46.** linear; $y = x + 2$

47. exponential; $y = 40(0.5)^x$

STANDARDIZED TEST PREP *page 573*

1. $3x^2 - 5x + 1 = 0$; $a = 3, b = -5, c = 1$; $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(1)}}{2(3)} = \frac{5 \pm \sqrt{13}}{6}$; $x = \frac{5 + \sqrt{13}}{6} \approx 1.43$ or $x = \frac{5 - \sqrt{13}}{6} \approx 0.23$; D

2. $y = 5x^2 - 2x + 3$; $x = -\frac{b}{2a} = -\frac{-2}{2(5)} = \frac{1}{5}$; F

3. $\sqrt{49} < \sqrt{52} < \sqrt{64}$, so $7 < \sqrt{52} < 8$; C

4. slope $-\frac{1}{3}$ and through $(0, 6)$: $y - 6 = -\frac{1}{3}(x - 0)$; $y = -\frac{1}{3}x + 6$; test $(9, 3)$: $3 \stackrel{?}{=} -\frac{1}{3}(9) + 6$, $3 \stackrel{?}{=} -3 + 6$, $3 = 3$; F **5.** $P(\text{not 1 or 2}) = P(3, 4, 5, \text{or } 6) = \frac{4}{6} = \frac{2}{3}$; D

6. $3x^2 - 4x - 3 = 0$; $b^2 - 4ac = (-4)^2 - 4(3)(-3) = 16 + 36 = 52$; I **7.** $(3x - 1)(5x + 3) = 15x^2 + 9x - 5x - 3 = 15x^2 + 4x - 3$; D **8.** $x^2 + 4x + 4 = 49$; $(x + 2)^2 = 49$; $x + 2 = \pm\sqrt{49} = \pm 7$; $x + 2 = 7$ or $x + 2 = -7$; $x = 5$ or $x = -9$; I **9.** Solve the second equation for y: $6y - x = 24$; $6y = x + 24$; $y = \frac{1}{6}x + 4$; the two lines have different slopes, so they will intersect in 1 point; B **10.** Add the two equations: $3y = 9$, $y = 3$; substitute 3 for y in the first equation: $x + 3 = 5$, $x = 2$; C

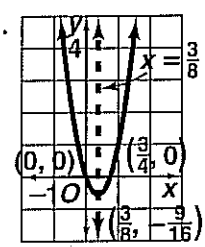
11. $x^2 + x - 20 = 0$; discriminant: $b^2 - 4ac = 1^2 - 4(1)(-20) = 81$; solutions: $(x + 5)(x - 4) = 0$, $x + 5 = 0$ or $x - 4 = 0$, $x = -5$ or $x = 4$, sum $= -5 + 4 = -1$; $81 > -1$; A **12.** slope $= \frac{6 - 3}{4 - (-1)} = \frac{3}{5} = 0.6$ **13.** percent of increase $= \frac{20 - 12}{12} \approx 0.67$, or 67%; 67

14. [2] $(2w - 6)w = 140$; $2w^2 - 6w - 140 = 0$; $w^2 - 3w - 70 = 0$; $(w - 10)(w + 7) = 0$; $w = 10$; 10 m by 14 m [1] correct answer but no work shown

15. [2] straight line, U-shaped curve that opens up or down, V-shaped curve that opens up or down (OR equivalent descriptions) [1] two correct descriptions

16. [2] $9a + 3b - 3 - 4a + 7 - 8b = (9 - 4)a + (3 - 8)b - 3 + 7 = 5a - 5b + 4$ [1] correct answer but no work shown

17.



vertex: $\left(\frac{3}{8}, -\frac{9}{16}\right)$
x-intercepts: 0 and $\frac{3}{4}$
axis of symmetry: $x = \frac{3}{8}$

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Activity 1 Answers may vary. Sample:

a. Let $R_1 = 31$ ft, $R_2 = 38$ ft. **b.** $d = 2\sqrt{(R_2)^2 - (R_1)^2} = 2\sqrt{38^2 - 31^2} = 2\sqrt{483} \approx 44$ ft **c.** Let $R_1 = 112$ ft, $R_2 = 119$ ft; $d = 2\sqrt{(R_2)^2 - (R_1)^2} = 2\sqrt{119^2 - 112^2} = 2\sqrt{1617} \approx 80$ ft.

Activity 2 **a–b.** Answers may vary. Sample:

a. Let $R = 118$ ft; $F = \frac{1}{8}(\pi^2 W N^2 R)$; $150 = \frac{1}{8}(\pi^2 \cdot 150 \cdot N^2 \cdot 118)$; $N \approx 0.082$ revolutions/s

b. $F = \frac{1}{8}(\pi^2 W N^2 R)$; $20 = \frac{1}{8}(\pi^2 \cdot 20 \cdot N^2 \cdot 118)$; $N \approx 0.082$ revolutions/s; $N \approx 298$ revolutions/h

c. They are the same.